# Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.1-Quadratic/1.2.1.6-g+h-x-^m-a+b-x+c-x^2-^p-d+e-x+f-x^2-^q

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# Contents

# Chapter 1

# Introduction

## Local contents

This report gives the result of running the computer algebra independent integration test. Elementary Algebraic integrals version.

The download section below contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [ 143 ]. This is test number [ 24 ].

## 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.3.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.45 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)
9. `IntegrateAlgebraic` under Mathematica 12.3.1 on windows 10. `https://github.com/stblake/algebraic_integration`. September 15, 2021 version.

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

| System | % solved | % Failed |
|---|---|---|
| Rubi | 100.00 ( 143 ) | 0.00 ( 0 ) |
| Mathematica | 99.30 ( 142 ) | 0.70 ( 1 ) |
| Maple | 98.60 ( 141 ) | 1.40 ( 2 ) |
| IntegrateAlgebraic | 91.61 ( 131 ) | 8.39 ( 12 ) |
| Fricas | 48.25 ( 69 ) | 51.75 ( 74 ) |
| Giac | 34.97 ( 50 ) | 65.03 ( 93 ) |
| Mupad | 13.29 ( 19 ) | 86.71 ( 124 ) |
| Maxima | 10.49 ( 15 ) | 89.51 ( 128 ) |
| Sympy | 7.69 ( 11 ) | % 92.31 ( 132 ) |

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

| grade | description |
|---|---|
| A | Integral was solved and antiderivative is optimal in quality and leaf size. |
| B | Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size. |
| C | Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons<br>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.<br>2. antiderivative contains a special function and the optimal antiderivative does not.<br>3. antiderivative contains the imaginary unit and the optimal antiderivative does not. |
| F | Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised. |

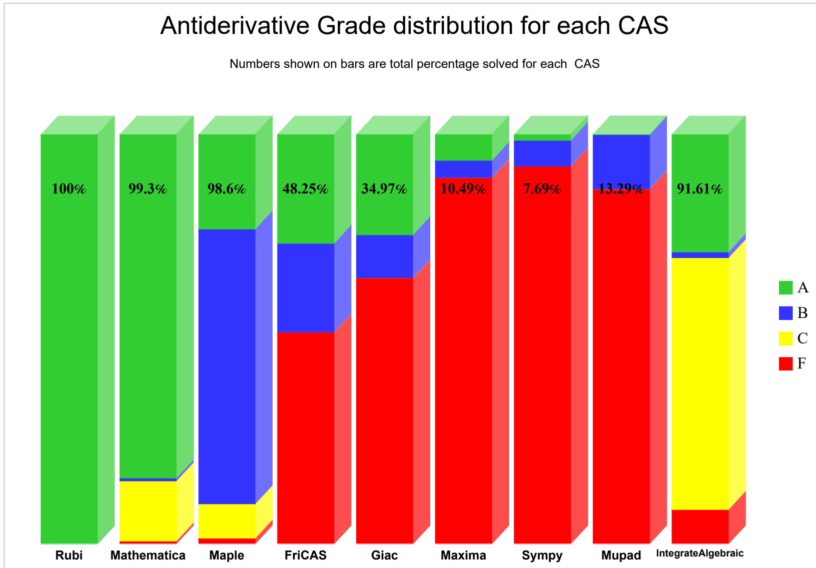
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

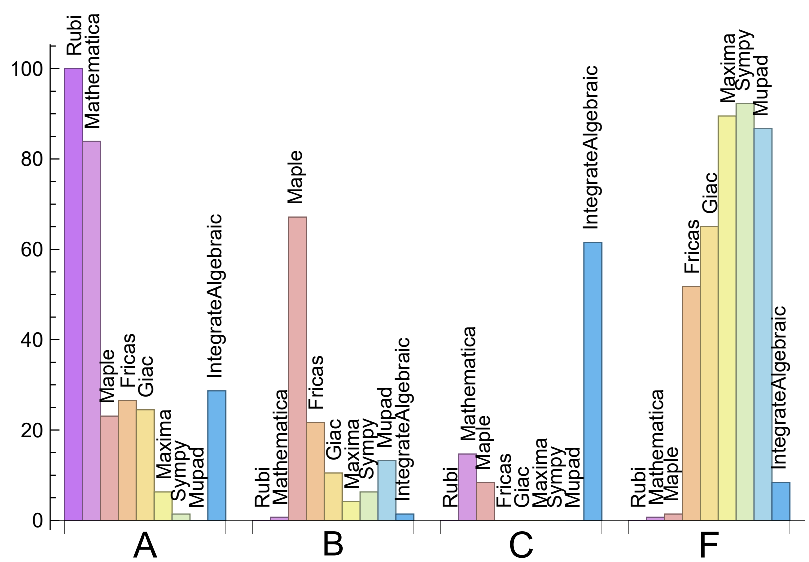
| System | % A grade | % B grade | % C grade | % F grade |
|---|---|---|---|---|
| Rubi | 100.00 | 0.00 | 0.00 | 0.00 |
| Mathematica | 83.92 | 0.70 | 14.69 | 0.70 |
| IntegrateAlgebraic | 28.67 | 1.40 | 61.54 | 8.39 |
| Fricas | 26.57 | 21.68 | 0.00 | 51.75 |
| Giac | 24.48 | 10.49 | 0.00 | 65.03 |
| Maple | 23.08 | 67.13 | 8.39 | 1.40 |
| Maxima | 6.29 | 4.20 | 0.00 | 89.51 |
| Sympy | 1.40 | 6.29 | 0.00 | 92.31 |
| Mupad | N/A | 13.29 | 0.00 | 86.71 |

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.




The figure below compares the CAS systems for each grade level.




The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

| System | Number failed | Percentage normal failure | Percentage timeout failure | Percentage exception failure |
|---|---|---|---|---|
| Rubi | 0 | 0.00 % | 0.00 % | 0.00 % |
| Mathematica | 1 | 100.00 % | 0.00 % | 0.00 % |
| Maple | 2 | 100.00 % | 0.00 % | 0.00 % |
| Fricas | 74 | 1.35 % | 98.65 % | 0.00 % |
| IntegrateAlgebraic | 12 | 91.67 % | 8.33 % | 0.00 % |
| Giac | 93 | 22.58 % | 18.28 % | 59.14 % |
| Maxima | 128 | 52.34 % | 0.00 % | 47.66 % |
| Sympy | 132 | 87.88 % | 12.12 % | 0.00 % |
| Mupad | 124 | 99.19 % | 0.81 % | 0.00 % |

Table 1.4: Failure statistics for each CAS

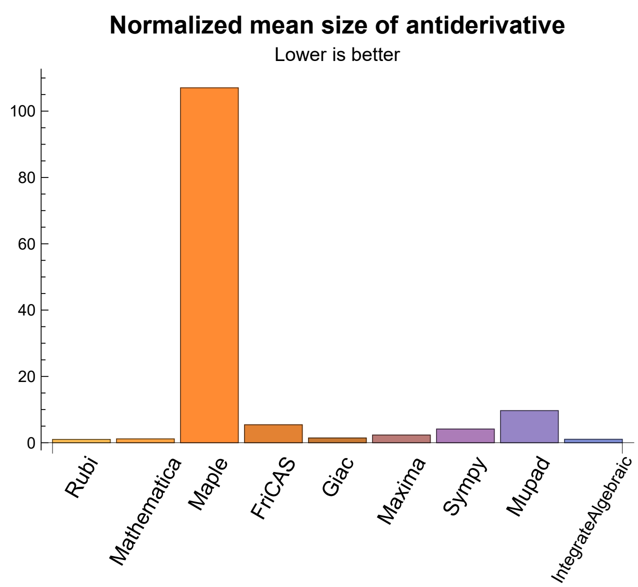
## 1.3 Performance

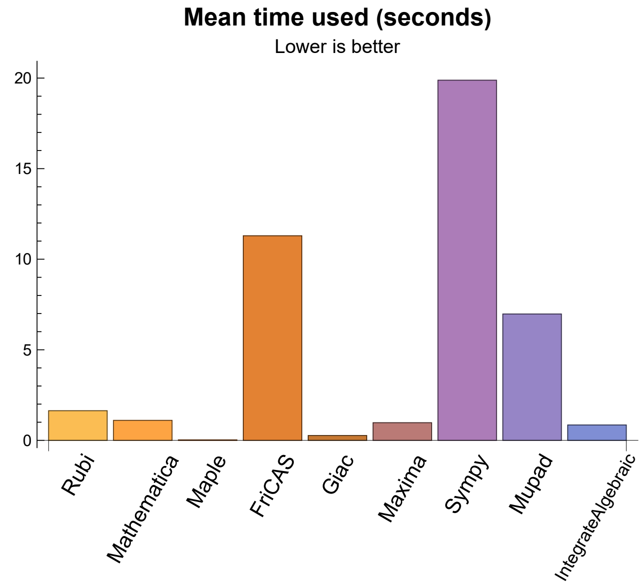
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

| System | Mean time (sec) | Mean size | Normalized mean | Median size | Normalized median |
|---|---|---|---|---|---|
| Rubi | 1.64 | 333.90 | 1.00 | 302.00 | 1.00 |
| Mathematica | 1.10 | 347.54 | 1.17 | 290.50 | 0.97 |
| Maple | 0.03 | 52015.57 | 107.04 | 1172.00 | 3.85 |
| Maxima | 0.97 | 388.27 | 2.32 | 220.00 | 1.11 |
| Fricas | 11.29 | 1373.01 | 5.42 | 344.00 | 2.21 |
| Sympy | 19.88 | 971.36 | 4.13 | 680.00 | 4.09 |
| Giac | 0.27 | 259.92 | 1.44 | 146.00 | 1.21 |
| Mupad | 6.98 | 7822.16 | 9.68 | 187.00 | 1.25 |
| IntegrateAl-gebraic | 0.85 | 346.69 | 1.03 | 273.00 | 0.88 |

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.




Mean time used (seconds)
Lower is better
20
15
10
5
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad
IntegrateAlgebraic

## 1.4 list of integrals that has no closed form antiderivative

{}

## 1.5 list of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

**Mupad** {}

**IntegrateAlgebraic** {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {}

**Mathematica** {25, 28, 142}

**IntegrateAlgebraic** {}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

**Mupad** Verification phase not implemented yet.

## 1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

### 1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

### 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
```

```
# 1.7 is a fudge factor since it is low side from actual leaf count
leafCount = round(1.7*count_ops(anti))

except Exception as ee:
       leafCount =1
```

### 1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

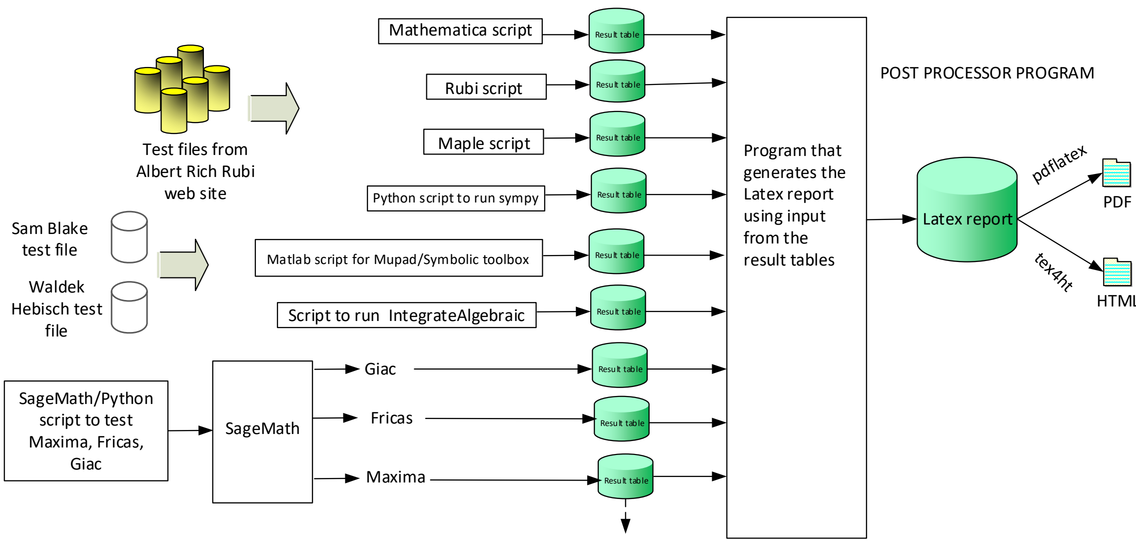
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.




**One record (line) per one integral result. The line is CSV comma separated. This is description of each record**

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"

    *The following field present only in Rubi and Mathematica Tables*

13. integer. 1 if result was verified or 0 if not verified.

    *The following fields present only in Rubi Tables*

14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,..}" which is list of the rules used by Rubi

**High level overview of the CAS independent integration test build system**

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May 11, 2021

# Chapter 2

# detailed summary tables of results

## Local contents

## 2.1 List of integrals sorted by grade for each CAS

### Local contents

### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143 }

B grade: { }

C grade: { }

F grade: { }

### 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 36, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 133, 134, 135, 136, 137, 138, 139, 140, 141 }

B grade: { 35 }

C grade: { 11, 12, 31, 32, 33, 34, 37, 38, 62, 63, 75, 76, 93, 126, 127, 128, 129, 130, 131, 132, 142 }

F grade: { 143 }

### 2.1.3 Maple

A grade: { 1, 2, 3, 10, 22, 28, 31, 32, 33, 34, 36, 38, 39, 92, 94, 95, 96, 99, 100, 101, 126, 127, 128, 129, 130, 131, 132, 133, 134, 138, 139, 140, 141 }

B grade: { 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 29, 30, 35, 37, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 97, 98, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 135, 136, 137 }

C grade: { 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51 }

F grade: { 142, 143 }

### 2.1.4 Maxima

A grade: { 1, 2, 3, 10, 92, 133, 134, 138, 139 }

B grade: { 25, 26, 27, 28, 29, 30 }

C grade: { }

F grade: { 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 135, 136, 137, 140, 141, 142, 143 }

### 2.1.5 FriCAS

A grade: { 1, 2, 3, 10, 13, 14, 33, 36, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 91, 92, 126, 127, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141 }

B grade: { 7, 11, 17, 18, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 37, 55, 66, 67, 80, 81, 82, 83, 93, 97, 98, 99, 116, 117, 135 }

C grade: { }

F grade: { 4, 5, 6, 8, 9, 12, 15, 16, 19, 20, 21, 35, 39, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 84, 85, 86, 87, 88, 89, 90, 94, 95, 96, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 118, 119, 120, 121, 122, 123, 124, 125, 142, 143 }

### 2.1.6 Sympy

A grade: { 33, 139 }

B grade: { 1, 2, 3, 13, 14, 17, 18, 31, 138 }

C grade: { }

F grade: { 4, 5, 6, 7, 8, 9, 10, 11, 12, 15, 16, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 140, 141, 142, 143 }

### 2.1.7 Giac

A grade: { 1, 2, 3, 4, 10, 13, 14, 15, 17, 18, 28, 29, 30, 36, 40, 41, 42, 43, 44, 45, 46, 47, 48, 92, 126, 127, 129, 131, 133, 134, 137, 138, 139, 140, 141 }

B grade: { 5, 11, 16, 31, 32, 33, 34, 37, 38, 51, 128, 130, 132, 135, 136 }

C grade: { }

F grade: { 6, 7, 8, 9, 12, 19, 20, 21, 22, 23, 24, 25, 26, 27, 35, 39, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 142, 143 }

### 2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 13, 14, 16, 17, 18, 31, 33, 36, 133, 134, 138, 139, 140, 141 }

C grade: { }

F grade: { 6, 7, 8, 9, 10, 11, 12, 15, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 34, 35, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 135, 136, 137, 142, 143 }

### 2.1.9 IntegrateAlgebraic

A grade: { 10, 28, 29, 30, 31, 32, 33, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 91, 92, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141 }

B grade: { 24, 142 }

C grade: { 6, 7, 8, 11, 12, 19, 20, 21, 22, 23, 25, 26, 27, 34, 35, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 143 }

F grade: { 1, 2, 3, 4, 5, 9, 13, 14, 15, 16, 17, 18 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA` and `IntegrateAlgebraic` to `I.A.`

| Problem 1 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 94 | 94 | 86 | 133 | 84 | 200 | 333 | 87 | 97 | 0 |
| N.S. | 1 | 1.00 | 0.91 | 1.41 | 0.89 | 2.13 | 3.54 | 0.93 | 1.03 | 0.00 |
| time (sec) | N/A | 0.113 | 0.072 | 0.031 | 0.968 | 0.404 | 1.854 | 0.151 | 3.436 | 0.001 |

| Problem 2 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 228 | 228 | 204 | 373 | 220 | 500 | 933 | 263 | 253 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 1.64 | 0.96 | 2.19 | 4.09 | 1.15 | 1.11 | 0.00 |
| time (sec) | N/A | 0.330 | 0.204 | 0.012 | 0.979 | 0.418 | 7.873 | 0.159 | 0.249 | 0.001 |

| Problem 3 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 441 | 441 | 422 | 822 | 471 | 1014 | 1962 | 623 | 552 | 0 |
| N.S. | 1 | 1.00 | 0.96 | 1.86 | 1.07 | 2.30 | 4.45 | 1.41 | 1.25 | 0.00 |
| time (sec) | N/A | 0.621 | 0.459 | 0.008 | 0.989 | 0.438 | 23.623 | 0.164 | 3.792 | 0.001 |

| Problem 4 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F(-1) | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 274 | 274 | 212 | 745 | 0 | 0 | 0 | 266 | 3888 | 0 |
| N.S. | 1 | 1.00 | 0.77 | 2.72 | 0.00 | 0.00 | 0.00 | 0.97 | 14.19 | 0.00 |
| time (sec) | N/A | 0.282 | 0.368 | 0.016 | 0.000 | 0.000 | 0.000 | 0.168 | 38.323 | 0.001 |

| Problem 5 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 596 | 596 | 523 | 9311 | 0 | 0 | 0 | 1313 | 23006 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 15.62 | 0.00 | 0.00 | 0.00 | 2.20 | 38.60 | 0.00 |
| time (sec) | N/A | 1.772 | 1.842 | 0.055 | 0.000 | 0.000 | 0.000 | 0.205 | 7.534 | 0.001 |

| Problem 6 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 331 | 331 | 322 | 3358 | 0 | 0 | 0 | 0 | -1 | 523 |
| N.S. | 1 | 1.00 | 0.97 | 10.15 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.58 |
| time (sec) | N/A | 0.601 | 0.801 | 0.085 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.782 |

| Problem 7 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 249 | 249 | 249 | 714 | 0 | 6113 | 0 | 0 | -1 | 219 |
| N.S. | 1 | 1.00 | 1.00 | 2.87 | 0.00 | 24.55 | 0.00 | 0.00 | -0.00 | 0.88 |
| time (sec) | N/A | 0.204 | 0.240 | 0.024 | 0.000 | 52.196 | 0.000 | 0.000 | 0.000 | 0.427 |

| Problem 8 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F(-1) | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 381 | 380 | 440 | 2758 | 0 | 0 | 0 | 0 | -1 | 658 |
| N.S. | 1 | 1.00 | 1.15 | 7.24 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.73 |
| time (sec) | N/A | 0.798 | 0.650 | 0.026 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.501 |

| Problem 9 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F(-1) | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 797 | 796 | 674 | 6422 | 0 | 0 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 8.06 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.00 |
| time (sec) | N/A | 1.870 | 4.060 | 0.031 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 180.082 |

| Problem 10 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 47 | 47 | 49 | 46 | 65 | 46 | 0 | 48 | -1 | 37 |
| N.S. | 1 | 1.00 | 1.04 | 0.98 | 1.38 | 0.98 | 0.00 | 1.02 | -0.02 | 0.79 |
| time (sec) | N/A | 0.034 | 0.008 | 0.018 | 0.973 | 0.629 | 0.000 | 0.246 | 0.000 | 0.225 |

| Problem 11 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | B | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 117 | 117 | 78 | 637 | 0 | 758 | 0 | 457 | -1 | 106 |
| N.S. | 1 | 1.00 | 0.67 | 5.44 | 0.00 | 6.48 | 0.00 | 3.91 | -0.01 | 0.91 |
| time (sec) | N/A | 0.169 | 0.033 | 0.176 | 0.000 | 0.560 | 0.000 | 0.376 | 0.000 | 0.239 |

| Problem 12 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | F(-1) | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 484 | 484 | 136 | 6871419 | 0 | 0 | 0 | 0 | -1 | 210 |
| N.S. | 1 | 1.00 | 0.28 | 14197.15 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.43 |
| time (sec) | N/A | 23.581 | 0.083 | 0.515 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.421 |

| Problem 13 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 184 | 182 | 175 | 510 | 0 | 583 | 1260 | 191 | 273 | 0 |
| N.S. | 1 | 0.99 | 0.95 | 2.77 | 0.00 | 3.17 | 6.85 | 1.04 | 1.48 | 0.00 |
| time (sec) | N/A | 0.346 | 0.211 | 0.008 | 0.000 | 0.444 | 16.651 | 0.169 | 3.852 | 0.001 |

| Problem 14 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 542 | 542 | 535 | 1672 | 0 | 1837 | 4663 | 738 | 893 | 0 |
| N.S. | 1 | 1.00 | 0.99 | 3.08 | 0.00 | 3.39 | 8.60 | 1.36 | 1.65 | 0.00 |
| time (sec) | N/A | 1.102 | 0.604 | 0.012 | 0.000 | 0.610 | 145.638 | 0.246 | 4.854 | 0.001 |

| Problem 15 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F(-1) | A | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 406 | 398 | 267 | 1698 | 0 | 0 | 0 | 416 | -1 | 0 |
| N.S. | 1 | 0.98 | 0.66 | 4.18 | 0.00 | 0.00 | 0.00 | 1.02 | -0.00 | 0.00 |
| time (sec) | N/A | 0.478 | 0.462 | 0.013 | 0.000 | 0.000 | 0.000 | 0.169 | 0.000 | 0.001 |

| Problem 16 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 1075 | 1067 | 952 | 51470 | 0 | 0 | 0 | 3226 | 118429 | 0 |
| N.S. | 1 | 0.99 | 0.89 | 47.88 | 0.00 | 0.00 | 0.00 | 3.00 | 110.17 | 0.00 |
| time (sec) | N/A | 4.177 | 6.700 | 0.054 | 0.000 | 0.000 | 0.000 | 0.451 | 30.314 | 0.001 |

| Problem 17 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 140 | 140 | 131 | 340 | 0 | 1150 | 709 | 219 | 395 | 0 |
| N.S. | 1 | 1.00 | 0.94 | 2.43 | 0.00 | 8.21 | 5.06 | 1.56 | 2.82 | 0.00 |
| time (sec) | N/A | 0.131 | 0.143 | 0.006 | 0.000 | 0.644 | 2.437 | 0.172 | 0.422 | 0.001 |

| Problem 18 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 140 | 140 | 131 | 340 | 0 | 1130 | 680 | 207 | 375 | 0 |
| N.S. | 1 | 1.00 | 0.94 | 2.43 | 0.00 | 8.07 | 4.86 | 1.48 | 2.68 | 0.00 |
| time (sec) | N/A | 0.104 | 0.027 | 0.006 | 0.000 | 0.611 | 2.259 | 0.170 | 4.020 | 0.001 |

| Problem 19 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 617 | 615 | 517 | 16209 | 0 | 0 | 0 | 0 | -1 | 893 |
| N.S. | 1 | 1.00 | 0.84 | 26.27 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.45 |
| time (sec) | N/A | 8.998 | 2.086 | 0.051 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.207 |

| Problem 20 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F(-1) | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 1092 | 1092 | 1627 | 59465 | 0 | 0 | 0 | 0 | -1 | 2787 |
| N.S. | 1 | 1.00 | 1.49 | 54.46 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 2.55 |
| time (sec) | N/A | 18.867 | 6.564 | 0.032 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 7.505 |

| Problem 21 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 416 | 416 | 393 | 2269 | 0 | 0 | 0 | 0 | -1 | 278 |
| N.S. | 1 | 1.00 | 0.94 | 5.45 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.67 |
| time (sec) | N/A | 2.702 | 4.175 | 0.044 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.533 |

| Problem 22 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 780 | 780 | 254 | 784 | 0 | 6861 | 0 | 0 | -1 | 218 |
| N.S. | 1 | 1.00 | 0.33 | 1.01 | 0.00 | 8.80 | 0.00 | 0.00 | -0.00 | 0.28 |
| time (sec) | N/A | 5.162 | 0.409 | 0.039 | 0.000 | 47.867 | 0.000 | 0.000 | 0.000 | 0.439 |

| Problem 23 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 302 | 302 | 283 | 1771 | 0 | 8977 | 0 | 0 | -1 | 195 |
| N.S. | 1 | 1.00 | 0.94 | 5.86 | 0.00 | 29.73 | 0.00 | 0.00 | -0.00 | 0.65 |
| time (sec) | N/A | 0.843 | 0.430 | 0.033 | 0.000 | 60.517 | 0.000 | 0.000 | 0.000 | 0.438 |

| Problem 24 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F | F(-1) | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 101 | 101 | 154 | 608 | 0 | 1515 | 0 | 0 | -1 | 563 |
| N.S. | 1 | 1.00 | 1.52 | 6.02 | 0.00 | 15.00 | 0.00 | 0.00 | -0.01 | 5.57 |
| time (sec) | N/A | 0.127 | 0.174 | 0.021 | 0.000 | 0.542 | 0.000 | 0.000 | 0.000 | 2.273 |

| Problem 25 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | F(-1) | F | C |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 139 | 139 | 140 | 324 | 361 | 322 | 0 | 0 | -1 | 149 |
| N.S. | 1 | 1.00 | 1.01 | 2.33 | 2.60 | 2.32 | 0.00 | 0.00 | -0.01 | 1.07 |
| time (sec) | N/A | 0.220 | 0.301 | 0.068 | 1.074 | 0.445 | 0.000 | 0.000 | 0.000 | 0.343 |

| Problem 26 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | F(-1) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 166 | 166 | 167 | 760 | 678 | 344 | 0 | 0 | -1 | 149 |
| N.S. | 1 | 1.00 | 1.01 | 4.58 | 4.08 | 2.07 | 0.00 | 0.00 | -0.01 | 0.90 |
| time (sec) | N/A | 0.221 | 0.378 | 0.027 | 1.112 | 0.442 | 0.000 | 0.000 | 0.000 | 0.394 |

| Problem 27 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | F(-1) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 193 | 193 | 185 | 1560 | 1276 | 439 | 0 | 0 | -1 | 195 |
| N.S. | 1 | 1.00 | 0.96 | 8.08 | 6.61 | 2.27 | 0.00 | 0.00 | -0.01 | 1.01 |
| time (sec) | N/A | 0.266 | 0.596 | 0.020 | 1.263 | 0.442 | 0.000 | 0.000 | 0.000 | 0.535 |

| Problem 28 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | F | A | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 151 | 151 | 148 | 186 | 363 | 245 | 0 | 93 | -1 | 109 |
| N.S. | 1 | 1.00 | 0.98 | 1.23 | 2.40 | 1.62 | 0.00 | 0.62 | -0.01 | 0.72 |
| time (sec) | N/A | 0.229 | 0.356 | 0.051 | 1.069 | 0.432 | 0.000 | 0.477 | 0.000 | 0.782 |

| Problem 29 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 174 | 174 | 172 | 466 | 668 | 365 | 0 | 112 | -1 | 148 |
| N.S. | 1 | 1.00 | 0.99 | 2.68 | 3.84 | 2.10 | 0.00 | 0.64 | -0.01 | 0.85 |
| time (sec) | N/A | 0.255 | 0.570 | 0.019 | 1.099 | 0.446 | 0.000 | 0.499 | 0.000 | 1.019 |

| Problem 30 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 197 | 197 | 190 | 878 | 1276 | 435 | 0 | 121 | -1 | 160 |
| N.S. | 1 | 1.00 | 0.96 | 4.46 | 6.48 | 2.21 | 0.00 | 0.61 | -0.01 | 0.81 |
| time (sec) | N/A | 0.303 | 0.710 | 0.017 | 1.241 | 0.433 | 0.000 | 0.541 | 0.000 | 0.974 |

| Problem 31 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 15 | 15 | 79 | 14 | 0 | 49 | 36 | 31 | 13 | 15 |
| N.S. | 1 | 1.00 | 5.27 | 0.93 | 0.00 | 3.27 | 2.40 | 2.07 | 0.87 | 1.00 |
| time (sec) | N/A | 0.017 | 0.045 | 0.020 | 0.000 | 0.410 | 6.999 | 0.347 | 3.763 | 0.300 |

| Problem 32 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 44 | 44 | 101 | 40 | 0 | 106 | 0 | 108 | -1 | 71 |
| N.S. | 1 | 1.00 | 2.30 | 0.91 | 0.00 | 2.41 | 0.00 | 2.45 | -0.02 | 1.61 |
| time (sec) | N/A | 0.052 | 0.055 | 0.012 | 0.000 | 0.410 | 0.000 | 0.374 | 0.000 | 0.347 |

| Problem 33 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 24 | 24 | 90 | 20 | 0 | 34 | 68 | 39 | 19 | 24 |
| N.S. | 1 | 1.00 | 3.75 | 0.83 | 0.00 | 1.42 | 2.83 | 1.62 | 0.79 | 1.00 |
| time (sec) | N/A | 0.019 | 0.063 | 0.018 | 0.000 | 0.416 | 6.813 | 0.355 | 3.781 | 0.373 |

| Problem 34 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F | B | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 56 | 56 | 114 | 45 | 0 | 307 | 0 | 133 | -1 | 92 |
| N.S. | 1 | 1.00 | 2.04 | 0.80 | 0.00 | 5.48 | 0.00 | 2.38 | -0.02 | 1.64 |
| time (sec) | N/A | 0.050 | 0.061 | 0.012 | 0.000 | 0.454 | 0.000 | 0.319 | 0.000 | 0.243 |

| Problem 35 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | F(-1) | F(-1) | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 249 | 249 | 767 | 3606 | 0 | 0 | 0 | 0 | -1 | 1019 |
| N.S. | 1 | 1.00 | 3.08 | 14.48 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 4.09 |
| time (sec) | N/A | 0.910 | 1.489 | 0.053 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 2.435 |

| Problem 36 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 48 | 48 | 46 | 48 | 0 | 85 | 0 | 81 | 49 | 47 |
| N.S. | 1 | 1.00 | 0.96 | 1.00 | 0.00 | 1.77 | 0.00 | 1.69 | 1.02 | 0.98 |
| time (sec) | N/A | 0.050 | 0.185 | 0.005 | 0.000 | 0.630 | 0.000 | 0.266 | 3.753 | 0.418 |

| Problem 37 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 165 | 94 | 0 | 56 | 0 | 98 | -1 | 17 |
| N.S. | 1 | 1.00 | 9.71 | 5.53 | 0.00 | 3.29 | 0.00 | 5.76 | -0.06 | 1.00 |
| time (sec) | N/A | 0.022 | 0.288 | 0.017 | 0.000 | 0.423 | 0.000 | 0.231 | 0.000 | 0.297 |

| Problem 38 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 86 | 86 | 150 | 123 | 0 | 132 | 0 | 163 | -1 | 56 |
| N.S. | 1 | 1.00 | 1.74 | 1.43 | 0.00 | 1.53 | 0.00 | 1.90 | -0.01 | 0.65 |
| time (sec) | N/A | 0.185 | 0.109 | 0.013 | 0.000 | 0.429 | 0.000 | 0.275 | 0.000 | 0.350 |

| Problem 39 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | F | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 136 | 136 | 108 | 121 | 0 | 0 | 0 | 0 | -1 | 127 |
| N.S. | 1 | 1.00 | 0.79 | 0.89 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.93 |
| time (sec) | N/A | 0.114 | 0.091 | 0.027 | 0.000 | 50.453 | 0.000 | 0.000 | 0.000 | 0.278 |

| Problem 40 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 212 | 212 | 129 | 103 | 0 | 175 | 0 | 117 | -1 | 111 |
| N.S. | 1 | 1.00 | 0.61 | 0.49 | 0.00 | 0.83 | 0.00 | 0.55 | -0.00 | 0.52 |
| time (sec) | N/A | 0.116 | 0.151 | 0.048 | 0.000 | 0.429 | 0.000 | 0.253 | 0.000 | 0.294 |

| Problem 41 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 161 | 161 | 117 | 83 | 0 | 157 | 0 | 98 | -1 | 96 |
| N.S. | 1 | 1.00 | 0.73 | 0.52 | 0.00 | 0.98 | 0.00 | 0.61 | -0.01 | 0.60 |
| time (sec) | N/A | 0.067 | 0.116 | 0.010 | 0.000 | 0.429 | 0.000 | 0.213 | 0.000 | 0.255 |

| Problem 42 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 148 | 148 | 85 | 65 | 0 | 128 | 0 | 79 | -1 | 87 |
| N.S. | 1 | 1.00 | 0.57 | 0.44 | 0.00 | 0.86 | 0.00 | 0.53 | -0.01 | 0.59 |
| time (sec) | N/A | 0.057 | 0.063 | 0.008 | 0.000 | 0.430 | 0.000 | 0.271 | 0.000 | 0.286 |

| Problem 43 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 160 | 160 | 139 | 94 | 0 | 341 | 0 | 102 | -1 | 114 |
| N.S. | 1 | 1.00 | 0.87 | 0.59 | 0.00 | 2.13 | 0.00 | 0.64 | -0.01 | 0.71 |
| time (sec) | N/A | 0.119 | 0.143 | 0.009 | 0.000 | 0.445 | 0.000 | 0.247 | 0.000 | 0.305 |

| Problem 44 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 156 | 156 | 118 | 118 | 0 | 333 | 0 | 126 | -1 | 111 |
| N.S. | 1 | 1.00 | 0.76 | 0.76 | 0.00 | 2.13 | 0.00 | 0.81 | -0.01 | 0.71 |
| time (sec) | N/A | 0.113 | 0.281 | 0.013 | 0.000 | 0.444 | 0.000 | 0.260 | 0.000 | 0.309 |

| Problem 45 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 161 | 161 | 126 | 141 | 0 | 377 | 0 | 199 | -1 | 115 |
| N.S. | 1 | 1.00 | 0.78 | 0.88 | 0.00 | 2.34 | 0.00 | 1.24 | -0.01 | 0.71 |
| time (sec) | N/A | 0.116 | 0.116 | 0.015 | 0.000 | 0.440 | 0.000 | 0.275 | 0.000 | 0.409 |

| Problem 46 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 317 | 317 | 198 | 530 | 0 | 517 | 0 | 368 | -1 | 263 |
| N.S. | 1 | 1.00 | 0.62 | 1.67 | 0.00 | 1.63 | 0.00 | 1.16 | -0.00 | 0.83 |
| time (sec) | N/A | 0.333 | 0.275 | 0.018 | 0.000 | 0.469 | 0.000 | 0.311 | 0.000 | 1.001 |

| Problem 47 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 227 | 227 | 147 | 381 | 0 | 391 | 0 | 268 | -1 | 196 |
| N.S. | 1 | 1.00 | 0.65 | 1.68 | 0.00 | 1.72 | 0.00 | 1.18 | -0.00 | 0.86 |
| time (sec) | N/A | 0.127 | 0.131 | 0.013 | 0.000 | 0.452 | 0.000 | 0.283 | 0.000 | 0.700 |

| Problem 48 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 198 | 198 | 134 | 257 | 0 | 287 | 0 | 185 | -1 | 150 |
| N.S. | 1 | 1.00 | 0.68 | 1.30 | 0.00 | 1.45 | 0.00 | 0.93 | -0.01 | 0.76 |
| time (sec) | N/A | 0.101 | 0.127 | 0.011 | 0.000 | 0.453 | 0.000 | 0.302 | 0.000 | 0.602 |

| Problem 49 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 211 | 211 | 149 | 214 | 0 | 651 | 0 | 0 | -1 | 154 |
| N.S. | 1 | 1.00 | 0.71 | 1.01 | 0.00 | 3.09 | 0.00 | 0.00 | -0.00 | 0.73 |
| time (sec) | N/A | 0.221 | 0.198 | 0.011 | 0.000 | 1.145 | 0.000 | 0.000 | 0.000 | 0.638 |

| Problem 50 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 202 | 202 | 155 | 249 | 0 | 647 | 0 | 0 | -1 | 136 |
| N.S. | 1 | 1.00 | 0.77 | 1.23 | 0.00 | 3.20 | 0.00 | 0.00 | -0.00 | 0.67 |
| time (sec) | N/A | 0.194 | 0.189 | 0.013 | 0.000 | 0.685 | 0.000 | 0.000 | 0.000 | 0.619 |

| Problem 51 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 215 | 215 | 161 | 358 | 0 | 693 | 0 | 450 | -1 | 150 |
| N.S. | 1 | 1.00 | 0.75 | 1.67 | 0.00 | 3.22 | 0.00 | 2.09 | -0.00 | 0.70 |
| time (sec) | N/A | 0.183 | 0.218 | 0.016 | 0.000 | 0.859 | 0.000 | 0.413 | 0.000 | 0.806 |

| Problem 52 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F(-1) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 452 | 452 | 516 | 7739 | 0 | 0 | 0 | 0 | -1 | 496 |
| N.S. | 1 | 1.00 | 1.14 | 17.12 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.10 |
| time (sec) | N/A | 1.967 | 2.500 | 0.039 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.672 |

| Problem 53 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F(-1) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 395 | 395 | 422 | 5581 | 0 | 0 | 0 | 0 | -1 | 383 |
| N.S. | 1 | 1.00 | 1.07 | 14.13 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.97 |
| time (sec) | N/A | 0.931 | 1.524 | 0.015 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.527 |

| Problem 54 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F(-1) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 298 | 298 | 282 | 3249 | 0 | 0 | 0 | 0 | -1 | 267 |
| N.S. | 1 | 1.00 | 0.95 | 10.90 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.90 |
| time (sec) | N/A | 0.376 | 0.363 | 0.013 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.446 |

| Problem 55 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 358 | 358 | 314 | 3544 | 0 | 2266 | 0 | 0 | -1 | 306 |
| N.S. | 1 | 1.00 | 0.88 | 9.90 | 0.00 | 6.33 | 0.00 | 0.00 | -0.00 | 0.85 |
| time (sec) | N/A | 1.314 | 0.651 | 0.019 | 0.000 | 38.141 | 0.000 | 0.000 | 0.000 | 0.488 |

| Problem 56 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F(-1) | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 382 | 382 | 569 | 3703 | 0 | 0 | 0 | 0 | -1 | 343 |
| N.S. | 1 | 1.00 | 1.49 | 9.69 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.90 |
| time (sec) | N/A | 1.417 | 3.033 | 0.025 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.577 |

| Problem 57 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F(-1) | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 507 | 507 | 642 | 3993 | 0 | 0 | 0 | 0 | -1 | 541 |
| N.S. | 1 | 1.00 | 1.27 | 7.88 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.07 |
| time (sec) | N/A | 1.880 | 2.607 | 0.019 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.995 |

| Problem 58 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F(-1) | F | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 795 | 795 | 793 | 19148 | 0 | 0 | 0 | 0 | -1 | 960 |
| N.S. | 1 | 1.00 | 1.00 | 24.09 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.21 |
| time (sec) | N/A | 4.264 | 3.449 | 0.030 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.215 |

| Problem 59 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 553 | 553 | 755 | 14709 | 0 | 0 | 0 | 0 | -1 | 771 |
| N.S. | 1 | 1.00 | 1.37 | 26.60 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.39 |
| time (sec) | N/A | 2.431 | 2.011 | 0.015 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.886 |

| Problem 60 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 484 | 482 | 603 | 8954 | 0 | 0 | 0 | 0 | -1 | 557 |
| N.S. | 1 | 1.00 | 1.25 | 18.50 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.15 |
| time (sec) | N/A | 4.240 | 1.073 | 0.014 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.703 |

| Problem 61 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F(-1) | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 496 | 496 | 746 | 9728 | 0 | 0 | 0 | 0 | -1 | 559 |
| N.S. | 1 | 1.00 | 1.50 | 19.61 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.13 |
| time (sec) | N/A | 2.569 | 1.575 | 0.018 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.684 |

| Problem 62 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | F(-1) | F | F | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 604 | 604 | 885 | 9912 | 0 | 0 | 0 | 0 | -1 | 502 |
| N.S. | 1 | 1.00 | 1.47 | 16.41 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.83 |
| time (sec) | N/A | 2.809 | 4.159 | 0.020 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.736 |

| Problem 63 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | F(-1) | F(-1) | F | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 668 | 668 | 904 | 10298 | 0 | 0 | 0 | 0 | -1 | 624 |
| N.S. | 1 | 1.00 | 1.35 | 15.42 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.93 |
| time (sec) | N/A | 3.465 | 3.223 | 0.019 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.954 |

| Problem 64 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 380 | 380 | 378 | 2397 | 0 | 0 | 0 | 0 | -1 | 315 |
| N.S. | 1 | 1.00 | 0.99 | 6.31 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.83 |
| time (sec) | N/A | 1.170 | 1.333 | 0.029 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.486 |

| Problem 65 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 344 | 344 | 334 | 1796 | 0 | 0 | 0 | 0 | -1 | 229 |
| N.S. | 1 | 1.00 | 0.97 | 5.22 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.67 |
| time (sec) | N/A | 0.541 | 0.701 | 0.015 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.432 |

| Problem 66 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 294 | 294 | 275 | 1172 | 0 | 5085 | 0 | 0 | -1 | 156 |
| N.S. | 1 | 1.00 | 0.94 | 3.99 | 0.00 | 17.30 | 0.00 | 0.00 | -0.00 | 0.53 |
| time (sec) | N/A | 0.235 | 0.320 | 0.013 | 0.000 | 1.645 | 0.000 | 0.000 | 0.000 | 0.384 |

| Problem 67 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 266 | 266 | 247 | 589 | 0 | 5073 | 0 | 0 | -1 | 131 |
| N.S. | 1 | 1.00 | 0.93 | 2.21 | 0.00 | 19.07 | 0.00 | 0.00 | -0.00 | 0.49 |
| time (sec) | N/A | 0.151 | 0.291 | 0.013 | 0.000 | 1.622 | 0.000 | 0.000 | 0.000 | 0.364 |

| Problem 68 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F(-1) | F | F(-1) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 330 | 330 | 319 | 681 | 0 | 0 | 0 | 0 | -1 | 242 |
| N.S. | 1 | 1.00 | 0.97 | 2.06 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.73 |
| time (sec) | N/A | 0.825 | 0.742 | 0.017 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.448 |

| Problem 69 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F(-1) | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 367 | 367 | 356 | 736 | 0 | 0 | 0 | 0 | -1 | 304 |
| N.S. | 1 | 1.00 | 0.97 | 2.01 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.83 |
| time (sec) | N/A | 1.199 | 0.838 | 0.018 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.552 |

| Problem 70 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F(-1) | F | F(-1) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 457 | 457 | 460 | 911 | 0 | 0 | 0 | 0 | -1 | 430 |
| N.S. | 1 | 1.00 | 1.01 | 1.99 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.94 |
| time (sec) | N/A | 1.863 | 1.569 | 0.018 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.840 |

| Problem 71 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 499 | 499 | 577 | 6124 | 0 | 0 | 0 | 0 | -1 | 425 |
| N.S. | 1 | 1.00 | 1.16 | 12.27 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.85 |
| time (sec) | N/A | 2.107 | 2.696 | 0.032 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.758 |

| Problem 72 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 410 | 410 | 509 | 4752 | 0 | 0 | 0 | 0 | -1 | 323 |
| N.S. | 1 | 1.00 | 1.24 | 11.59 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.79 |
| time (sec) | N/A | 0.709 | 2.073 | 0.019 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.632 |

| Problem 73 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 411 | 411 | 457 | 3000 | 0 | 0 | 0 | 0 | -1 | 352 |
| N.S. | 1 | 1.00 | 1.11 | 7.30 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.86 |
| time (sec) | N/A | 0.826 | 0.806 | 0.015 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.638 |

| Problem 74 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 416 | 416 | 320 | 1713 | 0 | 0 | 0 | 0 | -1 | 368 |
| N.S. | 1 | 1.00 | 0.77 | 4.12 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.88 |
| time (sec) | N/A | 0.617 | 2.061 | 0.015 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.644 |

| Problem 75 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | F(-1) | F | F(-1) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 526 | 526 | 497 | 1945 | 0 | 0 | 0 | 0 | -1 | 552 |
| N.S. | 1 | 1.00 | 0.94 | 3.70 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.05 |
| time (sec) | N/A | 2.183 | 3.497 | 0.017 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.951 |

| Problem 76 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | F(-1) | F | F | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 618 | 618 | 557 | 2046 | 0 | 0 | 0 | 0 | -1 | 720 |
| N.S. | 1 | 1.00 | 0.90 | 3.31 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.17 |
| time (sec) | N/A | 2.280 | 3.604 | 0.019 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.287 |

| Problem 77 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 392 | 392 | 327 | 1817 | 0 | 0 | 0 | 0 | -1 | 414 |
| N.S. | 1 | 1.00 | 0.83 | 4.64 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.06 |
| time (sec) | N/A | 0.952 | 0.889 | 0.018 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.053 |

| Problem 78 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 316 | 316 | 302 | 1810 | 0 | 0 | 0 | 0 | -1 | 327 |
| N.S. | 1 | 1.00 | 0.96 | 5.73 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.03 |
| time (sec) | N/A | 0.489 | 0.485 | 0.017 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.832 |

| Problem 79 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 282 | 282 | 272 | 1667 | 0 | 0 | 0 | 0 | -1 | 361 |
| N.S. | 1 | 1.00 | 0.96 | 5.91 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.28 |
| time (sec) | N/A | 0.295 | 0.294 | 0.014 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.585 |

| Problem 80 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 266 | 266 | 253 | 1669 | 0 | 1139 | 0 | 0 | -1 | 273 |
| N.S. | 1 | 1.00 | 0.95 | 6.27 | 0.00 | 4.28 | 0.00 | 0.00 | -0.00 | 1.03 |
| time (sec) | N/A | 0.226 | 0.166 | 0.014 | 0.000 | 98.756 | 0.000 | 0.000 | 0.000 | 0.420 |

| Problem 81 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F | F | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 267 | 267 | 255 | 1764 | 0 | 1253 | 0 | 0 | -1 | 342 |
| N.S. | 1 | 1.00 | 0.96 | 6.61 | 0.00 | 4.69 | 0.00 | 0.00 | -0.00 | 1.28 |
| time (sec) | N/A | 0.778 | 0.295 | 0.018 | 0.000 | 17.018 | 0.000 | 0.000 | 0.000 | 0.390 |

| Problem 82 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 286 | 286 | 275 | 1819 | 0 | 1094 | 0 | 0 | -1 | 299 |
| N.S. | 1 | 1.00 | 0.96 | 6.36 | 0.00 | 3.83 | 0.00 | 0.00 | -0.00 | 1.05 |
| time (sec) | N/A | 0.706 | 0.431 | 0.020 | 0.000 | 24.595 | 0.000 | 0.000 | 0.000 | 0.531 |

| Problem 83 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 353 | 353 | 316 | 1953 | 0 | 1485 | 0 | 0 | -1 | 392 |
| N.S. | 1 | 1.00 | 0.90 | 5.53 | 0.00 | 4.21 | 0.00 | 0.00 | -0.00 | 1.11 |
| time (sec) | N/A | 0.879 | 0.522 | 0.017 | 0.000 | 133.831 | 0.000 | 0.000 | 0.000 | 0.858 |

| Problem 84 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F(-1) | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 501 | 501 | 447 | 4884 | 0 | 0 | 0 | 0 | -1 | 774 |
| N.S. | 1 | 1.00 | 0.89 | 9.75 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.54 |
| time (sec) | N/A | 1.412 | 1.493 | 0.032 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 3.071 |

| Problem 85 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 417 | 417 | 395 | 4900 | 0 | 0 | 0 | 0 | -1 | 620 |
| N.S. | 1 | 1.00 | 0.95 | 11.75 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.49 |
| time (sec) | N/A | 1.015 | 1.017 | 0.018 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 2.563 |

| Problem 86 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 349 | 349 | 330 | 4567 | 0 | 0 | 0 | 0 | -1 | 626 |
| N.S. | 1 | 1.00 | 0.95 | 13.09 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.79 |
| time (sec) | N/A | 0.522 | 0.805 | 0.015 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.285 |

| Problem 87 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 315 | 315 | 298 | 4574 | 0 | 0 | 0 | 0 | -1 | 524 |
| N.S. | 1 | 1.00 | 0.95 | 14.52 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.66 |
| time (sec) | N/A | 0.517 | 0.812 | 0.015 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.996 |

| Problem 88 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F(-1) | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 469 | 469 | 755 | 4765 | 0 | 0 | 0 | 0 | -1 | 618 |
| N.S. | 1 | 1.00 | 1.61 | 10.16 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.32 |
| time (sec) | N/A | 1.274 | 0.505 | 0.018 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.023 |

| Problem 89 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F(-1) | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 463 | 463 | 765 | 4799 | 0 | 0 | 0 | 0 | -1 | 550 |
| N.S. | 1 | 1.00 | 1.65 | 10.37 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.19 |
| time (sec) | N/A | 1.201 | 0.638 | 0.018 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.890 |

| Problem 90 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F(-1) | F | F | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 614 | 614 | 303 | 5056 | 0 | 0 | 0 | 0 | -1 | 597 |
| N.S. | 1 | 1.00 | 0.49 | 8.23 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.97 |
| time (sec) | N/A | 1.436 | 0.970 | 0.020 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.171 |

| Problem 91 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 189 | 189 | 181 | 1346 | 0 | 2579 | 0 | 0 | -1 | 314 |
| N.S. | 1 | 1.00 | 0.96 | 7.12 | 0.00 | 13.65 | 0.00 | 0.00 | -0.01 | 1.66 |
| time (sec) | N/A | 0.285 | 0.583 | 0.018 | 0.000 | 164.299 | 0.000 | 0.000 | 0.000 | 0.823 |

| Problem 92 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 75 | 75 | 75 | 102 | 83 | 70 | 0 | 73 | -1 | 57 |
| N.S. | 1 | 1.00 | 1.00 | 1.36 | 1.11 | 0.93 | 0.00 | 0.97 | -0.01 | 0.76 |
| time (sec) | N/A | 0.058 | 0.038 | 0.015 | 0.975 | 0.416 | 0.000 | 0.195 | 0.000 | 0.144 |

| Problem 93 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 130 | 130 | 120 | 789 | 0 | 777 | 0 | 0 | -1 | 103 |
| N.S. | 1 | 1.00 | 0.92 | 6.07 | 0.00 | 5.98 | 0.00 | 0.00 | -0.01 | 0.79 |
| time (sec) | N/A | 0.160 | 0.161 | 0.137 | 0.000 | 0.460 | 0.000 | 0.000 | 0.000 | 0.431 |

| Problem 94 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | F(-1) | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 369 | 369 | 300 | 516 | 0 | 0 | 0 | 0 | -1 | 263 |
| N.S. | 1 | 1.00 | 0.81 | 1.40 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.71 |
| time (sec) | N/A | 0.809 | 1.709 | 0.027 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.790 |

| Problem 95 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | F(-1) | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 287 | 287 | 325 | 410 | 0 | 0 | 0 | 0 | -1 | 224 |
| N.S. | 1 | 1.00 | 1.13 | 1.43 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.78 |
| time (sec) | N/A | 0.633 | 1.158 | 0.018 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.497 |

| Problem 96 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | F(-1) | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 266 | 266 | 250 | 399 | 0 | 0 | 0 | 0 | -1 | 197 |
| N.S. | 1 | 1.00 | 0.94 | 1.50 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.74 |
| time (sec) | N/A | 0.216 | 0.425 | 0.015 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.394 |

| Problem 97 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 220 | 220 | 211 | 354 | 0 | 2753 | 0 | 0 | -1 | 149 |
| N.S. | 1 | 1.00 | 0.96 | 1.61 | 0.00 | 12.51 | 0.00 | 0.00 | -0.00 | 0.68 |
| time (sec) | N/A | 0.130 | 0.173 | 0.015 | 0.000 | 1.051 | 0.000 | 0.000 | 0.000 | 0.353 |

| Problem 98 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 220 | 220 | 209 | 358 | 0 | 2641 | 0 | 0 | -1 | 151 |
| N.S. | 1 | 1.00 | 0.95 | 1.63 | 0.00 | 12.00 | 0.00 | 0.00 | -0.00 | 0.69 |
| time (sec) | N/A | 0.118 | 0.096 | 0.015 | 0.000 | 1.242 | 0.000 | 0.000 | 0.000 | 0.351 |

| Problem 99 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F | F | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 267 | 267 | 252 | 391 | 0 | 5995 | 0 | 0 | -1 | 199 |
| N.S. | 1 | 1.00 | 0.94 | 1.46 | 0.00 | 22.45 | 0.00 | 0.00 | -0.00 | 0.75 |
| time (sec) | N/A | 0.663 | 0.443 | 0.016 | 0.000 | 70.193 | 0.000 | 0.000 | 0.000 | 0.394 |

| Problem 100 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F(-1) | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 291 | 291 | 325 | 427 | 0 | 0 | 0 | 0 | -1 | 227 |
| N.S. | 1 | 1.00 | 1.12 | 1.47 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.78 |
| time (sec) | N/A | 0.657 | 1.033 | 0.017 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.522 |

| Problem 101 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F(-1) | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 376 | 376 | 314 | 519 | 0 | 0 | 0 | 0 | -1 | 251 |
| N.S. | 1 | 1.00 | 0.84 | 1.38 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.67 |
| time (sec) | N/A | 0.734 | 2.009 | 0.022 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.837 |

| Problem 102 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 466 | 466 | 562 | 1648 | 0 | 0 | 0 | 0 | -1 | 469 |
| N.S. | 1 | 1.00 | 1.21 | 3.54 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.01 |
| time (sec) | N/A | 1.347 | 1.405 | 0.022 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.538 |

| Problem 103 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 341 | 341 | 414 | 1480 | 0 | 0 | 0 | 0 | -1 | 424 |
| N.S. | 1 | 1.00 | 1.21 | 4.34 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.24 |
| time (sec) | N/A | 1.041 | 1.197 | 0.019 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.958 |

| Problem 104 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 297 | 297 | 352 | 1427 | 0 | 0 | 0 | 0 | -1 | 388 |
| N.S. | 1 | 1.00 | 1.19 | 4.80 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.31 |
| time (sec) | N/A | 0.454 | 0.402 | 0.019 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.872 |

| Problem 105 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 299 | 299 | 356 | 1360 | 0 | 0 | 0 | 0 | -1 | 419 |
| N.S. | 1 | 1.00 | 1.19 | 4.55 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.40 |
| time (sec) | N/A | 0.400 | 0.353 | 0.016 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.909 |

| Problem 106 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 310 | 310 | 360 | 1376 | 0 | 0 | 0 | 0 | -1 | 392 |
| N.S. | 1 | 1.00 | 1.16 | 4.44 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.26 |
| time (sec) | N/A | 0.411 | 0.399 | 0.016 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.878 |

| Problem 107 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F(-1) | F | F | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 394 | 394 | 436 | 1518 | 0 | 0 | 0 | 0 | -1 | 498 |
| N.S. | 1 | 1.00 | 1.11 | 3.85 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.26 |
| time (sec) | N/A | 1.182 | 0.866 | 0.019 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.587 |

| Problem 108 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F(-1) | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 454 | 454 | 488 | 1656 | 0 | 0 | 0 | 0 | -1 | 649 |
| N.S. | 1 | 1.00 | 1.07 | 3.65 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.43 |
| time (sec) | N/A | 1.193 | 1.193 | 0.019 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 2.916 |

| Problem 109 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 761 | 761 | 552 | 14815 | 0 | 0 | 0 | 0 | -1 | 869 |
| N.S. | 1 | 1.00 | 0.73 | 19.47 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.14 |
| time (sec) | N/A | 3.135 | 2.312 | 0.019 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.448 |

| Problem 110 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 549 | 549 | 496 | 10138 | 0 | 0 | 0 | 0 | -1 | 646 |
| N.S. | 1 | 1.00 | 0.90 | 18.47 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.18 |
| time (sec) | N/A | 7.028 | 1.776 | 0.015 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.865 |

| Problem 111 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 431 | 431 | 417 | 6019 | 0 | 0 | 0 | 0 | -1 | 402 |
| N.S. | 1 | 1.00 | 0.97 | 13.97 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.93 |
| time (sec) | N/A | 0.650 | 0.738 | 0.015 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.001 |

| Problem 112 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F(-1) | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 523 | 521 | 454 | 6460 | 0 | 0 | 0 | 0 | -1 | 474 |
| N.S. | 1 | 1.00 | 0.87 | 12.35 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.91 |
| time (sec) | N/A | 3.701 | 1.309 | 0.019 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.639 |

| Problem 113 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F(-1) | F(-1) | F(-1) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 736 | 736 | 520 | 6765 | 0 | 0 | 0 | 0 | -1 | 589 |
| N.S. | 1 | 1.00 | 0.71 | 9.19 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.80 |
| time (sec) | N/A | 3.476 | 1.683 | 0.019 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.882 |

| Problem 114 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 545 | 545 | 550 | 3131 | 0 | 0 | 0 | 0 | -1 | 430 |
| N.S. | 1 | 1.00 | 1.01 | 5.74 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.79 |
| time (sec) | N/A | 3.722 | 2.201 | 0.032 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.740 |

| Problem 115 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F(-2) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 463 | 463 | 468 | 2321 | 0 | 0 | 0 | 0 | -1 | 324 |
| N.S. | 1 | 1.00 | 1.01 | 5.01 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.70 |
| time (sec) | N/A | 3.436 | 0.947 | 0.017 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.532 |

| Problem 116 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F | F(-1) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 402 | 402 | 407 | 1516 | 0 | 11311 | 0 | 0 | -1 | 204 |
| N.S. | 1 | 1.00 | 1.01 | 3.77 | 0.00 | 28.14 | 0.00 | 0.00 | -0.00 | 0.51 |
| time (sec) | N/A | 0.963 | 0.974 | 0.014 | 0.000 | 14.499 | 0.000 | 0.000 | 0.000 | 0.426 |

| Problem 117 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F | F(-1) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 374 | 374 | 376 | 761 | 0 | 11287 | 0 | 0 | -1 | 211 |
| N.S. | 1 | 1.00 | 1.01 | 2.03 | 0.00 | 30.18 | 0.00 | 0.00 | -0.00 | 0.56 |
| time (sec) | N/A | 0.314 | 0.789 | 0.014 | 0.000 | 17.215 | 0.000 | 0.000 | 0.000 | 0.002 |

| Problem 118 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F(-1) | F | F(-1) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 451 | 451 | 450 | 859 | 0 | 0 | 0 | 0 | -1 | 326 |
| N.S. | 1 | 1.00 | 1.00 | 1.90 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.72 |
| time (sec) | N/A | 2.632 | 2.414 | 0.019 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.539 |

| Problem 119 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F(-1) | F | F(-1) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 543 | 543 | 533 | 983 | 0 | 0 | 0 | 0 | -1 | 430 |
| N.S. | 1 | 1.00 | 0.98 | 1.81 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.79 |
| time (sec) | N/A | 4.592 | 1.408 | 0.017 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.767 |

| Problem 120 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F(-1) | F | F(-1) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 679 | 679 | 669 | 1296 | 0 | 0 | 0 | 0 | -1 | 558 |
| N.S. | 1 | 1.00 | 0.99 | 1.91 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.82 |
| time (sec) | N/A | 11.226 | 2.110 | 0.020 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.478 |

| Problem 121 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F(-1) | F(-1) | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 779 | 779 | 1066 | 14651 | 0 | 0 | 0 | 0 | -1 | 729 |
| N.S. | 1 | 1.00 | 1.37 | 18.81 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.94 |
| time (sec) | N/A | 14.170 | 2.624 | 0.035 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.691 |

| Problem 122 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F(-1) | F | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 609 | 609 | 1097 | 11341 | 0 | 0 | 0 | 0 | -1 | 570 |
| N.S. | 1 | 1.00 | 1.80 | 18.62 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.94 |
| time (sec) | N/A | 5.842 | 6.585 | 0.021 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.402 |

| Problem 123 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F(-1) | F | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 609 | 609 | 770 | 7163 | 0 | 0 | 0 | 0 | -1 | 608 |
| N.S. | 1 | 1.00 | 1.26 | 11.76 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.00 |
| time (sec) | N/A | 5.643 | 5.684 | 0.017 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.358 |

| Problem 124 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F(-1) | F | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 666 | 666 | 700 | 4099 | 0 | 0 | 0 | 0 | -1 | 730 |
| N.S. | 1 | 1.00 | 1.05 | 6.15 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.10 |
| time (sec) | N/A | 1.746 | 5.042 | 0.018 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.002 |

| Problem 125 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F(-1) | F | F | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 816 | 814 | 1121 | 4594 | 0 | 0 | 0 | 0 | -1 | 1054 |
| N.S. | 1 | 1.00 | 1.37 | 5.63 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.29 |
| time (sec) | N/A | 15.916 | 6.560 | 0.036 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 6.985 |

| Problem 126 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 140 | 140 | 210 | 159 | 0 | 178 | 0 | 188 | -1 | 108 |
| N.S. | 1 | 1.00 | 1.50 | 1.14 | 0.00 | 1.27 | 0.00 | 1.34 | -0.01 | 0.77 |
| time (sec) | N/A | 0.501 | 0.527 | 0.019 | 0.000 | 1.278 | 0.000 | 0.203 | 0.000 | 0.416 |

| Problem 127 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 115 | 115 | 192 | 144 | 0 | 175 | 0 | 185 | -1 | 99 |
| N.S. | 1 | 1.00 | 1.67 | 1.25 | 0.00 | 1.52 | 0.00 | 1.61 | -0.01 | 0.86 |
| time (sec) | N/A | 0.420 | 0.438 | 0.015 | 0.000 | 1.397 | 0.000 | 0.247 | 0.000 | 0.368 |

| Problem 128 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 98 | 98 | 159 | 130 | 0 | 161 | 0 | 171 | -1 | 82 |
| N.S. | 1 | 1.00 | 1.62 | 1.33 | 0.00 | 1.64 | 0.00 | 1.74 | -0.01 | 0.84 |
| time (sec) | N/A | 0.198 | 0.187 | 0.009 | 0.000 | 1.036 | 0.000 | 0.226 | 0.000 | 0.335 |

| Problem 129 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 68 | 69 | 174 | 92 | 0 | 50 | 0 | 68 | -1 | 38 |
| N.S. | 1 | 1.01 | 2.56 | 1.35 | 0.00 | 0.74 | 0.00 | 1.00 | -0.01 | 0.56 |
| time (sec) | N/A | 0.062 | 0.168 | 0.011 | 0.000 | 0.875 | 0.000 | 0.185 | 0.000 | 0.249 |

| Problem 130 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 95 | 95 | 150 | 121 | 0 | 132 | 0 | 165 | -1 | 62 |
| N.S. | 1 | 1.00 | 1.58 | 1.27 | 0.00 | 1.39 | 0.00 | 1.74 | -0.01 | 0.65 |
| time (sec) | N/A | 0.111 | 0.100 | 0.011 | 0.000 | 0.785 | 0.000 | 0.188 | 0.000 | 0.309 |

| Problem 131 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 130 | 130 | 200 | 152 | 0 | 170 | 0 | 199 | -1 | 97 |
| N.S. | 1 | 1.00 | 1.54 | 1.17 | 0.00 | 1.31 | 0.00 | 1.53 | -0.01 | 0.75 |
| time (sec) | N/A | 0.417 | 0.437 | 0.019 | 0.000 | 1.055 | 0.000 | 0.201 | 0.000 | 0.333 |

| Problem 132 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 151 | 151 | 225 | 169 | 0 | 194 | 0 | 269 | -1 | 118 |
| N.S. | 1 | 1.00 | 1.49 | 1.12 | 0.00 | 1.28 | 0.00 | 1.78 | -0.01 | 0.78 |
| time (sec) | N/A | 0.454 | 0.405 | 0.020 | 0.000 | 1.090 | 0.000 | 0.483 | 0.000 | 0.425 |

| Problem 133 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 149 | 149 | 87 | 147 | 155 | 88 | 0 | 85 | 187 | 92 |
| N.S. | 1 | 1.00 | 0.58 | 0.99 | 1.04 | 0.59 | 0.00 | 0.57 | 1.26 | 0.62 |
| time (sec) | N/A | 0.093 | 0.191 | 0.019 | 1.001 | 0.955 | 0.000 | 0.297 | 5.085 | 0.732 |

| Problem 134 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 103 | 103 | 72 | 96 | 104 | 73 | 0 | 70 | 136 | 77 |
| N.S. | 1 | 1.00 | 0.70 | 0.93 | 1.01 | 0.71 | 0.00 | 0.68 | 1.32 | 0.75 |
| time (sec) | N/A | 0.042 | 0.033 | 0.006 | 0.981 | 0.874 | 0.000 | 0.204 | 4.690 | 0.443 |

| Problem 135 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 28 | 28 | 37 | 163 | 0 | 53 | 0 | 63 | -1 | 30 |
| N.S. | 1 | 1.00 | 1.32 | 5.82 | 0.00 | 1.89 | 0.00 | 2.25 | -0.04 | 1.07 |
| time (sec) | N/A | 0.048 | 0.098 | 0.016 | 0.000 | 0.631 | 0.000 | 0.272 | 0.000 | 0.300 |

| Problem 136 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 84 | 84 | 114 | 245 | 0 | 126 | 0 | 159 | -1 | 80 |
| N.S. | 1 | 1.00 | 1.36 | 2.92 | 0.00 | 1.50 | 0.00 | 1.89 | -0.01 | 0.95 |
| time (sec) | N/A | 0.078 | 0.238 | 0.023 | 0.000 | 0.981 | 0.000 | 0.272 | 0.000 | 0.370 |

| Problem 137 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 139 | 139 | 131 | 306 | 0 | 186 | 0 | 232 | -1 | 95 |
| N.S. | 1 | 1.00 | 0.94 | 2.20 | 0.00 | 1.34 | 0.00 | 1.67 | -0.01 | 0.68 |
| time (sec) | N/A | 0.117 | 0.367 | 0.021 | 0.000 | 1.736 | 0.000 | 0.260 | 0.000 | 0.394 |

| Problem 138 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 15 | 15 | 13 | 16 | 11 | 11 | 31 | 11 | 15 | 13 |
| N.S. | 1 | 1.00 | 0.87 | 1.07 | 0.73 | 0.73 | 2.07 | 0.73 | 1.00 | 0.87 |
| time (sec) | N/A | 0.004 | 0.007 | 0.004 | 0.434 | 0.597 | 0.311 | 0.153 | 3.723 | 0.016 |

| Problem 139 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 16 | 16 | 13 | 14 | 9 | 11 | 10 | 11 | 13 | 13 |
| N.S. | 1 | 1.00 | 0.81 | 0.88 | 0.56 | 0.69 | 0.62 | 0.69 | 0.81 | 0.81 |
| time (sec) | N/A | 0.006 | 0.003 | 0.003 | 0.428 | 0.668 | 4.252 | 0.150 | 3.664 | 0.013 |

| Problem 140 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 15 | 15 | 13 | 20 | 0 | 11 | 0 | 11 | 15 | 13 |
| N.S. | 1 | 1.00 | 0.87 | 1.33 | 0.00 | 0.73 | 0.00 | 0.73 | 1.00 | 0.87 |
| time (sec) | N/A | 0.028 | 0.006 | 0.004 | 0.000 | 0.987 | 0.000 | 0.160 | 3.675 | 0.016 |

| Problem 141 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 15 | 15 | 13 | 18 | 0 | 11 | 0 | 11 | 13 | 13 |
| N.S. | 1 | 1.00 | 0.87 | 1.20 | 0.00 | 0.73 | 0.00 | 0.73 | 0.87 | 0.87 |
| time (sec) | N/A | 0.058 | 0.004 | 0.004 | 0.000 | 1.025 | 0.000 | 0.201 | 3.607 | 0.013 |

| Problem 142 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-1) | F | F | F | B |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 242 | 242 | 268 | 0 | 0 | 0 | 0 | 0 | -1 | 499 |
| N.S. | 1 | 1.00 | 1.11 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 2.06 |
| time (sec) | N/A | 0.093 | 0.560 | 0.135 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.077 |

| Problem 143 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F(-1) | F | F | F | C |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD | N/A |
| size | 488 | 488 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 1038 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 2.13 |
| time (sec) | N/A | 0.360 | 0.535 | 0.449 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 6.232 |

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [63] had the largest ratio of [.5556]

Table 2.1: Rubi specific breakdown of results for each integral

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1 | A | 5 | 4 | 1.00 | 25 | 0.160 |
| 2 | A | 5 | 4 | 1.00 | 27 | 0.148 |
| 3 | A | 5 | 4 | 1.00 | 27 | 0.148 |
| 4 | A | 8 | 8 | 1.00 | 27 | 0.296 |
| 5 | A | 9 | 9 | 1.00 | 27 | 0.333 |
| 6 | A | 9 | 6 | 1.00 | 30 | 0.200 |
| 7 | A | 5 | 3 | 1.00 | 30 | 0.100 |
| 8 | A | 6 | 4 | 1.00 | 30 | 0.133 |
| 9 | A | 7 | 5 | 1.00 | 30 | 0.167 |
| 10 | A | 5 | 4 | 1.00 | 23 | 0.174 |
| 11 | A | 5 | 4 | 1.00 | 23 | 0.174 |
| 12 | A | 5 | 4 | 1.00 | 30 | 0.133 |
| 13 | A | 6 | 5 | 0.99 | 28 | 0.179 |
| 14 | A | 6 | 5 | 1.00 | 30 | 0.167 |
| 15 | A | 9 | 5 | 0.98 | 30 | 0.167 |
| 16 | A | 10 | 6 | 0.99 | 30 | 0.200 |
| 17 | A | 5 | 5 | 1.00 | 34 | 0.147 |
| 18 | A | 5 | 5 | 1.00 | 34 | 0.147 |
| 19 | A | 9 | 6 | 1.00 | 32 | 0.188 |
| 20 | A | 10 | 7 | 1.00 | 32 | 0.219 |
| 21 | A | 5 | 3 | 1.00 | 32 | 0.094 |
| 22 | A | 5 | 3 | 1.00 | 29 | 0.103 |
| 23 | A | 5 | 3 | 1.00 | 29 | 0.103 |
| 24 | A | 6 | 6 | 1.00 | 26 | 0.231 |
| 25 | A | 5 | 4 | 1.00 | 30 | 0.133 |
| 26 | A | 7 | 6 | 1.00 | 30 | 0.200 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 27 | A | 7 | 6 | 1.00 | 30 | 0.200 |
| 28 | A | 5 | 3 | 1.00 | 30 | 0.100 |
| 29 | A | 6 | 4 | 1.00 | 30 | 0.133 |
| 30 | A | 7 | 5 | 1.00 | 30 | 0.167 |
| 31 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 32 | A | 5 | 5 | 1.00 | 26 | 0.192 |
| 33 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 34 | A | 5 | 5 | 1.00 | 20 | 0.250 |
| 35 | A | 6 | 5 | 1.00 | 36 | 0.139 |
| 36 | A | 2 | 2 | 1.00 | 36 | 0.056 |
| 37 | A | 2 | 2 | 1.00 | 32 | 0.062 |
| 38 | A | 13 | 9 | 1.00 | 32 | 0.281 |
| 39 | A | 5 | 5 | 1.00 | 38 | 0.132 |
| 40 | A | 6 | 6 | 1.00 | 35 | 0.171 |
| 41 | A | 5 | 5 | 1.00 | 33 | 0.152 |
| 42 | A | 5 | 5 | 1.00 | 32 | 0.156 |
| 43 | A | 8 | 8 | 1.00 | 35 | 0.229 |
| 44 | A | 8 | 8 | 1.00 | 35 | 0.229 |
| 45 | A | 8 | 8 | 1.00 | 35 | 0.229 |
| 46 | A | 6 | 6 | 1.00 | 38 | 0.158 |
| 47 | A | 5 | 5 | 1.00 | 36 | 0.139 |
| 48 | A | 5 | 5 | 1.00 | 35 | 0.143 |
| 49 | A | 7 | 6 | 1.00 | 38 | 0.158 |
| 50 | A | 7 | 6 | 1.00 | 38 | 0.158 |
| 51 | A | 7 | 6 | 1.00 | 38 | 0.158 |
| 52 | A | 9 | 6 | 1.00 | 27 | 0.222 |
| 53 | A | 9 | 6 | 1.00 | 25 | 0.240 |
| 54 | A | 8 | 5 | 1.00 | 24 | 0.208 |
| 55 | A | 12 | 9 | 1.00 | 27 | 0.333 |
| 56 | A | 18 | 12 | 1.00 | 27 | 0.444 |
| 57 | A | 22 | 13 | 1.00 | 27 | 0.482 |
| 58 | A | 10 | 7 | 1.00 | 27 | 0.259 |
| 59 | A | 10 | 7 | 1.00 | 25 | 0.280 |
| 60 | A | 9 | 6 | 1.00 | 24 | 0.250 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 61 | A | 17 | 11 | 1.00 | 27 | 0.407 |
| 62 | A | 21 | 14 | 1.00 | 27 | 0.518 |
| 63 | A | 26 | 15 | 1.00 | 27 | 0.556 |
| 64 | A | 10 | 6 | 1.00 | 27 | 0.222 |
| 65 | A | 8 | 5 | 1.00 | 27 | 0.185 |
| 66 | A | 5 | 3 | 1.00 | 25 | 0.120 |
| 67 | A | 5 | 3 | 1.00 | 24 | 0.125 |
| 68 | A | 10 | 7 | 1.00 | 27 | 0.259 |
| 69 | A | 11 | 8 | 1.00 | 27 | 0.296 |
| 70 | A | 15 | 9 | 1.00 | 27 | 0.333 |
| 71 | A | 10 | 7 | 1.00 | 27 | 0.259 |
| 72 | A | 6 | 4 | 1.00 | 27 | 0.148 |
| 73 | A | 6 | 4 | 1.00 | 25 | 0.160 |
| 74 | A | 6 | 4 | 1.00 | 24 | 0.167 |
| 75 | A | 12 | 9 | 1.00 | 27 | 0.333 |
| 76 | A | 14 | 11 | 1.00 | 27 | 0.407 |
| 77 | A | 15 | 9 | 1.00 | 28 | 0.321 |
| 78 | A | 9 | 6 | 1.00 | 28 | 0.214 |
| 79 | A | 9 | 6 | 1.00 | 26 | 0.231 |
| 80 | A | 8 | 5 | 1.00 | 25 | 0.200 |
| 81 | A | 17 | 9 | 1.00 | 28 | 0.321 |
| 82 | A | 16 | 8 | 1.00 | 28 | 0.286 |
| 83 | A | 20 | 10 | 1.00 | 28 | 0.357 |
| 84 | A | 17 | 10 | 1.00 | 28 | 0.357 |
| 85 | A | 10 | 7 | 1.00 | 28 | 0.250 |
| 86 | A | 10 | 7 | 1.00 | 26 | 0.269 |
| 87 | A | 9 | 6 | 1.00 | 25 | 0.240 |
| 88 | A | 19 | 11 | 1.00 | 28 | 0.393 |
| 89 | A | 18 | 10 | 1.00 | 28 | 0.357 |
| 90 | A | 26 | 13 | 1.00 | 28 | 0.464 |
| 91 | A | 9 | 6 | 1.00 | 24 | 0.250 |
| 92 | A | 8 | 6 | 1.00 | 22 | 0.273 |
| 93 | A | 10 | 9 | 1.00 | 17 | 0.529 |
| 94 | A | 13 | 7 | 1.00 | 28 | 0.250 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 95 | A | 10 | 6 | 1.00 | 28 | 0.214 |
| 96 | A | 8 | 5 | 1.00 | 28 | 0.179 |
| 97 | A | 5 | 3 | 1.00 | 26 | 0.115 |
| 98 | A | 5 | 3 | 1.00 | 25 | 0.120 |
| 99 | A | 9 | 4 | 1.00 | 28 | 0.143 |
| 100 | A | 10 | 5 | 1.00 | 28 | 0.179 |
| 101 | A | 13 | 6 | 1.00 | 28 | 0.214 |
| 102 | A | 13 | 9 | 1.00 | 28 | 0.321 |
| 103 | A | 9 | 6 | 1.00 | 28 | 0.214 |
| 104 | A | 6 | 4 | 1.00 | 28 | 0.143 |
| 105 | A | 6 | 4 | 1.00 | 26 | 0.154 |
| 106 | A | 6 | 4 | 1.00 | 25 | 0.160 |
| 107 | A | 12 | 7 | 1.00 | 28 | 0.250 |
| 108 | A | 12 | 7 | 1.00 | 28 | 0.250 |
| 109 | A | 9 | 6 | 1.00 | 30 | 0.200 |
| 110 | A | 9 | 6 | 1.00 | 28 | 0.214 |
| 111 | A | 8 | 5 | 1.00 | 27 | 0.185 |
| 112 | A | 17 | 9 | 1.00 | 30 | 0.300 |
| 113 | A | 23 | 10 | 1.00 | 30 | 0.333 |
| 114 | A | 12 | 6 | 1.00 | 30 | 0.200 |
| 115 | A | 8 | 5 | 1.00 | 30 | 0.167 |
| 116 | A | 5 | 3 | 1.00 | 28 | 0.107 |
| 117 | A | 5 | 3 | 1.00 | 27 | 0.111 |
| 118 | A | 9 | 4 | 1.00 | 30 | 0.133 |
| 119 | A | 12 | 5 | 1.00 | 30 | 0.167 |
| 120 | A | 16 | 7 | 1.00 | 30 | 0.233 |
| 121 | A | 10 | 7 | 1.00 | 30 | 0.233 |
| 122 | A | 6 | 4 | 1.00 | 30 | 0.133 |
| 123 | A | 6 | 4 | 1.00 | 28 | 0.143 |
| 124 | A | 6 | 4 | 1.00 | 27 | 0.148 |
| 125 | A | 12 | 7 | 1.00 | 30 | 0.233 |
| 126 | A | 24 | 14 | 1.00 | 30 | 0.467 |
| 127 | A | 20 | 13 | 1.00 | 30 | 0.433 |
| 128 | A | 16 | 12 | 1.00 | 30 | 0.400 |

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| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 129 | A | 6 | 4 | 1.01 | 28 | 0.143 |
| 130 | A | 10 | 8 | 1.00 | 27 | 0.296 |
| 131 | A | 17 | 11 | 1.00 | 30 | 0.367 |
| 132 | A | 20 | 12 | 1.00 | 30 | 0.400 |
| 133 | A | 8 | 6 | 1.00 | 34 | 0.176 |
| 134 | A | 6 | 5 | 1.00 | 30 | 0.167 |
| 135 | A | 3 | 3 | 1.00 | 34 | 0.088 |
| 136 | A | 5 | 5 | 1.00 | 34 | 0.147 |
| 137 | A | 7 | 7 | 1.00 | 34 | 0.206 |
| 138 | A | 1 | 1 | 1.00 | 17 | 0.059 |
| 139 | A | 1 | 1 | 1.00 | 15 | 0.067 |
| 140 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 141 | A | 3 | 3 | 1.00 | 21 | 0.143 |
| 142 | A | 2 | 2 | 1.00 | 40 | 0.050 |
| 143 | A | 2 | 2 | 1.00 | 104 | 0.019 |

# Chapter 3

# Listing of integrals

## Local contents

## 3.1 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{d+fx^2}\,dx$

**Optimal**. Leaf size=94

$$-\frac{\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{d}}\right)(-aAf+Acd+bBd)}{\sqrt{d}\,f^{3/2}}-\frac{\log\left(d+fx^2\right)(-aBf-Abf+Bcd)}{2f^2}+\frac{x(Ac+bB)}{f}+\frac{Bcx^2}{2f}$$

**Rubi** [A] time = 0.11, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1629, 635, 205, 260}

$$-\frac{\log\left(d+fx^2\right)(-aBf-Abf+Bcd)}{2f^2}-\frac{\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{d}}\right)(-aAf+Acd+bBd)}{\sqrt{d}\,f^{3/2}}+\frac{x(Ac+bB)}{f}+\frac{Bcx^2}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x + c*x^2))/(d + f*x^2),x]
```

```
[Out] ((b*B + A*c)*x)/f + (B*c*x^2)/(2*f) - ((b*B*d + A*c*d - a*A*f)*ArcTan[(Sqrt
[f]*x)/Sqrt[d]])/(Sqrt[d]*f^(3/2)) - ((B*c*d - A*b*f - a*B*f)*Log[d + f*x^2
])/(2*f^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
```

```
d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{d+fx^2}\,dx &= \int \left(\frac{bB+Ac}{f}+\frac{Bcx}{f}-\frac{bBd+Acd-aAf+(Bcd-Abf-aBf)x}{f\left(d+fx^2\right)}\right)dx \\
&= \frac{(bB+Ac)x}{f}+\frac{Bcx^2}{2f}-\frac{\int \frac{bBd+Acd-aAf+(Bcd-Abf-aBf)x}{d+fx^2}\,dx}{f} \\
&= \frac{(bB+Ac)x}{f}+\frac{Bcx^2}{2f}-\frac{(bBd+Acd-aAf)\int \frac{1}{d+fx^2}\,dx}{f}-\frac{(Bcd-Abf-aBf)\int}{f} \\
&= \frac{(bB+Ac)x}{f}+\frac{Bcx^2}{2f}-\frac{(bBd+Acd-aAf)\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,f^{3/2}}-\frac{(Bcd-Abf-aBf)}{2f^2}
\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 86, normalized size = 0.91

$$\frac{\log\left(d+fx^2\right)(aBf+Abf-Bcd)-\frac{2\sqrt{f}\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{d}}\right)(-aAf+Acd+bBd)}{\sqrt{d}}+fx(2Ac+2bB+Bcx)}{2f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x + c*x^2))/(d + f*x^2),x]
```

```
[Out] (f*x*(2*b*B + 2*A*c + B*c*x) - (2*Sqrt[f]*(b*B*d + A*c*d - a*A*f)*ArcTan[(S
qrt[f]*x)/Sqrt[d]])/Sqrt[d] + (-(B*c*d) + A*b*f + a*B*f)*Log[d + f*x^2])/(2
*f^2)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{d+fx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((A + B*x)*(a + b*x + c*x^2))/(d + f*x^2),x]
```

```
[Out] IntegrateAlgebraic[((A + B*x)*(a + b*x + c*x^2))/(d + f*x^2), x]
```

**fricas [A]** time = 0.40, size = 200, normalized size = 2.13

$$\left[\frac{Bcdfx^2 + 2(Bb + Ac)dfx - \left(Aaf - (Bb + Ac)d\right)\sqrt{-df}\log\left(\frac{fx^2 - 2\sqrt{-df}\,x - d}{fx^2 + d}\right) - \left(Bcd^2 - (Ba + Ab)df\right)\log\left(fx^2 + d\right)}{2df^2}, \frac{Bcdfx^2 + 2(Bb + Ac)dfx + 2\left(Aaf - (Bb + Ac)d\right)\sqrt{df}\arctan\left(\frac{\sqrt{df}\,x}{d}\right) - \left(Bcd^2 - (Ba + Ab)df\right)\log\left(fx^2 + d\right)}{2df^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/2*(B*c*d*f*x^2 + 2*(B*b + A*c)*d*f*x - (A*a*f - (B*b + A*c)*d)*sqrt(-d*f
)*log((f*x^2 - 2*sqrt(-d*f)*x - d)/(f*x^2 + d)) - (B*c*d^2 - (B*a + A*b)*d*
f)*log(f*x^2 + d))/(d*f^2), 1/2*(B*c*d*f*x^2 + 2*(B*b + A*c)*d*f*x + 2*(A*a
*f - (B*b + A*c)*d)*sqrt(d*f)*arctan(sqrt(d*f)*x/d) - (B*c*d^2 - (B*a + A*b
)*d*f)*log(f*x^2 + d))/(d*f^2)]
```

**giac [A]** time = 0.15, size = 87, normalized size = 0.93

$$-\frac{\left(Bbd + Acd - Aaf\right)\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}\,f} - \frac{\left(Bcd - Baf - Abf\right)\log\left(fx^2 + d\right)}{2f^2} + \frac{Bcfx^2 + 2Bbfx + 2Acfx}{2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+d),x, algorithm="giac")
```

```
[Out] -(B*b*d + A*c*d - A*a*f)*arctan(f*x/sqrt(d*f))/(sqrt(d*f)*f) - 1/2*(B*c*d -
 B*a*f - A*b*f)*log(f*x^2 + d)/f^2 + 1/2*(B*c*f*x^2 + 2*B*b*f*x + 2*A*c*f*x
)/f^2
```

**maple [A]** time = 0.03, size = 133, normalized size = 1.41

$$\frac{Aa\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}} - \frac{Acd\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}\,f} - \frac{Bbd\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}\,f} + \frac{Bcx^2}{2f} + \frac{Ab\ln\left(fx^2 + d\right)}{2f} + \frac{Acx}{f} + \frac{Ba\ln\left(fx^2 + d\right)}{2f} + \frac{Bbx}{f} - \frac{Bcd\ln\left(fx^2 + d\right)}{2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(c*x^2+b*x+a)/(f*x^2+d),x)
```

```
[Out] 1/2*B*c*x^2/f+1/f*A*c*x+1/f*b*B*x+1/2/f*ln(f*x^2+d)*A*b+1/2/f*ln(f*x^2+d)*B
*a-1/2/f^2*ln(f*x^2+d)*B*c*d+1/(d*f)^(1/2)*arctan(x*f/(d*f)^(1/2))*A*a-1/f/
(d*f)^(1/2)*arctan(x*f/(d*f)^(1/2))*A*c*d-1/f/(d*f)^(1/2)*arctan(x*f/(d*f)^
(1/2))*B*b*d
```

**maxima [A]** time = 0.97, size = 84, normalized size = 0.89

$$\frac{\left(Aaf - (Bb + Ac)d\right)\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}\,f} + \frac{Bcx^2 + 2(Bb + Ac)x}{2f} - \frac{\left(Bcd - (Ba + Ab)f\right)\log\left(fx^2 + d\right)}{2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+d),x, algorithm="maxima")
```

```
[Out] (A*a*f - (B*b + A*c)*d)*arctan(f*x/sqrt(d*f))/(sqrt(d*f)*f) + 1/2*(B*c*x^2
+ 2*(B*b + A*c)*x)/f - 1/2*(B*c*d - (B*a + A*b)*f)*log(f*x^2 + d)/f^2
```

**mupad [B]** time = 3.44, size = 97, normalized size = 1.03

$$\frac{x\,(A\,c+B\,b)}{f}-\frac{\operatorname{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{d}}\right)\,\left(A\,c\,d-A\,a\,f+B\,b\,d\right)}{\sqrt{d}\,f^{3/2}}+\frac{B\,c\,x^2}{2\,f}+\frac{\ln\left(f\,x^2+d\right)\,\left(4\,A\,b\,d\,f^3+4\,B\,a\,d\,f^3-4\,B\,c\,d^2\,f^2\right)}{8\,d\,f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x)*(a + b*x + c*x^2))/(d + f*x^2),x)
```

```
[Out] (x*(A*c + B*b))/f - (atan((f^(1/2)*x)/d^(1/2))*(A*c*d - A*a*f + B*b*d))/(d^
(1/2)*f^(3/2)) + (B*c*x^2)/(2*f) + (log(d + f*x^2)*(4*A*b*d*f^3 + 4*B*a*d*f
^3 - 4*B*c*d^2*f^2))/(8*d*f^4)
```

**sympy [B]** time = 1.85, size = 333, normalized size = 3.54

$$\frac{Bcx^2}{2f}+x\left(\frac{Ac}{f}+\frac{Bb}{f}\right)+\left(\frac{Abf+Baf-Bcd}{2f^2}-\frac{\sqrt{-df^5}\,\left(Aaf-Acd-Bbd\right)}{2df^4}\right)\log\left(x+\frac{-Abdf-Badf+Bcd^2+2df^2\left(\frac{Abf+Baf-Bcd}{2f^2}-\frac{\sqrt{-df^5}\left(Aaf-Acd-Bbd\right)}{2df^4}\right)}{Aaf^2-Acdf-Bbdf}\right)+\left(\frac{Abf+Baf-Bcd}{2f^2}+\frac{\sqrt{-df^5}\,\left(Aaf-Acd-Bbd\right)}{2df^4}\right)\log\left(x+\frac{-Abdf-Badf+Bcd^2+2df^2\left(\frac{Abf+Baf-Bcd}{2f^2}+\frac{\sqrt{-df^5}\left(Aaf-Acd-Bbd\right)}{2df^4}\right)}{Aaf^2-Acdf-Bbdf}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x**2+b*x+a)/(f*x**2+d),x)
```

```
[Out] B*c*x**2/(2*f) + x*(A*c/f + B*b/f) + ((A*b*f + B*a*f - B*c*d)/(2*f**2) - sq
rt(-d*f**5)*(A*a*f - A*c*d - B*b*d)/(2*d*f**4))*log(x + (-A*b*d*f - B*a*d*f
 + B*c*d**2 + 2*d*f**2*((A*b*f + B*a*f - B*c*d)/(2*f**2) - sqrt(-d*f**5)*(A
*a*f - A*c*d - B*b*d)/(2*d*f**4)))/(A*a*f**2 - A*c*d*f - B*b*d*f)) + ((A*b*
f + B*a*f - B*c*d)/(2*f**2) + sqrt(-d*f**5)*(A*a*f - A*c*d - B*b*d)/(2*d*f*
*4))*log(x + (-A*b*d*f - B*a*d*f + B*c*d**2 + 2*d*f**2*((A*b*f + B*a*f - B*
c*d)/(2*f**2) + sqrt(-d*f**5)*(A*a*f - A*c*d - B*b*d)/(2*d*f**4)))/(A*a*f**
2 - A*c*d*f - B*b*d*f))
```

## 3.2 $\int \frac{(A+Bx)(a+bx+cx^2)^2}{d+fx^2}\,dx$

**Optimal**. Leaf size=228

$$-\frac{\log\left(d+fx^2\right)\left(2Abf(cd-af)-B\left(-f\left(b^2d-a^2f\right)-2acdf+c^2d^2\right)\right)}{2f^3}+\frac{x^2\left(2Abcf-B\left(-2acf+b^2(-f)+c^2d\right)\right)}{2f^2}$$

**Rubi [A]** time = 0.33, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1012, 635, 205, 260}

$$-\frac{\log\left(d+fx^2\right)\left(2Abf(cd-af)-B\left(-f\left(b^2d-a^2f\right)-2acdf+c^2d^2\right)\right)}{2f^3}+\frac{x^2\left(2Abcf-B\left(-2acf+b^2(-f)+c^2d\right)\right)}{2f^2}+\frac{x\left(-Ac(cd-2af)-bB(2cd-2af)+Ab^2f\right)}{f^2}-\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)\left(-A(cd-af)^2-2bBd(cd-af)+Ab^2df\right)}{\sqrt{d}f^{5/2}}+\frac{cx^3(Ac+2bB)}{3f}+\frac{Bc^2x^4}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x + c*x^2)^2)/(d + f*x^2),x]
```

```
[Out] ((A*b^2*f - A*c*(c*d - 2*a*f) - b*B*(2*c*d - 2*a*f))*x)/f^2 + ((2*A*b*c*f -
 B*(c^2*d - b^2*f - 2*a*c*f))*x^2)/(2*f^2) + (c*(2*b*B + A*c)*x^3)/(3*f) +
(B*c^2*x^4)/(4*f) - ((A*b^2*d*f - 2*b*B*d*(c*d - a*f) - A*(c*d - a*f)^2)*Ar
cTan[(Sqrt[f]*x)/Sqrt[d]])/(Sqrt[d]*f^(5/2)) - ((2*A*b*f*(c*d - a*f) - B*(c
^2*d^2 - 2*a*c*d*f - f*(b^2*d - a^2*f)))*Log[d + f*x^2])/(2*f^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1012

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + c*x^2)^p*(d + e*x +
```

```
f*x^2)^q*(g + h*x), x], x] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 -
4*d*f, 0] && IntegersQ[p, q] && (GtQ[p, 0] || GtQ[q, 0])
```

Rubi steps

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+fx^2}\,dx = \int \left(\frac{Ab^2f-Ac(cd-2af)-bB(2cd-2af)}{f^2}+\frac{\left(2Abcf-B\left(c^2d-b^2f-2acf\right)\right.}{f^2}\right.$$

$$=\frac{\left(Ab^2f-Ac(cd-2af)-bB(2cd-2af)\right)x}{f^2}+\frac{\left(2Abcf-B\left(c^2d-b^2f-2acf\right)\right)}{2f^2}$$

$$=\frac{\left(Ab^2f-Ac(cd-2af)-bB(2cd-2af)\right)x}{f^2}+\frac{\left(2Abcf-B\left(c^2d-b^2f-2acf\right)\right)}{2f^2}$$

$$=\frac{\left(Ab^2f-Ac(cd-2af)-bB(2cd-2af)\right)x}{f^2}+\frac{\left(2Abcf-B\left(c^2d-b^2f-2acf\right)\right)}{2f^2}$$

**Mathematica [A]** time = 0.20, size = 204, normalized size = 0.89

$$\frac{6\log\left(d+fx^2\right)\left(B\left(a^2f^2-2acdf+b^2(-d)f+c^2d^2\right)+2Abf(af-cd)\right)+fx\left(4Ac\left(6af-3cd+cfx^2\right)+4bB\left(6af-6cd+2cfx^2\right)+3Bcx\left(4af-2cd+cfx^2\right)+6b^2f(2A+Bx)+12Abcfx\right)}{12f^3}+\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)\left(A(cd-af)^2+2bBd(cd-af)-Ab^2df\right)}{\sqrt{d}\,f^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x + c*x^2)^2)/(d + f*x^2),x]
```

```
[Out] ((-(A*b^2*d*f) + 2*b*B*d*(c*d - a*f) + A*(c*d - a*f)^2)*ArcTan[(Sqrt[f]*x)/
Sqrt[d]])/(Sqrt[d]*f^(5/2)) + (f*x*(12*A*b*c*f*x + 6*b^2*f*(2*A + B*x) + 3*
B*c*x*(-2*c*d + 4*a*f + c*f*x^2) + 4*A*c*(-3*c*d + 6*a*f + c*f*x^2) + 4*b*B
*(-6*c*d + 6*a*f + 2*c*f*x^2)) + 6*(2*A*b*f*(-(c*d) + a*f) + B*(c^2*d^2 - b
^2*d*f - 2*a*c*d*f + a^2*f^2))*Log[d + f*x^2])/(12*f^3)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+fx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((A + B*x)*(a + b*x + c*x^2)^2)/(d + f*x^2),x]
```

```
[Out] IntegrateAlgebraic[((A + B*x)*(a + b*x + c*x^2)^2)/(d + f*x^2), x]
```

**fricas [A]** time = 0.42, size = 500, normalized size = 2.19

$$\left[\frac{3 Bc^2 df^2 x^4 + 4 \left(2 Bbc + Ac^2\right) df^2 x^3 - 6 \left(Bc^2 d^2 f - \left(Bb^2 + 2 (Ba + Ab)c\right) df^2\right) x^2 - 6 \left(Aa^2 f^2 + \left(2 Bbc + Ac^2\right) d^2 - \left(2 Bab + Ab^2 + 2 Aac\right) df\right) \sqrt{-df} \log\left(\frac{fx^2 - 2 \sqrt{-df}\, x - d}{fx^2 + d}\right) - 12 \left(\left(2 Bbc + Ac^2\right) d^2 f - \left(2 Bab + Ab^2 + 2 Aac\right) df^2\right) x + 6 \left(Bc^2 d^3 - \left(Bb^2 + 2 (Ba + Ab)c\right) d^2 f + \left(Ba^2 + 2 Aab\right) df^2\right) \log\left(fx^2 + d\right)}{12 df^3}, \frac{3 Bc^2 df^2 x^4 + 4 \left(2 Bbc + Ac^2\right) df^2 x^3 - 6 \left(Bc^2 d^2 f - \left(Bb^2 + 2 (Ba + Ab)c\right) df^2\right) x^2 + 12 \left(Aa^2 f^2 + \left(2 Bbc + Ac^2\right) d^2 - \left(2 Bab + Ab^2 + 2 Aac\right) df\right) \sqrt{df} \arctan\left(\frac{\sqrt{df}\, x}{d}\right) - 12 \left(\left(2 Bbc + Ac^2\right) d^2 f - \left(2 Bab + Ab^2 + 2 Aac\right) df^2\right) x + 6 \left(Bc^2 d^3 - \left(Bb^2 + 2 (Ba + Ab)c\right) d^2 f + \left(Ba^2 + 2 Aab\right) df^2\right) \log\left(fx^2 + d\right)}{12 df^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/12*(3*B*c^2*d*f^2*x^4 + 4*(2*B*b*c + A*c^2)*d*f^2*x^3 - 6*(B*c^2*d^2*f -
 (B*b^2 + 2*(B*a + A*b)*c)*d*f^2)*x^2 - 6*(A*a^2*f^2 + (2*B*b*c + A*c^2)*d^
2 - (2*B*a*b + A*b^2 + 2*A*a*c)*d*f)*sqrt(-d*f)*log((f*x^2 - 2*sqrt(-d*f)*x
 - d)/(f*x^2 + d)) - 12*((2*B*b*c + A*c^2)*d^2*f - (2*B*a*b + A*b^2 + 2*A*a
*c)*d*f^2)*x + 6*(B*c^2*d^3 - (B*b^2 + 2*(B*a + A*b)*c)*d^2*f + (B*a^2 + 2*
A*a*b)*d*f^2)*log(f*x^2 + d))/(d*f^3), 1/12*(3*B*c^2*d*f^2*x^4 + 4*(2*B*b*c
 + A*c^2)*d*f^2*x^3 - 6*(B*c^2*d^2*f - (B*b^2 + 2*(B*a + A*b)*c)*d*f^2)*x^2
 + 12*(A*a^2*f^2 + (2*B*b*c + A*c^2)*d^2 - (2*B*a*b + A*b^2 + 2*A*a*c)*d*f)
*sqrt(d*f)*arctan(sqrt(d*f)*x/d) - 12*((2*B*b*c + A*c^2)*d^2*f - (2*B*a*b +
 A*b^2 + 2*A*a*c)*d*f^2)*x + 6*(B*c^2*d^3 - (B*b^2 + 2*(B*a + A*b)*c)*d^2*f
 + (B*a^2 + 2*A*a*b)*d*f^2)*log(f*x^2 + d))/(d*f^3)]
```

**giac [A]** time = 0.16, size = 263, normalized size = 1.15

$$\frac{\left(2 Bbcd^2 + Ac^2 d^2 - 2 Babdf - Ab^2 df - 2 Aacdf + Aa^2 f^2\right) \arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}\, f^2} + \frac{\left(Bc^2 d^2 - Bb^2 df - 2 Bacdf - 2 Abcdf + Ba^2 f^2 + 2 Aabf^2\right) \log\left(fx^2 + d\right)}{2 f^3} + \frac{3 Bc^2 f^3 x^4 + 8 Bbcf^3 x^3 + 4 Ac^2 f^3 x^3 - 6 Bc^2 df^2 x^2 + 6 Bb^2 f^3 x^2 + 12 Bacf^3 x^2 + 12 Abcf^3 x^2 - 24 Bbcdf^2 x - 12 Ac^2 df^2 x + 24 Babf^3 x + 12 Ab^2 f^3 x + 24 Aacf^3 x}{12 f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="giac")
```

```
[Out] (2*B*b*c*d^2 + A*c^2*d^2 - 2*B*a*b*d*f - A*b^2*d*f - 2*A*a*c*d*f + A*a^2*f^
2)*arctan(f*x/sqrt(d*f))/(sqrt(d*f)*f^2) + 1/2*(B*c^2*d^2 - B*b^2*d*f - 2*B
*a*c*d*f - 2*A*b*c*d*f + B*a^2*f^2 + 2*A*a*b*f^2)*log(f*x^2 + d)/f^3 + 1/12
*(3*B*c^2*f^3*x^4 + 8*B*b*c*f^3*x^3 + 4*A*c^2*f^3*x^3 - 6*B*c^2*d*f^2*x^2 +
 6*B*b^2*f^3*x^2 + 12*B*a*c*f^3*x^2 + 12*A*b*c*f^3*x^2 - 24*B*b*c*d*f^2*x -
 12*A*c^2*d*f^2*x + 24*B*a*b*f^3*x + 12*A*b^2*f^3*x + 24*A*a*c*f^3*x)/f^4
```

**maple [A]** time = 0.01, size = 373, normalized size = 1.64

$$\frac{B c^2 x^4}{4f} + \frac{A c^2 x^3}{3f} + \frac{2Bbc x^3}{3f} + \frac{A a^2 \arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}} - \frac{2Aacd \arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}\, f} - \frac{A b^2 d \arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}\, f} + \frac{Abc x^2}{f} + \frac{A c^2 d^2 \arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}\, f^2} - \frac{2Babd \arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}\, f} + \frac{Bac x^2}{f} + \frac{B b^2 x^2}{2f} + \frac{2Bbc d^2 \arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}\, f^2} - \frac{B c^2 d x^2}{2f^2} + \frac{Aab \ln\left(f x^2 + d\right)}{f} + \frac{2Aacx}{f} + \frac{A b^2 x}{f} - \frac{Abcd \ln\left(f x^2 + d\right)}{f^2} - \frac{A c^2 dx}{f^2} + \frac{B a^2 \ln\left(f x^2 + d\right)}{2f} + \frac{2Babx}{f} - \frac{Bacd \ln\left(f x^2 + d\right)}{f^2} - \frac{B b^2 d \ln\left(f x^2 + d\right)}{2f^2} - \frac{2Bbcdx}{f^2} + \frac{B c^2 d^2 \ln\left(f x^2 + d\right)}{2f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+d),x)
```

```
[Out] 1/4*B*c^2*x^4/f+1/3/f*A*x^3*c^2+2/3/f*B*x^3*b*c+1/f*A*x^2*b*c+1/f*B*x^2*a*c
+1/2/f*B*x^2*b^2-1/2/f^2*B*x^2*c^2*d+2/f*A*a*c*x+1/f*A*b^2*x-1/f^2*A*c^2*d*
x+2/f*B*a*b*x-2/f^2*B*b*c*d*x+1/f*ln(f*x^2+d)*A*a*b-1/f^2*ln(f*x^2+d)*A*b*c
*d+1/2/f*ln(f*x^2+d)*B*a^2-1/f^2*ln(f*x^2+d)*B*a*c*d-1/2/f^2*ln(f*x^2+d)*B*
```

```
b^2*d+1/2/f^3*ln(f*x^2+d)*B*c^2*d^2+1/(d*f)^(1/2)*arctan(1/(d*f)^(1/2)*f*x)
*A*a^2-2/f/(d*f)^(1/2)*arctan(1/(d*f)^(1/2)*f*x)*A*a*c*d-1/f/(d*f)^(1/2)*ar
ctan(1/(d*f)^(1/2)*f*x)*A*b^2*d+1/f^2/(d*f)^(1/2)*arctan(1/(d*f)^(1/2)*f*x)
*A*c^2*d^2-2/f/(d*f)^(1/2)*arctan(1/(d*f)^(1/2)*f*x)*B*a*b*d+2/f^2/(d*f)^(1
/2)*arctan(1/(d*f)^(1/2)*f*x)*B*b*c*d^2
```

**maxima [A]** time = 0.98, size = 220, normalized size = 0.96

$$\frac{\left(Aa^2f^2+\left(2\,Bbc+Ac^2\right)d^2-\left(2\,Bab+Ab^2+2\,Aac\right)df\right)\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}\,f^2}+\frac{3\,Bc^2fx^4+4\left(2\,Bbc+Ac^2\right)fx^3-6\left(Bc^2d-\left(Bb^2+2\,(Ba+Ab)c\right)f\right)x^2-12\left(\left(2\,Bbc+Ac^2\right)d-\left(2\,Bab+Ab^2+2\,Aac\right)f\right)x}{12\,f^2}+\frac{\left(Bc^2d^2-\left(Bb^2+2\,(Ba+Ab)c\right)df+\left(Ba^2+2\,Aab\right)f^2\right)\log\left(fx^2+d\right)}{2\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="maxima")
```

```
[Out] (A*a^2*f^2 + (2*B*b*c + A*c^2)*d^2 - (2*B*a*b + A*b^2 + 2*A*a*c)*d*f)*arcta
n(f*x/sqrt(d*f))/(sqrt(d*f)*f^2) + 1/12*(3*B*c^2*f*x^4 + 4*(2*B*b*c + A*c^2
)*f*x^3 - 6*(B*c^2*d - (B*b^2 + 2*(B*a + A*b)*c)*f)*x^2 - 12*((2*B*b*c + A*
c^2)*d - (2*B*a*b + A*b^2 + 2*A*a*c)*f)*x)/f^2 + 1/2*(B*c^2*d^2 - (B*b^2 +
2*(B*a + A*b)*c)*d*f + (B*a^2 + 2*A*a*b)*f^2)*log(f*x^2 + d)/f^3
```

**mupad [B]** time = 0.25, size = 253, normalized size = 1.11

$$x\left(\frac{Ab^2+2Bab+2Aac}{f}-\frac{d\left(Ac^2+2Bbc\right)}{f^2}\right)+x^2\left(\frac{Bb^2+2Acb+2Bac}{2f}-\frac{Bc^2d}{2f^2}\right)+\frac{x^3\left(Ac^2+2Bbc\right)}{3f}+\frac{Bc^2x^4}{4f}+\frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{d}}\right)\left(Aa^2f^2-2Babdf-2Aacdf-Ab^2df+2Bbcd^2+Ac^2d^2\right)}{\sqrt{d}\,f^{5/2}}+\frac{\ln\left(fx^2+d\right)\left(4Ba^2df^5+8Aabdf^5-8Bacd^2f^4-4Bb^2d^2f^4-8Abcd^2f^4+4Bc^2d^3f^3\right)}{8df^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x)*(a + b*x + c*x^2)^2)/(d + f*x^2),x)
```

```
[Out] x*((A*b^2 + 2*A*a*c + 2*B*a*b)/f - (d*(A*c^2 + 2*B*b*c))/f^2) + x^2*((B*b^2
 + 2*A*b*c + 2*B*a*c)/(2*f) - (B*c^2*d)/(2*f^2)) + (x^3*(A*c^2 + 2*B*b*c))/
(3*f) + (B*c^2*x^4)/(4*f) + (atan((f^(1/2)*x)/d^(1/2))*(A*a^2*f^2 + A*c^2*d
^2 + 2*B*b*c*d^2 - A*b^2*d*f - 2*A*a*c*d*f - 2*B*a*b*d*f))/(d^(1/2)*f^(5/2)
) + (log(d + f*x^2)*(4*B*a^2*d*f^5 - 4*B*b^2*d^2*f^4 + 4*B*c^2*d^3*f^3 + 8*
A*a*b*d*f^5 - 8*A*b*c*d^2*f^4 - 8*B*a*c*d^2*f^4))/(8*d*f^6)
```

**sympy [B]** time = 7.87, size = 933, normalized size = 4.09

$$\frac{Bc^2x^4}{4f}+x^3\left(\frac{Ac^2}{3f}+\frac{2Bbc}{3f}\right)+x^2\left(\frac{Abc}{f}+\frac{Bac}{f}+\frac{Bb^2}{2f}-\frac{Bc^2d}{2f^2}\right)+x\left(\frac{2Aac}{f}+\frac{Ab^2}{f}-\frac{Ac^2d}{f^2}+\frac{2Bab}{f}-\frac{2Bbcd}{f^2}\right)+\left(\frac{2Aabf^2-2Abcdf+Ba^2f^2-2Bacdf-Bb^2df+Bc^2d^2}{2f^3}-\frac{\sqrt{-df^7}\left(Aa^2f^2-2Aacdf-Ab^2df+Ac^2d^2-2Babdf+2Bbcd^2\right)}{2df^6}\right)\log\left(x+\frac{-2Aabdf^2+2Abcd^2f-Ba^2df^2+2Bacd^2f+Bb^2d^2f-Bc^2d^3+2df^3\left(\frac{2Aabf^2-2Abcdf+Ba^2f^2-2Bacdf-Bb^2df+Bc^2d^2}{2f^3}-\frac{\sqrt{-df^7}\left(Aa^2f^2-2Aacdf-Ab^2df+Ac^2d^2-2Babdf+2Bbcd^2\right)}{2df^6}\right)}{Aa^2f^3-2Aacdf^2-Ab^2df^2+Ac^2d^2f-2Babdf^2+2Bbcd^2f}\right)+\left(\frac{2Aabf^2-2Abcdf+Ba^2f^2-2Bacdf-Bb^2df+Bc^2d^2}{2f^3}+\frac{\sqrt{-df^7}\left(Aa^2f^2-2Aacdf-Ab^2df+Ac^2d^2-2Babdf+2Bbcd^2\right)}{2df^6}\right)\log\left(x+\frac{-2Aabdf^2+2Abcd^2f-Ba^2df^2+2Bacd^2f+Bb^2d^2f-Bc^2d^3+2df^3\left(\frac{2Aabf^2-2Abcdf+Ba^2f^2-2Bacdf-Bb^2df+Bc^2d^2}{2f^3}+\frac{\sqrt{-df^7}\left(Aa^2f^2-2Aacdf-Ab^2df+Ac^2d^2-2Babdf+2Bbcd^2\right)}{2df^6}\right)}{Aa^2f^3-2Aacdf^2-Ab^2df^2+Ac^2d^2f-2Babdf^2+2Bbcd^2f}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x**2+b*x+a)**2/(f*x**2+d),x)
```

```
[Out] B*c**2*x**4/(4*f) + x**3*(A*c**2/(3*f) + 2*B*b*c/(3*f)) + x**2*(A*b*c/f + B
*a*c/f + B*b**2/(2*f) - B*c**2*d/(2*f**2)) + x*(2*A*a*c/f + A*b**2/f - A*c*
*2*d/f**2 + 2*B*a*b/f - 2*B*b*c*d/f**2) + ((2*A*a*b*f**2 - 2*A*b*c*d*f + B*
a**2*f**2 - 2*B*a*c*d*f - B*b**2*d*f + B*c**2*d**2)/(2*f**3) - sqrt(-d*f**7
```

```
)*(A*a**2*f**2 - 2*A*a*c*d*f - A*b**2*d*f + A*c**2*d**2 - 2*B*a*b*d*f + 2*B
*b*c*d**2)/(2*d*f**6))*log(x + (-2*A*a*b*d*f**2 + 2*A*b*c*d**2*f - B*a**2*d
*f**2 + 2*B*a*c*d**2*f + B*b**2*d**2*f - B*c**2*d**3 + 2*d*f**3*((2*A*a*b*f
**2 - 2*A*b*c*d*f + B*a**2*f**2 - 2*B*a*c*d*f - B*b**2*d*f + B*c**2*d**2)/(
2*f**3) - sqrt(-d*f**7)*(A*a**2*f**2 - 2*A*a*c*d*f - A*b**2*d*f + A*c**2*d*
*2 - 2*B*a*b*d*f + 2*B*b*c*d**2)/(2*d*f**6)))/(A*a**2*f**3 - 2*A*a*c*d*f**2
 - A*b**2*d*f**2 + A*c**2*d**2*f - 2*B*a*b*d*f**2 + 2*B*b*c*d**2*f)) + ((2*
A*a*b*f**2 - 2*A*b*c*d*f + B*a**2*f**2 - 2*B*a*c*d*f - B*b**2*d*f + B*c**2*
d**2)/(2*f**3) + sqrt(-d*f**7)*(A*a**2*f**2 - 2*A*a*c*d*f - A*b**2*d*f + A*
c**2*d**2 - 2*B*a*b*d*f + 2*B*b*c*d**2)/(2*d*f**6))*log(x + (-2*A*a*b*d*f**
2 + 2*A*b*c*d**2*f - B*a**2*d*f**2 + 2*B*a*c*d**2*f + B*b**2*d**2*f - B*c**
2*d**3 + 2*d*f**3*((2*A*a*b*f**2 - 2*A*b*c*d*f + B*a**2*f**2 - 2*B*a*c*d*f
- B*b**2*d*f + B*c**2*d**2)/(2*f**3) + sqrt(-d*f**7)*(A*a**2*f**2 - 2*A*a*c
*d*f - A*b**2*d*f + A*c**2*d**2 - 2*B*a*b*d*f + 2*B*b*c*d**2)/(2*d*f**6)))/
(A*a**2*f**3 - 2*A*a*c*d*f**2 - A*b**2*d*f**2 + A*c**2*d**2*f - 2*B*a*b*d*f
**2 + 2*B*b*c*d**2*f))
```

## 3.3 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{d+fx^2}\,dx$

**Optimal**. Leaf size=441

$$\frac{\log\left(d+fx^2\right)\left(Abf\left(-f\left(b^2d-3a^2f\right)-6acdf+3c^2d^2\right)-B(cd-af)\left(-f\left(3b^2d-a^2f\right)-2acdf+c^2d^2\right)\right)}{2f^4}-\frac{x^2\left(A\right.}{}$$

**Rubi [A]** time = 0.62, antiderivative size = 441, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1012, 635, 205, 260}

$$-\frac{x^2\left(Abf\left(-6acf+b^2(-f)+3c^2d\right)-B\left(-3cf\left(b^2d-a^2f\right)+3ab^2f^2-3ac^2df+c^3d^2\right)\right)}{2f^3}+\frac{\log\left(d+fx^2\right)\left(Abf\left(-f\left(b^2d-3a^2f\right)-6acdf+3c^2d^2\right)-B(cd-af)\left(-f\left(3b^2d-a^2f\right)-2acdf+c^2d^2\right)\right)}{2f^4}-\frac{x\left(-Ac\left(3a^2f^2-3acdf+c^2d^2\right)+3Ab^2f(cd-af)-3bB(cd-af)^2+b^3Bdf\right)}{f^3}+\frac{cx^4\left(3Abcf-B\left(-3acf-3b^2f+c^2d\right)\right)}{4f^2}+\frac{x^3\left(-Ac^2(cd-3af)-3bBc(cd-2af)+3Ab^2cf+b^3Bf\right)}{3f^2}+\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)\left(3Ab^2df(cd-af)-A(cd-af)^3-3bBd(cd-af)^2+b^3Bd^2f\right)}{\sqrt{d}\,f^{7/2}}+\frac{c^2x^5(Ac+3bB)}{5f}+\frac{Bc^3x^6}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x + c*x^2)^3)/(d + f*x^2),x]
```

```
[Out] -(((b^3*B*d*f + 3*A*b^2*f*(c*d - a*f) - 3*b*B*(c*d - a*f)^2 - A*c*(c^2*d^2
- 3*a*c*d*f + 3*a^2*f^2))*x)/f^3) - ((A*b*f*(3*c^2*d - b^2*f - 6*a*c*f) - B
*(c^3*d^2 - 3*a*c^2*d*f + 3*a*b^2*f^2 - 3*c*f*(b^2*d - a^2*f)))*x^2)/(2*f^3
) + ((b^3*B*f + 3*A*b^2*c*f - A*c^2*(c*d - 3*a*f) - 3*b*B*c*(c*d - 2*a*f))*
x^3)/(3*f^2) + (c*(3*A*b*c*f - B*(c^2*d - 3*b^2*f - 3*a*c*f))*x^4)/(4*f^2)
+ (c^2*(3*b*B + A*c)*x^5)/(5*f) + (B*c^3*x^6)/(6*f) + ((b^3*B*d^2*f + 3*A*b
^2*d*f*(c*d - a*f) - 3*b*B*d*(c*d - a*f)^2 - A*(c*d - a*f)^3)*ArcTan[(Sqrt[
f]*x)/Sqrt[d]])/(Sqrt[d]*f^(7/2)) + ((A*b*f*(3*c^2*d^2 - 6*a*c*d*f - f*(b^2
*d - 3*a^2*f)) - B*(c*d - a*f)*(c^2*d^2 - 2*a*c*d*f - f*(3*b^2*d - a^2*f)))
*Log[d + f*x^2])/(2*f^4)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1012

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + c*x^2)^p*(d + e*x +
 f*x^2)^q*(g + h*x), x], x] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 -
 4*d*f, 0] && IntegersQ[p, q] && (GtQ[p, 0] || GtQ[q, 0])
```

Rubi steps

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{d+fx^2}\,dx = \int \left(-\frac{b^3Bdf+3Ab^2f(cd-af)-3bB(cd-af)^2-Ac\left(c^2d^2-3acdf+3a^2f^2\right)}{f^3}\right.$$

$$= -\frac{\left(b^3Bdf+3Ab^2f(cd-af)-3bB(cd-af)^2-Ac\left(c^2d^2-3acdf+3a^2f^2\right)\right)x}{f^3}$$

$$= -\frac{\left(b^3Bdf+3Ab^2f(cd-af)-3bB(cd-af)^2-Ac\left(c^2d^2-3acdf+3a^2f^2\right)\right)x}{f^3}$$

$$= -\frac{\left(b^3Bdf+3Ab^2f(cd-af)-3bB(cd-af)^2-Ac\left(c^2d^2-3acdf+3a^2f^2\right)\right)x}{f^3}$$

**Mathematica** [A] time = 0.46, size = 422, normalized size = 0.96

$$\frac{fx\left(3b\left(4B\left(15a^2f^2+10acf\left(fx^2-3d\right)+c^2\left(15d^2-5dfx^2+3f^2x^4\right)\right)+15Acfx\left(4af-2cd+cfx^2\right)\right)+c\left(4A\left(45a^2f^2+15acf\left(fx^2-3d\right)+c^2\left(15d^2-5dfx^2+3f^2x^4\right)\right)+5Bx\left(18a^2f^2+9acf\left(fx^2-2d\right)+c^2\left(6d^2-3dfx^2+2f^2x^4\right)\right)\right)+15b^2f\left(4A\left(3af-3cd+cfx^2\right)+3Bx\left(2af-2cd+cfx^2\right)\right)+10b^3f\left(3Afx-6Bd+2Bfx^2\right)\right)-30\log\left(d+fx^2\right)\left(Abf\left(-3a^2f^2+6acdf+b^2df-3c^2d^2\right)+B(cd-af)\left(a^2f^2-2acdf-3b^2df+c^2d^2\right)\right)}{60f^4}+\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)\left(3Ab^2df(cd-af)-A(cd-af)^3-3bBd(cd-af)^2+b^3Bd^2f\right)}{\sqrt{d}\,f^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x + c*x^2)^3)/(d + f*x^2),x]
```

```
[Out] ((b^3*B*d^2*f + 3*A*b^2*d*f*(c*d - a*f) - 3*b*B*d*(c*d - a*f)^2 - A*(c*d -
a*f)^3)*ArcTan[(Sqrt[f]*x)/Sqrt[d]])/(Sqrt[d]*f^(7/2)) + (f*x*(10*b^3*f*(-6
*B*d + 3*A*f*x + 2*B*f*x^2) + 15*b^2*f*(3*B*x*(-2*c*d + 2*a*f + c*f*x^2) +
4*A*(-3*c*d + 3*a*f + c*f*x^2)) + 3*b*(15*A*c*f*x*(-2*c*d + 4*a*f + c*f*x^2
) + 4*B*(15*a^2*f^2 + 10*a*c*f*(-3*d + f*x^2) + c^2*(15*d^2 - 5*d*f*x^2 + 3
*f^2*x^4))) + c*(5*B*x*(18*a^2*f^2 + 9*a*c*f*(-2*d + f*x^2) + c^2*(6*d^2 -
3*d*f*x^2 + 2*f^2*x^4)) + 4*A*(45*a^2*f^2 + 15*a*c*f*(-3*d + f*x^2) + c^2*(
15*d^2 - 5*d*f*x^2 + 3*f^2*x^4)))) - 30*(A*b*f*(-3*c^2*d^2 + b^2*d*f + 6*a*
c*d*f - 3*a^2*f^2) + B*(c*d - a*f)*(c^2*d^2 - 3*b^2*d*f - 2*a*c*d*f + a^2*f
^2))*Log[d + f*x^2])/(60*f^4)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{d+fx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((A + B*x)*(a + b*x + c*x^2)^3)/(d + f*x^2),x]
```

```
[Out] IntegrateAlgebraic[((A + B*x)*(a + b*x + c*x^2)^3)/(d + f*x^2), x]
```

**fricas [A]** time = 0.44, size = 1014, normalized size = 2.30

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^3/(f*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/60*(10*B*c^3*d*f^3*x^6 + 12*(3*B*b*c^2 + A*c^3)*d*f^3*x^5 - 15*(B*c^3*d^
2*f^2 - 3*(B*b^2*c + (B*a + A*b)*c^2)*d*f^3)*x^4 - 20*((3*B*b*c^2 + A*c^3)*
d^2*f^2 - (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*d*f^3)*x^3 + 30*(B*c^
3*d^3*f - 3*(B*b^2*c + (B*a + A*b)*c^2)*d^2*f^2 + (3*B*a*b^2 + A*b^3 + 3*(B
*a^2 + 2*A*a*b)*c)*d*f^3)*x^2 - 30*(A*a^3*f^3 - (3*B*b*c^2 + A*c^3)*d^3 + (
B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*d^2*f - 3*(B*a^2*b + A*a*b^2 + A
*a^2*c)*d*f^2)*sqrt(-d*f)*log((f*x^2 - 2*sqrt(-d*f)*x - d)/(f*x^2 + d)) + 6
0*((3*B*b*c^2 + A*c^3)*d^3*f - (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*
d^2*f^2 + 3*(B*a^2*b + A*a*b^2 + A*a^2*c)*d*f^3)*x - 30*(B*c^3*d^4 - 3*(B*b
^2*c + (B*a + A*b)*c^2)*d^3*f + (3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)
*d^2*f^2 - (B*a^3 + 3*A*a^2*b)*d*f^3)*log(f*x^2 + d))/(d*f^4), 1/60*(10*B*c
^3*d*f^3*x^6 + 12*(3*B*b*c^2 + A*c^3)*d*f^3*x^5 - 15*(B*c^3*d^2*f^2 - 3*(B*
b^2*c + (B*a + A*b)*c^2)*d*f^3)*x^4 - 20*((3*B*b*c^2 + A*c^3)*d^2*f^2 - (B*
b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*d*f^3)*x^3 + 30*(B*c^3*d^3*f - 3*(
B*b^2*c + (B*a + A*b)*c^2)*d^2*f^2 + (3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*
b)*c)*d*f^3)*x^2 + 60*(A*a^3*f^3 - (3*B*b*c^2 + A*c^3)*d^3 + (B*b^3 + 3*A*a
*c^2 + 3*(2*B*a*b + A*b^2)*c)*d^2*f - 3*(B*a^2*b + A*a*b^2 + A*a^2*c)*d*f^2
)*sqrt(d*f)*arctan(sqrt(d*f)*x/d) + 60*((3*B*b*c^2 + A*c^3)*d^3*f - (B*b^3
+ 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*d^2*f^2 + 3*(B*a^2*b + A*a*b^2 + A*a^2
*c)*d*f^3)*x - 30*(B*c^3*d^4 - 3*(B*b^2*c + (B*a + A*b)*c^2)*d^3*f + (3*B*a
*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*d^2*f^2 - (B*a^3 + 3*A*a^2*b)*d*f^3)*
log(f*x^2 + d))/(d*f^4)]
```

**giac [A]** time = 0.16, size = 623, normalized size = 1.41

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^3/(f*x^2+d),x, algorithm="giac")
```

```
[Out] -(3*B*b*c^2*d^3 + A*c^3*d^3 - B*b^3*d^2*f - 6*B*a*b*c*d^2*f - 3*A*b^2*c*d^2
*f - 3*A*a*c^2*d^2*f + 3*B*a^2*b*d*f^2 + 3*A*a*b^2*d*f^2 + 3*A*a^2*c*d*f^2
- A*a^3*f^3)*arctan(f*x/sqrt(d*f))/(sqrt(d*f)*f^3) - 1/2*(B*c^3*d^3 - 3*B*b
^2*c*d^2*f - 3*B*a*c^2*d^2*f - 3*A*b*c^2*d^2*f + 3*B*a*b^2*d*f^2 + A*b^3*d*
f^2 + 3*B*a^2*c*d*f^2 + 6*A*a*b*c*d*f^2 - B*a^3*f^3 - 3*A*a^2*b*f^3)*log(f*
x^2 + d)/f^4 + 1/60*(10*B*c^3*f^5*x^6 + 36*B*b*c^2*f^5*x^5 + 12*A*c^3*f^5*x
^5 - 15*B*c^3*d*f^4*x^4 + 45*B*b^2*c*f^5*x^4 + 45*B*a*c^2*f^5*x^4 + 45*A*b*
c^2*f^5*x^4 - 60*B*b*c^2*d*f^4*x^3 - 20*A*c^3*d*f^4*x^3 + 20*B*b^3*f^5*x^3
+ 120*B*a*b*c*f^5*x^3 + 60*A*b^2*c*f^5*x^3 + 60*A*a*c^2*f^5*x^3 + 30*B*c^3*
d^2*f^3*x^2 - 90*B*b^2*c*d*f^4*x^2 - 90*B*a*c^2*d*f^4*x^2 - 90*A*b*c^2*d*f^
4*x^2 + 90*B*a*b^2*f^5*x^2 + 30*A*b^3*f^5*x^2 + 90*B*a^2*c*f^5*x^2 + 180*A*
a*b*c*f^5*x^2 + 180*B*b*c^2*d^2*f^3*x + 60*A*c^3*d^2*f^3*x - 60*B*b^3*d*f^4
*x - 360*B*a*b*c*d*f^4*x - 180*A*b^2*c*d*f^4*x - 180*A*a*c^2*d*f^4*x + 180*
B*a^2*b*f^5*x + 180*A*a*b^2*f^5*x + 180*A*a^2*c*f^5*x)/f^6
```

**maple [A]** time = 0.01, size = 822, normalized size = 1.86

$$\frac{Bc^3x^6}{6f}+\frac{Ac^3x^5}{5f}+\frac{3Bbc^2x^5}{5f}+\frac{3Abc^2x^4}{4f}+\frac{3Bac^2x^4}{4f}+\frac{3Bb^2cx^4}{4f}-\frac{Bc^3dx^4}{4f^2}+\frac{Aac^2x^3}{f}+\frac{Ab^2cx^3}{f}-\frac{Ac^3dx^3}{3f^2}+\frac{2Babcx^3}{f}+\frac{Bb^3x^3}{3f}-\frac{Bbc^2dx^3}{f^2}+\frac{Aa^3\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}}-\frac{3Aa^2cd\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}\,f}-\frac{3Aab^2d\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}\,f}+\frac{3Aabcx^2}{f}+\frac{3Aac^2d^2\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}\,f^2}+\frac{Ab^3x^2}{2f}+\frac{3Ab^2cd^2\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}\,f^2}-\frac{3Abc^2dx^2}{2f^2}-\frac{Ac^3d^3\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}\,f^3}-\frac{3Ba^2bd\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}\,f}+\frac{3Ba^2cx^2}{2f}+\frac{3Bab^2x^2}{2f}+\frac{6Babcd^2\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}\,f^2}-\frac{3Bac^2dx^2}{2f^2}+\frac{Bb^3d^2\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}\,f^2}-\frac{3Bb^2cdx^2}{2f^2}-\frac{3Bbc^2d^3\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}\,f^3}+\frac{Bc^3d^2x^2}{2f^3}+\frac{3Aa^2b\ln\left(fx^2+d\right)}{2f}+\frac{3Aa^2cx}{f}+\frac{3Aab^2x}{f}-\frac{3Aabcd\ln\left(fx^2+d\right)}{f^2}-\frac{3Aac^2dx}{f^2}-\frac{Ab^3d\ln\left(fx^2+d\right)}{2f^2}-\frac{3Ab^2cdx}{f^2}+\frac{3Abc^2d^2\ln\left(fx^2+d\right)}{2f^3}+\frac{Ac^3d^2x}{f^3}+\frac{Ba^3\ln\left(fx^2+d\right)}{2f}+\frac{3Ba^2bx}{f}-\frac{3Ba^2cd\ln\left(fx^2+d\right)}{2f^2}-\frac{3Bab^2d\ln\left(fx^2+d\right)}{2f^2}-\frac{6Babcdx}{f^2}+\frac{3Bac^2d^2\ln\left(fx^2+d\right)}{2f^3}-\frac{Bb^3dx}{f^2}+\frac{3Bb^2cd^2\ln\left(fx^2+d\right)}{2f^3}+\frac{3Bbc^2d^2x}{f^3}-\frac{Bc^3d^3\ln\left(fx^2+d\right)}{2f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(c*x^2+b*x+a)^3/(f*x^2+d),x)
```

```
[Out] 6/f^2/(d*f)^(1/2)*arctan(1/(d*f)^(1/2)*f*x)*B*a*b*c*d^2-1/f^2*b^3*B*d*x+3/f
*A*a*b^2*x+1/6*B*c^3*x^6/f-3/2/f^2*ln(f*x^2+d)*B*a^2*c*d-3/2/f^2*ln(f*x^2+d
)*B*a*b^2*d+3/2/f^3*ln(f*x^2+d)*B*a*c^2*d^2+3/2/f^3*ln(f*x^2+d)*B*b^2*c*d^2
-1/f^3/(d*f)^(1/2)*arctan(1/(d*f)^(1/2)*f*x)*A*c^3*d^3+1/f^2/(d*f)^(1/2)*ar
ctan(1/(d*f)^(1/2)*f*x)*b^3*B*d^2+3/f^3*B*b*c^2*d^2*x+2/f*B*x^3*a*b*c-1/f^2
*B*x^3*b*c^2*d+3/f*A*x^2*a*b*c-3/2/f^2*A*x^2*b*c^2*d-3/2/f^2*B*x^2*a*c^2*d-
3/2/f^2*B*x^2*b^2*c*d-3/f^2*A*a*c^2*d*x-3/f^2*A*b^2*c*d*x+3/2/f^3*ln(f*x^2+
d)*A*b*c^2*d^2+1/2/f*ln(f*x^2+d)*B*a^3+1/(d*f)^(1/2)*arctan(1/(d*f)^(1/2)*f
*x)*A*a^3+1/3/f*B*x^3*b^3+1/2/f*A*x^2*b^3+1/5/f*A*x^5*c^3-3/f^2*ln(f*x^2+d)
*A*a*b*c*d-3/f/(d*f)^(1/2)*arctan(1/(d*f)^(1/2)*f*x)*A*a^2*c*d-3/f/(d*f)^(1
/2)*arctan(1/(d*f)^(1/2)*f*x)*A*a*b^2*d+3/f^2/(d*f)^(1/2)*arctan(1/(d*f)^(1
/2)*f*x)*A*a*c^2*d^2+3/f^2/(d*f)^(1/2)*arctan(1/(d*f)^(1/2)*f*x)*A*b^2*c*d^
2-3/f/(d*f)^(1/2)*arctan(1/(d*f)^(1/2)*f*x)*B*a^2*b*d-3/f^3/(d*f)^(1/2)*arc
tan(1/(d*f)^(1/2)*f*x)*B*b*c^2*d^3-6/f^2*B*a*b*c*d*x+3/5/f*B*x^5*b*c^2+3/4/
f*A*x^4*b*c^2+1/f*A*x^3*a*c^2+3/2/f*ln(f*x^2+d)*A*a^2*b-1/2/f^2*ln(f*x^2+d)
*A*b^3*d-1/2/f^4*ln(f*x^2+d)*B*c^3*d^3+1/f*A*x^3*b^2*c+1/2/f^3*B*x^2*c^3*d^
2+1/f^3*A*c^3*d^2*x+3/f*B*a^2*b*x-1/3/f^2*A*x^3*c^3*d+3/2/f*B*x^2*a^2*c+3/2
/f*B*x^2*a*b^2+3/f*A*a^2*c*x+3/4/f*B*x^4*a*c^2+3/4/f*B*x^4*b^2*c-1/4/f^2*B*
x^4*c^3*d
```

**maxima [A]** time = 0.99, size = 471, normalized size = 1.07

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^3/(f*x^2+d),x, algorithm="maxima")
```

```
[Out] (A*a^3*f^3 - (3*B*b*c^2 + A*c^3)*d^3 + (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*
b^2)*c)*d^2*f - 3*(B*a^2*b + A*a*b^2 + A*a^2*c)*d*f^2)*arctan(f*x/sqrt(d*f)
)/(sqrt(d*f)*f^3) + 1/60*(10*B*c^3*f^2*x^6 + 12*(3*B*b*c^2 + A*c^3)*f^2*x^5
 - 15*(B*c^3*d*f - 3*(B*b^2*c + (B*a + A*b)*c^2)*f^2)*x^4 - 20*((3*B*b*c^2
+ A*c^3)*d*f - (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*f^2)*x^3 + 30*(B
*c^3*d^2 - 3*(B*b^2*c + (B*a + A*b)*c^2)*d*f + (3*B*a*b^2 + A*b^3 + 3*(B*a^
2 + 2*A*a*b)*c)*f^2)*x^2 + 60*((3*B*b*c^2 + A*c^3)*d^2 - (B*b^3 + 3*A*a*c^2
 + 3*(2*B*a*b + A*b^2)*c)*d*f + 3*(B*a^2*b + A*a*b^2 + A*a^2*c)*f^2)*x)/f^3
 - 1/2*(B*c^3*d^3 - 3*(B*b^2*c + (B*a + A*b)*c^2)*d^2*f + (3*B*a*b^2 + A*b^
3 + 3*(B*a^2 + 2*A*a*b)*c)*d*f^2 - (B*a^3 + 3*A*a^2*b)*f^3)*log(f*x^2 + d)/
f^4
```

**mupad [B]** time = 3.79, size = 552, normalized size = 1.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x)*(a + b*x + c*x^2)^3)/(d + f*x^2),x)
```

```
[Out] x^2*((A*b^3 + 3*B*a*b^2 + 3*B*a^2*c + 6*A*a*b*c)/(2*f) - (d*((3*A*b*c^2 + 3
*B*a*c^2 + 3*B*b^2*c)/f - (B*c^3*d)/f^2))/(2*f)) + x*((3*A*a*b^2 + 3*A*a^2*
c + 3*B*a^2*b)/f - (d*((B*b^3 + 3*A*a*c^2 + 3*A*b^2*c + 6*B*a*b*c)/f - (d*(
A*c^3 + 3*B*b*c^2))/f^2))/f) + x^3*((B*b^3 + 3*A*a*c^2 + 3*A*b^2*c + 6*B*a*
b*c)/(3*f) - (d*(A*c^3 + 3*B*b*c^2))/(3*f^2)) + x^4*((3*A*b*c^2 + 3*B*a*c^2
 + 3*B*b^2*c)/(4*f) - (B*c^3*d)/(4*f^2)) + (x^5*(A*c^3 + 3*B*b*c^2))/(5*f)
+ (B*c^3*x^6)/(6*f) + (log(d + f*x^2)*(4*B*a^3*d*f^7 - 4*A*b^3*d^2*f^6 - 4*
B*c^3*d^4*f^4 - 12*B*a*b^2*d^2*f^6 + 12*A*b*c^2*d^3*f^5 + 12*B*a*c^2*d^3*f^
5 - 12*B*a^2*c*d^2*f^6 + 12*B*b^2*c*d^3*f^5 + 12*A*a^2*b*d*f^7 - 24*A*a*b*c
*d^2*f^6))/(8*d*f^8) + (atan((f^(1/2)*x)/d^(1/2))*(A*a^3*f^3 - A*c^3*d^3 -
3*B*b*c^2*d^3 + B*b^3*d^2*f - 3*A*a*b^2*d*f^2 + 3*A*a*c^2*d^2*f - 3*A*a^2*c
*d*f^2 - 3*B*a^2*b*d*f^2 + 3*A*b^2*c*d^2*f + 6*B*a*b*c*d^2*f))/(d^(1/2)*f^(
7/2))
```

**sympy [B]** time = 23.62, size = 1962, normalized size = 4.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x**2+b*x+a)**3/(f*x**2+d),x)
```

```
[Out] B*c**3*x**6/(6*f) + x**5*(A*c**3/(5*f) + 3*B*b*c**2/(5*f)) + x**4*(3*A*b*c*
*2/(4*f) + 3*B*a*c**2/(4*f) + 3*B*b**2*c/(4*f) - B*c**3*d/(4*f**2)) + x**3*
(A*a*c**2/f + A*b**2*c/f - A*c**3*d/(3*f**2) + 2*B*a*b*c/f + B*b**3/(3*f) -
 B*b*c**2*d/f**2) + x**2*(3*A*a*b*c/f + A*b**3/(2*f) - 3*A*b*c**2*d/(2*f**2
) + 3*B*a**2*c/(2*f) + 3*B*a*b**2/(2*f) - 3*B*a*c**2*d/(2*f**2) - 3*B*b**2*
c*d/(2*f**2) + B*c**3*d**2/(2*f**3)) + x*(3*A*a**2*c/f + 3*A*a*b**2/f - 3*A
*a*c**2*d/f**2 - 3*A*b**2*c*d/f**2 + A*c**3*d**2/f**3 + 3*B*a**2*b/f - 6*B*
a*b*c*d/f**2 - B*b**3*d/f**2 + 3*B*b*c**2*d**2/f**3) + ((3*A*a**2*b*f**3 -
6*A*a*b*c*d*f**2 - A*b**3*d*f**2 + 3*A*b*c**2*d**2*f + B*a**3*f**3 - 3*B*a*
*2*c*d*f**2 - 3*B*a*b**2*d*f**2 + 3*B*a*c**2*d**2*f + 3*B*b**2*c*d**2*f - B
*c**3*d**3)/(2*f**4) - sqrt(-d*f**9)*(A*a**3*f**3 - 3*A*a**2*c*d*f**2 - 3*A
*a*b**2*d*f**2 + 3*A*a*c**2*d**2*f + 3*A*b**2*c*d**2*f - A*c**3*d**3 - 3*B*
a**2*b*d*f**2 + 6*B*a*b*c*d**2*f + B*b**3*d**2*f - 3*B*b*c**2*d**3)/(2*d*f*
*8))*log(x + (-3*A*a**2*b*d*f**3 + 6*A*a*b*c*d**2*f**2 + A*b**3*d**2*f**2 -
 3*A*b*c**2*d**3*f - B*a**3*d*f**3 + 3*B*a**2*c*d**2*f**2 + 3*B*a*b**2*d**2
*f**2 - 3*B*a*c**2*d**3*f - 3*B*b**2*c*d**3*f + B*c**3*d**4 + 2*d*f**4*((3*
A*a**2*b*f**3 - 6*A*a*b*c*d*f**2 - A*b**3*d*f**2 + 3*A*b*c**2*d**2*f + B*a*
*3*f**3 - 3*B*a**2*c*d*f**2 - 3*B*a*b**2*d*f**2 + 3*B*a*c**2*d**2*f + 3*B*b
**2*c*d**2*f - B*c**3*d**3)/(2*f**4) - sqrt(-d*f**9)*(A*a**3*f**3 - 3*A*a**
2*c*d*f**2 - 3*A*a*b**2*d*f**2 + 3*A*a*c**2*d**2*f + 3*A*b**2*c*d**2*f - A*
c**3*d**3 - 3*B*a**2*b*d*f**2 + 6*B*a*b*c*d**2*f + B*b**3*d**2*f - 3*B*b*c*
*2*d**3)/(2*d*f**8)))/(A*a**3*f**4 - 3*A*a**2*c*d*f**3 - 3*A*a*b**2*d*f**3
+ 3*A*a*c**2*d**2*f**2 + 3*A*b**2*c*d**2*f**2 - A*c**3*d**3*f - 3*B*a**2*b*
d*f**3 + 6*B*a*b*c*d**2*f**2 + B*b**3*d**2*f**2 - 3*B*b*c**2*d**3*f)) + ((3
*A*a**2*b*f**3 - 6*A*a*b*c*d*f**2 - A*b**3*d*f**2 + 3*A*b*c**2*d**2*f + B*a
**3*f**3 - 3*B*a**2*c*d*f**2 - 3*B*a*b**2*d*f**2 + 3*B*a*c**2*d**2*f + 3*B*
b**2*c*d**2*f - B*c**3*d**3)/(2*f**4) + sqrt(-d*f**9)*(A*a**3*f**3 - 3*A*a*
*2*c*d*f**2 - 3*A*a*b**2*d*f**2 + 3*A*a*c**2*d**2*f + 3*A*b**2*c*d**2*f - A
*c**3*d**3 - 3*B*a**2*b*d*f**2 + 6*B*a*b*c*d**2*f + B*b**3*d**2*f - 3*B*b*c
**2*d**3)/(2*d*f**8))*log(x + (-3*A*a**2*b*d*f**3 + 6*A*a*b*c*d**2*f**2 + A
*b**3*d**2*f**2 - 3*A*b*c**2*d**3*f - B*a**3*d*f**3 + 3*B*a**2*c*d**2*f**2
+ 3*B*a*b**2*d**2*f**2 - 3*B*a*c**2*d**3*f - 3*B*b**2*c*d**3*f + B*c**3*d**
4 + 2*d*f**4*((3*A*a**2*b*f**3 - 6*A*a*b*c*d*f**2 - A*b**3*d*f**2 + 3*A*b*c
**2*d**2*f + B*a**3*f**3 - 3*B*a**2*c*d*f**2 - 3*B*a*b**2*d*f**2 + 3*B*a*c*
*2*d**2*f + 3*B*b**2*c*d**2*f - B*c**3*d**3)/(2*f**4) + sqrt(-d*f**9)*(A*a*
*3*f**3 - 3*A*a**2*c*d*f**2 - 3*A*a*b**2*d*f**2 + 3*A*a*c**2*d**2*f + 3*A*b
**2*c*d**2*f - A*c**3*d**3 - 3*B*a**2*b*d*f**2 + 6*B*a*b*c*d**2*f + B*b**3*
d**2*f - 3*B*b*c**2*d**3)/(2*d*f**8)))/(A*a**3*f**4 - 3*A*a**2*c*d*f**3 - 3
*A*a*b**2*d*f**3 + 3*A*a*c**2*d**2*f**2 + 3*A*b**2*c*d**2*f**2 - A*c**3*d**
3*f - 3*B*a**2*b*d*f**3 + 6*B*a*b*c*d**2*f**2 + B*b**3*d**2*f**2 - 3*B*b*c*
*2*d**3*f))
```

## 3.4 $\int \frac{A+Bx}{\left(a+bx+cx^2\right)\left(d+fx^2\right)} \, dx$

**Optimal**. Leaf size=274

$$\frac{\log\left(a+bx+cx^2\right)(-aBf+Abf+Bcd)}{2\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)}-\frac{\log\left(d+fx^2\right)(-aBf+Abf+Bcd)}{2\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)}+\frac{\sqrt{f}\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)(aAf-Acd+}{\sqrt{d}\left(f\left(a^2f+b^2d\right)-2acdf+c\right.}$$

**Rubi** [A] time = 0.28, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {1023, 634, 618, 206, 628, 635, 205, 260}

$$\frac{\log\left(a+bx+cx^2\right)(-aBf+Abf+Bcd)}{2\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)}-\frac{\log\left(d+fx^2\right)(-aBf+Abf+Bcd)}{2\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)}+\frac{\sqrt{f}\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)(aAf-Acd+bBd)}{\sqrt{d}\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)}-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2Ac(cd-af)-bB(af+cd)+Ab^2f\right)}{\sqrt{b^2-4ac}\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/((a + b*x + c*x^2)*(d + f*x^2)),x]
```

```
[Out] (Sqrt[f]*(b*B*d - A*c*d + a*A*f)*ArcTan[(Sqrt[f]*x)/Sqrt[d]])/(Sqrt[d]*(c^2
*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))) - ((A*b^2*f + 2*A*c*(c*d - a*f) - b*
B*(c*d + a*f))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(
c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))) + ((B*c*d + A*b*f - a*B*f)*Log[a
+ b*x + c*x^2])/(2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))) - ((B*c*d + A
*b*f - a*B*f)*Log[d + f*x^2])/(2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f)))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1023

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.)*
(x_)^2)), x_Symbol] :> With[{q = Simplify[c^2*d^2 + b^2*d*f - 2*a*c*d*f + a
^2*f^2]}, Dist[1/q, Int[Simp[g*c^2*d + g*b^2*f - a*b*h*f - a*g*c*f + c*(h*c
*d + g*b*f - a*h*f)*x, x]/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[Simp[b*
h*d*f - g*c*d*f + a*g*f^2 - f*(h*c*d + g*b*f - a*h*f)*x, x]/(d + f*x^2), x]
, x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
]
```

Rubi steps

$$\int \frac{A+Bx}{\left(a+bx+cx^2\right)\left(d+fx^2\right)}\,dx = \frac{\int \frac{-abBf+A\left(c^2d+b^2f-acf\right)+c(Bcd+Abf-aBf)x}{a+bx+cx^2}\,dx}{c^2d^2-2acdf+f\left(b^2d+a^2f\right)} + \frac{\int \frac{f(bBd-Acd+aAf)-f(Bcd+Abf-aBf)x}{d+fx^2}\,d}{c^2d^2-2acdf+f\left(b^2d+a^2f\right)}$$

$$= \frac{(f(bBd-Acd+aAf))\int \frac{1}{d+fx^2}\,dx}{c^2d^2-2acdf+f\left(b^2d+a^2f\right)} + \frac{(Bcd+Abf-aBf)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2\left(c^2d^2-2acdf+f\left(b^2d+a^2f\right)\right)} - \frac{(f(B}{c^2}$$

$$= \frac{\sqrt{f}\,(bBd-Acd+aAf)\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,\left(c^2d^2-2acdf+f\left(b^2d+a^2f\right)\right)} + \frac{(Bcd+Abf-aBf)\log\left(a+bx+cx^2\right)}{2\left(c^2d^2-2acdf+f\left(b^2d+a^2f\right)\right)}$$

$$= \frac{\sqrt{f}\,(bBd-Acd+aAf)\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,\left(c^2d^2-2acdf+f\left(b^2d+a^2f\right)\right)} - \frac{\left(Ab^2f+2Ac(cd-af)-bB(cd+af)\right)}{\sqrt{b^2-4ac}\,\left(c^2d^2-2acdf+f\left(b\right.\right.}$$

**Mathematica [A]** time = 0.37, size = 212, normalized size = 0.77

$$\frac{\sqrt{d}\left(\sqrt{4ac-b^2}\,(-aBf+Abf+Bcd)\left(\log(a+x(b+cx))-\log\left(d+fx^2\right)\right)+2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)\left(2Ac(cd-af)-bB(af+cd)+Ab^2f\right)\right)+2\sqrt{f}\,\sqrt{4ac-b^2}\,\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{d}}\right)(aAf-Acd+bBd)}{2\sqrt{d}\,\sqrt{4ac-b^2}\,\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/((a + b*x + c*x^2)*(d + f*x^2)),x]
```

```
[Out] (2*Sqrt[-b^2 + 4*a*c]*Sqrt[f]*(b*B*d - A*c*d + a*A*f)*ArcTan[(Sqrt[f]*x)/Sq
rt[d]] + Sqrt[d]*(2*(A*b^2*f + 2*A*c*(c*d - a*f) - b*B*(c*d + a*f))*ArcTan[
(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a*c]*(B*c*d + A*b*f - a*B*f
)*(-Log[d + f*x^2] + Log[a + x*(b + c*x)])))/(2*Sqrt[-b^2 + 4*a*c]*Sqrt[d]*
(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f)))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx}{\left(a+bx+cx^2\right)\left(d+fx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x)/((a + b*x + c*x^2)*(d + f*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x)/((a + b*x + c*x^2)*(d + f*x^2)), x]
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [A]** time = 0.17, size = 266, normalized size = 0.97

$$\frac{\left(Bcd - Baf + Abf\right)\log\left(cx^2 + bx + a\right)}{2\left(c^2d^2 + b^2df - 2\,acdf + a^2f^2\right)} - \frac{\left(Bcd - Baf + Abf\right)\log\left(fx^2 + d\right)}{2\left(c^2d^2 + b^2df - 2\,acdf + a^2f^2\right)} + \frac{\left(Bbdf - Acdf + Aaf^2\right)\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\left(c^2d^2 + b^2df - 2\,acdf + a^2f^2\right)\sqrt{df}} - \frac{\left(Bbcd - 2\,Ac^2d + Babf - Ab^2f + 2\,Aacf\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(c^2d^2 + b^2df - 2\,acdf + a^2f^2\right)\sqrt{-b^2 + 4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d),x, algorithm="giac")
```

```
[Out] 1/2*(B*c*d - B*a*f + A*b*f)*log(c*x^2 + b*x + a)/(c^2*d^2 + b^2*d*f - 2*a*c
*d*f + a^2*f^2) - 1/2*(B*c*d - B*a*f + A*b*f)*log(f*x^2 + d)/(c^2*d^2 + b^2
*d*f - 2*a*c*d*f + a^2*f^2) + (B*b*d*f - A*c*d*f + A*a*f^2)*arctan(f*x/sqrt
(d*f))/((c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2)*sqrt(d*f)) - (B*b*c*d - 2
*A*c^2*d + B*a*b*f - A*b^2*f + 2*A*a*c*f)*arctan((2*c*x + b)/sqrt(-b^2 + 4*
a*c))/((c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2)*sqrt(-b^2 + 4*a*c))
```

**maple [B]** time = 0.02, size = 745, normalized size = 2.72

$$-\frac{2Aacf\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(a^2f^2-2acdf+b^2df+c^2d^2\right)\sqrt{4ac-b^2}}+\frac{Aaf^2\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\left(a^2f^2-2acdf+b^2df+c^2d^2\right)\sqrt{df}}+\frac{Ab^2f\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(a^2f^2-2acdf+b^2df+c^2d^2\right)\sqrt{4ac-b^2}}+\frac{2Ac^2d\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(a^2f^2-2acdf+b^2df+c^2d^2\right)\sqrt{4ac-b^2}}-\frac{Acdf\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\left(a^2f^2-2acdf+b^2df+c^2d^2\right)\sqrt{df}}-\frac{Babf\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(a^2f^2-2acdf+b^2df+c^2d^2\right)\sqrt{4ac-b^2}}-\frac{Bbcd\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(a^2f^2-2acdf+b^2df+c^2d^2\right)\sqrt{4ac-b^2}}+\frac{Bbdf\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\left(a^2f^2-2acdf+b^2df+c^2d^2\right)\sqrt{df}}-\frac{Abf\ln\left(fx^2+d\right)}{2\left(a^2f^2-2acdf+b^2df+c^2d^2\right)}+\frac{Abf\ln\left(cx^2+bx+a\right)}{2a^2f^2-4acdf+2b^2df+2c^2d^2}+\frac{Baf\ln\left(fx^2+d\right)}{2a^2f^2-4acdf+2b^2df+2c^2d^2}-\frac{Baf\ln\left(cx^2+bx+a\right)}{2\left(a^2f^2-2acdf+b^2df+c^2d^2\right)}-\frac{Bcd\ln\left(fx^2+d\right)}{2\left(a^2f^2-2acdf+b^2df+c^2d^2\right)}+\frac{Bcd\ln\left(cx^2+bx+a\right)}{2a^2f^2-4acdf+2b^2df+2c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d),x)
```

```
[Out] -1/2*f/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)*ln(f*x^2+d)*A*b+1/2*f/(a^2*f^2-2
*a*c*d*f+b^2*d*f+c^2*d^2)*ln(f*x^2+d)*B*a-1/2/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^
2*d^2)*ln(f*x^2+d)*B*c*d+f^2/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)/(d*f)^(1/2
)*arctan(1/(d*f)^(1/2)*f*x)*A*a-f/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)/(d*f)
^(1/2)*arctan(1/(d*f)^(1/2)*f*x)*A*c*d+f/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2
)/(d*f)^(1/2)*arctan(1/(d*f)^(1/2)*f*x)*B*b*d+1/2/(a^2*f^2-2*a*c*d*f+b^2*d*
f+c^2*d^2)*ln(c*x^2+b*x+a)*A*b*f-1/2/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)*ln
(c*x^2+b*x+a)*B*a*f+1/2/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)*c*ln(c*x^2+b*x+
a)*B*d-2/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*
x+b)/(4*a*c-b^2)^(1/2))*A*a*c*f+1/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)/(4*a*
c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*A*b^2*f+2/(a^2*f^2-2*a*c*d
*f+b^2*d*f+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*A
*c^2*d-1/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*
x+b)/(4*a*c-b^2)^(1/2))*B*a*b*f-1/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)/(4*a*
c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*B*b*c*d
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

**mupad [B]** time = 38.32, size = 3888, normalized size = 14.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/((d + f*x^2)*(a + b*x + c*x^2)),x)
```

```
[Out] (log(B^3*c^2*f^2*x + (((B*c*d^2)/2 + (A*b*d*f)/2 - (B*a*d*f)/2 - (A*a*f*(-d
*f)^(1/2))/2 + (A*c*d*(-d*f)^(1/2))/2 - (B*b*d*(-d*f)^(1/2))/2)*((((B*c*d^2
)/2 + (A*b*d*f)/2 - (B*a*d*f)/2 - (A*a*f*(-d*f)^(1/2))/2 + (A*c*d*(-d*f)^(1
/2))/2 - (B*b*d*(-d*f)^(1/2))/2)*(4*A*a^2*c^2*f^4 + 4*A*c^4*d^2*f^2 - c*f^2
*x*(3*A*b^3*f^2 + 4*B*c^3*d^2 - B*a*b^2*f^2 + 4*B*a^2*c*f^2 - 12*A*a*b*c*f^
2 + 12*A*b*c^2*d*f - 8*B*a*c^2*d*f - 3*B*b^2*c*d*f) - 3*A*b^2*c^2*d*f^3 - 4
*B*b*c^3*d^2*f^2 - A*a*b^2*c*f^4 - 8*A*a*c^3*d*f^3 + B*b^3*c*d*f^3 + (2*c*f
^2*((B*c*d^2)/2 + (A*b*d*f)/2 - (B*a*d*f)/2 - (A*a*f*(-d*f)^(1/2))/2 + (A*c
*d*(-d*f)^(1/2))/2 - (B*b*d*(-d*f)^(1/2))/2)*(2*b*c^3*d^3 + 4*c^4*d^3*x - a
^2*b^2*f^3*x + 4*a*b^3*d*f^2 - 2*b^3*c*d^2*f + 4*a^3*c*f^3*x + 3*b^4*d*f^2*
x + 12*a*b*c^2*d^2*f - 14*a^2*b*c*d*f^2 - 4*a*c^3*d^2*f*x - 4*a^2*c^2*d*f^2
*x + 3*b^2*c^2*d^2*f*x - 10*a*b^2*c*d*f^2*x))/(d*(a^2*f^2 + c^2*d^2 + b^2*d
*f - 2*a*c*d*f)) + 4*B*a*b*c^2*d*f^3))/(d*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*
a*c*d*f)) - c*f^2*x*(2*B^2*c^2*d - 4*A^2*c^2*f - B^2*b^2*f + 2*B^2*a*c*f +
2*A*B*b*c*f) + A^2*b*c^2*f^3 + B^2*b*c^2*d*f^2 - 4*A*B*a*c^2*f^3 + A*B*b^2*
c*f^3 - 4*A*B*c^3*d*f^2))/(d*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f)) + A
*B^2*c^2*f^2)*(f*((B*a*d)/2 - (A*b*d)/2 + (A*a*(-d*f)^(1/2))/2) - (B*c*d^2)
/2 - (A*c*d*(-d*f)^(1/2))/2 + (B*b*d*(-d*f)^(1/2))/2))/(c^2*d^3 + a^2*d*f^2
 + b^2*d^2*f - 2*a*c*d^2*f) - (log(B^3*c^2*f^2*x + (((B*c*d^2)/2 + (A*b*d*f
)/2 - (B*a*d*f)/2 + (A*a*f*(-d*f)^(1/2))/2 - (A*c*d*(-d*f)^(1/2))/2 + (B*b*
d*(-d*f)^(1/2))/2)*((((B*c*d^2)/2 + (A*b*d*f)/2 - (B*a*d*f)/2 + (A*a*f*(-d*
f)^(1/2))/2 - (A*c*d*(-d*f)^(1/2))/2 + (B*b*d*(-d*f)^(1/2))/2)*(4*A*a^2*c^2
*f^4 + 4*A*c^4*d^2*f^2 - c*f^2*x*(3*A*b^3*f^2 + 4*B*c^3*d^2 - B*a*b^2*f^2 +
 4*B*a^2*c*f^2 - 12*A*a*b*c*f^2 + 12*A*b*c^2*d*f - 8*B*a*c^2*d*f - 3*B*b^2*
c*d*f) - 3*A*b^2*c^2*d*f^3 - 4*B*b*c^3*d^2*f^2 - A*a*b^2*c*f^4 - 8*A*a*c^3*
d*f^3 + B*b^3*c*d*f^3 + (2*c*f^2*((B*c*d^2)/2 + (A*b*d*f)/2 - (B*a*d*f)/2 +
 (A*a*f*(-d*f)^(1/2))/2 - (A*c*d*(-d*f)^(1/2))/2 + (B*b*d*(-d*f)^(1/2))/2)*
(2*b*c^3*d^3 + 4*c^4*d^3*x - a^2*b^2*f^3*x + 4*a*b^3*d*f^2 - 2*b^3*c*d^2*f
+ 4*a^3*c*f^3*x + 3*b^4*d*f^2*x + 12*a*b*c^2*d^2*f - 14*a^2*b*c*d*f^2 - 4*a
*c^3*d^2*f*x - 4*a^2*c^2*d*f^2*x + 3*b^2*c^2*d^2*f*x - 10*a*b^2*c*d*f^2*x))
```

```
/(d*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f)) + 4*B*a*b*c^2*d*f^3))/(d*(a^
2*f^2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f)) - c*f^2*x*(2*B^2*c^2*d - 4*A^2*c^2*
f - B^2*b^2*f + 2*B^2*a*c*f + 2*A*B*b*c*f) + A^2*b*c^2*f^3 + B^2*b*c^2*d*f^
2 - 4*A*B*a*c^2*f^3 + A*B*b^2*c*f^3 - 4*A*B*c^3*d*f^2))/(d*(a^2*f^2 + c^2*d
^2 + b^2*d*f - 2*a*c*d*f)) + A*B^2*c^2*f^2)*(f*((A*b*d)/2 - (B*a*d)/2 + (A*
a*(-d*f)^(1/2))/2) + (B*c*d^2)/2 - (A*c*d*(-d*f)^(1/2))/2 + (B*b*d*(-d*f)^(
1/2))/2))/(c^2*d^3 + a^2*d*f^2 + b^2*d^2*f - 2*a*c*d^2*f) - (log(B^3*c^2*f^
2*x + A*B^2*c^2*f^2 - ((((A*f*(b^2 - 4*a*c)^(3/2) + 2*A*b*f*(4*a*c - b^2) -
 2*B*a*f*(4*a*c - b^2) + 2*B*c*d*(4*a*c - b^2) + 4*A*c^2*d*(b^2 - 4*a*c)^(1
/2) + A*b^2*f*(b^2 - 4*a*c)^(1/2) - 2*B*a*b*f*(b^2 - 4*a*c)^(1/2) - 2*B*b*c
*d*(b^2 - 4*a*c)^(1/2))*(c*f^2*x*(3*A*b^3*f^2 + 4*B*c^3*d^2 - B*a*b^2*f^2 +
 4*B*a^2*c*f^2 - 12*A*a*b*c*f^2 + 12*A*b*c^2*d*f - 8*B*a*c^2*d*f - 3*B*b^2*
c*d*f) - 4*A*c^4*d^2*f^2 - 4*A*a^2*c^2*f^4 + 3*A*b^2*c^2*d*f^3 + 4*B*b*c^3*
d^2*f^2 + A*a*b^2*c*f^4 + 8*A*a*c^3*d*f^3 - B*b^3*c*d*f^3 - 4*B*a*b*c^2*d*f
^3 + (c*f^2*(A*f*(b^2 - 4*a*c)^(3/2) + 2*A*b*f*(4*a*c - b^2) - 2*B*a*f*(4*a
*c - b^2) + 2*B*c*d*(4*a*c - b^2) + 4*A*c^2*d*(b^2 - 4*a*c)^(1/2) + A*b^2*f
*(b^2 - 4*a*c)^(1/2) - 2*B*a*b*f*(b^2 - 4*a*c)^(1/2) - 2*B*b*c*d*(b^2 - 4*a
*c)^(1/2))*(2*b*c^3*d^3 + 4*c^4*d^3*x - a^2*b^2*f^3*x + 4*a*b^3*d*f^2 - 2*b
^3*c*d^2*f + 4*a^3*c*f^3*x + 3*b^4*d*f^2*x + 12*a*b*c^2*d^2*f - 14*a^2*b*c*
d*f^2 - 4*a*c^3*d^2*f*x - 4*a^2*c^2*d*f^2*x + 3*b^2*c^2*d^2*f*x - 10*a*b^2*
c*d*f^2*x))/(2*(4*a*c - b^2)*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f))))/(
4*(4*a*c - b^2)*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f)) - c*f^2*x*(2*B^2
*c^2*d - 4*A^2*c^2*f - B^2*b^2*f + 2*B^2*a*c*f + 2*A*B*b*c*f) + A^2*b*c^2*f
^3 + B^2*b*c^2*d*f^2 - 4*A*B*a*c^2*f^3 + A*B*b^2*c*f^3 - 4*A*B*c^3*d*f^2)*(
A*f*(b^2 - 4*a*c)^(3/2) + 2*A*b*f*(4*a*c - b^2) - 2*B*a*f*(4*a*c - b^2) + 2
*B*c*d*(4*a*c - b^2) + 4*A*c^2*d*(b^2 - 4*a*c)^(1/2) + A*b^2*f*(b^2 - 4*a*c
)^(1/2) - 2*B*a*b*f*(b^2 - 4*a*c)^(1/2) - 2*B*b*c*d*(b^2 - 4*a*c)^(1/2)))/(
4*(4*a*c - b^2)*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f)))*(A*f*(b^2 - 4*a
*c)^(3/2) + 2*A*b*f*(4*a*c - b^2) - 2*B*a*f*(4*a*c - b^2) + 2*B*c*d*(4*a*c
- b^2) + 4*A*c^2*d*(b^2 - 4*a*c)^(1/2) + A*b^2*f*(b^2 - 4*a*c)^(1/2) - 2*B*
a*b*f*(b^2 - 4*a*c)^(1/2) - 2*B*b*c*d*(b^2 - 4*a*c)^(1/2)))/(b^2*(4*a^2*f^2
 + 4*c^2*d^2 + 4*b^2*d*f - 24*a*c*d*f) - 4*a*c*(4*a^2*f^2 + 4*c^2*d^2 - 8*a
*c*d*f)) + (log(B^3*c^2*f^2*x + A*B^2*c^2*f^2 + ((((A*f*(b^2 - 4*a*c)^(3/2)
 - 2*A*b*f*(4*a*c - b^2) + 2*B*a*f*(4*a*c - b^2) - 2*B*c*d*(4*a*c - b^2) +
4*A*c^2*d*(b^2 - 4*a*c)^(1/2) + A*b^2*f*(b^2 - 4*a*c)^(1/2) - 2*B*a*b*f*(b^
2 - 4*a*c)^(1/2) - 2*B*b*c*d*(b^2 - 4*a*c)^(1/2))*(4*A*a^2*c^2*f^4 + 4*A*c^
4*d^2*f^2 - c*f^2*x*(3*A*b^3*f^2 + 4*B*c^3*d^2 - B*a*b^2*f^2 + 4*B*a^2*c*f^
2 - 12*A*a*b*c*f^2 + 12*A*b*c^2*d*f - 8*B*a*c^2*d*f - 3*B*b^2*c*d*f) - 3*A*
b^2*c^2*d*f^3 - 4*B*b*c^3*d^2*f^2 - A*a*b^2*c*f^4 - 8*A*a*c^3*d*f^3 + B*b^3
*c*d*f^3 + 4*B*a*b*c^2*d*f^3 + (c*f^2*(A*f*(b^2 - 4*a*c)^(3/2) - 2*A*b*f*(4
*a*c - b^2) + 2*B*a*f*(4*a*c - b^2) - 2*B*c*d*(4*a*c - b^2) + 4*A*c^2*d*(b^
2 - 4*a*c)^(1/2) + A*b^2*f*(b^2 - 4*a*c)^(1/2) - 2*B*a*b*f*(b^2 - 4*a*c)^(1
/2) - 2*B*b*c*d*(b^2 - 4*a*c)^(1/2))*(2*b*c^3*d^3 + 4*c^4*d^3*x - a^2*b^2*f
^3*x + 4*a*b^3*d*f^2 - 2*b^3*c*d^2*f + 4*a^3*c*f^3*x + 3*b^4*d*f^2*x + 12*a
*b*c^2*d^2*f - 14*a^2*b*c*d*f^2 - 4*a*c^3*d^2*f*x - 4*a^2*c^2*d*f^2*x + 3*b
```

```
^2*c^2*d^2*f*x - 10*a*b^2*c*d*f^2*x))/(2*(4*a*c - b^2)*(a^2*f^2 + c^2*d^2 +
 b^2*d*f - 2*a*c*d*f))))/(4*(4*a*c - b^2)*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*
a*c*d*f)) - c*f^2*x*(2*B^2*c^2*d - 4*A^2*c^2*f - B^2*b^2*f + 2*B^2*a*c*f +
2*A*B*b*c*f) + A^2*b*c^2*f^3 + B^2*b*c^2*d*f^2 - 4*A*B*a*c^2*f^3 + A*B*b^2*
c*f^3 - 4*A*B*c^3*d*f^2)*(A*f*(b^2 - 4*a*c)^(3/2) - 2*A*b*f*(4*a*c - b^2) +
 2*B*a*f*(4*a*c - b^2) - 2*B*c*d*(4*a*c - b^2) + 4*A*c^2*d*(b^2 - 4*a*c)^(1
/2) + A*b^2*f*(b^2 - 4*a*c)^(1/2) - 2*B*a*b*f*(b^2 - 4*a*c)^(1/2) - 2*B*b*c
*d*(b^2 - 4*a*c)^(1/2)))/(4*(4*a*c - b^2)*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*
a*c*d*f)))*(A*f*(b^2 - 4*a*c)^(3/2) - 2*A*b*f*(4*a*c - b^2) + 2*B*a*f*(4*a*
c - b^2) - 2*B*c*d*(4*a*c - b^2) + 4*A*c^2*d*(b^2 - 4*a*c)^(1/2) + A*b^2*f*
(b^2 - 4*a*c)^(1/2) - 2*B*a*b*f*(b^2 - 4*a*c)^(1/2) - 2*B*b*c*d*(b^2 - 4*a*
c)^(1/2)))/(b^2*(4*a^2*f^2 + 4*c^2*d^2 + 4*b^2*d*f - 24*a*c*d*f) - 4*a*c*(4
*a^2*f^2 + 4*c^2*d^2 - 8*a*c*d*f))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)/(f*x**2+d),x)
```

```
[Out] Timed out
```

## 3.5 $\int \frac{A+Bx}{\left(a+bx+cx^2\right)^2\left(d+fx^2\right)} \, dx$

**Optimal**. Leaf size=596

$$-\frac{f^{3/2}\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{d}}\right)\left(-A(cd-af)^2+2bBd(cd-af)+Ab^2df\right)}{\sqrt{d}\,\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)^2}-\frac{f\log\left(a+bx+cx^2\right)\left(B\left(-f\left(b^2d-a^2f\right)-2acdf+\right.\right.}{2\left(f\left(a^2f+b^2d\right)-2acdf+c^2\right.}$$

**Rubi [A]** time = 1.77, antiderivative size = 596, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1018, 1074, 634, 618, 206, 628, 635, 205, 260}

$$-\frac{f^{3/2}\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{d}}\right)\left(-A(cd-af)^2+2bBd(cd-af)+Ab^2df\right)}{\sqrt{d}\,\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)^2}-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-4Ab^2cf\left(3a^2f^2-3acdf+2c^2d^2\right)+b^3Bf\left(-a^2f^2-4acdf+5c^2d^2\right)+2bBc\left(3a^2cdf^2+3a^3f^3-7ac^2d^2f+c^3d^3\right)-2Ab^4f^2(cd-af)-4Ac^2(cd-3af)(cd-af)^2+b^5Bdf^2\right)}{\left(b^2-4ac\right)^{3/2}\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)^2}-\frac{f\log\left(a+bx+cx^2\right)\left(B\left(-f\left(b^2d-a^2f\right)-2acdf+c^2d^2\right)+2Abf(cd-af)\right)}{2\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)^2}+\frac{f\log\left(d+fx^2\right)\left(B\left(-f\left(b^2d-a^2f\right)-2acdf+c^2d^2\right)+2Abf(cd-af)\right)}{2\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)^2}+\frac{-(Ab-aB)\left(-2acf+b^2f+2c^2d\right)-cx\left(2Ac(cd-af)-bB(af+cd)+Ab^2f\right)+Abc(af+cd)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)\left((cd-af)^2+b^2df\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/((a + b*x + c*x^2)^2*(d + f*x^2)),x]
```

```
[Out] (A*b*c*(c*d + a*f) - (A*b - a*B)*(2*c^2*d + b^2*f - 2*a*c*f) - c*(A*b^2*f +
 2*A*c*(c*d - a*f) - b*B*(c*d + a*f))*x)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a
*f)^2)*(a + b*x + c*x^2)) - (f^(3/2)*(A*b^2*d*f + 2*b*B*d*(c*d - a*f) - A*(
c*d - a*f)^2)*ArcTan[(Sqrt[f]*x)/Sqrt[d]])/(Sqrt[d]*(c^2*d^2 - 2*a*c*d*f +
f*(b^2*d + a^2*f))^2) - ((b^5*B*d*f^2 - 2*A*b^4*f^2*(c*d - a*f) - 4*A*c^2*(
c*d - 3*a*f)*(c*d - a*f)^2 + b^3*B*f*(5*c^2*d^2 - 4*a*c*d*f - a^2*f^2) - 4*
A*b^2*c*f*(2*c^2*d^2 - 3*a*c*d*f + 3*a^2*f^2) + 2*b*B*c*(c^3*d^3 - 7*a*c^2*
d^2*f + 3*a^2*c*d*f^2 + 3*a^3*f^3))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])
/((b^2 - 4*a*c)^(3/2)*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))^2) - (f*(2*
A*b*f*(c*d - a*f) + B*(c^2*d^2 - 2*a*c*d*f - f*(b^2*d - a^2*f)))*Log[a + b*
x + c*x^2])/(2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))^2) + (f*(2*A*b*f*(
c*d - a*f) + B*(c^2*d^2 - 2*a*c*d*f - f*(b^2*d - a^2*f)))*Log[d + f*x^2])/(
2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1018

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^
(q + 1)*((g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f
)) + c*(g*(2*c^2*d + b^2*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x))/((b^2 - 4*
a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1074

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 + b^2*d*f - 2*a*c*d
*f + a^2*f^2}, Dist[1/q, Int[(A*c^2*d - a*c*C*d + A*b^2*f - a*b*B*f - a*A*c
*f + a^2*C*f + c*(B*c*d - b*C*d + A*b*f - a*B*f)*x)/(a + b*x + c*x^2), x],
x] + Dist[1/q, Int[(c*C*d^2 + b*B*d*f - A*c*d*f - a*C*d*f + a*A*f^2 - f*(B*
c*d - b*C*d + A*b*f - a*B*f)*x)/(d + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[
{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{A+Bx}{\left(a+bx+cx^2\right)^2\left(d+fx^2\right)}\,dx = \frac{Abc(cd+af)-(Ab-aB)\left(2c^2d+b^2f-2acf\right)-c\left(Ab^2f+2Ac(cd-af)-bI\right.}{\left(b^2-4ac\right)\left(b^2df+(cd-af)^2\right)\left(a+bx+cx^2\right)}$$

$$= \frac{Abc(cd+af)-(Ab-aB)\left(2c^2d+b^2f-2acf\right)-c\left(Ab^2f+2Ac(cd-af)-bI\right.}{\left(b^2-4ac\right)\left(b^2df+(cd-af)^2\right)\left(a+bx+cx^2\right)}$$

$$= \frac{Abc(cd+af)-(Ab-aB)\left(2c^2d+b^2f-2acf\right)-c\left(Ab^2f+2Ac(cd-af)-bI\right.}{\left(b^2-4ac\right)\left(b^2df+(cd-af)^2\right)\left(a+bx+cx^2\right)}$$

$$= \frac{Abc(cd+af)-(Ab-aB)\left(2c^2d+b^2f-2acf\right)-c\left(Ab^2f+2Ac(cd-af)-bI\right.}{\left(b^2-4ac\right)\left(b^2df+(cd-af)^2\right)\left(a+bx+cx^2\right)}$$

$$= \frac{Abc(cd+af)-(Ab-aB)\left(2c^2d+b^2f-2acf\right)-c\left(Ab^2f+2Ac(cd-af)-bI\right.}{\left(b^2-4ac\right)\left(b^2df+(cd-af)^2\right)\left(a+bx+cx^2\right)}$$

**Mathematica [A]** time = 1.84, size = 523, normalized size = 0.88

$$\frac{f\log\left(d+fx^2\right)\left(B\left(f\left(a^2f-b^2d\right)-2acdf+c^2d^2\right)+2Abf(cd-af)\right)+f\log(a+x(b+cx))\left(B\left(f\left(b^2d-a^2f\right)+2acdf-c^2d^2\right)+2Abf(af-cd)\right)-\frac{2\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)\left(B\left(2a^2cf-a\left(b^2f+bcfx+2c^2d\right)-bc^2dx\right)+A\left(bc(cd-3af)+2c^2x(cd-af)+b^3f+b^2cfx\right)\right)}{\left(b^2-4ac\right)(a+x(b+cx))}-\frac{2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)\left(-4Ab^2cf\left(3a^2f^2-3acdf+2c^2d^2\right)-b^3Bf\left(a^2f^2+4acdf-5c^2d^2\right)+2bBc\left(3a^3f^3+3a^2cdf^2-7ac^2d^2f+c^3d^3\right)+2Ab^4f^2(af-cd)-4Ac^2(cd-3af)(cd-af)^2+b^5Bdf^2\right)}{\left(4ac-b^2\right)^{3/2}}+\frac{2f^{3/2}\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)\left(A(cd-af)^2+2bBd(af-cd)-Ab^2df\right)}{\sqrt{d}}}{2\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/((a + b*x + c*x^2)^2*(d + f*x^2)),x]
```

```
[Out] ((-2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))*(A*(b^3*f + b*c*(c*d - 3*a*f
) + b^2*c*f*x + 2*c^2*(c*d - a*f)*x) + B*(2*a^2*c*f - b*c^2*d*x - a*(2*c^2*
d + b^2*f + b*c*f*x))))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*f^(3/2)*(-(A
```

```
*b^2*d*f) + A*(c*d - a*f)^2 + 2*b*B*d*(-(c*d) + a*f))*ArcTan[(Sqrt[f]*x)/Sq
rt[d]])/Sqrt[d] - (2*(b^5*B*d*f^2 - 4*A*c^2*(c*d - 3*a*f)*(c*d - a*f)^2 + 2
*A*b^4*f^2*(-(c*d) + a*f) - b^3*B*f*(-5*c^2*d^2 + 4*a*c*d*f + a^2*f^2) - 4*
A*b^2*c*f*(2*c^2*d^2 - 3*a*c*d*f + 3*a^2*f^2) + 2*b*B*c*(c^3*d^3 - 7*a*c^2*
d^2*f + 3*a^2*c*d*f^2 + 3*a^3*f^3))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])
/(-b^2 + 4*a*c)^(3/2) + f*(2*A*b*f*(c*d - a*f) + B*(c^2*d^2 - 2*a*c*d*f + f
*(-(b^2*d) + a^2*f)))*Log[d + f*x^2] + f*(2*A*b*f*(-(c*d) + a*f) + B*(-(c^2
*d^2) + 2*a*c*d*f + f*(b^2*d - a^2*f)))*Log[a + x*(b + c*x)])/(2*(c^2*d^2 -
 2*a*c*d*f + f*(b^2*d + a^2*f))^2)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx}{\left(a+bx+cx^2\right)^2\left(d+fx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x)/((a + b*x + c*x^2)^2*(d + f*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x)/((a + b*x + c*x^2)^2*(d + f*x^2)), x]
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [B]** time = 0.21, size = 1313, normalized size = 2.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="giac")
```

```
[Out] -1/2*(B*c^2*d^2*f - B*b^2*d*f^2 - 2*B*a*c*d*f^2 + 2*A*b*c*d*f^2 + B*a^2*f^3
 - 2*A*a*b*f^3)*log(c*x^2 + b*x + a)/(c^4*d^4 + 2*b^2*c^2*d^3*f - 4*a*c^3*d
^3*f + b^4*d^2*f^2 - 4*a*b^2*c*d^2*f^2 + 6*a^2*c^2*d^2*f^2 + 2*a^2*b^2*d*f^
3 - 4*a^3*c*d*f^3 + a^4*f^4) + 1/2*(B*c^2*d^2*f - B*b^2*d*f^2 - 2*B*a*c*d*f
^2 + 2*A*b*c*d*f^2 + B*a^2*f^3 - 2*A*a*b*f^3)*log(f*x^2 + d)/(c^4*d^4 + 2*b
^2*c^2*d^3*f - 4*a*c^3*d^3*f + b^4*d^2*f^2 - 4*a*b^2*c*d^2*f^2 + 6*a^2*c^2*
d^2*f^2 + 2*a^2*b^2*d*f^3 - 4*a^3*c*d*f^3 + a^4*f^4) - (2*B*b*c*d^2*f^2 - A
*c^2*d^2*f^2 - 2*B*a*b*d*f^3 + A*b^2*d*f^3 + 2*A*a*c*d*f^3 - A*a^2*f^4)*arc
tan(f*x/sqrt(d*f))/((c^4*d^4 + 2*b^2*c^2*d^3*f - 4*a*c^3*d^3*f + b^4*d^2*f^
```

```
2 - 4*a*b^2*c*d^2*f^2 + 6*a^2*c^2*d^2*f^2 + 2*a^2*b^2*d*f^3 - 4*a^3*c*d*f^3
 + a^4*f^4)*sqrt(d*f)) + (2*B*b*c^4*d^3 - 4*A*c^5*d^3 + 5*B*b^3*c^2*d^2*f -
 14*B*a*b*c^3*d^2*f - 8*A*b^2*c^3*d^2*f + 20*A*a*c^4*d^2*f + B*b^5*d*f^2 -
4*B*a*b^3*c*d*f^2 - 2*A*b^4*c*d*f^2 + 6*B*a^2*b*c^2*d*f^2 + 12*A*a*b^2*c^2*
d*f^2 - 28*A*a^2*c^3*d*f^2 - B*a^2*b^3*f^3 + 2*A*a*b^4*f^3 + 6*B*a^3*b*c*f^
3 - 12*A*a^2*b^2*c*f^3 + 12*A*a^3*c^2*f^3)*arctan((2*c*x + b)/sqrt(-b^2 + 4
*a*c))/((b^2*c^4*d^4 - 4*a*c^5*d^4 + 2*b^4*c^2*d^3*f - 12*a*b^2*c^3*d^3*f +
 16*a^2*c^4*d^3*f + b^6*d^2*f^2 - 8*a*b^4*c*d^2*f^2 + 22*a^2*b^2*c^2*d^2*f^
2 - 24*a^3*c^3*d^2*f^2 + 2*a^2*b^4*d*f^3 - 12*a^3*b^2*c*d*f^3 + 16*a^4*c^2*
d*f^3 + a^4*b^2*f^4 - 4*a^5*c*f^4)*sqrt(-b^2 + 4*a*c)) + (2*B*a*c^4*d^3 - A
*b*c^4*d^3 + 3*B*a*b^2*c^2*d^2*f - 2*A*b^3*c^2*d^2*f - 6*B*a^2*c^3*d^2*f +
5*A*a*b*c^3*d^2*f + B*a*b^4*d*f^2 - A*b^5*d*f^2 - 4*B*a^2*b^2*c*d*f^2 + 5*A
*a*b^3*c*d*f^2 + 6*B*a^3*c^2*d*f^2 - 7*A*a^2*b*c^2*d*f^2 + B*a^3*b^2*f^3 -
A*a^2*b^3*f^3 - 2*B*a^4*c*f^3 + 3*A*a^3*b*c*f^3 + (B*b*c^4*d^3 - 2*A*c^5*d^
3 + B*b^3*c^2*d^2*f - B*a*b*c^3*d^2*f - 3*A*b^2*c^3*d^2*f + 6*A*a*c^4*d^2*f
 + B*a*b^3*c*d*f^2 - A*b^4*c*d*f^2 - B*a^2*b*c^2*d*f^2 + 4*A*a*b^2*c^2*d*f^
2 - 6*A*a^2*c^3*d*f^2 + B*a^3*b*c*f^3 - A*a^2*b^2*c*f^3 + 2*A*a^3*c^2*f^3)*
x)/((c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2)^2*(c*x^2 + b*x + a)*(b^2 - 4*
a*c))
```

**maple** [B] time = 0.06, size = 9311, normalized size = 15.62

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+d),x)
```

```
[Out] result too large to display
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

**mupad** [B] time = 7.53, size = 23006, normalized size = 38.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/((d + f*x^2)*(a + b*x + c*x^2)^2),x)

[Out] ((A*b^3*f + A*b*c^2*d - 2*B*a*c^2*d - B*a*b^2*f + 2*B*a^2*c*f - 3*A*a*b*c*f
)/((4*a*c - b^2)*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f)) - (x*(2*A*a*c^2
*f - 2*A*c^3*d + B*b*c^2*d - A*b^2*c*f + B*a*b*c*f))/((4*a*c - b^2)*(a^2*f^
2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f)))/(a + b*x + c*x^2) + symsum(log((x*(4*A
^3*b^3*c^4*f^6 + 16*B^3*a^3*c^4*f^6 - 3*B^3*a^2*b^2*c^3*f^6 + B^3*b^2*c^5*d
^2*f^4 - 16*A^3*a*b*c^5*f^6 + 20*A^2*B*a^2*c^5*f^6 - 3*A^2*B*b^4*c^3*f^6 +
4*A^2*B*c^7*d^2*f^4 - 16*B^3*a^2*c^5*d*f^5 + 6*B^3*a*b^2*c^4*d*f^5 - 24*A^2
*B*a*c^6*d*f^5 + 6*A*B^2*a*b^3*c^3*f^6 - 28*A*B^2*a^2*b*c^4*f^6 + 8*A^2*B*a
*b^2*c^4*f^6 - 4*A*B^2*b*c^6*d^2*f^4 - 6*A*B^2*b^3*c^4*d*f^5 + 8*A^2*B*b^2*
c^5*d*f^5 + 16*A*B^2*a*b*c^5*d*f^5))/(16*a^2*c^6*d^4 + a^4*b^4*f^4 + b^4*c^
4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 +
2*a^2*b^6*d*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 + 2*b^6*c^2*d^3*f + 9
6*a^4*c^4*d^2*f^2 + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 - 20*a
*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 + 64*a^2*b^2*c^4*d
^3*f + 64*a^4*b^2*c^2*d*f^3) - root(2560*a^3*b^2*c^9*d^8*f*z^4 - 1152*a^2*b
^4*c^8*d^8*f*z^4 + 384*a^5*b^8*c*d^3*f^6*z^4 + 384*a*b^8*c^5*d^7*f^2*z^4 +
288*a^3*b^10*c*d^4*f^5*z^4 + 288*a*b^10*c^3*d^6*f^3*z^4 + 224*a^7*b^6*c*d^2
*f^7*z^4 - 192*a^10*b^2*c^2*d*f^8*z^4 + 224*a*b^6*c^7*d^8*f*z^4 + 80*a*b^12
*c*d^5*f^4*z^4 + 48*a^9*b^4*c*d*f^8*z^4 - 33920*a^6*b^2*c^6*d^5*f^4*z^4 + 2
7936*a^5*b^4*c^5*d^5*f^4*z^4 + 26112*a^7*b^2*c^5*d^4*f^5*z^4 + 26112*a^5*b^
2*c^7*d^6*f^3*z^4 - 20352*a^6*b^4*c^4*d^4*f^5*z^4 - 20352*a^4*b^4*c^6*d^6*f
^3*z^4 - 13080*a^4*b^6*c^4*d^5*f^4*z^4 - 11520*a^8*b^2*c^4*d^3*f^6*z^4 - 11
520*a^4*b^2*c^8*d^7*f^2*z^4 + 8736*a^5*b^6*c^3*d^4*f^5*z^4 + 8736*a^3*b^6*c
^5*d^6*f^3*z^4 + 7488*a^7*b^4*c^3*d^3*f^6*z^4 + 7488*a^3*b^4*c^7*d^7*f^2*z^
4 + 3840*a^3*b^8*c^3*d^5*f^4*z^4 + 2560*a^9*b^2*c^3*d^2*f^7*z^4 - 2416*a^6*
b^6*c^2*d^3*f^6*z^4 - 2416*a^2*b^6*c^6*d^7*f^2*z^4 - 2160*a^4*b^8*c^2*d^4*f
^5*z^4 - 2160*a^2*b^8*c^4*d^6*f^3*z^4 - 1152*a^8*b^4*c^2*d^2*f^7*z^4 - 720*
a^2*b^10*c^2*d^5*f^4*z^4 - 16*b^8*c^6*d^8*f*z^4 - 2048*a^4*c^10*d^8*f*z^4 +
 256*a^11*c^3*d*f^8*z^4 - 4*a^8*b^6*d*f^8*z^4 + 48*a*b^4*c^9*d^9*z^4 - 24*b
^10*c^4*d^7*f^2*z^4 - 16*b^12*c^2*d^6*f^3*z^4 + 17920*a^7*c^7*d^5*f^4*z^4 -
 14336*a^8*c^6*d^4*f^5*z^4 - 14336*a^6*c^8*d^6*f^3*z^4 + 7168*a^9*c^5*d^3*f
^6*z^4 + 7168*a^5*c^9*d^7*f^2*z^4 - 2048*a^10*c^4*d^2*f^7*z^4 - 24*a^4*b^10
*d^3*f^6*z^4 - 16*a^6*b^8*d^2*f^7*z^4 - 16*a^2*b^12*d^4*f^5*z^4 - 192*a^2*b
^2*c^10*d^9*z^4 - 4*b^14*d^5*f^4*z^4 - 4*b^6*c^8*d^9*z^4 + 256*a^3*c^11*d^9
*z^4 + 912*A*B*a^6*b*c^3*d*f^6*z^2 + 192*A*B*a^4*b^5*c*d*f^6*z^2 + 920*A*B*
a^4*b^3*c^3*d^2*f^5*z^2 - 480*A*B*a^2*b^5*c^3*d^3*f^4*z^2 - 336*A*B*a^2*b^3
*c^5*d^4*f^3*z^2 - 272*A*B*a^3*b^3*c^4*d^3*f^4*z^2 + 240*A*B*a^3*b^5*c^2*d^
2*f^5*z^2 + 192*A*B*a*b*c^8*d^6*f*z^2 - 2496*A*B*a^5*b*c^4*d^2*f^5*z^2 + 18
72*A*B*a^4*b*c^5*d^3*f^4*z^2 - 744*A*B*a^5*b^3*c^2*d*f^6*z^2 - 720*A*B*a^2*
b*c^7*d^5*f^2*z^2 + 504*A*B*a*b^3*c^6*d^5*f^2*z^2 + 256*A*B*a^3*b*c^6*d^4*f
^3*z^2 + 168*A*B*a*b^7*c^2*d^3*f^4*z^2 - 144*A*B*a^2*b^7*c*d^2*f^5*z^2 + 14
4*A*B*a*b^5*c^4*d^4*f^3*z^2 - 56*B^2*a*b^2*c^7*d^6*f*z^2 - 36*B^2*a^5*b^4*c
*d*f^6*z^2 - 16*B^2*a*b^8*c*d^3*f^4*z^2 - 164*A^2*a^3*b^6*c*d*f^6*z^2 - 16*
```

A^2*a*b^8*c*d^2*f^5*z^2 - 96*A*B*b^5*c^5*d^5*f^2*z^2 - 24*A*B*b^7*c^3*d^4*f
^3*z^2 - 580*B^2*a^4*b^2*c^4*d^3*f^4*z^2 + 536*B^2*a^3*b^4*c^3*d^3*f^4*z^2
- 348*B^2*a^4*b^4*c^2*d^2*f^5*z^2 + 316*B^2*a^2*b^2*c^6*d^5*f^2*z^2 + 200*B
^2*a^5*b^2*c^3*d^2*f^5*z^2 - 120*B^2*a^2*b^4*c^4*d^4*f^3*z^2 - 66*B^2*a^2*b
^6*c^2*d^3*f^4*z^2 - 16*B^2*a^3*b^2*c^5*d^4*f^3*z^2 + 1952*A^2*a^4*b^2*c^4*
d^2*f^5*z^2 - 1792*A^2*a^3*b^2*c^5*d^3*f^4*z^2 - 1272*A^2*a^3*b^4*c^3*d^2*f
^5*z^2 + 976*A^2*a^2*b^2*c^6*d^4*f^3*z^2 + 960*A^2*a^2*b^4*c^4*d^3*f^4*z^2
+ 282*A^2*a^2*b^6*c^2*d^2*f^5*z^2 - 72*A*B*b^3*c^7*d^6*f*z^2 - 16*A*B*b^9*c
*d^3*f^4*z^2 - 16*A*B*a^3*b^7*d*f^6*z^2 + 16*A*B*a*b^9*d^2*f^5*z^2 - 180*B^
2*a*b^4*c^5*d^5*f^2*z^2 + 132*B^2*a^6*b^2*c^2*d*f^6*z^2 + 108*B^2*a^3*b^6*c
*d^2*f^5*z^2 + 20*B^2*a*b^6*c^3*d^4*f^3*z^2 - 736*A^2*a^5*b^2*c^3*d*f^6*z^2
+ 624*A^2*a^4*b^4*c^2*d*f^6*z^2 - 416*A^2*a*b^2*c^7*d^5*f^2*z^2 - 276*A^2*
a*b^4*c^5*d^4*f^3*z^2 - 196*A^2*a*b^6*c^3*d^3*f^4*z^2 + 31*B^2*b^6*c^4*d^5*
f^2*z^2 + 2*B^2*b^8*c^2*d^4*f^3*z^2 - 768*B^2*a^5*c^5*d^3*f^4*z^2 + 512*B^2
*a^6*c^4*d^2*f^5*z^2 + 512*B^2*a^4*c^6*d^4*f^3*z^2 - 128*B^2*a^3*c^7*d^5*f^
2*z^2 + 80*A^2*b^4*c^6*d^5*f^2*z^2 + 31*A^2*b^6*c^4*d^4*f^3*z^2 + 14*A^2*b^
8*c^2*d^3*f^4*z^2 - 1152*A^2*a^3*c^7*d^4*f^3*z^2 + 1008*A^2*a^4*c^6*d^3*f^4
*z^2 + 624*A^2*a^2*c^8*d^5*f^2*z^2 - 288*A^2*a^5*c^5*d^2*f^5*z^2 - 10*B^2*a
^2*b^8*d^2*f^5*z^2 - 48*A^2*a^6*b^2*c^2*f^7*z^2 - 16*A*B*b*c^9*d^7*z^2 + 20
*B^2*b^4*c^6*d^6*f*z^2 - 128*B^2*a^7*c^3*d*f^6*z^2 + 64*A^2*b^2*c^8*d^6*f*z
^2 - 112*A^2*a^6*c^4*d*f^6*z^2 + 3*B^2*a^4*b^6*d*f^6*z^2 + 14*A^2*a^2*b^8*d
*f^6*z^2 + 12*A^2*a^5*b^4*c*f^7*z^2 - 160*A^2*a*c^9*d^6*f*z^2 + 3*B^2*b^10*
d^3*f^4*z^2 - A^2*b^10*d^2*f^5*z^2 + 64*A^2*a^7*c^3*f^7*z^2 + 4*B^2*b^2*c^8
*d^7*z^2 - A^2*a^4*b^6*f^7*z^2 + 16*A^2*c^10*d^7*z^2 - 160*A*B^2*a*b*c^6*d^
4*f^2*z + 112*A*B^2*a^4*b*c^3*d*f^5*z - 24*A*B^2*a^2*b^5*c*d*f^5*z + 480*A^
2*B*a^2*b^2*c^4*d^2*f^4*z - 176*A*B^2*a^2*b^3*c^3*d^2*f^4*z - 10*A^2*B*a*b^
6*c*d*f^5*z + 384*A*B^2*a^2*b*c^5*d^3*f^3*z - 352*A*B^2*a^3*b*c^4*d^2*f^4*z
- 288*A^2*B*a*b^2*c^5*d^3*f^3*z - 160*A^2*B*a^3*b^2*c^3*d*f^5*z - 148*A^2*
B*a*b^4*c^3*d^2*f^4*z + 112*A*B^2*a*b^3*c^4*d^3*f^3*z + 72*A^2*B*a^2*b^4*c^
2*d*f^5*z + 72*A*B^2*a*b^5*c^2*d^2*f^4*z + 48*A*B^2*a^3*b^3*c^2*d*f^5*z + 4
8*B^3*a*b^2*c^5*d^4*f^2*z - 36*B^3*a^4*b^2*c^2*d*f^5*z - 4*B^3*a*b^4*c^3*d^
3*f^3*z - 480*A^3*a^2*b*c^5*d^2*f^4*z - 160*A^3*a^2*b^3*c^3*d*f^5*z + 128*A
^3*a*b^3*c^4*d^2*f^4*z + 112*A^2*B*b^4*c^4*d^3*f^3*z - 64*A*B^2*b^5*c^3*d^3
*f^3*z + 16*A^2*B*b^2*c^6*d^4*f^2*z + 16*A*B^2*b^3*c^5*d^4*f^2*z - A^2*B*b^
6*c^2*d^2*f^4*z + 448*A^2*B*a^3*c^5*d^2*f^4*z - 352*A^2*B*a^2*c^6*d^3*f^3*z
- 48*A^2*B*a^4*b^2*c^2*f^6*z + 12*B^3*a^3*b^4*c*d*f^5*z - 10*B^3*a*b^6*c*d
^2*f^4*z + 416*A^3*a^3*b*c^4*d*f^5*z + 224*A^3*a*b*c^6*d^3*f^3*z + 24*A^3*a
*b^5*c^2*d*f^5*z - 2*A*B^2*b^7*c*d^2*f^4*z - 272*A^2*B*a^4*c^4*d*f^5*z + 12
8*A^2*B*a*c^7*d^4*f^2*z + 12*A^2*B*a^3*b^4*c*f^6*z - 120*B^3*a^2*b^2*c^4*d^
3*f^3*z + 112*B^3*a^3*b^2*c^3*d^2*f^4*z + 16*A*B^2*b*c^7*d^5*f*z + 2*A*B^2*
a*b^7*d*f^5*z - 2*A^3*b^7*c*d*f^5*z - 16*A^2*B*c^8*d^5*f*z + 11*B^3*b^6*c^2
*d^3*f^3*z - 8*B^3*b^4*c^4*d^4*f^2*z - 64*A^3*b^3*c^5*d^3*f^3*z + 96*A^3*a^
3*b^3*c^2*f^6*z - 4*B^3*b^2*c^6*d^5*f*z - 32*A^3*b*c^7*d^4*f^2*z - B^3*a^2*
b^6*d*f^5*z - 128*A^3*a^4*b*c^3*f^6*z - 24*A^3*a^2*b^5*c*f^6*z + 64*A^2*B*a
^5*c^3*f^6*z - A^2*B*a^2*b^6*f^6*z + A^2*B*b^8*d*f^5*z + 2*A^3*a*b^7*f^6*z

+ B^3*b^8*d^2*f^4*z + 32*A^3*B*a*b*c^4*d*f^4 - 18*A^2*B^2*a*b^2*c^3*d*f^4 + 32*A*B^3*a*b*c^4*d^2*f^3 - 28*A*B^3*a^2*b*c^3*d*f^4 + 6*A*B^3*a*b^3*c^2*d*f^4 - 10*A^3*B*b^3*c^3*d*f^4 - 4*A^3*B*b*c^5*d^2*f^3 - 4*A*B^3*b*c^5*d^3*f^2 - 28*A^3*B*a^2*b*c^3*f^5 + 6*A^3*B*a*b^3*c^2*f^5 + 9*A^2*B^2*b^2*c^4*d^2*f^3 - 3*A^2*B^2*a^2*b^2*c^2*f^5 - 10*B^4*a*b^2*c^3*d^2*f^3 - 3*B^4*a^2*b^2*c^2*d*f^4 - 10*A*B^3*b^3*c^3*d^2*f^3 + 3*A^2*B^2*b^4*c^2*d*f^4 + 36*A^2*B^2*a^2*c^4*d*f^4 - 24*A^2*B^2*a*c^5*d^2*f^3 + 4*A^2*B^2*c^6*d^3*f^2 + 16*A^2*B^2*a^3*c^3*f^5 + 16*B^4*a^3*c^3*d*f^4 + 8*A^4*b^2*c^4*d*f^4 - 8*A^4*a*b^2*c^3*f^5 - 24*A^4*a*c^5*d*f^4 + 3*B^4*b^4*c^2*d^2*f^3 + 4*A^4*c^6*d^2*f^3 + 36*A^4*a^2*c^4*f^5 + B^4*b^2*c^4*d^3*f^2, z, k)*(root(2560*a^3*b^2*c^9*d^8*f*z^4 - 1152*a^2*b^4*c^8*d^8*f*z^4 + 384*a^5*b^8*c*d^3*f^6*z^4 + 384*a*b^8*c^5*d^7*f^2*z^4 + 288*a^3*b^10*c*d^4*f^5*z^4 + 288*a*b^10*c^3*d^6*f^3*z^4 + 224*a^7*b^6*c*d^2*f^7*z^4 - 192*a^10*b^2*c^2*d*f^8*z^4 + 224*a*b^6*c^7*d^8*f*z^4 + 80*a*b^12*c*d^5*f^4*z^4 + 48*a^9*b^4*c*d*f^8*z^4 - 33920*a^6*b^2*c^6*d^5*f^4*z^4 + 27936*a^5*b^4*c^5*d^5*f^4*z^4 + 26112*a^7*b^2*c^5*d^4*f^5*z^4 + 26112*a^5*b^2*c^7*d^6*f^3*z^4 - 20352*a^6*b^4*c^4*d^4*f^5*z^4 - 20352*a^4*b^4*c^6*d^6*f^3*z^4 - 13080*a^4*b^6*c^4*d^5*f^4*z^4 - 11520*a^8*b^2*c^4*d^3*f^6*z^4 - 11520*a^4*b^2*c^8*d^7*f^2*z^4 + 8736*a^5*b^6*c^3*d^4*f^5*z^4 + 8736*a^3*b^6*c^5*d^6*f^3*z^4 + 7488*a^7*b^4*c^3*d^3*f^6*z^4 + 7488*a^3*b^4*c^7*d^7*f^2*z^4 + 3840*a^3*b^8*c^3*d^5*f^4*z^4 + 2560*a^9*b^2*c^3*d^2*f^7*z^4 - 2416*a^6*b^6*c^2*d^3*f^6*z^4 - 2416*a^2*b^6*c^6*d^7*f^2*z^4 - 2160*a^4*b^8*c^2*d^4*f^5*z^4 - 2160*a^2*b^8*c^4*d^6*f^3*z^4 - 1152*a^8*b^4*c^2*d^2*f^7*z^4 - 720*a^2*b^10*c^2*d^5*f^4*z^4 - 16*b^8*c^6*d^8*f*z^4 - 2048*a^4*c^10*d^8*f*z^4 + 256*a^11*c^3*d*f^8*z^4 - 4*a^8*b^6*d*f^8*z^4 + 48*a*b^4*c^9*d^9*z^4 - 24*b^10*c^4*d^7*f^2*z^4 - 16*b^12*c^2*d^6*f^3*z^4 + 17920*a^7*c^7*d^5*f^4*z^4 - 14336*a^8*c^6*d^4*f^5*z^4 - 14336*a^6*c^8*d^6*f^3*z^4 + 7168*a^9*c^5*d^3*f^6*z^4 + 7168*a^5*c^9*d^7*f^2*z^4 - 2048*a^10*c^4*d^2*f^7*z^4 - 24*a^4*b^10*d^3*f^6*z^4 - 16*a^6*b^8*d^2*f^7*z^4 - 16*a^2*b^12*d^4*f^5*z^4 - 192*a^2*b^2*c^10*d^9*z^4 - 4*b^14*d^5*f^4*z^4 - 4*b^6*c^8*d^9*z^4 + 256*a^3*c^11*d^9*z^4 + 912*A*B*a^6*b*c^3*d*f^6*z^2 + 192*A*B*a^4*b^5*c*d*f^6*z^2 + 920*A*B*a^4*b^3*c^3*d^2*f^5*z^2 - 480*A*B*a^2*b^5*c^3*d^3*f^4*z^2 - 336*A*B*a^2*b^3*c^5*d^4*f^3*z^2 - 272*A*B*a^3*b^3*c^4*d^3*f^4*z^2 + 240*A*B*a^3*b^5*c^2*d^2*f^5*z^2 + 192*A*B*a*b*c^8*d^6*f*z^2 - 2496*A*B*a^5*b*c^4*d^2*f^5*z^2 + 1872*A*B*a^4*b*c^5*d^3*f^4*z^2 - 744*A*B*a^5*b^3*c^2*d*f^6*z^2 - 720*A*B*a^2*b*c^7*d^5*f^2*z^2 + 504*A*B*a*b^3*c^6*d^5*f^2*z^2 + 256*A*B*a^3*b*c^6*d^4*f^3*z^2 + 168*A*B*a*b^7*c^2*d^3*f^4*z^2 - 144*A*B*a^2*b^7*c*d^2*f^5*z^2 + 144*A*B*a*b^5*c^4*d^4*f^3*z^2 - 56*B^2*a*b^2*c^7*d^6*f*z^2 - 36*B^2*a^5*b^4*c*d*f^6*z^2 - 16*B^2*a*b^8*c*d^3*f^4*z^2 - 164*A^2*a^3*b^6*c*d*f^6*z^2 - 16*A^2*a*b^8*c*d^2*f^5*z^2 - 96*A*B*b^5*c^5*d^5*f^2*z^2 - 24*A*B*b^7*c^3*d^4*f^3*z^2 - 580*B^2*a^4*b^2*c^4*d^3*f^4*z^2 + 536*B^2*a^3*b^4*c^3*d^3*f^4*z^2 - 348*B^2*a^4*b^4*c^2*d^2*f^5*z^2 + 316*B^2*a^2*b^2*c^6*d^5*f^2*z^2 + 200*B^2*a^5*b^2*c^3*d^2*f^5*z^2 - 120*B^2*a^2*b^4*c^4*d^4*f^3*z^2 - 66*B^2*a^2*b^6*c^2*d^3*f^4*z^2 - 16*B^2*a^3*b^2*c^5*d^4*f^3*z^2 + 1952*A^2*a^4*b^2*c^4*d^2*f^5*z^2 - 1792*A^2*a^3*b^2*c^5*d^3*f^4*z^2 - 1272*A^2*a^3*b^4*c^3*d^2*f^5*z^2 + 976*A^2*a^2*b^2*c^6*d^4*f^3*z^2 + 960*A^2*a^2*b^

4*c^4*d^3*f^4*z^2 + 282*A^2*a^2*b^6*c^2*d^2*f^5*z^2 - 72*A*B*b^3*c^7*d^6*f*
z^2 - 16*A*B*b^9*c*d^3*f^4*z^2 - 16*A*B*a^3*b^7*d*f^6*z^2 + 16*A*B*a*b^9*d^
2*f^5*z^2 - 180*B^2*a*b^4*c^5*d^5*f^2*z^2 + 132*B^2*a^6*b^2*c^2*d*f^6*z^2 +
108*B^2*a^3*b^6*c*d^2*f^5*z^2 + 20*B^2*a*b^6*c^3*d^4*f^3*z^2 - 736*A^2*a^5
*b^2*c^3*d*f^6*z^2 + 624*A^2*a^4*b^4*c^2*d*f^6*z^2 - 416*A^2*a*b^2*c^7*d^5*
f^2*z^2 - 276*A^2*a*b^4*c^5*d^4*f^3*z^2 - 196*A^2*a*b^6*c^3*d^3*f^4*z^2 + 3
1*B^2*b^6*c^4*d^5*f^2*z^2 + 2*B^2*b^8*c^2*d^4*f^3*z^2 - 768*B^2*a^5*c^5*d^3
*f^4*z^2 + 512*B^2*a^6*c^4*d^2*f^5*z^2 + 512*B^2*a^4*c^6*d^4*f^3*z^2 - 128*
B^2*a^3*c^7*d^5*f^2*z^2 + 80*A^2*b^4*c^6*d^5*f^2*z^2 + 31*A^2*b^6*c^4*d^4*f
^3*z^2 + 14*A^2*b^8*c^2*d^3*f^4*z^2 - 1152*A^2*a^3*c^7*d^4*f^3*z^2 + 1008*A
^2*a^4*c^6*d^3*f^4*z^2 + 624*A^2*a^2*c^8*d^5*f^2*z^2 - 288*A^2*a^5*c^5*d^2*
f^5*z^2 - 10*B^2*a^2*b^8*d^2*f^5*z^2 - 48*A^2*a^6*b^2*c^2*f^7*z^2 - 16*A*B*
b*c^9*d^7*z^2 + 20*B^2*b^4*c^6*d^6*f*z^2 - 128*B^2*a^7*c^3*d*f^6*z^2 + 64*A
^2*b^2*c^8*d^6*f*z^2 - 112*A^2*a^6*c^4*d*f^6*z^2 + 3*B^2*a^4*b^6*d*f^6*z^2
+ 14*A^2*a^2*b^8*d*f^6*z^2 + 12*A^2*a^5*b^4*c*f^7*z^2 - 160*A^2*a*c^9*d^6*f
*z^2 + 3*B^2*b^10*d^3*f^4*z^2 - A^2*b^10*d^2*f^5*z^2 + 64*A^2*a^7*c^3*f^7*z
^2 + 4*B^2*b^2*c^8*d^7*z^2 - A^2*a^4*b^6*f^7*z^2 + 16*A^2*c^10*d^7*z^2 - 16
0*A*B^2*a*b*c^6*d^4*f^2*z + 112*A*B^2*a^4*b*c^3*d*f^5*z - 24*A*B^2*a^2*b^5*
c*d*f^5*z + 480*A^2*B*a^2*b^2*c^4*d^2*f^4*z - 176*A*B^2*a^2*b^3*c^3*d^2*f^4
*z - 10*A^2*B*a*b^6*c*d*f^5*z + 384*A*B^2*a^2*b*c^5*d^3*f^3*z - 352*A*B^2*a
^3*b*c^4*d^2*f^4*z - 288*A^2*B*a*b^2*c^5*d^3*f^3*z - 160*A^2*B*a^3*b^2*c^3*
d*f^5*z - 148*A^2*B*a*b^4*c^3*d^2*f^4*z + 112*A*B^2*a*b^3*c^4*d^3*f^3*z + 7
2*A^2*B*a^2*b^4*c^2*d*f^5*z + 72*A*B^2*a*b^5*c^2*d^2*f^4*z + 48*A*B^2*a^3*b
^3*c^2*d*f^5*z + 48*B^3*a*b^2*c^5*d^4*f^2*z - 36*B^3*a^4*b^2*c^2*d*f^5*z -
4*B^3*a*b^4*c^3*d^3*f^3*z - 480*A^3*a^2*b*c^5*d^2*f^4*z - 160*A^3*a^2*b^3*c
^3*d*f^5*z + 128*A^3*a*b^3*c^4*d^2*f^4*z + 112*A^2*B*b^4*c^4*d^3*f^3*z - 64
*A*B^2*b^5*c^3*d^3*f^3*z + 16*A^2*B*b^2*c^6*d^4*f^2*z + 16*A*B^2*b^3*c^5*d^
4*f^2*z - A^2*B*b^6*c^2*d^2*f^4*z + 448*A^2*B*a^3*c^5*d^2*f^4*z - 352*A^2*B
*a^2*c^6*d^3*f^3*z - 48*A^2*B*a^4*b^2*c^2*f^6*z + 12*B^3*a^3*b^4*c*d*f^5*z
- 10*B^3*a*b^6*c*d^2*f^4*z + 416*A^3*a^3*b*c^4*d*f^5*z + 224*A^3*a*b*c^6*d^
3*f^3*z + 24*A^3*a*b^5*c^2*d*f^5*z - 2*A*B^2*b^7*c*d^2*f^4*z - 272*A^2*B*a^
4*c^4*d*f^5*z + 128*A^2*B*a*c^7*d^4*f^2*z + 12*A^2*B*a^3*b^4*c*f^6*z - 120*
B^3*a^2*b^2*c^4*d^3*f^3*z + 112*B^3*a^3*b^2*c^3*d^2*f^4*z + 16*A*B^2*b*c^7*
d^5*f*z + 2*A*B^2*a*b^7*d*f^5*z - 2*A^3*b^7*c*d*f^5*z - 16*A^2*B*c^8*d^5*f*
z + 11*B^3*b^6*c^2*d^3*f^3*z - 8*B^3*b^4*c^4*d^4*f^2*z - 64*A^3*b^3*c^5*d^3
*f^3*z + 96*A^3*a^3*b^3*c^2*f^6*z - 4*B^3*b^2*c^6*d^5*f*z - 32*A^3*b*c^7*d^
4*f^2*z - B^3*a^2*b^6*d*f^5*z - 128*A^3*a^4*b*c^3*f^6*z - 24*A^3*a^2*b^5*c*
f^6*z + 64*A^2*B*a^5*c^3*f^6*z - A^2*B*a^2*b^6*f^6*z + A^2*B*b^8*d*f^5*z +
2*A^3*a*b^7*f^6*z + B^3*b^8*d^2*f^4*z + 32*A^3*B*a*b*c^4*d*f^4 - 18*A^2*B^2
*a*b^2*c^3*d*f^4 + 32*A*B^3*a*b*c^4*d^2*f^3 - 28*A*B^3*a^2*b*c^3*d*f^4 + 6*
A*B^3*a*b^3*c^2*d*f^4 - 10*A^3*B*b^3*c^3*d*f^4 - 4*A^3*B*b*c^5*d^2*f^3 - 4*
A*B^3*b*c^5*d^3*f^2 - 28*A^3*B*a^2*b*c^3*f^5 + 6*A^3*B*a*b^3*c^2*f^5 + 9*A^
2*B^2*b^2*c^4*d^2*f^3 - 3*A^2*B^2*a^2*b^2*c^2*f^5 - 10*B^4*a*b^2*c^3*d^2*f^
3 - 3*B^4*a^2*b^2*c^2*d*f^4 - 10*A*B^3*b^3*c^3*d^2*f^3 + 3*A^2*B^2*b^4*c^2*
d*f^4 + 36*A^2*B^2*a^2*c^4*d*f^4 - 24*A^2*B^2*a*c^5*d^2*f^3 + 4*A^2*B^2*c^6

*d^3*f^2 + 16*A^2*B^2*a^3*c^3*f^5 + 16*B^4*a^3*c^3*d*f^4 + 8*A^4*b^2*c^4*d*
f^4 - 8*A^4*a*b^2*c^3*f^5 - 24*A^4*a*c^5*d*f^4 + 3*B^4*b^4*c^2*d^2*f^3 + 4*
A^4*c^6*d^2*f^3 + 36*A^4*a^2*c^4*f^5 + B^4*b^2*c^4*d^3*f^2, z, k)*(root(256
0*a^3*b^2*c^9*d^8*f*z^4 - 1152*a^2*b^4*c^8*d^8*f*z^4 + 384*a^5*b^8*c*d^3*f^
6*z^4 + 384*a*b^8*c^5*d^7*f^2*z^4 + 288*a^3*b^10*c*d^4*f^5*z^4 + 288*a*b^10
*c^3*d^6*f^3*z^4 + 224*a^7*b^6*c*d^2*f^7*z^4 - 192*a^10*b^2*c^2*d*f^8*z^4 +
 224*a*b^6*c^7*d^8*f*z^4 + 80*a*b^12*c*d^5*f^4*z^4 + 48*a^9*b^4*c*d*f^8*z^4
 - 33920*a^6*b^2*c^6*d^5*f^4*z^4 + 27936*a^5*b^4*c^5*d^5*f^4*z^4 + 26112*a^
7*b^2*c^5*d^4*f^5*z^4 + 26112*a^5*b^2*c^7*d^6*f^3*z^4 - 20352*a^6*b^4*c^4*d
^4*f^5*z^4 - 20352*a^4*b^4*c^6*d^6*f^3*z^4 - 13080*a^4*b^6*c^4*d^5*f^4*z^4
- 11520*a^8*b^2*c^4*d^3*f^6*z^4 - 11520*a^4*b^2*c^8*d^7*f^2*z^4 + 8736*a^5*
b^6*c^3*d^4*f^5*z^4 + 8736*a^3*b^6*c^5*d^6*f^3*z^4 + 7488*a^7*b^4*c^3*d^3*f
^6*z^4 + 7488*a^3*b^4*c^7*d^7*f^2*z^4 + 3840*a^3*b^8*c^3*d^5*f^4*z^4 + 2560
*a^9*b^2*c^3*d^2*f^7*z^4 - 2416*a^6*b^6*c^2*d^3*f^6*z^4 - 2416*a^2*b^6*c^6*
d^7*f^2*z^4 - 2160*a^4*b^8*c^2*d^4*f^5*z^4 - 2160*a^2*b^8*c^4*d^6*f^3*z^4 -
 1152*a^8*b^4*c^2*d^2*f^7*z^4 - 720*a^2*b^10*c^2*d^5*f^4*z^4 - 16*b^8*c^6*d
^8*f*z^4 - 2048*a^4*c^10*d^8*f*z^4 + 256*a^11*c^3*d*f^8*z^4 - 4*a^8*b^6*d*f
^8*z^4 + 48*a*b^4*c^9*d^9*z^4 - 24*b^10*c^4*d^7*f^2*z^4 - 16*b^12*c^2*d^6*f
^3*z^4 + 17920*a^7*c^7*d^5*f^4*z^4 - 14336*a^8*c^6*d^4*f^5*z^4 - 14336*a^6*
c^8*d^6*f^3*z^4 + 7168*a^9*c^5*d^3*f^6*z^4 + 7168*a^5*c^9*d^7*f^2*z^4 - 204
8*a^10*c^4*d^2*f^7*z^4 - 24*a^4*b^10*d^3*f^6*z^4 - 16*a^6*b^8*d^2*f^7*z^4 -
 16*a^2*b^12*d^4*f^5*z^4 - 192*a^2*b^2*c^10*d^9*z^4 - 4*b^14*d^5*f^4*z^4 -
4*b^6*c^8*d^9*z^4 + 256*a^3*c^11*d^9*z^4 + 912*A*B*a^6*b*c^3*d*f^6*z^2 + 19
2*A*B*a^4*b^5*c*d*f^6*z^2 + 920*A*B*a^4*b^3*c^3*d^2*f^5*z^2 - 480*A*B*a^2*b
^5*c^3*d^3*f^4*z^2 - 336*A*B*a^2*b^3*c^5*d^4*f^3*z^2 - 272*A*B*a^3*b^3*c^4*
d^3*f^4*z^2 + 240*A*B*a^3*b^5*c^2*d^2*f^5*z^2 + 192*A*B*a*b*c^8*d^6*f*z^2 -
 2496*A*B*a^5*b*c^4*d^2*f^5*z^2 + 1872*A*B*a^4*b*c^5*d^3*f^4*z^2 - 744*A*B*
a^5*b^3*c^2*d*f^6*z^2 - 720*A*B*a^2*b*c^7*d^5*f^2*z^2 + 504*A*B*a*b^3*c^6*d
^5*f^2*z^2 + 256*A*B*a^3*b*c^6*d^4*f^3*z^2 + 168*A*B*a*b^7*c^2*d^3*f^4*z^2
- 144*A*B*a^2*b^7*c*d^2*f^5*z^2 + 144*A*B*a*b^5*c^4*d^4*f^3*z^2 - 56*B^2*a*
b^2*c^7*d^6*f*z^2 - 36*B^2*a^5*b^4*c*d*f^6*z^2 - 16*B^2*a*b^8*c*d^3*f^4*z^2
 - 164*A^2*a^3*b^6*c*d*f^6*z^2 - 16*A^2*a*b^8*c*d^2*f^5*z^2 - 96*A*B*b^5*c^
5*d^5*f^2*z^2 - 24*A*B*b^7*c^3*d^4*f^3*z^2 - 580*B^2*a^4*b^2*c^4*d^3*f^4*z^
2 + 536*B^2*a^3*b^4*c^3*d^3*f^4*z^2 - 348*B^2*a^4*b^4*c^2*d^2*f^5*z^2 + 316
*B^2*a^2*b^2*c^6*d^5*f^2*z^2 + 200*B^2*a^5*b^2*c^3*d^2*f^5*z^2 - 120*B^2*a^
2*b^4*c^4*d^4*f^3*z^2 - 66*B^2*a^2*b^6*c^2*d^3*f^4*z^2 - 16*B^2*a^3*b^2*c^5
*d^4*f^3*z^2 + 1952*A^2*a^4*b^2*c^4*d^2*f^5*z^2 - 1792*A^2*a^3*b^2*c^5*d^3*
f^4*z^2 - 1272*A^2*a^3*b^4*c^3*d^2*f^5*z^2 + 976*A^2*a^2*b^2*c^6*d^4*f^3*z^
2 + 960*A^2*a^2*b^4*c^4*d^3*f^4*z^2 + 282*A^2*a^2*b^6*c^2*d^2*f^5*z^2 - 72*
A*B*b^3*c^7*d^6*f*z^2 - 16*A*B*b^9*c*d^3*f^4*z^2 - 16*A*B*a^3*b^7*d*f^6*z^2
 + 16*A*B*a*b^9*d^2*f^5*z^2 - 180*B^2*a*b^4*c^5*d^5*f^2*z^2 + 132*B^2*a^6*b
^2*c^2*d*f^6*z^2 + 108*B^2*a^3*b^6*c*d^2*f^5*z^2 + 20*B^2*a*b^6*c^3*d^4*f^3
*z^2 - 736*A^2*a^5*b^2*c^3*d*f^6*z^2 + 624*A^2*a^4*b^4*c^2*d*f^6*z^2 - 416*
A^2*a*b^2*c^7*d^5*f^2*z^2 - 276*A^2*a*b^4*c^5*d^4*f^3*z^2 - 196*A^2*a*b^6*c
^3*d^3*f^4*z^2 + 31*B^2*b^6*c^4*d^5*f^2*z^2 + 2*B^2*b^8*c^2*d^4*f^3*z^2 - 7

68*B^2*a^5*c^5*d^3*f^4*z^2 + 512*B^2*a^6*c^4*d^2*f^5*z^2 + 512*B^2*a^4*c^6*
d^4*f^3*z^2 - 128*B^2*a^3*c^7*d^5*f^2*z^2 + 80*A^2*b^4*c^6*d^5*f^2*z^2 + 31
*A^2*b^6*c^4*d^4*f^3*z^2 + 14*A^2*b^8*c^2*d^3*f^4*z^2 - 1152*A^2*a^3*c^7*d^
4*f^3*z^2 + 1008*A^2*a^4*c^6*d^3*f^4*z^2 + 624*A^2*a^2*c^8*d^5*f^2*z^2 - 28
8*A^2*a^5*c^5*d^2*f^5*z^2 - 10*B^2*a^2*b^8*d^2*f^5*z^2 - 48*A^2*a^6*b^2*c^2
*f^7*z^2 - 16*A*B*b*c^9*d^7*z^2 + 20*B^2*b^4*c^6*d^6*f*z^2 - 128*B^2*a^7*c^
3*d*f^6*z^2 + 64*A^2*b^2*c^8*d^6*f*z^2 - 112*A^2*a^6*c^4*d*f^6*z^2 + 3*B^2*
a^4*b^6*d*f^6*z^2 + 14*A^2*a^2*b^8*d*f^6*z^2 + 12*A^2*a^5*b^4*c*f^7*z^2 - 1
60*A^2*a*c^9*d^6*f*z^2 + 3*B^2*b^10*d^3*f^4*z^2 - A^2*b^10*d^2*f^5*z^2 + 64
*A^2*a^7*c^3*f^7*z^2 + 4*B^2*b^2*c^8*d^7*z^2 - A^2*a^4*b^6*f^7*z^2 + 16*A^2
*c^10*d^7*z^2 - 160*A*B^2*a*b*c^6*d^4*f^2*z + 112*A*B^2*a^4*b*c^3*d*f^5*z -
 24*A*B^2*a^2*b^5*c*d*f^5*z + 480*A^2*B*a^2*b^2*c^4*d^2*f^4*z - 176*A*B^2*a
^2*b^3*c^3*d^2*f^4*z - 10*A^2*B*a*b^6*c*d*f^5*z + 384*A*B^2*a^2*b*c^5*d^3*f
^3*z - 352*A*B^2*a^3*b*c^4*d^2*f^4*z - 288*A^2*B*a*b^2*c^5*d^3*f^3*z - 160*
A^2*B*a^3*b^2*c^3*d*f^5*z - 148*A^2*B*a*b^4*c^3*d^2*f^4*z + 112*A*B^2*a*b^3
*c^4*d^3*f^3*z + 72*A^2*B*a^2*b^4*c^2*d*f^5*z + 72*A*B^2*a*b^5*c^2*d^2*f^4*
z + 48*A*B^2*a^3*b^3*c^2*d*f^5*z + 48*B^3*a*b^2*c^5*d^4*f^2*z - 36*B^3*a^4*
b^2*c^2*d*f^5*z - 4*B^3*a*b^4*c^3*d^3*f^3*z - 480*A^3*a^2*b*c^5*d^2*f^4*z -
 160*A^3*a^2*b^3*c^3*d*f^5*z + 128*A^3*a*b^3*c^4*d^2*f^4*z + 112*A^2*B*b^4*
c^4*d^3*f^3*z - 64*A*B^2*b^5*c^3*d^3*f^3*z + 16*A^2*B*b^2*c^6*d^4*f^2*z + 1
6*A*B^2*b^3*c^5*d^4*f^2*z - A^2*B*b^6*c^2*d^2*f^4*z + 448*A^2*B*a^3*c^5*d^2
*f^4*z - 352*A^2*B*a^2*c^6*d^3*f^3*z - 48*A^2*B*a^4*b^2*c^2*f^6*z + 12*B^3*
a^3*b^4*c*d*f^5*z - 10*B^3*a*b^6*c*d^2*f^4*z + 416*A^3*a^3*b*c^4*d*f^5*z +
224*A^3*a*b*c^6*d^3*f^3*z + 24*A^3*a*b^5*c^2*d*f^5*z - 2*A*B^2*b^7*c*d^2*f^
4*z - 272*A^2*B*a^4*c^4*d*f^5*z + 128*A^2*B*a*c^7*d^4*f^2*z + 12*A^2*B*a^3*
b^4*c*f^6*z - 120*B^3*a^2*b^2*c^4*d^3*f^3*z + 112*B^3*a^3*b^2*c^3*d^2*f^4*z
 + 16*A*B^2*b*c^7*d^5*f*z + 2*A*B^2*a*b^7*d*f^5*z - 2*A^3*b^7*c*d*f^5*z - 1
6*A^2*B*c^8*d^5*f*z + 11*B^3*b^6*c^2*d^3*f^3*z - 8*B^3*b^4*c^4*d^4*f^2*z -
64*A^3*b^3*c^5*d^3*f^3*z + 96*A^3*a^3*b^3*c^2*f^6*z - 4*B^3*b^2*c^6*d^5*f*z
 - 32*A^3*b*c^7*d^4*f^2*z - B^3*a^2*b^6*d*f^5*z - 128*A^3*a^4*b*c^3*f^6*z -
 24*A^3*a^2*b^5*c*f^6*z + 64*A^2*B*a^5*c^3*f^6*z - A^2*B*a^2*b^6*f^6*z + A^
2*B*b^8*d*f^5*z + 2*A^3*a*b^7*f^6*z + B^3*b^8*d^2*f^4*z + 32*A^3*B*a*b*c^4*
d*f^4 - 18*A^2*B^2*a*b^2*c^3*d*f^4 + 32*A*B^3*a*b*c^4*d^2*f^3 - 28*A*B^3*a^
2*b*c^3*d*f^4 + 6*A*B^3*a*b^3*c^2*d*f^4 - 10*A^3*B*b^3*c^3*d*f^4 - 4*A^3*B*
b*c^5*d^2*f^3 - 4*A*B^3*b*c^5*d^3*f^2 - 28*A^3*B*a^2*b*c^3*f^5 + 6*A^3*B*a*
b^3*c^2*f^5 + 9*A^2*B^2*b^2*c^4*d^2*f^3 - 3*A^2*B^2*a^2*b^2*c^2*f^5 - 10*B^
4*a*b^2*c^3*d^2*f^3 - 3*B^4*a^2*b^2*c^2*d*f^4 - 10*A*B^3*b^3*c^3*d^2*f^3 +
3*A^2*B^2*b^4*c^2*d*f^4 + 36*A^2*B^2*a^2*c^4*d*f^4 - 24*A^2*B^2*a*c^5*d^2*f
^3 + 4*A^2*B^2*c^6*d^3*f^2 + 16*A^2*B^2*a^3*c^3*f^5 + 16*B^4*a^3*c^3*d*f^4
+ 8*A^4*b^2*c^4*d*f^4 - 8*A^4*a*b^2*c^3*f^5 - 24*A^4*a*c^5*d*f^4 + 3*B^4*b^
4*c^2*d^2*f^3 + 4*A^4*c^6*d^2*f^3 + 36*A^4*a^2*c^4*f^5 + B^4*b^2*c^4*d^3*f^
2, z, k)*((4*b^5*c^8*d^7*f^2 + 4*b^7*c^6*d^6*f^3 - 4*b^9*c^4*d^5*f^4 - 4*b^
11*c^2*d^4*f^5 - 612*a^2*b^5*c^6*d^5*f^4 - 712*a^2*b^7*c^4*d^4*f^5 - 132*a^
2*b^9*c^2*d^3*f^6 + 1696*a^3*b^3*c^7*d^5*f^4 + 2736*a^3*b^5*c^5*d^4*f^5 + 8
96*a^3*b^7*c^3*d^3*f^6 - 5120*a^4*b^3*c^6*d^4*f^5 - 3140*a^4*b^5*c^4*d^3*f^

6 - 220*a^4*b^7*c^2*d^2*f^7 + 5664*a^5*b^3*c^5*d^3*f^6 + 1128*a^5*b^5*c^3*d
^2*f^7 - 2560*a^6*b^3*c^4*d^2*f^7 + 8*a*b^11*c*d^3*f^6 + 8*a^5*b^7*c*d*f^8
- 448*a^8*b*c^4*d*f^8 - 32*a*b^3*c^9*d^7*f^2 - 24*a*b^5*c^7*d^6*f^3 + 88*a*
b^7*c^5*d^5*f^4 + 88*a*b^9*c^3*d^4*f^5 + 64*a^2*b*c^10*d^7*f^2 + 128*a^3*b*
c^9*d^6*f^3 + 16*a^3*b^9*c*d^2*f^7 - 1600*a^4*b*c^8*d^5*f^4 + 3840*a^5*b*c^
7*d^4*f^5 - 4160*a^6*b*c^6*d^3*f^6 - 92*a^6*b^5*c^2*d*f^8 + 2176*a^7*b*c^5*
d^2*f^7 + 352*a^7*b^3*c^3*d*f^8)/(16*a^2*c^6*d^4 + a^4*b^4*f^4 + b^4*c^4*d^
4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^
2*b^6*d*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 + 2*b^6*c^2*d^3*f + 96*a^
4*c^4*d^2*f^2 + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 - 20*a*b^4
*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 + 64*a^2*b^2*c^4*d^3*f
+ 64*a^4*b^2*c^2*d*f^3) + (x*(128*a^9*c^4*f^9 - 2*a^6*b^6*c*f^9 - 640*a^8*
c^5*d*f^8 + 6*b^12*c*d^3*f^6 + 24*a^7*b^4*c^2*f^9 - 96*a^8*b^2*c^3*f^9 + 12
8*a^2*c^11*d^7*f^2 - 640*a^3*c^10*d^6*f^3 + 1152*a^4*c^9*d^5*f^4 - 640*a^5*
c^8*d^4*f^5 - 640*a^6*c^7*d^3*f^6 + 1152*a^7*c^6*d^2*f^7 + 8*b^4*c^9*d^7*f^
2 + 22*b^6*c^7*d^6*f^3 + 26*b^8*c^5*d^5*f^4 + 18*b^10*c^3*d^4*f^5 + 672*a^2
*b^2*c^9*d^6*f^3 + 1224*a^2*b^4*c^7*d^5*f^4 + 1202*a^2*b^6*c^5*d^4*f^5 + 56
4*a^2*b^8*c^3*d^3*f^6 - 2048*a^3*b^2*c^8*d^5*f^4 - 2744*a^3*b^4*c^6*d^4*f^5
- 1736*a^3*b^6*c^4*d^3*f^6 - 128*a^3*b^8*c^2*d^2*f^7 + 2656*a^4*b^2*c^7*d^
4*f^5 + 2648*a^4*b^4*c^5*d^3*f^6 + 570*a^4*b^6*c^3*d^2*f^7 - 1344*a^5*b^2*c
^6*d^3*f^6 - 904*a^5*b^4*c^4*d^2*f^7 - 160*a^6*b^2*c^5*d^2*f^7 + 2*a^4*b^8*
c*d*f^8 - 64*a*b^2*c^10*d^7*f^2 - 216*a*b^4*c^8*d^6*f^3 - 300*a*b^6*c^6*d^5
*f^4 - 240*a*b^8*c^4*d^4*f^5 - 92*a*b^10*c^2*d^3*f^6 + 10*a^2*b^10*c*d^2*f^
7 - 12*a^5*b^6*c^2*d*f^8 - 40*a^6*b^4*c^3*d*f^8 + 384*a^7*b^2*c^4*d*f^8))/(
16*a^2*c^6*d^4 + a^4*b^4*f^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 -
8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 64*a^3*c^5*d^3*f - 6
4*a^5*c^3*d*f^3 + 2*b^6*c^2*d^3*f + 96*a^4*c^4*d^2*f^2 + 54*a^2*b^4*c^2*d^2
*f^2 - 112*a^3*b^2*c^3*d^2*f^2 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 -
20*a^3*b^4*c*d*f^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3)) + (A*a^4
*b^6*c*f^8 - 64*A*a^7*c^4*f^8 - 32*A*a*c^10*d^6*f^2 + 352*A*a^6*c^5*d*f^7 +
A*b^10*c*d^2*f^6 - 12*A*a^5*b^4*c^2*f^8 + 48*A*a^6*b^2*c^3*f^8 + 224*A*a^2
*c^9*d^5*f^3 - 640*A*a^3*c^8*d^4*f^4 + 960*A*a^4*c^7*d^3*f^5 - 800*A*a^5*c^
6*d^2*f^6 + 8*A*b^2*c^9*d^6*f^2 + 16*A*b^4*c^7*d^5*f^3 + A*b^6*c^5*d^4*f^4
- 6*A*b^8*c^3*d^3*f^5 - 4*B*b^3*c^8*d^6*f^2 - 12*B*b^5*c^6*d^5*f^3 - 4*B*b^
7*c^4*d^4*f^4 + 4*B*b^9*c^2*d^3*f^5 - 120*A*a*b^2*c^8*d^5*f^3 - 60*A*a*b^4*
c^6*d^4*f^4 + 36*A*a*b^6*c^4*d^3*f^5 - 8*A*a*b^8*c^2*d^2*f^6 - 20*A*a^3*b^6
*c^2*d*f^7 + 80*A*a^4*b^4*c^3*d*f^7 - 216*A*a^5*b^2*c^4*d*f^7 + 92*B*a*b^3*
c^7*d^5*f^3 + 72*B*a*b^5*c^5*d^4*f^4 - 20*B*a*b^7*c^3*d^3*f^5 - 176*B*a^2*b
*c^8*d^5*f^3 + 544*B*a^3*b*c^7*d^4*f^4 - 736*B*a^4*b*c^6*d^3*f^5 - 4*B*a^4*
b^5*c^2*d*f^7 + 464*B*a^5*b*c^5*d^2*f^6 + 44*B*a^5*b^3*c^3*d*f^7 + 384*A*a^
2*b^2*c^7*d^4*f^4 + 32*A*a^2*b^4*c^5*d^3*f^5 + 14*A*a^2*b^6*c^3*d^2*f^6 - 5
60*A*a^3*b^2*c^6*d^3*f^5 - 56*A*a^3*b^4*c^4*d^2*f^6 + 456*A*a^4*b^2*c^5*d^2
*f^6 - 360*B*a^2*b^3*c^6*d^4*f^4 - 64*B*a^2*b^5*c^4*d^3*f^5 + 504*B*a^3*b^3
*c^5*d^3*f^5 + 40*B*a^3*b^5*c^3*d^2*f^6 - 276*B*a^4*b^3*c^4*d^2*f^6 + 2*A*a
^2*b^8*c*d*f^7 + 16*B*a*b*c^9*d^6*f^2 - 112*B*a^6*b*c^4*d*f^7)/(16*a^2*c^6*

```
d^4 + a^4*b^4*f^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^
5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d
*f^3 + 2*b^6*c^2*d^3*f + 96*a^4*c^4*d^2*f^2 + 54*a^2*b^4*c^2*d^2*f^2 - 112*
a^3*b^2*c^3*d^2*f^2 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*
c*d*f^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3) + (x*(64*B*a^7*c^4*f
^8 + 4*A*a^3*b^7*c*f^8 - 256*A*a^6*b*c^4*f^8 - B*a^4*b^6*c*f^8 - 320*B*a^6*
c^5*d*f^7 + 3*B*b^10*c*d^2*f^6 - 48*A*a^4*b^5*c^2*f^8 + 192*A*a^5*b^3*c^3*f
^8 + 12*B*a^5*b^4*c^2*f^8 - 48*B*a^6*b^2*c^3*f^8 - 16*A*b^3*c^8*d^5*f^3 - 4
8*A*b^5*c^6*d^4*f^4 - 36*A*b^7*c^4*d^3*f^5 - 4*A*b^9*c^2*d^2*f^6 - 64*B*a^2
*c^9*d^5*f^3 + 320*B*a^3*c^8*d^4*f^4 - 640*B*a^4*c^7*d^3*f^5 + 640*B*a^5*c^
6*d^2*f^6 + 4*B*b^4*c^7*d^5*f^3 + 23*B*b^6*c^5*d^4*f^4 + 22*B*b^8*c^3*d^3*f
^5 + 320*A*a*b^3*c^7*d^4*f^4 + 352*A*a*b^5*c^5*d^3*f^5 + 76*A*a*b^7*c^3*d^2
*f^6 - 512*A*a^2*b*c^8*d^4*f^4 - 60*A*a^2*b^7*c^2*d*f^7 + 1408*A*a^3*b*c^7*
d^3*f^5 + 352*A*a^3*b^5*c^3*d*f^7 - 1792*A*a^4*b*c^6*d^2*f^6 - 976*A*a^4*b^
3*c^4*d*f^7 - 132*B*a*b^4*c^6*d^4*f^4 - 196*B*a*b^6*c^4*d^3*f^5 - 40*B*a*b^
8*c^2*d^2*f^6 - 20*B*a^3*b^6*c^2*d*f^7 + 52*B*a^4*b^4*c^3*d*f^7 + 64*B*a^5*
b^2*c^4*d*f^7 + 4*A*a*b^9*c*d*f^7 - 1184*A*a^2*b^3*c^6*d^3*f^5 - 544*A*a^2*
b^5*c^4*d^2*f^6 + 1664*A*a^3*b^3*c^5*d^2*f^6 + 80*B*a^2*b^2*c^7*d^4*f^4 + 5
20*B*a^2*b^4*c^5*d^3*f^5 + 210*B*a^2*b^6*c^3*d^2*f^6 - 192*B*a^3*b^2*c^6*d^
3*f^5 - 456*B*a^3*b^4*c^4*d^2*f^6 + 96*B*a^4*b^2*c^5*d^2*f^6 + 64*A*a*b*c^9
*d^5*f^3 + 1088*A*a^5*b*c^5*d*f^7 + 2*B*a^2*b^8*c*d*f^7))/(16*a^2*c^6*d^4 +
 a^4*b^4*f^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4
 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3
+ 2*b^6*c^2*d^3*f + 96*a^4*c^4*d^2*f^2 + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b
^2*c^3*d^2*f^2 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f
^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3)) + (13*A^2*a^2*b^5*c^2*f^
7 - 56*A^2*a^3*b^3*c^3*f^7 + 16*A^2*b^3*c^6*d^3*f^4 + A^2*b^5*c^4*d^2*f^5 +
 8*B^2*b^3*c^6*d^4*f^3 + 9*B^2*b^5*c^4*d^3*f^4 - 64*A*B*a^5*c^4*f^7 - A^2*a
*b^7*c*f^7 + 80*A^2*a^4*b*c^4*f^7 + 16*A^2*b*c^8*d^4*f^3 + 2*A^2*b^7*c^2*d*
f^6 - 48*A^2*a*b*c^7*d^3*f^4 - 22*A^2*a*b^5*c^3*d*f^6 - 48*A^2*a^3*b*c^5*d*
f^6 - 16*B^2*a*b*c^7*d^4*f^3 - 64*B^2*a^4*b*c^4*d*f^6 - A*B*b^8*c*d*f^6 - 8
*A^2*a*b^3*c^5*d^2*f^5 + 64*A^2*a^2*b^3*c^4*d*f^6 - 56*B^2*a*b^3*c^5*d^3*f^
4 + 2*B^2*a*b^5*c^3*d^2*f^5 + 96*B^2*a^2*b*c^6*d^3*f^4 - 11*B^2*a^2*b^5*c^2
*d*f^6 - 16*B^2*a^3*b*c^5*d^2*f^5 + 40*B^2*a^3*b^3*c^3*d*f^6 + A*B*a^2*b^6*
c*f^7 + 32*A*B*a*c^8*d^4*f^3 + 32*A*B*a^4*c^5*d*f^6 + B^2*a*b^7*c*d*f^6 - 8
*B^2*a^2*b^3*c^4*d^2*f^5 - 12*A*B*a^3*b^4*c^2*f^7 + 48*A*B*a^4*b^2*c^3*f^7
- 160*A*B*a^2*c^7*d^3*f^4 + 160*A*B*a^3*c^6*d^2*f^5 - 24*A*B*b^2*c^7*d^4*f^
3 - 24*A*B*b^4*c^5*d^3*f^4 + A*B*b^6*c^3*d^2*f^5 + 120*A*B*a*b^2*c^6*d^3*f^
4 - 4*A*B*a*b^4*c^4*d^2*f^5 - 24*A*B*a^2*b^4*c^3*d*f^6 + 8*A*B*a^3*b^2*c^4*
d*f^6 - 24*A*B*a^2*b^2*c^5*d^2*f^5 + 10*A*B*a*b^6*c^2*d*f^6)/(16*a^2*c^6*d^
4 + a^4*b^4*f^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*
d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f
^3 + 2*b^6*c^2*d^3*f + 96*a^4*c^4*d^2*f^2 + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^
3*b^2*c^3*d^2*f^2 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*
d*f^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3) + (x*(104*A^2*a^4*c^5*
```

f^7 - 32*B^2*a^5*c^4*f^7 + 8*A^2*c^9*d^4*f^3 + A^2*b^8*c*f^7 + 50*A^2*a^2*b
^4*c^3*f^7 - 96*A^2*a^3*b^2*c^4*f^7 - 12*B^2*a^3*b^4*c^2*f^7 + 42*B^2*a^4*b
^2*c^3*f^7 + 208*A^2*a^2*c^7*d^2*f^5 + 8*A^2*b^2*c^7*d^3*f^4 + 18*A^2*b^4*c
^5*d^2*f^5 - 32*B^2*a^2*c^7*d^3*f^4 + 32*B^2*a^3*c^6*d^2*f^5 + 2*B^2*b^2*c^
7*d^4*f^3 - 6*B^2*b^4*c^5*d^3*f^4 + 9*B^2*b^6*c^3*d^2*f^5 - 12*A^2*a*b^6*c^
2*f^7 + B^2*a^2*b^6*c*f^7 - 64*A^2*a*c^8*d^3*f^4 - 256*A^2*a^3*c^6*d*f^6 +
2*A^2*b^6*c^3*d*f^6 + 32*B^2*a^4*c^5*d*f^6 - 36*A^2*a*b^4*c^4*d*f^6 - 2*B^2
*a*b^6*c^2*d*f^6 - 2*A*B*a*b^7*c*f^7 - 144*A^2*a*b^2*c^6*d^2*f^5 + 168*A^2*
a^2*b^2*c^5*d*f^6 + 24*B^2*a*b^2*c^6*d^3*f^4 - 64*B^2*a*b^4*c^4*d^2*f^5 + 2
6*B^2*a^2*b^4*c^3*d*f^6 - 88*B^2*a^3*b^2*c^4*d*f^6 + 72*A*B*a^4*b*c^4*f^7 -
8*A*B*b*c^8*d^4*f^3 + 2*A*B*b^7*c^2*d*f^6 + 84*B^2*a^2*b^2*c^5*d^2*f^5 + 2
4*A*B*a^2*b^5*c^2*f^7 - 84*A*B*a^3*b^3*c^3*f^7 + 4*A*B*b^3*c^6*d^3*f^4 - 20
*A*B*b^5*c^4*d^2*f^5 + 148*A*B*a*b^3*c^5*d^2*f^5 - 192*A*B*a^2*b*c^6*d^2*f^
5 - 4*A*B*a^2*b^3*c^4*d*f^6 + 16*A*B*a*b*c^7*d^3*f^4 - 12*A*B*a*b^5*c^3*d*f
^6 + 112*A*B*a^3*b*c^5*d*f^6))/(16*a^2*c^6*d^4 + a^4*b^4*f^4 + b^4*c^4*d^4
+ 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*
b^6*d*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 + 2*b^6*c^2*d^3*f + 96*a^4*
c^4*d^2*f^2 + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 - 20*a*b^4*c
^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 + 64*a^2*b^2*c^4*d^3*f +
64*a^4*b^2*c^2*d*f^3)) - (16*A^3*a*c^6*d*f^5 - 4*A^3*c^7*d^2*f^4 - B^3*b^3
*c^4*d^2*f^4 - 12*A^3*a^2*c^5*f^6 - 16*A*B^2*a^3*c^4*f^6 + 2*A^3*a*b^2*c^4*
f^6 - 6*A^3*b^2*c^5*d*f^5 + 4*B^3*a*b*c^5*d^2*f^4 + 3*B^3*a*b^3*c^3*d*f^5 -
12*B^3*a^2*b*c^4*d*f^5 + 3*A*B^2*a^2*b^2*c^3*f^6 + A*B^2*b^2*c^5*d^2*f^4 -
3*A^2*B*a*b^3*c^3*f^6 + 16*A^2*B*a^2*b*c^4*f^6 - 8*A*B^2*a*c^6*d^2*f^4 + 2
4*A*B^2*a^2*c^5*d*f^5 - 3*A*B^2*b^4*c^3*d*f^5 + 4*A^2*B*b*c^6*d^2*f^4 + 9*A
^2*B*b^3*c^4*d*f^5 + 4*A*B^2*a*b^2*c^4*d*f^5 - 28*A^2*B*a*b*c^5*d*f^5)/(16*
a^2*c^6*d^4 + a^4*b^4*f^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*
a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 64*a^3*c^5*d^3*f - 64*a
^5*c^3*d*f^3 + 2*b^6*c^2*d^3*f + 96*a^4*c^4*d^2*f^2 + 54*a^2*b^4*c^2*d^2*f^
2 - 112*a^3*b^2*c^3*d^2*f^2 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*
a^3*b^4*c*d*f^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3))*root(2560*a
^3*b^2*c^9*d^8*f*z^4 - 1152*a^2*b^4*c^8*d^8*f*z^4 + 384*a^5*b^8*c*d^3*f^6*z
^4 + 384*a*b^8*c^5*d^7*f^2*z^4 + 288*a^3*b^10*c*d^4*f^5*z^4 + 288*a*b^10*c^
3*d^6*f^3*z^4 + 224*a^7*b^6*c*d^2*f^7*z^4 - 192*a^10*b^2*c^2*d*f^8*z^4 + 22
4*a*b^6*c^7*d^8*f*z^4 + 80*a*b^12*c*d^5*f^4*z^4 + 48*a^9*b^4*c*d*f^8*z^4 -
33920*a^6*b^2*c^6*d^5*f^4*z^4 + 27936*a^5*b^4*c^5*d^5*f^4*z^4 + 26112*a^7*b
^2*c^5*d^4*f^5*z^4 + 26112*a^5*b^2*c^7*d^6*f^3*z^4 - 20352*a^6*b^4*c^4*d^4*
f^5*z^4 - 20352*a^4*b^4*c^6*d^6*f^3*z^4 - 13080*a^4*b^6*c^4*d^5*f^4*z^4 - 1
1520*a^8*b^2*c^4*d^3*f^6*z^4 - 11520*a^4*b^2*c^8*d^7*f^2*z^4 + 8736*a^5*b^6
*c^3*d^4*f^5*z^4 + 8736*a^3*b^6*c^5*d^6*f^3*z^4 + 7488*a^7*b^4*c^3*d^3*f^6*
z^4 + 7488*a^3*b^4*c^7*d^7*f^2*z^4 + 3840*a^3*b^8*c^3*d^5*f^4*z^4 + 2560*a^
9*b^2*c^3*d^2*f^7*z^4 - 2416*a^6*b^6*c^2*d^3*f^6*z^4 - 2416*a^2*b^6*c^6*d^7
*f^2*z^4 - 2160*a^4*b^8*c^2*d^4*f^5*z^4 - 2160*a^2*b^8*c^4*d^6*f^3*z^4 - 11
52*a^8*b^4*c^2*d^2*f^7*z^4 - 720*a^2*b^10*c^2*d^5*f^4*z^4 - 16*b^8*c^6*d^8*
f*z^4 - 2048*a^4*c^10*d^8*f*z^4 + 256*a^11*c^3*d*f^8*z^4 - 4*a^8*b^6*d*f^8*

z^4 + 48*a*b^4*c^9*d^9*z^4 - 24*b^10*c^4*d^7*f^2*z^4 - 16*b^12*c^2*d^6*f^3*
z^4 + 17920*a^7*c^7*d^5*f^4*z^4 - 14336*a^8*c^6*d^4*f^5*z^4 - 14336*a^6*c^8
*d^6*f^3*z^4 + 7168*a^9*c^5*d^3*f^6*z^4 + 7168*a^5*c^9*d^7*f^2*z^4 - 2048*a
^10*c^4*d^2*f^7*z^4 - 24*a^4*b^10*d^3*f^6*z^4 - 16*a^6*b^8*d^2*f^7*z^4 - 16
*a^2*b^12*d^4*f^5*z^4 - 192*a^2*b^2*c^10*d^9*z^4 - 4*b^14*d^5*f^4*z^4 - 4*b
^6*c^8*d^9*z^4 + 256*a^3*c^11*d^9*z^4 + 912*A*B*a^6*b*c^3*d*f^6*z^2 + 192*A
*B*a^4*b^5*c*d*f^6*z^2 + 920*A*B*a^4*b^3*c^3*d^2*f^5*z^2 - 480*A*B*a^2*b^5*
c^3*d^3*f^4*z^2 - 336*A*B*a^2*b^3*c^5*d^4*f^3*z^2 - 272*A*B*a^3*b^3*c^4*d^3
*f^4*z^2 + 240*A*B*a^3*b^5*c^2*d^2*f^5*z^2 + 192*A*B*a*b*c^8*d^6*f*z^2 - 24
96*A*B*a^5*b*c^4*d^2*f^5*z^2 + 1872*A*B*a^4*b*c^5*d^3*f^4*z^2 - 744*A*B*a^5
*b^3*c^2*d*f^6*z^2 - 720*A*B*a^2*b*c^7*d^5*f^2*z^2 + 504*A*B*a*b^3*c^6*d^5*
f^2*z^2 + 256*A*B*a^3*b*c^6*d^4*f^3*z^2 + 168*A*B*a*b^7*c^2*d^3*f^4*z^2 - 1
44*A*B*a^2*b^7*c*d^2*f^5*z^2 + 144*A*B*a*b^5*c^4*d^4*f^3*z^2 - 56*B^2*a*b^2
*c^7*d^6*f*z^2 - 36*B^2*a^5*b^4*c*d*f^6*z^2 - 16*B^2*a*b^8*c*d^3*f^4*z^2 -
164*A^2*a^3*b^6*c*d*f^6*z^2 - 16*A^2*a*b^8*c*d^2*f^5*z^2 - 96*A*B*b^5*c^5*d
^5*f^2*z^2 - 24*A*B*b^7*c^3*d^4*f^3*z^2 - 580*B^2*a^4*b^2*c^4*d^3*f^4*z^2 +
536*B^2*a^3*b^4*c^3*d^3*f^4*z^2 - 348*B^2*a^4*b^4*c^2*d^2*f^5*z^2 + 316*B^
2*a^2*b^2*c^6*d^5*f^2*z^2 + 200*B^2*a^5*b^2*c^3*d^2*f^5*z^2 - 120*B^2*a^2*b
^4*c^4*d^4*f^3*z^2 - 66*B^2*a^2*b^6*c^2*d^3*f^4*z^2 - 16*B^2*a^3*b^2*c^5*d^
4*f^3*z^2 + 1952*A^2*a^4*b^2*c^4*d^2*f^5*z^2 - 1792*A^2*a^3*b^2*c^5*d^3*f^4
*z^2 - 1272*A^2*a^3*b^4*c^3*d^2*f^5*z^2 + 976*A^2*a^2*b^2*c^6*d^4*f^3*z^2 +
960*A^2*a^2*b^4*c^4*d^3*f^4*z^2 + 282*A^2*a^2*b^6*c^2*d^2*f^5*z^2 - 72*A*B
*b^3*c^7*d^6*f*z^2 - 16*A*B*b^9*c*d^3*f^4*z^2 - 16*A*B*a^3*b^7*d*f^6*z^2 +
16*A*B*a*b^9*d^2*f^5*z^2 - 180*B^2*a*b^4*c^5*d^5*f^2*z^2 + 132*B^2*a^6*b^2*
c^2*d*f^6*z^2 + 108*B^2*a^3*b^6*c*d^2*f^5*z^2 + 20*B^2*a*b^6*c^3*d^4*f^3*z^
2 - 736*A^2*a^5*b^2*c^3*d*f^6*z^2 + 624*A^2*a^4*b^4*c^2*d*f^6*z^2 - 416*A^2
*a*b^2*c^7*d^5*f^2*z^2 - 276*A^2*a*b^4*c^5*d^4*f^3*z^2 - 196*A^2*a*b^6*c^3*
d^3*f^4*z^2 + 31*B^2*b^6*c^4*d^5*f^2*z^2 + 2*B^2*b^8*c^2*d^4*f^3*z^2 - 768*
B^2*a^5*c^5*d^3*f^4*z^2 + 512*B^2*a^6*c^4*d^2*f^5*z^2 + 512*B^2*a^4*c^6*d^4
*f^3*z^2 - 128*B^2*a^3*c^7*d^5*f^2*z^2 + 80*A^2*b^4*c^6*d^5*f^2*z^2 + 31*A^
2*b^6*c^4*d^4*f^3*z^2 + 14*A^2*b^8*c^2*d^3*f^4*z^2 - 1152*A^2*a^3*c^7*d^4*f
^3*z^2 + 1008*A^2*a^4*c^6*d^3*f^4*z^2 + 624*A^2*a^2*c^8*d^5*f^2*z^2 - 288*A
^2*a^5*c^5*d^2*f^5*z^2 - 10*B^2*a^2*b^8*d^2*f^5*z^2 - 48*A^2*a^6*b^2*c^2*f^
7*z^2 - 16*A*B*b*c^9*d^7*z^2 + 20*B^2*b^4*c^6*d^6*f*z^2 - 128*B^2*a^7*c^3*d
*f^6*z^2 + 64*A^2*b^2*c^8*d^6*f*z^2 - 112*A^2*a^6*c^4*d*f^6*z^2 + 3*B^2*a^4
*b^6*d*f^6*z^2 + 14*A^2*a^2*b^8*d*f^6*z^2 + 12*A^2*a^5*b^4*c*f^7*z^2 - 160*
A^2*a*c^9*d^6*f*z^2 + 3*B^2*b^10*d^3*f^4*z^2 - A^2*b^10*d^2*f^5*z^2 + 64*A^
2*a^7*c^3*f^7*z^2 + 4*B^2*b^2*c^8*d^7*z^2 - A^2*a^4*b^6*f^7*z^2 + 16*A^2*c^
10*d^7*z^2 - 160*A*B^2*a*b*c^6*d^4*f^2*z + 112*A*B^2*a^4*b*c^3*d*f^5*z - 24
*A*B^2*a^2*b^5*c*d*f^5*z + 480*A^2*B*a^2*b^2*c^4*d^2*f^4*z - 176*A*B^2*a^2*
b^3*c^3*d^2*f^4*z - 10*A^2*B*a*b^6*c*d*f^5*z + 384*A*B^2*a^2*b*c^5*d^3*f^3*
z - 352*A*B^2*a^3*b*c^4*d^2*f^4*z - 288*A^2*B*a*b^2*c^5*d^3*f^3*z - 160*A^2
*B*a^3*b^2*c^3*d*f^5*z - 148*A^2*B*a*b^4*c^3*d^2*f^4*z + 112*A*B^2*a*b^3*c^
4*d^3*f^3*z + 72*A^2*B*a^2*b^4*c^2*d*f^5*z + 72*A*B^2*a*b^5*c^2*d^2*f^4*z +
48*A*B^2*a^3*b^3*c^2*d*f^5*z + 48*B^3*a*b^2*c^5*d^4*f^2*z - 36*B^3*a^4*b^2

```
*c^2*d*f^5*z - 4*B^3*a*b^4*c^3*d^3*f^3*z - 480*A^3*a^2*b*c^5*d^2*f^4*z - 16
0*A^3*a^2*b^3*c^3*d*f^5*z + 128*A^3*a*b^3*c^4*d^2*f^4*z + 112*A^2*B*b^4*c^4
*d^3*f^3*z - 64*A*B^2*b^5*c^3*d^3*f^3*z + 16*A^2*B*b^2*c^6*d^4*f^2*z + 16*A
*B^2*b^3*c^5*d^4*f^2*z - A^2*B*b^6*c^2*d^2*f^4*z + 448*A^2*B*a^3*c^5*d^2*f^
4*z - 352*A^2*B*a^2*c^6*d^3*f^3*z - 48*A^2*B*a^4*b^2*c^2*f^6*z + 12*B^3*a^3
*b^4*c*d*f^5*z - 10*B^3*a*b^6*c*d^2*f^4*z + 416*A^3*a^3*b*c^4*d*f^5*z + 224
*A^3*a*b*c^6*d^3*f^3*z + 24*A^3*a*b^5*c^2*d*f^5*z - 2*A*B^2*b^7*c*d^2*f^4*z
 - 272*A^2*B*a^4*c^4*d*f^5*z + 128*A^2*B*a*c^7*d^4*f^2*z + 12*A^2*B*a^3*b^4
*c*f^6*z - 120*B^3*a^2*b^2*c^4*d^3*f^3*z + 112*B^3*a^3*b^2*c^3*d^2*f^4*z +
16*A*B^2*b*c^7*d^5*f*z + 2*A*B^2*a*b^7*d*f^5*z - 2*A^3*b^7*c*d*f^5*z - 16*A
^2*B*c^8*d^5*f*z + 11*B^3*b^6*c^2*d^3*f^3*z - 8*B^3*b^4*c^4*d^4*f^2*z - 64*
A^3*b^3*c^5*d^3*f^3*z + 96*A^3*a^3*b^3*c^2*f^6*z - 4*B^3*b^2*c^6*d^5*f*z -
32*A^3*b*c^7*d^4*f^2*z - B^3*a^2*b^6*d*f^5*z - 128*A^3*a^4*b*c^3*f^6*z - 24
*A^3*a^2*b^5*c*f^6*z + 64*A^2*B*a^5*c^3*f^6*z - A^2*B*a^2*b^6*f^6*z + A^2*B
*b^8*d*f^5*z + 2*A^3*a*b^7*f^6*z + B^3*b^8*d^2*f^4*z + 32*A^3*B*a*b*c^4*d*f
^4 - 18*A^2*B^2*a*b^2*c^3*d*f^4 + 32*A*B^3*a*b*c^4*d^2*f^3 - 28*A*B^3*a^2*b
*c^3*d*f^4 + 6*A*B^3*a*b^3*c^2*d*f^4 - 10*A^3*B*b^3*c^3*d*f^4 - 4*A^3*B*b*c
^5*d^2*f^3 - 4*A*B^3*b*c^5*d^3*f^2 - 28*A^3*B*a^2*b*c^3*f^5 + 6*A^3*B*a*b^3
*c^2*f^5 + 9*A^2*B^2*b^2*c^4*d^2*f^3 - 3*A^2*B^2*a^2*b^2*c^2*f^5 - 10*B^4*a
*b^2*c^3*d^2*f^3 - 3*B^4*a^2*b^2*c^2*d*f^4 - 10*A*B^3*b^3*c^3*d^2*f^3 + 3*A
^2*B^2*b^4*c^2*d*f^4 + 36*A^2*B^2*a^2*c^4*d*f^4 - 24*A^2*B^2*a*c^5*d^2*f^3
+ 4*A^2*B^2*c^6*d^3*f^2 + 16*A^2*B^2*a^3*c^3*f^5 + 16*B^4*a^3*c^3*d*f^4 + 8
*A^4*b^2*c^4*d*f^4 - 8*A^4*a*b^2*c^3*f^5 - 24*A^4*a*c^5*d*f^4 + 3*B^4*b^4*c
^2*d^2*f^3 + 4*A^4*c^6*d^2*f^3 + 36*A^4*a^2*c^4*f^5 + B^4*b^2*c^4*d^3*f^2,
z, k), k, 1, 4)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)**2/(f*x**2+d),x)
```

```
[Out] Timed out
```

## 3.6 $\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$

**Optimal**. Leaf size=331

$$-\frac{\left(B\sqrt{d}-A\sqrt{f}\right)\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}\,f^{3/2}}+\frac{\left(A\sqrt{f}+B\sqrt{d}\right)\sqrt{af+b\sqrt{d}\sqrt{}}}{}$$

**Rubi [A]** time = 0.60, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1021, 1078, 621, 206, 1033, 724}

$$-\frac{\left(B\sqrt{d}-A\sqrt{f}\right)\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}\,f^{3/2}}+\frac{\left(A\sqrt{f}+B\sqrt{d}\right)\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}\,f^{3/2}}-\frac{(2Ac+bB)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\,f}-\frac{B\sqrt{a+bx+cx^2}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] -((B*Sqrt[a + b*x + c*x^2])/f) - ((b*B + 2*A*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt
[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*f) - ((B*Sqrt[d] - A*Sqrt[f])*Sqrt[
c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt
[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c
*x^2])])/(2*Sqrt[d]*f^(3/2)) + ((B*Sqrt[d] + A*Sqrt[f])*Sqrt[c*d + b*Sqrt[d
]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f
])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sq
rt[d]*f^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1021

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rubi steps

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = -\frac{B\sqrt{a+bx+cx^2}}{f} + \frac{\int \frac{\frac{1}{2}(bBd+2aAf)+(Bcd+Abf+aBf)x+\frac{1}{2}(bB+2Ac)fx^2}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{f}$$

$$= -\frac{B\sqrt{a+bx+cx^2}}{f} - \frac{\int \frac{-\frac{1}{2}(bB+2Ac)df-\frac{1}{2}f(bBd+2aAf)-f(Bcd+Abf+aBf)x}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{f^2} - \frac{(bB+2A}{}$$

$$= -\frac{B\sqrt{a+bx+cx^2}}{f} - \frac{(bB+2Ac)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f} + \frac{\left(\left(B\sqrt{d} - A\right.\right.}{}$$

$$= -\frac{B\sqrt{a+bx+cx^2}}{f} - \frac{(bB+2Ac)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f} - \frac{\left(\left(B\sqrt{d} - A\sqrt{f}\right)\left(cd\right.\right.}{}$$

$$= -\frac{B\sqrt{a+bx+cx^2}}{f} - \frac{(bB+2Ac)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f} - \frac{\left(B\sqrt{d} - A\sqrt{f}\right)\sqrt{cd}}{}$$

**Mathematica** [A] time = 0.80, size = 322, normalized size = 0.97

$$\frac{\left(A\sqrt{f}+B\sqrt{d}\right)\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)-\left(B\sqrt{d}-A\sqrt{f}\right)\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)-2B\sqrt{d}\sqrt{f}\sqrt{a+x(b+cx)}}{2\sqrt{d}f^{3/2}} - \frac{(2Ac+bB)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{2\sqrt{c}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] -1/2*((b*B + 2*A*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])
/(Sqrt[c]*f) + (-2*B*Sqrt[d]*Sqrt[f]*Sqrt[a + x*(b + c*x)] + (B*Sqrt[d] + A
*Sqrt[f])*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt
[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*S
qrt[a + x*(b + c*x)])] - (B*Sqrt[d] - A*Sqrt[f])*Sqrt[c*d - b*Sqrt[d]*Sqrt[
f] + a*f]*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/
(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(2*Sqrt[d]*
f^(3/2))
```

**IntegrateAlgebraic** [C] time = 0.78, size = 523, normalized size = 1.58

$$-\frac{\text{RootSum}\left[\#1^4(-f)+2\#1^2af+4\#1^2cd-4\#1b\sqrt{c}d-a^2f+b^2d\&, \frac{\#1^2Abf\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+\#1^2Bcd\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+\#1^2aBf\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+a^2(-B)f\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2\#1Ac^{3/2}d\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+Abcd\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2\#1aA\sqrt{c}f\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+b^2Bd\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-aBcd\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2\#1bB\sqrt{c}d\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)}{\#1^3f-\#1af-2\#1cd+b\sqrt{c}d}\&\right]}{2f} + \frac{(2Ac+bB)\log\left(-2\sqrt{c}f\sqrt{a+bx+cx^2}+bf+2cfx\right)}{2\sqrt{c}f} - \frac{B\sqrt{a+bx+cx^2}}{f}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((A + B*x)*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] -((B*Sqrt[a + b*x + c*x^2])/f) + ((b*B + 2*A*c)*Log[b*f + 2*c*f*x - 2*Sqrt[
c]*f*Sqrt[a + b*x + c*x^2]])/(2*Sqrt[c]*f) - RootSum[b^2*d - a^2*f - 4*b*Sq
rt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (b^2*B*d*Log[-(Sqrt[c]*x)
 + Sqrt[a + b*x + c*x^2] - #1] + A*b*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x +
c*x^2] - #1] - a*B*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a^2
*B*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*b*B*Sqrt[c]*d*Log[-
(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*A*c^(3/2)*d*Log[-(Sqrt[c]*
x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*a*A*Sqrt[c]*f*Log[-(Sqrt[c]*x) + Sq
rt[a + b*x + c*x^2] - #1]*#1 + B*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^
2] - #1]*#1^2 + A*b*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 +
 a*B*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d -
2*c*d*#1 - a*f*#1 + f*#1^3) & ]/(2*f)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

**maple [B]** time = 0.08, size = 3358, normalized size = 10.15

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] 1/2/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^
(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*A-1/2/f*((x+(d*f)^(1/2)/f)^2*c
+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))
^(1/2)*B-1/2/f*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2
```

```
)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(
-b*(d*f)^(1/2)+a*f+c*d))^(1/2))*c^(1/2)*A+1/2*(d*f)^(1/2)/f^2*ln((1/2/f*(-2
*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f
*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/
2))*c^(1/2)*B+1/4/(d*f)^(1/2)*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(
1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)
^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/c^(1/2)*b*A-1/4/f*ln((1/2/f*
(-2*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+
1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^
(1/2))/c^(1/2)*b*B+1/2/f/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(
d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b
*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b
*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f
))*b*A-1/2*(d*f)^(1/2)/f^2/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b
*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(
-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)
+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)
/f))*b*B-1/2/(d*f)^(1/2)/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(
d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b
*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b
*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f
))*a*A+1/2/f/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a
*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)
+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)
^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*a*B-1/2/(
d*f)^(1/2)/f/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a
*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)
+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)
^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*c*d*A+1/2
/f^2/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+
1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d
))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f
)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*c*d*B-1/2/(d*f)^(
1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d
*f)^(1/2)+a*f+c*d)/f)^(1/2)*A-1/2/f*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)
+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*B-1/2/f*ln((1/2*
(2*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c
+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)
)*c^(1/2)*A-1/2*(d*f)^(1/2)/f^2*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^
(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(
1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))*c^(1/2)*B-1/4/(d*f)^(1/2)*ln((1/2
*(2*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*
c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2
))/c^(1/2)*b*A-1/4/f*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c
^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*
```

```
(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/c^(1/2)*b*B+1/2/f/((b*(d*f)^(1/2)+a*f+c*d)/f
)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1
/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f
)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)
^(1/2)/f))*b*A+1/2*(d*f)^(1/2)/f^2/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*
(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(
d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f
*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*b*B
+1/2/(d*f)^(1/2)/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f
+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d
)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)
+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a*A+1/2/f/((b*(d*f)^(
1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f
)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f
)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^
(1/2))/(x-(d*f)^(1/2)/f))*a*B+1/2/(d*f)^(1/2)/f/((b*(d*f)^(1/2)+a*f+c*d)/f)
^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/
2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)
^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^
(1/2)/f))*c*d*A+1/2/f^2/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1
/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+
a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(
1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c*d*B
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                  /(2*f)                 +(c*d)/f+a))
     /f^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,x)\,\sqrt{c\,x^2+b\,x+a}}{d-f\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/(d - f*x^2),x)
```

```
[Out] int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/(d - f*x^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{A\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx - \int \frac{Bx\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(A*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(B*x*sqrt(a
+ b*x + c*x**2)/(-d + f*x**2), x)
```

## 3.7 $\int \frac{A+Bx}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx$

**Optimal**. Leaf size=249

$$\frac{\left(\frac{A\sqrt{f}}{\sqrt{d}}+B\right)\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\left(B-\frac{A\sqrt{f}}{\sqrt{d}}\right)\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}$$

**Rubi** [A] time = 0.20, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1033, 724, 206}

$$\frac{\left(\frac{A\sqrt{f}}{\sqrt{d}}+B\right)\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\left(B-\frac{A\sqrt{f}}{\sqrt{d}}\right)\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -((B - (A*Sqrt[f])/Sqrt[d])*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d]
 - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^
2])])/(2*Sqrt[f]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + ((B + (A*Sqrt[f])/S
qrt[d])*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*
Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[f]*Sqr
t[c*d + b*Sqrt[d]*Sqrt[f] + a*f])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\int \frac{A+Bx}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx = \frac{1}{2}\left(B-\frac{A\sqrt{f}}{\sqrt{d}}\right)\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx + \frac{1}{2}\left(B+\frac{A\sqrt{f}}{\sqrt{d}}\right)\int \frac{}{\left(\sqrt{d}\right.}$$

$$= \left(-B-\frac{A\sqrt{f}}{\sqrt{d}}\right)\text{Subst}\left(\int \frac{1}{4cdf+4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{-b\sqrt{d}\sqrt{f}-2af}{\sqrt{a+}}\right.$$

$$= -\frac{\left(B-\frac{A\sqrt{f}}{\sqrt{d}}\right)\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{f}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} + \frac{\left(B+\frac{A\sqrt{f}}{\sqrt{d}}\right)\tanh^{-1}\left(\frac{b\sqrt{d}+2a}{2\sqrt{cd+b}}\right.}{2\sqrt{f}\sqrt{cd+b\sqrt{d}}}$$

**Mathematica [A]** time = 0.24, size = 249, normalized size = 1.00

$$\frac{-\frac{\left(B\sqrt{d}-A\sqrt{f}\right)\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}-\frac{\left(A\sqrt{f}+B\sqrt{d}\right)\tanh^{-1}\left(\frac{-2\left(a\sqrt{f}+c\sqrt{d}x\right)-b\left(\sqrt{d}+\sqrt{f}x\right)}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] (-(((B*Sqrt[d] - A*Sqrt[f])*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt
[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*
x)])])/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) - ((B*Sqrt[d] + A*Sqrt[f])*ArcT
anh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d +
b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d + b*Sqrt[d]*Sqrt
[f] + a*f])/(2*Sqrt[d]*Sqrt[f])
```

**IntegrateAlgebraic [C]** time = 0.43, size = 219, normalized size = 0.88

$$-\frac{1}{2}\text{RootSum}\left[\#1^4(-f)+2\#1^2af+4\#1^2cd-4\#1b\sqrt{c}d-a^2f+b^2d\&, \frac{\#1^2B\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2\#1A\sqrt{c}\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+Ab\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-aB\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)}{\#1^3f-\#1af-2\#1cd+b\sqrt{c}d}\&\right]$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x)/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -1/2*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f
*#1^4 & , (A*b*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*B*Log[-(S
qrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*A*Sqrt[c]*Log[-(Sqrt[c]*x) + Sq
rt[a + b*x + c*x^2] - #1]*#1 + B*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] -
 #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ]
```

**fricas [B]** time = 52.20, size = 6113, normalized size = 24.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt((B^2*c*d^2 + A^2*a*f^2 + (B^2*a - 2*A*B*b + A^2*c)*d*f + (c^2*d^3*
f + a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2
*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)
*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/(c^4*d^5*f + a^4*d*f^5
- 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^3 - 2
*(a^2*b^2 - 2*a^3*c)*d^2*f^4)))/(c^2*d^3*f + a^2*d*f^3 - (b^2 - 2*a*c)*d^2*
f^2))*log(-((B^4*b^2 - 2*A*B^3*b*c)*d^2 - 2*(A*B^3*a*b - A^3*B*b*c)*d*f + (
2*A^3*B*a*b - A^4*b^2)*f^2 + 2*((2*A^3*B*a - A^4*b)*c*f^2 + (B^4*b*c - 2*A*
B^3*c^2)*d^2 - 2*(A*B^3*a*c - A^3*B*c^2)*d*f)*x + 2*((B^3*b^2 - 3*A*B^2*b*c
 + 2*A^2*B*c^2)*d^2*f - (3*A*B^2*a*b - A^2*B*b^2 - (4*A^2*B*a - A^3*b)*c)*d
*f^2 + (2*A^2*B*a^2 - A^3*a*b)*f^3 - (B*c^3*d^4*f - (B*b^2*c - (3*B*a - A*b
)*c^2)*d^3*f^2 - (B*a*b^2 - A*b^3 - (3*B*a^2 - 2*A*a*b)*c)*d^2*f^3 + (B*a^3
 - A*a^2*b)*d*f^4)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2
*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^
2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c
^3)*d^4*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)
*d^2*f^4)))*sqrt(c*x^2 + b*x + a)*sqrt((B^2*c*d^2 + A^2*a*f^2 + (B^2*a - 2*
A*B*b + A^2*c)*d*f + (c^2*d^3*f + a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2)*sqrt((
(B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2
- 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2
)*f^2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*
b^2*c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4)))/(c^2*d^3*f +
a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2)) - (2*B^2*a*c^2*d^3*f - 2*A^2*a^3*f^4 -
2*(B^2*a*b^2 - 2*B^2*a^2*c + A^2*a*c^2)*d^2*f^2 + 2*(B^2*a^3 + A^2*a*b^2 -
2*A^2*a^2*c)*d*f^3 + (B^2*b*c^2*d^3*f - A^2*a^2*b*f^4 - (B^2*b^3 - 2*B^2*a*
b*c + A^2*b*c^2)*d^2*f^2 + (B^2*a^2*b + A^2*b^3 - 2*A^2*a*b*c)*d*f^3)*x)*sq
rt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*
b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4
*b^2)*f^2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 -
```

```
4*a*b^2*c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4)))/x) - 1/4*
sqrt((B^2*c*d^2 + A^2*a*f^2 + (B^2*a - 2*A*B*b + A^2*c)*d*f + (c^2*d^3*f +
a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2
*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f
 + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/(c^4*d^5*f + a^4*d*f^5 - 2*
(b^2*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^3 - 2*(a^
2*b^2 - 2*a^3*c)*d^2*f^4)))/(c^2*d^3*f + a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2)
)*log(-((B^4*b^2 - 2*A*B^3*b*c)*d^2 - 2*(A*B^3*a*b - A^3*B*b*c)*d*f + (2*A^
3*B*a*b - A^4*b^2)*f^2 + 2*((2*A^3*B*a - A^4*b)*c*f^2 + (B^4*b*c - 2*A*B^3*
c^2)*d^2 - 2*(A*B^3*a*c - A^3*B*c^2)*d*f)*x - 2*((B^3*b^2 - 3*A*B^2*b*c + 2
*A^2*B*c^2)*d^2*f - (3*A*B^2*a*b - A^2*B*b^2 - (4*A^2*B*a - A^3*b)*c)*d*f^2
 + (2*A^2*B*a^2 - A^3*a*b)*f^3 - (B*c^3*d^4*f - (B*b^2*c - (3*B*a - A*b)*c^
2)*d^3*f^2 - (B*a*b^2 - A*b^3 - (3*B*a^2 - 2*A*a*b)*c)*d^2*f^3 + (B*a^3 - A
*a^2*b)*d*f^4)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B
^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 -
4*A^3*B*a*b + A^4*b^2)*f^2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*
d^4*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2
*f^4)))*sqrt(c*x^2 + b*x + a)*sqrt((B^2*c*d^2 + A^2*a*f^2 + (B^2*a - 2*A*B*
b + A^2*c)*d*f + (c^2*d^3*f + a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2)*sqrt(((B^4
*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*
(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^
2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*b^2*
c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4)))/(c^2*d^3*f + a^2*
d*f^3 - (b^2 - 2*a*c)*d^2*f^2)) - (2*B^2*a*c^2*d^3*f - 2*A^2*a^3*f^4 - 2*(B
^2*a*b^2 - 2*B^2*a^2*c + A^2*a*c^2)*d^2*f^2 + 2*(B^2*a^3 + A^2*a*b^2 - 2*A^
2*a^2*c)*d*f^3 + (B^2*b*c^2*d^3*f - A^2*a^2*b*f^4 - (B^2*b^3 - 2*B^2*a*b*c
+ A^2*b*c^2)*d^2*f^2 + (B^2*a^2*b + A^2*b^3 - 2*A^2*a*b*c)*d*f^3)*x)*sqrt((
(B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2
- 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2
)*f^2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*
b^2*c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4)))/x) + 1/4*sqrt
((B^2*c*d^2 + A^2*a*f^2 + (B^2*a - 2*A*B*b + A^2*c)*d*f - (c^2*d^3*f + a^2*
d*f^3 - (b^2 - 2*a*c)*d^2*f^2)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2
)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (
4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2
*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^
2 - 2*a^3*c)*d^2*f^4)))/(c^2*d^3*f + a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2))*lo
g(-((B^4*b^2 - 2*A*B^3*b*c)*d^2 - 2*(A*B^3*a*b - A^3*B*b*c)*d*f + (2*A^3*B*
a*b - A^4*b^2)*f^2 + 2*((2*A^3*B*a - A^4*b)*c*f^2 + (B^4*b*c - 2*A*B^3*c^2)
*d^2 - 2*(A*B^3*a*c - A^3*B*c^2)*d*f)*x + 2*((B^3*b^2 - 3*A*B^2*b*c + 2*A^2
*B*c^2)*d^2*f - (3*A*B^2*a*b - A^2*B*b^2 - (4*A^2*B*a - A^3*b)*c)*d*f^2 + (
2*A^2*B*a^2 - A^3*a*b)*f^3 + (B*c^3*d^4*f - (B*b^2*c - (3*B*a - A*b)*c^2)*d
^3*f^2 - (B*a*b^2 - A*b^3 - (3*B*a^2 - 2*A*a*b)*c)*d^2*f^3 + (B*a^3 - A*a^2
*b)*d*f^4)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a
*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^
```

```
3*B*a*b + A^4*b^2)*f^2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*
f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4
)))*sqrt(c*x^2 + b*x + a)*sqrt((B^2*c*d^2 + A^2*a*f^2 + (B^2*a - 2*A*B*b +
A^2*c)*d*f - (c^2*d^3*f + a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2)*sqrt(((B^4*b^2
 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A
^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/(
c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*b^2*c +
6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4)))/(c^2*d^3*f + a^2*d*f^
3 - (b^2 - 2*a*c)*d^2*f^2)) + (2*B^2*a*c^2*d^3*f - 2*A^2*a^3*f^4 - 2*(B^2*a
*b^2 - 2*B^2*a^2*c + A^2*a*c^2)*d^2*f^2 + 2*(B^2*a^3 + A^2*a*b^2 - 2*A^2*a^
2*c)*d*f^3 + (B^2*b*c^2*d^3*f - A^2*a^2*b*f^4 - (B^2*b^3 - 2*B^2*a*b*c + A^
2*b*c^2)*d^2*f^2 + (B^2*a^2*b + A^2*b^3 - 2*A^2*a*b*c)*d*f^3)*x)*sqrt(((B^4
*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*
(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^
2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*b^2*
c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4)))/x) - 1/4*sqrt((B^
2*c*d^2 + A^2*a*f^2 + (B^2*a - 2*A*B*b + A^2*c)*d*f - (c^2*d^3*f + a^2*d*f^
3 - (b^2 - 2*a*c)*d^2*f^2)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^
2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^
2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2
 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 -
2*a^3*c)*d^2*f^4)))/(c^2*d^3*f + a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2))*log(-(
(B^4*b^2 - 2*A*B^3*b*c)*d^2 - 2*(A*B^3*a*b - A^3*B*b*c)*d*f + (2*A^3*B*a*b
- A^4*b^2)*f^2 + 2*((2*A^3*B*a - A^4*b)*c*f^2 + (B^4*b*c - 2*A*B^3*c^2)*d^2
 - 2*(A*B^3*a*c - A^3*B*c^2)*d*f)*x - 2*((B^3*b^2 - 3*A*B^2*b*c + 2*A^2*B*c
^2)*d^2*f - (3*A*B^2*a*b - A^2*B*b^2 - (4*A^2*B*a - A^3*b)*c)*d*f^2 + (2*A^
2*B*a^2 - A^3*a*b)*f^3 + (B*c^3*d^4*f - (B*b^2*c - (3*B*a - A*b)*c^2)*d^3*f
^2 - (B*a*b^2 - A*b^3 - (3*B*a^2 - 2*A*a*b)*c)*d^2*f^3 + (B*a^3 - A*a^2*b)*
d*f^4)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b -
 A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*
a*b + A^4*b^2)*f^2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2
+ (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4)))*
sqrt(c*x^2 + b*x + a)*sqrt((B^2*c*d^2 + A^2*a*f^2 + (B^2*a - 2*A*B*b + A^2*
c)*d*f - (c^2*d^3*f + a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2)*sqrt(((B^4*b^2 - 4
*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B
^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/(c^4*
d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*b^2*c + 6*a^
2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4)))/(c^2*d^3*f + a^2*d*f^3 -
(b^2 - 2*a*c)*d^2*f^2)) + (2*B^2*a*c^2*d^3*f - 2*A^2*a^3*f^4 - 2*(B^2*a*b^2
 - 2*B^2*a^2*c + A^2*a*c^2)*d^2*f^2 + 2*(B^2*a^3 + A^2*a*b^2 - 2*A^2*a^2*c)
*d*f^3 + (B^2*b*c^2*d^3*f - A^2*a^2*b*f^4 - (B^2*b^3 - 2*B^2*a*b*c + A^2*b*
c^2)*d^2*f^2 + (B^2*a^2*b + A^2*b^3 - 2*A^2*a*b*c)*d*f^3)*x)*sqrt(((B^4*b^2
 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A
^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/(
c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*b^2*c +
```

```
6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4)))/x)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument ValueConj Error: Bad Argument Typ
e
```

**maple [B]** time = 0.02, size = 714, normalized size = 2.87

$$-\frac{A \ln\left(\frac{\frac{2af+2cd-2\sqrt{df}\,b}{f}+\frac{\left(bf-2\sqrt{df}\,c\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{af+cd-\sqrt{df}\,b}{f}}\,\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^{2} c+\frac{\left(bf-2\sqrt{df}\,c\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+\frac{af+cd-\sqrt{df}\,b}{f}}}{x+\frac{\sqrt{df}}{f}}\right)}{2\sqrt{df}\,\sqrt{\frac{af+cd-\sqrt{df}\,b}{f}}}+\frac{A \ln\left(\frac{\frac{2af+2cd+2\sqrt{df}\,b}{f}+\frac{\left(bf+2\sqrt{df}\,c\right)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{af+cd+\sqrt{df}\,b}{f}}\,\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^{2} c+\frac{\left(bf+2\sqrt{df}\,c\right)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+\frac{af+cd+\sqrt{df}\,b}{f}}}{x-\frac{\sqrt{df}}{f}}\right)}{2\sqrt{df}\,\sqrt{\frac{af+cd+\sqrt{df}\,b}{f}}}+\frac{B \ln\left(\frac{\frac{2af+2cd-2\sqrt{df}\,b}{f}+\frac{\left(bf-2\sqrt{df}\,c\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{af+cd-\sqrt{df}\,b}{f}}\,\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^{2} c+\frac{\left(bf-2\sqrt{df}\,c\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+\frac{af+cd-\sqrt{df}\,b}{f}}}{x+\frac{\sqrt{df}}{f}}\right)}{2\sqrt{\frac{af+cd-\sqrt{df}\,b}{f}}\,f}+\frac{B \ln\left(\frac{\frac{2af+2cd+2\sqrt{df}\,b}{f}+\frac{\left(bf+2\sqrt{df}\,c\right)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{af+cd+\sqrt{df}\,b}{f}}\,\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^{2} c+\frac{\left(bf+2\sqrt{df}\,c\right)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+\frac{af+cd+\sqrt{df}\,b}{f}}}{x-\frac{\sqrt{df}}{f}}\right)}{2\sqrt{\frac{af+cd+\sqrt{df}\,b}{f}}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -1/2/(d*f)^(1/2)/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/
2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(
1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(
d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*A+1/2
/f/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/
f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))
^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+
1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*B+1/2/(d*f)^(1/2)/(
(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)
^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*
f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*
f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*A+1/2/f/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/
2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f
)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/
2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2
)/f))*B
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                  /(2*f)                  +(c*d)/f+a))
     /f^2 zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+Bx}{\left(d-fx^2\right)\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((A + B*x)/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{A}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx-\int \frac{Bx}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(A/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)), x)
 - Integral(B*x/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2))
, x)
```

## 3.8 $\int \frac{A+Bx}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx$

**Optimal**. Leaf size=381

$$-\frac{2\left(A\left(b^3f-bc(3af+cd)\right)+cx\left(-2Ac(af+cd)+bB(cd-af)+Ab^2f\right)+aB\left(2acf+b^2(-f)+2c^2d\right)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}-\frac{\sqrt{f}\,(B\cdots}{\cdots}$$

**Rubi [A]** time = 0.80, antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1018, 1033, 724, 206}

$$-\frac{2\left(cx\left(-2Ac(af+cd)+bB(cd-af)+Ab^2f\right)-Abc(3af+cd)+aB\left(2acf+b^2(-f)+2c^2d\right)+Ab^3f\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}-\frac{\sqrt{f}\left(B\sqrt{d}-A\sqrt{f}\right)\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{\sqrt{f}\left(A\sqrt{f}+B\sqrt{d}\right)\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (-2*(A*b^3*f - A*b*c*(c*d + 3*a*f) + a*B*(2*c^2*d - b^2*f + 2*a*c*f) + c*(A
*b^2*f + b*B*(c*d - a*f) - 2*A*c*(c*d + a*f))*x))/((b^2 - 4*a*c)*(b^2*d*f -
 (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) - ((B*Sqrt[d] - A*Sqrt[f])*Sqrt[f]*A
rcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d
- b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*(c*d - b*Sqr
t[d]*Sqrt[f] + a*f)^(3/2)) + ((B*Sqrt[d] + A*Sqrt[f])*Sqrt[f]*ArcTanh[(b*Sq
rt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*
Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*(c*d + b*Sqrt[d]*Sqrt[f]
 + a*f)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1018

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^
(q + 1)*((g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f
)) + c*(g*(2*c^2*d + b^2*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x))/((b^2 - 4*
a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\int \frac{A+Bx}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx = -\frac{2\left(Ab^3f - Abc(cd+3af) + aB\left(2c^2d - b^2f + 2acf\right) + c\left(Ab^2f + bB(cd - \right.\right.}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}}$$

$$= -\frac{2\left(Ab^3f - Abc(cd+3af) + aB\left(2c^2d - b^2f + 2acf\right) + c\left(Ab^2f + bB(cd - \right.\right.}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}}$$

$$= -\frac{2\left(Ab^3f - Abc(cd+3af) + aB\left(2c^2d - b^2f + 2acf\right) + c\left(Ab^2f + bB(cd - \right.\right.}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}}$$

$$= -\frac{2\left(Ab^3f - Abc(cd+3af) + aB\left(2c^2d - b^2f + 2acf\right) + c\left(Ab^2f + bB(cd - \right.\right.}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}}$$

**Mathematica [A]** time = 0.65, size = 440, normalized size = 1.15

$$\frac{2\left(\frac{B\left(2a^2cf+a\left(b^2(-f)-bcfx+2c^2d\right)+bc^2dx\right)+A\left(-bc(3af+cd)-2c^2x(af+cd)+b^3f+b^2cfx\right)}{\sqrt{a+x(b+cx)}}+\frac{\sqrt{f}\left(b^2-4ac\right)\left(A\sqrt{f}-B\sqrt{d}\right)\left(af+b\sqrt{d}\sqrt{f}+cd\right)\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{4\sqrt{d}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\sqrt{f}\left(4ac-b^2\right)\left(A\sqrt{f}+B\sqrt{d}\right)\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)\tanh^{-1}\left(\frac{-2\left(a\sqrt{f}+c\sqrt{d}x\right)-b\left(\sqrt{d}+\sqrt{f}x\right)}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{4\sqrt{d}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\left(b^2-4ac\right)\left((af+cd)^2-b^2df\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (2*((A*(b^3*f - b*c*(c*d + 3*a*f) + b^2*c*f*x - 2*c^2*(c*d + a*f)*x) + B*(2
*a^2*c*f + b*c^2*d*x + a*(2*c^2*d - b^2*f - b*c*f*x)))/Sqrt[a + x*(b + c*x)
] + ((b^2 - 4*a*c)*(-(B*Sqrt[d]) + A*Sqrt[f])*Sqrt[f]*(c*d + b*Sqrt[d]*Sqrt
[f] + a*f)*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))
/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(4*Sqrt[d]
*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + ((-b^2 + 4*a*c)*(B*Sqrt[d] + A*Sqrt
[f])*Sqrt[f]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(-2*(a*Sqrt[f] + c*Sqr
t[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*S
qrt[a + x*(b + c*x)])])/(4*Sqrt[d]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])))/(
(b^2 - 4*a*c)*(-(b^2*d*f) + (c*d + a*f)^2))
```

**IntegrateAlgebraic [C]** time = 1.50, size = 658, normalized size = 1.73

$$\frac{f\,\text{RootSum}\left[\#1^4(-f)+2\#1^2af+4\#1^2cd-4\#1b\sqrt{c}d-a^2f+b^2d\&,\frac{\#1^2Abf\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-\#1^2Bcd\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-\#1^2aBf\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+a^2Bf\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+2\#1Ac^{3/2}d\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-Abcd\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2aAbf\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+2\#1aA\sqrt{c}f\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+b^2Bd\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+aBcd\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2\#1bB\sqrt{c}d\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)}{\#1^3f-\#1af-2\#1cd+b\sqrt{c}d}\&\right]}{2\left(a^2f^2+2acdf+b^2(-d)f+c^2d^2\right)}-\frac{2\left(2a^2Bcf-3aAbcf-2aAc^2fx-ab^2Bf-abBcfx+2aBc^2d+Ab^3f+Ab^2cfx-Abc^2d-2Ac^3dx+bBc^2dx\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(-a^2f^2-2acdf+b^2df-c^2d^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x)/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (-2*(-(A*b*c^2*d) + 2*a*B*c^2*d + A*b^3*f - a*b^2*B*f - 3*a*A*b*c*f + 2*a^2
*B*c*f + b*B*c^2*d*x - 2*A*c^3*d*x + A*b^2*c*f*x - a*b*B*c*f*x - 2*a*A*c^2*
f*x))/((b^2 - 4*a*c)*(-(c^2*d^2) + b^2*d*f - 2*a*c*d*f - a^2*f^2)*Sqrt[a +
b*x + c*x^2]) + (f*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 +
2*a*f*#1^2 - f*#1^4 & , (b^2*B*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] -
 #1] - A*b*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a*B*c*d*Log
[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*a*A*b*f*Log[-(Sqrt[c]*x) +
Sqrt[a + b*x + c*x^2] - #1] + a^2*B*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x
^2] - #1] - 2*b*B*Sqrt[c]*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*
#1 + 2*A*c^(3/2)*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*a*
A*Sqrt[c]*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - B*c*d*Log[-
(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + A*b*f*Log[-(Sqrt[c]*x) + S
qrt[a + b*x + c*x^2] - #1]*#1^2 - a*B*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c
*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ])/(2*(c^2*d
^2 - b^2*d*f + 2*a*c*d*f + a^2*f^2))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 10.66Conj E
rror: Bad Argument Type
```

**maple [B]** time = 0.03, size = 2758, normalized size = 7.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] 1/2/(d*f)^(1/2)*f/(a*f+c*d-(d*f)^(1/2)*b)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*
f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*A-1/2/(a*f
+c*d-(d*f)^(1/2)*b)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(
1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*B+2/(a*f+c*d-(d*f)^(1/2)*b)/(4*a
*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a
*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*c^2*A-2*(d*f)^(1/2)/f/(a*f+c*d-(d*f)^(1/2)
*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)
/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*c^2*B-1/(d*f)^(1/2)*f/(a*f+c*d-(d*
f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*
f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*b*c*A+1/(a*f+c*d-(d*f)^(1/
2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/
2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*b*c*B+1/(a*f+c*d-(d*f)^(1/2)*b)/
(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/
f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b*c*A-(d*f)^(1/2)/f/(a*f+c*d-(d*f)^(1/2)
*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)
/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b*c*B-1/2/(d*f)^(1/2)*f/(a*f+c*d-(d*
f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*
f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b^2*A+1/2/(a*f+c*d-(d*f)^(1/
2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/
2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b^2*B-1/2/(d*f)^(1/2)*f/(a*f+c*d-(
d*f)^(1/2)*b)/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*
```

```
b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f
)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a
*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*A+1/2/(a*f+c*d-(d*f)^(1/
2)*b)/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*
f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*
((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(
d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*B-1/2/(d*f)^(1/2)*f/(a*f+c*d+(d*
f)^(1/2)*b)/((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/
f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*A-1/2/(a*f+c*d+(d*f)^(1/2)*b)/((x-(d*f)^
(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)
*b)/f)^(1/2)*B+2/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c
+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)
*x*c^2*A+2*(d*f)^(1/2)/f/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2
)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/
f)^(1/2)*x*c^2*B+1/(d*f)^(1/2)*f/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d
*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(
1/2)*b)/f)^(1/2)*x*b*c*A+1/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1
/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b
)/f)^(1/2)*x*b*c*B+1/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)
^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(
1/2)*b*c*A+(d*f)^(1/2)/f/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2
)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/
f)^(1/2)*b*c*B+1/2/(d*f)^(1/2)*f/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d
*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(
1/2)*b)/f)^(1/2)*b^2*A+1/2/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1
/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b
)/f)^(1/2)*b^2*B+1/2/(d*f)^(1/2)*f/(a*f+c*d+(d*f)^(1/2)*b)/((a*f+c*d+(d*f)^
(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-
(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c
+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)
)/(x-(d*f)^(1/2)/f))*A+1/2/(a*f+c*d+(d*f)^(1/2)*b)/((a*f+c*d+(d*f)^(1/2)*b)
/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1
/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*
(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*
f)^(1/2)/f))*B
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                       /(2*f)                  +(c*d)/f+a))
    /f^2 zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B\,x}{\left(d - f\,x^2\right)\,\left(c\,x^2 + b\,x + a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int((A + B*x)/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] Timed out
```

## 3.9 $\int \frac{A+Bx}{\left(a+bx+cx^2\right)^{5/2}\left(d-fx^2\right)}\,dx$

**Optimal**. Leaf size=797

$$-\frac{\left(B\sqrt{d}-A\sqrt{f}\right)\tanh^{-1}\left(\frac{-2\sqrt{f}\,a+\left(2c\sqrt{d}-b\sqrt{f}\right)x+b\sqrt{d}}{2\sqrt{-\sqrt{d}\,\sqrt{f}\,b+cd+af}\,\sqrt{cx^2+bx+a}}\right)f^{3/2}}{2\sqrt{d}\,\left(-\sqrt{d}\,\sqrt{f}\,b+cd+af\right)^{5/2}}+\frac{\left(\sqrt{f}\,A+B\sqrt{d}\right)\tanh^{-1}\left(\frac{2\sqrt{f}\,a+\left(\sqrt{f}\,b+2c\sqrt{d}\right)x+b\sqrt{d}}{2\sqrt{\sqrt{d}\,\sqrt{f}\,b+cd+af}\,\sqrt{cx^2+bx+a}}\right)f^{3}}{2\sqrt{d}\,\left(\sqrt{d}\,\sqrt{f}\,b+cd+af\right)^{5/2}}$$

**Rubi [A]** time = 1.87, antiderivative size = 796, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1018, 1064, 1033, 724, 206}

$$-\frac{\left(B\sqrt{d}-A\sqrt{f}\right)\tanh^{-1}\left(\frac{-2\sqrt{f}a+\left(2c\sqrt{d}-b\sqrt{f}\right)x+b\sqrt{d}}{2\sqrt{-\sqrt{d}\sqrt{f}b+cd+af}\sqrt{cx^2+bx+a}}\right)f^{3/2}}{2\sqrt{d}\left(-\sqrt{d}\sqrt{f}b+cd+af\right)^{5/2}}+\frac{\left(\sqrt{f}A+B\sqrt{d}\right)\tanh^{-1}\left(\frac{2\sqrt{f}a+\left(\sqrt{f}b+2c\sqrt{d}\right)x+b\sqrt{d}}{2\sqrt{\sqrt{d}\sqrt{f}b+cd+af}\sqrt{cx^2+bx+a}}\right)f^{3/2}}{2\sqrt{d}\left(\sqrt{d}\sqrt{f}b+cd+af\right)^{5/2}}-\frac{2\left(3Bdf^2b^6-Af^2(7cd+6af)b^5-Bf\left(7c^2d^2+14acfd-3a^2f^2\right)b^4+Acf\left(15c^2d^2+46acfd+43a^2f^2\right)b^3+2Bc\left(2c^3d^3+5ac^2fd^2+4a^2cf^2d-11a^3f^3\right)b^2-4Ac^2\left(2c^3d^3+9ac^2fd^2+24a^2cf^2d+17a^3f^3\right)b+24a^2Bc^2f(cd+af)^2+c\left(3Bdf^2b^5-2Af^2(4cd+3af)b^4-Bf\left(17c^2d^2+10acfd-3a^2f^2\right)b^3+2Acf\left(15c^2d^2+22acfd+19a^2f^2\right)b^2+4Bc\left(2c^3d^3+11ac^2fd^2+4a^2cf^2d-5a^3f^3\right)b-8Ac^2(cd+af)^2(2cd+5af)\right)x\right)}{3\left(b^2-4ac\right)^2\left(c^2d^2+2acfd-f\left(b^2d-a^2f\right)\right)^2\sqrt{cx^2+bx+a}}-\frac{2\left(Afb^3-Ac(cd+3af)b+aB\left(-fb^2+2c^2d+2acf\right)+c\left(Afb^2+B(cd-af)b-2Ac(cd+af)\right)x\right)}{3\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\left(cx^2+bx+a\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/((a + b*x + c*x^2)^(5/2)*(d - f*x^2)),x]
```

```
[Out] (-2*(A*b^3*f - A*b*c*(c*d + 3*a*f) + a*B*(2*c^2*d - b^2*f + 2*a*c*f) + c*(A
*b^2*f + b*B*(c*d - a*f) - 2*A*c*(c*d + a*f))*x))/(3*(b^2 - 4*a*c)*(b^2*d*f
 - (c*d + a*f)^2)*(a + b*x + c*x^2)^(3/2)) - (2*(3*b^6*B*d*f^2 + 24*a^2*B*c
^2*f*(c*d + a*f)^2 - A*b^5*f^2*(7*c*d + 6*a*f) - b^4*B*f*(7*c^2*d^2 + 14*a*
c*d*f - 3*a^2*f^2) + A*b^3*c*f*(15*c^2*d^2 + 46*a*c*d*f + 43*a^2*f^2) + 2*b
^2*B*c*(2*c^3*d^3 + 5*a*c^2*d^2*f + 4*a^2*c*d*f^2 - 11*a^3*f^3) - 4*A*b*c^2
*(2*c^3*d^3 + 9*a*c^2*d^2*f + 24*a^2*c*d*f^2 + 17*a^3*f^3) + c*(3*b^5*B*d*f
^2 - 2*A*b^4*f^2*(4*c*d + 3*a*f) - 8*A*c^2*(c*d + a*f)^2*(2*c*d + 5*a*f) -
b^3*B*f*(17*c^2*d^2 + 10*a*c*d*f - 3*a^2*f^2) + 2*A*b^2*c*f*(15*c^2*d^2 + 2
2*a*c*d*f + 19*a^2*f^2) + 4*b*B*c*(2*c^3*d^3 + 11*a*c^2*d^2*f + 4*a^2*c*d*f
^2 - 5*a^3*f^3))*x))/(3*(b^2 - 4*a*c)^2*(c^2*d^2 + 2*a*c*d*f - f*(b^2*d - a
^2*f))^2*Sqrt[a + b*x + c*x^2]) - ((B*Sqrt[d] - A*Sqrt[f])*f^(3/2)*ArcTanh[
(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqr
t[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*(c*d - b*Sqrt[d]*Sq
rt[f] + a*f)^(5/2)) + ((B*Sqrt[d] + A*Sqrt[f])*f^(3/2)*ArcTanh[(b*Sqrt[d] +
 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f]
 + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)
^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1018

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^
(q + 1)*((g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f
)) + c*(g*(2*c^2*d + b^2*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x))/((b^2 - 4*
a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f))))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1064

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p +
1)*(d + f*x^2)^(q + 1)*((A*c - a*C)*(-(b*(c*d + a*f))) + (A*b - a*B)*(2*c^2
*d + b^2*f - c*(2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(2*a*f)) - B*(b*c*d + a
*b*f) + C*(b^2*d - 2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a
*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p +
1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)
*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + a
*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)
) - (2*f*((A*c - a*C)*(-(b*(c*d + a*f))) + (A*b - a*B)*(2*c^2*d + b^2*f - c
*(2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + a*B*f) + 2*(A*c*(c*d
```

```
 - a*f) - a*(c*C*d - a*C*f)))*(b*f*(p + 1)))*x - c*f*(b^2*(C*d + A*f) - b*(
B*c*d + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(2*p + 2*q + 5)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] &&
 ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rubi steps

$$\int \frac{A+Bx}{\left(a+bx+cx^2\right)^{5/2}\left(d-fx^2\right)}\,dx = -\frac{2\left(Ab^3f-Abc(cd+3af)+aB\left(2c^2d-b^2f+2acf\right)+c\left(Ab^2f+bB(cd-a\right.\right.}{3\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\left(a+bx+cx^2\right)^{3/2}}$$

$$= -\frac{2\left(Ab^3f-Abc(cd+3af)+aB\left(2c^2d-b^2f+2acf\right)+c\left(Ab^2f+bB(cd-a\right.\right.}{3\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\left(a+bx+cx^2\right)^{3/2}}$$

$$= -\frac{2\left(Ab^3f-Abc(cd+3af)+aB\left(2c^2d-b^2f+2acf\right)+c\left(Ab^2f+bB(cd-a\right.\right.}{3\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\left(a+bx+cx^2\right)^{3/2}}$$

$$= -\frac{2\left(Ab^3f-Abc(cd+3af)+aB\left(2c^2d-b^2f+2acf\right)+c\left(Ab^2f+bB(cd-a\right.\right.}{3\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\left(a+bx+cx^2\right)^{3/2}}$$

$$= -\frac{2\left(Ab^3f-Abc(cd+3af)+aB\left(2c^2d-b^2f+2acf\right)+c\left(Ab^2f+bB(cd-a\right.\right.}{3\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\left(a+bx+cx^2\right)^{3/2}}$$

**Mathematica [A]** time = 4.06, size = 674, normalized size = 0.85

$$\frac{2\left(\frac{B\left(2a^2cf+a\left(b^2(-f)-bcfx+2c^2d\right)+bc^2dx\right)+A\left(-bc(3af+cd)-2c^2x(af+cd)+b^3f+b^2cfx\right)}{(a+x(b+cx))^{3/2}}-\frac{3f\left(-b^2\left(B\left(-a^2f^2+2acdf+c^2d^2\right)+2aAcf^2x\right)+b^3f(Bcdx-A(2af+cd))+bc(af+cd)(5aAf+aBfx+Acd-3Bcdx)+2c(af+cd)^2(Acx-aB)+b^4Bdf\right)}{\sqrt{a+x(b+cx)}\left(f\left(a^2f-b^2d\right)+2acdf+c^2d^2\right)}+\frac{3f^{3/2}\left(b^2-4ac\right)\left(\frac{\left(A\sqrt{f}-B\sqrt{d}\right)\left(af+b\sqrt{d}\sqrt{f}+cd\right)^2\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}-\frac{\left(A\sqrt{f}+B\sqrt{d}\right)\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^2\tanh^{-1}\left(\frac{-2\left(a\sqrt{f}+c\sqrt{d}x\right)-b\left(\sqrt{d}+\sqrt{f}x\right)}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{4\sqrt{d}\left((af+cd)^2-b^2df\right)}+\frac{4c(b+2cx)\left(2Ac(af+cd)+bB(af-cd)-Ab^2f\right)}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}}\right)}{3\left(b^2-4ac\right)\left((af+cd)^2-b^2df\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/((a + b*x + c*x^2)^(5/2)*(d - f*x^2)),x]
```

```
[Out] (2*((4*c*(-(A*b^2*f) + b*B*(-(c*d) + a*f) + 2*A*c*(c*d + a*f))*(b + 2*c*x))
/((b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]) - (3*f*(b^4*B*d*f + 2*c*(c*d + a*f)^
```

```
2*(-(a*B) + A*c*x) + b^3*f*(-(A*(c*d + 2*a*f)) + B*c*d*x) + b*c*(c*d + a*f)
*(A*c*d + 5*a*A*f - 3*B*c*d*x + a*B*f*x) - b^2*(B*(c^2*d^2 + 2*a*c*d*f - a^
2*f^2) + 2*a*A*c*f^2*x)))/((c^2*d^2 + 2*a*c*d*f + f*(-(b^2*d) + a^2*f))*Sqr
t[a + x*(b + c*x)]) + (A*(b^3*f - b*c*(c*d + 3*a*f) + b^2*c*f*x - 2*c^2*(c*
d + a*f)*x) + B*(2*a^2*c*f + b*c^2*d*x + a*(2*c^2*d - b^2*f - b*c*f*x)))/(a
 + x*(b + c*x))^(3/2) + (3*(b^2 - 4*a*c)*f^(3/2)*(((-(B*Sqrt[d]) + A*Sqrt[f
])*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^2*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x
+ b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a +
x*(b + c*x)])])/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f] - ((B*Sqrt[d] + A*Sqrt[
f])*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^2*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x)
 - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a +
 x*(b + c*x)])])/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]))/(4*Sqrt[d]*(-(b^2*d*
f) + (c*d + a*f)^2))))/(3*(b^2 - 4*a*c)*(-(b^2*d*f) + (c*d + a*f)^2))
```

**IntegrateAlgebraic [F]** time = 180.08, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x)/((a + b*x + c*x^2)^(5/2)*(d - f*x^2)),x]
```

```
[Out] $Aborted
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(5/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(5/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 5.84Conj Er
ror: Bad Argument Type
```

**maple [B]** time = 0.03, size = 6422, normalized size = 8.06

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(c*x^2+b*x+a)^(5/2)/(-f*x^2+d),x)
```

```
[Out] result too large to display
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(5/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                   /(2*f)                 +(c*d)/f+a))
     /f^2 zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,x}{\left(d-f\,x^2\right)\,{\left(c\,x^2+b\,x+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/((d - f*x^2)*(a + b*x + c*x^2)^(5/2)),x)
```

```
[Out] int((A + B*x)/((d - f*x^2)*(a + b*x + c*x^2)^(5/2)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)**(5/2)/(-f*x**2+d),x)
```

```
[Out] Timed out
```

## 3.10 $\int \frac{1+2x}{(-1+x^2)\sqrt{-1+x+x^2}}\,dx$

**Optimal**. Leaf size=47

$$\frac{3}{2}\tanh^{-1}\left(\frac{1-3x}{2\sqrt{x^2+x-1}}\right)-\frac{1}{2}\tan^{-1}\left(\frac{x+3}{2\sqrt{x^2+x-1}}\right)$$

**Rubi [A]** time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1033, 724, 206, 204}

$$\frac{3}{2}\tanh^{-1}\left(\frac{1-3x}{2\sqrt{x^2+x-1}}\right)-\frac{1}{2}\tan^{-1}\left(\frac{x+3}{2\sqrt{x^2+x-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)/((-1 + x^2)*Sqrt[-1 + x + x^2]),x]
```

```
[Out] -ArcTan[(3 + x)/(2*Sqrt[-1 + x + x^2])]/2 + (3*ArcTanh[(1 - 3*x)/(2*Sqrt[-1
 + x + x^2])])/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
```

```
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\int \frac{1+2x}{\left(-1+x^2\right)\sqrt{-1+x+x^2}}\,dx = \frac{1}{2}\int \frac{1}{(1+x)\sqrt{-1+x+x^2}}\,dx + \frac{3}{2}\int \frac{1}{(-1+x)\sqrt{-1+x+x^2}}\,dx$$
$$= -\left(3\,\mathrm{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{-1+3x}{\sqrt{-1+x+x^2}}\right)\right) - \mathrm{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, \frac{-3}{\sqrt{-1+}}\right.$$
$$= \frac{1}{2}\tan^{-1}\left(\frac{-3-x}{2\sqrt{-1+x+x^2}}\right) + \frac{3}{2}\tanh^{-1}\left(\frac{1-3x}{2\sqrt{-1+x+x^2}}\right)$$

**Mathematica [A]** time = 0.01, size = 49, normalized size = 1.04

$$\frac{1}{2}\tan^{-1}\left(\frac{-x-3}{2\sqrt{x^2+x-1}}\right) - \frac{3}{2}\tanh^{-1}\left(\frac{3x-1}{2\sqrt{x^2+x-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)/((-1 + x^2)*Sqrt[-1 + x + x^2]),x]
```

```
[Out] ArcTan[(-3 - x)/(2*Sqrt[-1 + x + x^2])]/2 - (3*ArcTanh[(-1 + 3*x)/(2*Sqrt[-
1 + x + x^2])])/2
```

**IntegrateAlgebraic [A]** time = 0.22, size = 37, normalized size = 0.79

$$-\tan^{-1}\left(-\sqrt{x^2+x-1}+x+1\right) - 3\tanh^{-1}\left(\sqrt{x^2+x-1}-x+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 2*x)/((-1 + x^2)*Sqrt[-1 + x + x^2]),x]
```

```
[Out] -ArcTan[1 + x - Sqrt[-1 + x + x^2]] - 3*ArcTanh[1 - x + Sqrt[-1 + x + x^2]]
```

**fricas [A]** time = 0.63, size = 46, normalized size = 0.98

$$\arctan\left(-x+\sqrt{x^2+x-1}-1\right) - \frac{3}{2}\log\left(-x+\sqrt{x^2+x-1}+2\right) + \frac{3}{2}\log\left(-x+\sqrt{x^2+x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2-1)/(x^2+x-1)^(1/2),x, algorithm="fricas")
```

```
[Out] arctan(-x + sqrt(x^2 + x - 1) - 1) - 3/2*log(-x + sqrt(x^2 + x - 1) + 2) +
3/2*log(-x + sqrt(x^2 + x - 1))
```

**giac [A]** time = 0.25, size = 48, normalized size = 1.02

$$\arctan\left(-x+\sqrt{x^2+x-1}-1\right)-\frac{3}{2}\log\left(\left|-x+\sqrt{x^2+x-1}+2\right|\right)+\frac{3}{2}\log\left(\left|-x+\sqrt{x^2+x-1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2-1)/(x^2+x-1)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(-x + sqrt(x^2 + x - 1) - 1) - 3/2*log(abs(-x + sqrt(x^2 + x - 1) + 2
)) + 3/2*log(abs(-x + sqrt(x^2 + x - 1)))
```

**maple [A]** time = 0.02, size = 46, normalized size = 0.98

$$-\frac{3\,\mathrm{arctanh}\left(\frac{3x-1}{2\sqrt{3x+(x-1)^2-2}}\right)}{2}+\frac{\arctan\left(\frac{-x-3}{2\sqrt{-x+(x+1)^2-2}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)/(x^2-1)/(x^2+x-1)^(1/2),x)
```

```
[Out] -3/2*arctanh(1/2*(-1+3*x)/((x-1)^2+3*x-2)^(1/2))+1/2*arctan(1/2*(-3-x)/((1+
x)^2-x-2)^(1/2))
```

**maxima [A]** time = 0.97, size = 65, normalized size = 1.38

$$-\frac{1}{2}\arcsin\left(\frac{2\sqrt{5}\,x}{5\,|2\,x+2|}+\frac{6\sqrt{5}}{5\,|2\,x+2|}\right)-\frac{3}{2}\log\left(\frac{2\sqrt{x^2+x-1}}{|2\,x-2|}+\frac{2}{|2\,x-2|}+\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2-1)/(x^2+x-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*arcsin(2/5*sqrt(5)*x/abs(2*x + 2) + 6/5*sqrt(5)/abs(2*x + 2)) - 3/2*lo
g(2*sqrt(x^2 + x - 1)/abs(2*x - 2) + 2/abs(2*x - 2) + 3/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{2\,x+1}{\left(x^2-1\right)\,\sqrt{x^2+x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)/((x^2 - 1)*(x + x^2 - 1)^(1/2)),x)
```

```
[Out] int((2*x + 1)/((x^2 - 1)*(x + x^2 - 1)^(1/2)), x)
```

**sympy** **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+1}{(x-1)(x+1)\sqrt{x^2+x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x**2-1)/(x**2+x-1)**(1/2),x)
```

```
[Out] Integral((2*x + 1)/((x - 1)*(x + 1)*sqrt(x**2 + x - 1)), x)
```

## 3.11 $\int \frac{1+2x}{(1+x^2)\sqrt{-1+x+x^2}}\,dx$

**Optimal**. Leaf size=117

$$\sqrt{\frac{1}{2}\left(\sqrt{5}-2\right)}\tanh^{-1}\left(\frac{\sqrt{5}x-2\sqrt{5}+5}{\sqrt{10\left(\sqrt{5}-2\right)}\sqrt{x^2+x-1}}\right)-\sqrt{\frac{1}{2}\left(2+\sqrt{5}\right)}\tan^{-1}\left(\frac{-\sqrt{5}x+2\sqrt{5}+5}{\sqrt{10\left(2+\sqrt{5}\right)}\sqrt{x^2+x-1}}\right)$$

**Rubi** [A] time = 0.17, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1036, 1030, 207, 203}

$$\sqrt{\frac{1}{2}\left(\sqrt{5}-2\right)}\tanh^{-1}\left(\frac{\sqrt{5}x-2\sqrt{5}+5}{\sqrt{10\left(\sqrt{5}-2\right)}\sqrt{x^2+x-1}}\right)-\sqrt{\frac{1}{2}\left(2+\sqrt{5}\right)}\tan^{-1}\left(\frac{-\sqrt{5}x+2\sqrt{5}+5}{\sqrt{10\left(2+\sqrt{5}\right)}\sqrt{x^2+x-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)/((1 + x^2)*Sqrt[-1 + x + x^2]),x]
```

```
[Out] -(Sqrt[(2 + Sqrt[5])/2]*ArcTan[(5 + 2*Sqrt[5] - Sqrt[5]*x)/(Sqrt[10*(2 + Sq
rt[5])]*Sqrt[-1 + x + x^2])]) + Sqrt[(-2 + Sqrt[5])/2]*ArcTanh[(5 - 2*Sqrt[
5] + Sqrt[5]*x)/(Sqrt[10*(-2 + Sqrt[5])]*Sqrt[-1 + x + x^2])]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1030

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1036

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[-(a*h*e) - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[-(a*h*e) - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[-(a*c)]
```

Rubi steps

$$\int \frac{1+2x}{(1+x^2)\sqrt{-1+x+x^2}}\,dx = -\frac{\int \frac{-\sqrt{5}+\left(-5-2\sqrt{5}\right)x}{(1+x^2)\sqrt{-1+x+x^2}}\,dx}{2\sqrt{5}} + \frac{\int \frac{\sqrt{5}+\left(-5+2\sqrt{5}\right)x}{(1+x^2)\sqrt{-1+x+x^2}}\,dx}{2\sqrt{5}}$$

$$= -\left(\left(-5+2\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{10\left(2-\sqrt{5}\right)+x^2}\,dx, x, \frac{-5+2\sqrt{5}-\sqrt{5}\,x}{\sqrt{-1+x+x^2}}\right)\right) + \left(5+2\sqrt{\vphantom{5}}\right.$$

$$= -\sqrt{\frac{1}{2}\left(2+\sqrt{5}\right)}\tan^{-1}\left(\frac{5+2\sqrt{5}-\sqrt{5}\,x}{\sqrt{10\left(2+\sqrt{5}\right)}\sqrt{-1+x+x^2}}\right) + \sqrt{\frac{1}{2}\left(-2+\sqrt{5}\right)}\tanh^{-1}$$

**Mathematica [C]** time = 0.03, size = 78, normalized size = 0.67

$$-\frac{1}{2}i\left(\sqrt{2+i}\tanh^{-1}\left(\frac{\sqrt{2+i}\,(x-i)}{2\sqrt{x^2+x-1}}\right)-\sqrt{2-i}\tanh^{-1}\left(\frac{\sqrt{2-i}\,(x+i)}{2\sqrt{x^2+x-1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)/((1 + x^2)*Sqrt[-1 + x + x^2]),x]
```

```
[Out] (-1/2*I)*(Sqrt[2 + I]*ArcTanh[(Sqrt[2 + I]*(-I + x))/(2*Sqrt[-1 + x + x^2])
] - Sqrt[2 - I]*ArcTanh[(Sqrt[2 - I]*(I + x))/(2*Sqrt[-1 + x + x^2])])
```

**IntegrateAlgebraic [C]** time = 0.24, size = 106, normalized size = 0.91

$$\frac{1}{2}\text{RootSum}\left[\#1^4+6\#1^2-4\#1+2\&, \frac{2\#1^2\log\left(-\#1+\sqrt{x^2+x-1}-x\right)-2\#1\log\left(-\#1+\sqrt{x^2+x-1}-x\right)+3\log\left(-\#1+\sqrt{x^2+x-1}-x\right)}{\#1^3+3\#1-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 2*x)/((1 + x^2)*Sqrt[-1 + x + x^2]),x]
```

```
[Out] RootSum[2 - 4*#1 + 6*#1^2 + #1^4 & , (3*Log[-x + Sqrt[-1 + x + x^2] - #1] -
 2*Log[-x + Sqrt[-1 + x + x^2] - #1]*#1 + 2*Log[-x + Sqrt[-1 + x + x^2] - #
1]*#1^2)/(-1 + 3*#1 + #1^3) & ]/2
```

**fricas [B]** time = 0.56, size = 758, normalized size = 6.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+1)/(x^2+x-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/20*5^(1/4)*sqrt(4*sqrt(5) + 10)*(2*sqrt(5) - 5)*log(2*x^2 - 2*sqrt(x^2 +
x - 1)*x + 1/5*(5^(1/4)*sqrt(x^2 + x - 1)*(2*sqrt(5) - 5) - 5^(1/4)*(sqrt(5
)*(2*x + 1) - 5*x))*sqrt(4*sqrt(5) + 10) + x + sqrt(5)) - 1/20*5^(1/4)*sqrt
(4*sqrt(5) + 10)*(2*sqrt(5) - 5)*log(2*x^2 - 2*sqrt(x^2 + x - 1)*x - 1/5*(5
^(1/4)*sqrt(x^2 + x - 1)*(2*sqrt(5) - 5) - 5^(1/4)*(sqrt(5)*(2*x + 1) - 5*x
))*sqrt(4*sqrt(5) + 10) + x + sqrt(5)) - 1/5*5^(3/4)*sqrt(4*sqrt(5) + 10)*a
rctan(2/55*sqrt(5)*(sqrt(5)*(2*x - 1) + 3*x + 4) + 1/275*sqrt(10*x^2 - 10*s
qrt(x^2 + x - 1)*x + (5^(1/4)*sqrt(x^2 + x - 1)*(2*sqrt(5) - 5) - 5^(1/4)*(
sqrt(5)*(2*x + 1) - 5*x))*sqrt(4*sqrt(5) + 10) + 5*x + 5*sqrt(5))*((5^(3/4)
*(2*sqrt(5) + 3) + 2*5^(1/4)*(4*sqrt(5) - 5))*sqrt(4*sqrt(5) + 10) + 2*sqrt
(5)*(3*sqrt(5) + 10) - 20*sqrt(5) + 80) - 2/55*sqrt(x^2 + x - 1)*(sqrt(5)*(
2*sqrt(5) + 3) + 8*sqrt(5) - 10) + 1/55*sqrt(5)*(16*x + 3) + 1/275*(5^(3/4)
*(sqrt(5)*(3*x + 4) + 10*x - 5) - sqrt(x^2 + x - 1)*(5^(3/4)*(3*sqrt(5) + 1
0) - 10*5^(1/4)*(sqrt(5) - 4)) - 10*5^(1/4)*(sqrt(5)*(x - 6) - 4*x + 13))*s
qrt(4*sqrt(5) + 10) - 4/11*x + 2/11) - 1/5*5^(3/4)*sqrt(4*sqrt(5) + 10)*arc
tan(-2/55*sqrt(5)*(sqrt(5)*(2*x - 1) + 3*x + 4) + 1/275*sqrt(10*x^2 - 10*sq
rt(x^2 + x - 1)*x - (5^(1/4)*sqrt(x^2 + x - 1)*(2*sqrt(5) - 5) - 5^(1/4)*(s
qrt(5)*(2*x + 1) - 5*x))*sqrt(4*sqrt(5) + 10) + 5*x + 5*sqrt(5))*((5^(3/4)*
(2*sqrt(5) + 3) + 2*5^(1/4)*(4*sqrt(5) - 5))*sqrt(4*sqrt(5) + 10) - 2*sqrt(
5)*(3*sqrt(5) + 10) + 20*sqrt(5) - 80) + 2/55*sqrt(x^2 + x - 1)*(sqrt(5)*(2
*sqrt(5) + 3) + 8*sqrt(5) - 10) - 1/55*sqrt(5)*(16*x + 3) + 1/275*(5^(3/4)*
(sqrt(5)*(3*x + 4) + 10*x - 5) - sqrt(x^2 + x - 1)*(5^(3/4)*(3*sqrt(5) + 10
) - 10*5^(1/4)*(sqrt(5) - 4)) - 10*5^(1/4)*(sqrt(5)*(x - 6) - 4*x + 13))*sq
rt(4*sqrt(5) + 10) + 4/11*x - 2/11)
```

**giac [B]** time = 0.38, size = 457, normalized size = 3.91

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+1)/(x^2+x-1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2*sqrt(5) - 4)*log(16*(15*sqrt(5)*(x - sqrt(x^2 + x - 1)) + 33*x +
 5*sqrt(5) - 33*sqrt(x^2 + x - 1) + 2*sqrt(5*sqrt(5) + 11) + 11)^2 + 16*(5*
sqrt(5)*(x - sqrt(x^2 + x - 1)) + 11*x - 5*sqrt(5)*sqrt(5*sqrt(5) + 11) - 1
```

```
5*sqrt(5) - 11*sqrt(x^2 + x - 1) - 11*sqrt(5*sqrt(5) + 11) - 33)^2) - 1/4*s
qrt(2*sqrt(5) - 4)*log(16*(15*sqrt(5)*(x - sqrt(x^2 + x - 1)) + 33*x + 5*sq
rt(5) - 33*sqrt(x^2 + x - 1) - 2*sqrt(5*sqrt(5) + 11) + 11)^2 + 16*(5*sqrt(
5)*(x - sqrt(x^2 + x - 1)) + 11*x + 5*sqrt(5)*sqrt(5*sqrt(5) + 11) - 15*sqr
t(5) - 11*sqrt(x^2 + x - 1) + 11*sqrt(5*sqrt(5) + 11) - 33)^2) + 1/2*sqrt(2
*sqrt(5) - 4)*(arctan(3) + arctan(1/10*(x - sqrt(x^2 + x - 1))*(sqrt(5)*sqr
t(5*sqrt(5) + 11) + 4*sqrt(5) - 5*sqrt(5*sqrt(5) + 11)) - 7/10*sqrt(5)*sqrt
(5*sqrt(5) + 11) + 1/5*sqrt(5) + 3/2*sqrt(5*sqrt(5) + 11)))/(sqrt(5) - 2) -
 1/2*sqrt(2*sqrt(5) - 4)*(arctan(3) + arctan(-1/10*(x - sqrt(x^2 + x - 1))*
(sqrt(5)*sqrt(5*sqrt(5) + 11) - 4*sqrt(5) - 5*sqrt(5*sqrt(5) + 11)) + 7/10*
sqrt(5)*sqrt(5*sqrt(5) + 11) + 1/5*sqrt(5) - 3/2*sqrt(5*sqrt(5) + 11)))/(sq
rt(5) - 2)
```

**maple [B]** time = 0.18, size = 637, normalized size = 5.44

$$\frac{\sqrt{\frac{10(x-\sqrt{5}-2)^2}{(-x-\sqrt{5}+2)^2}-\frac{5\sqrt{5}\,(x-\sqrt{5}-2)^2}{(-x-\sqrt{5}+2)^2}+10+5\sqrt{5}}\,\sqrt{5}\left(\operatorname{arctanh}\left(\frac{\sqrt{\frac{10(x-\sqrt{5}-2)^2}{(-x-\sqrt{5}+2)^2}-\frac{5\sqrt{5}\,(x-\sqrt{5}-2)^2}{(-x-\sqrt{5}+2)^2}+10+5\sqrt{5}}}{\sqrt{20+10\sqrt{5}}}\right)+\sqrt{5}\,\arctan\left(\frac{\sqrt{5}\,\sqrt{(\sqrt{5}-2)\left(-\frac{(x-\sqrt{5}-2)^2}{(-x-\sqrt{5}+2)^2}+4\sqrt{5}+9\right)}\,\sqrt{20+10\sqrt{5}}\,\left(\frac{\sqrt{5}\,(x-\sqrt{5}-2)^2}{(-x-\sqrt{5}+2)^2}+\frac{2(x-\sqrt{5}-2)^2}{(-x-\sqrt{5}+2)^2}-\sqrt{5}+2\right)(x-\sqrt{5}-2)(\sqrt{5}-2)}{5(-x-\sqrt{5}+2)\left(\frac{(x-\sqrt{5}-2)^4}{(-x-\sqrt{5}+2)^4}-\frac{18(x-\sqrt{5}-2)^2}{(-x-\sqrt{5}+2)^2}+1\right)}\right)+2\arctan\left(\frac{\sqrt{5}\,\sqrt{(\sqrt{5}-2)\left(-\frac{(x-\sqrt{5}-2)^2}{(-x-\sqrt{5}+2)^2}+4\sqrt{5}+9\right)}\,\sqrt{20+10\sqrt{5}}\,\left(\frac{\sqrt{5}\,(x-\sqrt{5}-2)^2}{(-x-\sqrt{5}+2)^2}+\frac{2(x-\sqrt{5}-2)^2}{(-x-\sqrt{5}+2)^2}-\sqrt{5}+2\right)(x-\sqrt{5}-2)(\sqrt{5}-2)}{5(-x-\sqrt{5}+2)\left(\frac{(x-\sqrt{5}-2)^4}{(-x-\sqrt{5}+2)^4}-\frac{18(x-\sqrt{5}-2)^2}{(-x-\sqrt{5}+2)^2}+1\right)}\right)\right)}{\sqrt{-\frac{5\left(\frac{\sqrt{5}\,(x-\sqrt{5}-2)^2}{(-x-\sqrt{5}+2)^2}-\frac{2(x-\sqrt{5}-2)^2}{(-x-\sqrt{5}+2)^2}-\sqrt{5}-2\right)}{\left(\frac{x-\sqrt{5}-2}{-x-\sqrt{5}+2}+1\right)^2}}\,\left(\frac{x-\sqrt{5}-2}{-x-\sqrt{5}+2}+1\right)\sqrt{20+10\sqrt{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)/(x^2+1)/(x^2+x-1)^(1/2),x)
```

```
[Out] (10*(-5^(1/2)-2+x)^2/(-5^(1/2)+2-x)^2-5*5^(1/2)*(-5^(1/2)-2+x)^2/(-5^(1/2)+
2-x)^2+10+5*5^(1/2))^(1/2)*5^(1/2)*(arctan(1/5*5^(1/2)*((5^(1/2)-2)*(-(-5^(
1/2)-2+x)^2/(-5^(1/2)+2-x)^2+4*5^(1/2)+9))^(1/2)*(20+10*5^(1/2))^(1/2)*(5^(
1/2)*(-5^(1/2)-2+x)^2/(-5^(1/2)+2-x)^2+2*(-5^(1/2)-2+x)^2/(-5^(1/2)+2-x)^2-
5^(1/2)+2)*(-5^(1/2)-2+x)/(-5^(1/2)+2-x)*(5^(1/2)-2)/((-5^(1/2)-2+x)^4/(-5^
(1/2)+2-x)^4-18*(-5^(1/2)-2+x)^2/(-5^(1/2)+2-x)^2+1))*5^(1/2)+arctanh((10*(
-5^(1/2)-2+x)^2/(-5^(1/2)+2-x)^2-5*5^(1/2)*(-5^(1/2)-2+x)^2/(-5^(1/2)+2-x)^
2+10+5*5^(1/2))^(1/2)/(20+10*5^(1/2))^(1/2))+2*arctan(1/5*5^(1/2)*((5^(1/2)
-2)*(-(-5^(1/2)-2+x)^2/(-5^(1/2)+2-x)^2+4*5^(1/2)+9))^(1/2)*(20+10*5^(1/2))
^(1/2)*(5^(1/2)*(-5^(1/2)-2+x)^2/(-5^(1/2)+2-x)^2+2*(-5^(1/2)-2+x)^2/(-5^(1
/2)+2-x)^2-5^(1/2)+2)*(-5^(1/2)-2+x)/(-5^(1/2)+2-x)*(5^(1/2)-2)/((-5^(1/2)-
2+x)^4/(-5^(1/2)+2-x)^4-18*(-5^(1/2)-2+x)^2/(-5^(1/2)+2-x)^2+1)))/(-5*(5^(1
/2)*(-5^(1/2)-2+x)^2/(-5^(1/2)+2-x)^2-2*(-5^(1/2)-2+x)^2/(-5^(1/2)+2-x)^2-5
^(1/2)-2)/((-5^(1/2)-2+x)/(-5^(1/2)+2-x)+1)^2)^(1/2)/((-5^(1/2)-2+x)/(-5^(1
/2)+2-x)+1)/(20+10*5^(1/2))^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x+1}{\sqrt{x^2+x-1}\,\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+1)/(x^2+x-1)^(1/2),x, algorithm="maxima")
[Out] integrate((2*x + 1)/(sqrt(x^2 + x - 1)*(x^2 + 1)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{2\,x+1}{\left(x^2+1\right)\,\sqrt{x^2+x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)/((x^2 + 1)*(x + x^2 - 1)^(1/2)),x)
[Out] int((2*x + 1)/((x^2 + 1)*(x + x^2 - 1)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+1}{\left(x^2+1\right)\sqrt{x^2+x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x**2+1)/(x**2+x-1)**(1/2),x)
[Out] Integral((2*x + 1)/((x**2 + 1)*sqrt(x**2 + x - 1)), x)
```

## 3.12 $\int \frac{a-c+bx}{\left(1+x^2\right)\sqrt{a+bx+cx^2}}\,dx$

**Optimal**. Leaf size=484

$$-\frac{\sqrt{-a\left(2c-\sqrt{a^2-2ac+b^2+c^2}\right)+c\left(c-\sqrt{a^2-2ac+b^2+c^2}\right)+a^2+b^2}\,\tan^{-1}\left(\frac{b\sqrt{a^2-2ac+b^2+c^2}-x\cdots}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+c^2}\sqrt{-a\left(2c-\sqrt{a^2-2ac\cdots}\right.}}\right.}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+c^2}}$$

**Rubi [A]** time = 23.58, antiderivative size = 484, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1036, 1030, 208, 205}

$$-\frac{\sqrt{-a\left(2c-\sqrt{a^2-2ac+b^2+c^2}\right)+c\left(c-\sqrt{a^2-2ac+b^2+c^2}\right)+a^2+b^2}\,\tan^{-1}\left(\frac{b\sqrt{a^2-2ac+b^2+c^2}-x\left((a-c)\left(\sqrt{a^2-2ac+b^2+c^2}+a-c\right)+b^2\right)}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+c^2}\sqrt{-a\left(2c-\sqrt{a^2-2ac+b^2+c^2}\right)+c\left(c-\sqrt{a^2-2ac+b^2+c^2}\right)+a^2+b^2}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+c^2}}-\frac{\sqrt{-a\left(\sqrt{a^2-2ac+b^2+c^2}+2c\right)+c\left(\sqrt{a^2-2ac+b^2+c^2}+c\right)+a^2+b^2}\,\tanh^{-1}\left(\frac{x\left((a-c)\left(-\sqrt{a^2-2ac+b^2+c^2}+a-c\right)+b^2\right)+b\sqrt{a^2-2ac+b^2+c^2}}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+c^2}\sqrt{-a\left(\sqrt{a^2-2ac+b^2+c^2}+2c\right)+c\left(\sqrt{a^2-2ac+b^2+c^2}+c\right)+a^2+b^2}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - c + b*x)/((1 + x^2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -((Sqrt[a^2 + b^2 + c*(c - Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c - Sqrt[a
^2 + b^2 - 2*a*c + c^2])]*ArcTan[(b*Sqrt[a^2 + b^2 - 2*a*c + c^2] - (b^2 +
(a - c)*(a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2]))*x)/(Sqrt[2]*(a^2 + b^2 - 2
*a*c + c^2)^(1/4)*Sqrt[a^2 + b^2 + c*(c - Sqrt[a^2 + b^2 - 2*a*c + c^2]) -
a*(2*c - Sqrt[a^2 + b^2 - 2*a*c + c^2])]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*
(a^2 + b^2 - 2*a*c + c^2)^(1/4))) - (Sqrt[a^2 + b^2 + c*(c + Sqrt[a^2 + b^2
 - 2*a*c + c^2]) - a*(2*c + Sqrt[a^2 + b^2 - 2*a*c + c^2])]*ArcTanh[(b*Sqrt
[a^2 + b^2 - 2*a*c + c^2] + (b^2 + (a - c)*(a - c - Sqrt[a^2 + b^2 - 2*a*c
+ c^2]))*x)/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4)*Sqrt[a^2 + b^2 + c*(c
+ Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c + Sqrt[a^2 + b^2 - 2*a*c + c^2])]
*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1030

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1036

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[-(a*h*e) - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[-(a*h*e) - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[-(a*c)]
```

Rubi steps

$$\int \frac{a-c+bx}{\left(1+x^2\right)\sqrt{a+bx+cx^2}}\,dx = -\frac{\int \frac{-b^2-(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)-b\sqrt{a^2+b^2-2ac+c^2}\,x}{\left(1+x^2\right)\sqrt{a+bx+cx^2}}\,dx}{2\sqrt{a^2+b^2-2ac+c^2}} + \frac{\int \frac{-b^2-(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c}\right.}{\left(1+x^2\right)\sqrt{a+bx}}}{2\sqrt{a^2+b^2-2}}$$

$$= \left(b\left(b^2+(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)\right)\text{Subst}\left(\int \frac{}{-2b\sqrt{a^2+b^2-2ac}}\right.$$

$$= -\frac{\sqrt{a^2+b^2+c\left(c-\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c-\sqrt{a^2+b^2-2ac+c^2}\right)}\tan^{-}}{\sqrt{2}\sqrt[4]{a^2+b^2}}$$

**Mathematica [C]** time = 0.08, size = 136, normalized size = 0.28

$$\frac{1}{2}i\left(\sqrt{a+ib-c}\tanh^{-1}\left(\frac{2a+b(x+i)+2icx}{2\sqrt{a+ib-c}\sqrt{a+x(b+cx)}}\right)-\sqrt{a-ib-c}\tanh^{-1}\left(\frac{2a+b(x-i)-2icx}{2\sqrt{a-ib-c}\sqrt{a+x(b+cx)}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - c + b*x)/((1 + x^2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (I/2)*(-(Sqrt[a - I*b - c]*ArcTanh[(2*a - (2*I)*c*x + b*(-I + x))/(2*Sqrt[a
 - I*b - c]*Sqrt[a + x*(b + c*x)])]) + Sqrt[a + I*b - c]*ArcTanh[(2*a + (2*
I)*c*x + b*(I + x))/(2*Sqrt[a + I*b - c]*Sqrt[a + x*(b + c*x)])])
```

**IntegrateAlgebraic [C]** time = 0.42, size = 210, normalized size = 0.43

$$\frac{1}{2}\text{RootSum}\left[\#1^4-2\#1^2a+4\#1^2c-4\#1b\sqrt{c}+a^2+b^2\&,\frac{\#1^2(-b)\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)-2\#1c^{3/2}\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)+2\#1a\sqrt{c}\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)+bc\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)}{-\#1^3+\#1a-2\#1c+b\sqrt{c}}\&\right]$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - c + b*x)/((1 + x^2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] RootSum[a^2 + b^2 - 4*b*Sqrt[c]*#1 - 2*a*#1^2 + 4*c*#1^2 + #1^4 & , (b*c*Lo
g[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*a*Sqrt[c]*Log[-(Sqrt[c]*x)
 + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*c^(3/2)*Log[-(Sqrt[c]*x) + Sqrt[a + b
*x + c*x^2] - #1]*#1 - b*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^
2)/(b*Sqrt[c] + a*#1 - 2*c*#1 - #1^3) & ]/2
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a-c)/(x^2+1)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a-c)/(x^2+1)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b,c]=[42,62,91]Warning, choosing root of [1,0,%%%{-2,[2,0,0]%%%
}+%%%{4,[1,0,1]%%%}+%%%{4,[1,0,0]%%%}+%%%{-2,[0,2,0]%%%}+%%%{-2,[0,0,2]%%%}
+%%%{-4,[0,0,1]%%%},%%%{-8,[2,0,0]%%%}+%%%{16,[1,0,1]%%%}+%%%{-8,[0,2,0]%%%
}+%%%{-8,[0,0,2]%%%},%%%{1,[4,0,0]%%%}+%%%{-4,[3,0,1]%%%}+%%%{4,[3,0,0]%%%}
+%%%{2,[2,2,0]%%%}+%%%{6,[2,0,2]%%%}+%%%{-12,[2,0,1]%%%}+%%%{-4,[1,2,1]%%%}
+%%%{4,[1,2,0]%%%}+%%%{-4,[1,0,3]%%%}+%%%{12,[1,0,2]%%%}+%%%{1,[0,4,0]%%%}+
%%%{2,[0,2,2]%%%}+%%%{-4,[0,2,1]%%%}+%%%{-4,[0,2,0]%%%}+%%%{1,[0,0,4]%%%}+%
%%{-4,[0,0,3]%%%}] at parameters values [-27,26,-89]Warning, need to choose
 a branch for the root of a polynomial with parameters. This might be wrong
.The choice was done assuming [a,b]=[-66,-52]Evaluation time: 7.83Unable to
 convert to real %%%{1.0,[2]%%%}+%%%{132.0,[1]%%%}+%%%{7060.0,[0]%%%} Error
: Bad Argument Value
```

**maple [B]** time = 0.52, size = 6871419, normalized size = 14197.15

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a-c)/(x^2+1)/(c*x^2+b*x+a)^(1/2),x)
[Out] result too large to display
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx + a - c}{\sqrt{cx^2 + bx + a}\left(x^2 + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a-c)/(x^2+1)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x + a - c)/(sqrt(c*x^2 + b*x + a)*(x^2 + 1)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a - c + b\,x}{\left(x^2 + 1\right)\,\sqrt{c\,x^2 + b\,x + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - c + b*x)/((x^2 + 1)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((a - c + b*x)/((x^2 + 1)*(a + b*x + c*x^2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx - c}{\left(x^2 + 1\right)\sqrt{a + bx + cx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a-c)/(x**2+1)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((a + b*x - c)/((x**2 + 1)*sqrt(a + b*x + c*x**2)), x)
```

## 3.13 $\int \frac{(A+Bx)(a+bx+cx^2)}{d+ex+fx^2}\,dx$

**Optimal**. Leaf size=184

$$-\frac{\log\left(d+ex+fx^2\right)\left(Af(ce-bf)-B\left(af^2-bef-cdf+ce^2\right)\right)}{2f^3}-\frac{\tanh^{-1}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)\left(Af\left(2af^2-bef-2cdf+ce^2\right.\right.}{f^3\sqrt{e^2}}$$

**Rubi** [A] time = 0.35, antiderivative size = 182, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1628, 634, 618, 206, 628}

$$-\frac{\log\left(d+ex+fx^2\right)\left(Bf(be-af)+Af(ce-bf)-Bc\left(e^2-df\right)\right)}{2f^3}-\frac{\tanh^{-1}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)\left(Af\left(2af^2-bef-2cdf+ce^2\right)+Bf\left(-aef-2bdf+be^2\right)-Bc\left(e^3-3def\right)\right)}{f^3\sqrt{e^2-4df}}-\frac{x(-Acf-bBf+Bce)}{f^2}+\frac{Bcx^2}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x + c*x^2))/(d + e*x + f*x^2),x]
```

```
[Out] -(((B*c*e - b*B*f - A*c*f)*x)/f^2) + (B*c*x^2)/(2*f) - ((B*f*(b*e^2 - 2*b*d
*f - a*e*f) - B*c*(e^3 - 3*d*e*f) + A*f*(c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2)
)*ArcTanh[(e + 2*f*x)/Sqrt[e^2 - 4*d*f]])/(f^3*Sqrt[e^2 - 4*d*f]) - ((B*f*(
b*e - a*f) + A*f*(c*e - b*f) - B*c*(e^2 - d*f))*Log[d + e*x + f*x^2])/(2*f^
3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{d+ex+fx^2}\,dx = \int \left(-\frac{Bce-bBf-Acf}{f^2}+\frac{Bcx}{f}+\frac{-Af(cd-af)+Bd(ce-bf)-\left(Bf(be-af)\right.}{f^2\left(d+ex+fx\right.}\right.$$

$$= -\frac{(Bce-bBf-Acf)x}{f^2}+\frac{Bcx^2}{2f}+\frac{\int \frac{-Af(cd-af)+Bd(ce-bf)-\left(Bf(be-af)+Af(ce-bf)-Bc\left(e^2\right.\right.}{d+ex+fx^2}}{f^2}$$

$$= -\frac{(Bce-bBf-Acf)x}{f^2}+\frac{Bcx^2}{2f}+\frac{\left(-Bf(be-af)-Af(ce-bf)+Bc\left(e^2-df\right)\right)}{2f^3}$$

$$= -\frac{(Bce-bBf-Acf)x}{f^2}+\frac{Bcx^2}{2f}-\frac{\left(Bf(be-af)+Af(ce-bf)-Bc\left(e^2-df\right)\right)\mathrm{l}}{2f^3}$$

$$= -\frac{(Bce-bBf-Acf)x}{f^2}+\frac{Bcx^2}{2f}-\frac{\left(Bf\left(be^2-2bdf-aef\right)-Bc\left(e^3-3def\right)+A\right.}{f^3\sqrt{e^2}}$$

**Mathematica [A]** time = 0.21, size = 175, normalized size = 0.95

$$\frac{\log(d+x(e+fx))\left(Bf(af-be)+Af(bf-ce)+Bc\left(e^2-df\right)\right)-\frac{2\tan^{-1}\left(\frac{e+2fx}{\sqrt{4df-e^2}}\right)\left(Af\left(-2af^2+bef+2cdf-ce^2\right)+Bf\left(aef+2bdf-be^2\right)+Bc\left(e^3-3def\right)\right)}{\sqrt{4df-e^2}}+2fx(Acf+bBf-Bce)+Bcf^2x^2}{2f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x + c*x^2))/(d + e*x + f*x^2),x]
```

```
[Out] (2*f*(-(B*c*e) + b*B*f + A*c*f)*x + B*c*f^2*x^2 - (2*(B*f*(-(b*e^2) + 2*b*d
*f + a*e*f) + B*c*(e^3 - 3*d*e*f) + A*f*(-(c*e^2) + 2*c*d*f + b*e*f - 2*a*f
^2))*ArcTan[(e + 2*f*x)/Sqrt[-e^2 + 4*d*f]])/Sqrt[-e^2 + 4*d*f] + (B*f*(-(b
```

```
*e) + a*f) + A*f*(-(c*e) + b*f) + B*c*(e^2 - d*f))*Log[d + x*(e + f*x)])/(2
*f^3)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{d+ex+fx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((A + B*x)*(a + b*x + c*x^2))/(d + e*x + f*x^2),x]
```

```
[Out] IntegrateAlgebraic[((A + B*x)*(a + b*x + c*x^2))/(d + e*x + f*x^2), x]
```

**fricas [A]** time = 0.44, size = 583, normalized size = 3.17

$$\left[\frac{\left(Bce^2f^2-4\,Bcdf^3\right)x^2-\left(Bce^3-2\,Aaf^3+(2\,(Bb+Ac)d+(Ba+Ab)e)f^2-\left(3\,Bcde+(Bb+Ac)e^2\right)f\right)\sqrt{e^2-4\,df}\,\log\left(\frac{2\,f^2x^2+2\,efx+e^2-2\,df-\sqrt{e^2-4\,df}\,(2\,fx+e)}{fx^2+ex+d}\right)-2\left(Bce^3f+4\,(Bb+Ac)df^3-\left(4\,Bcde+(Bb+Ac)e^2\right)f^2\right)x+\left(Bce^4-4\,(Ba+Ab)df^3+\left(4\,Bcd^2+4\,(Bb+Ac)de+(Ba+Ab)e^2\right)f^2-\left(5\,Bcde^2+(Bb+Ac)e^3\right)f\right)\log\left(fx^2+ex+d\right)}{2\left(e^2f^3-4\,df^4\right)},\ \frac{\left(Bce^2f^2-4\,Bcdf^3\right)x^2+2\left(Bce^3-2\,Aaf^3+(2\,(Bb+Ac)d+(Ba+Ab)e)f^2-\left(3\,Bcde+(Bb+Ac)e^2\right)f\right)\sqrt{-e^2+4\,df}\,\arctan\left(-\frac{\sqrt{-e^2+4\,df}\,(2\,fx+e)}{e^2-4\,df}\right)-2\left(Bce^3f+4\,(Bb+Ac)df^3-\left(4\,Bcde+(Bb+Ac)e^2\right)f^2\right)x+\left(Bce^4-4\,(Ba+Ab)df^3+\left(4\,Bcd^2+4\,(Bb+Ac)de+(Ba+Ab)e^2\right)f^2-\left(5\,Bcde^2+(Bb+Ac)e^3\right)f\right)\log\left(fx^2+ex+d\right)}{2\left(e^2f^3-4\,df^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*((B*c*e^2*f^2 - 4*B*c*d*f^3)*x^2 - (B*c*e^3 - 2*A*a*f^3 + (2*(B*b + A*
c)*d + (B*a + A*b)*e)*f^2 - (3*B*c*d*e + (B*b + A*c)*e^2)*f)*sqrt(e^2 - 4*d
*f)*log((2*f^2*x^2 + 2*e*f*x + e^2 - 2*d*f - sqrt(e^2 - 4*d*f)*(2*f*x + e))
/(f*x^2 + e*x + d)) - 2*(B*c*e^3*f + 4*(B*b + A*c)*d*f^3 - (4*B*c*d*e + (B*
b + A*c)*e^2)*f^2)*x + (B*c*e^4 - 4*(B*a + A*b)*d*f^3 + (4*B*c*d^2 + 4*(B*b
 + A*c)*d*e + (B*a + A*b)*e^2)*f^2 - (5*B*c*d*e^2 + (B*b + A*c)*e^3)*f)*log
(f*x^2 + e*x + d))/(e^2*f^3 - 4*d*f^4), 1/2*((B*c*e^2*f^2 - 4*B*c*d*f^3)*x^
2 + 2*(B*c*e^3 - 2*A*a*f^3 + (2*(B*b + A*c)*d + (B*a + A*b)*e)*f^2 - (3*B*c
*d*e + (B*b + A*c)*e^2)*f)*sqrt(-e^2 + 4*d*f)*arctan(-sqrt(-e^2 + 4*d*f)*(2
*f*x + e)/(e^2 - 4*d*f)) - 2*(B*c*e^3*f + 4*(B*b + A*c)*d*f^3 - (4*B*c*d*e
+ (B*b + A*c)*e^2)*f^2)*x + (B*c*e^4 - 4*(B*a + A*b)*d*f^3 + (4*B*c*d^2 + 4
*(B*b + A*c)*d*e + (B*a + A*b)*e^2)*f^2 - (5*B*c*d*e^2 + (B*b + A*c)*e^3)*f
)*log(f*x^2 + e*x + d))/(e^2*f^3 - 4*d*f^4)]
```

**giac [A]** time = 0.17, size = 191, normalized size = 1.04

$$\frac{Bcfx^2+2\,Bbfx+2\,Acfx-2\,Bcxe}{2\,f^2}-\frac{\left(Bcdf-Baf^2-Abf^2+Bbfe+Acfe-Bce^2\right)\log\left(fx^2+xe+d\right)}{2\,f^3}-\frac{\left(2\,Bbdf^2+2\,Acdf^2-2\,Aaf^3-3\,Bcdfe+Baf^2e+Abf^2e-Bbfe^2-Acfe^2+Bce^3\right)\arctan\left(\frac{2\,fx+e}{\sqrt{4\,df-e^2}}\right)}{\sqrt{4\,df-e^2}\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/2*(B*c*f*x^2 + 2*B*b*f*x + 2*A*c*f*x - 2*B*c*x*e)/f^2 - 1/2*(B*c*d*f - B*
a*f^2 - A*b*f^2 + B*b*f*e + A*c*f*e - B*c*e^2)*log(f*x^2 + x*e + d)/f^3 - (
```

```
2*B*b*d*f^2 + 2*A*c*d*f^2 - 2*A*a*f^3 - 3*B*c*d*f*e + B*a*f^2*e + A*b*f^2*e
 - B*b*f*e^2 - A*c*f*e^2 + B*c*e^3)*arctan((2*f*x + e)/sqrt(4*d*f - e^2))/(
sqrt(4*d*f - e^2)*f^3)
```

**maple [B]** time = 0.01, size = 510, normalized size = 2.77

$$\frac{2Aa\arctan\left(\frac{2fx+e}{\sqrt{4df-e^2}}\right)}{\sqrt{4df-e^2}}-\frac{Abe\arctan\left(\frac{2fx+e}{\sqrt{4df-e^2}}\right)}{\sqrt{4df-e^2}\,f}-\frac{2Acd\arctan\left(\frac{2fx+e}{\sqrt{4df-e^2}}\right)}{\sqrt{4df-e^2}\,f}+\frac{Ac\,e^2\arctan\left(\frac{2fx+e}{\sqrt{4df-e^2}}\right)}{\sqrt{4df-e^2}\,f^2}-\frac{Bae\arctan\left(\frac{2fx+e}{\sqrt{4df-e^2}}\right)}{\sqrt{4df-e^2}\,f}-\frac{2Bbd\arctan\left(\frac{2fx+e}{\sqrt{4df-e^2}}\right)}{\sqrt{4df-e^2}\,f}+\frac{Bb\,e^2\arctan\left(\frac{2fx+e}{\sqrt{4df-e^2}}\right)}{\sqrt{4df-e^2}\,f^2}+\frac{3Bcde\arctan\left(\frac{2fx+e}{\sqrt{4df-e^2}}\right)}{\sqrt{4df-e^2}\,f^2}-\frac{Bc\,e^3\arctan\left(\frac{2fx+e}{\sqrt{4df-e^2}}\right)}{\sqrt{4df-e^2}\,f^3}+\frac{Bc\,x^2}{2f}+\frac{Ab\ln\left(f\,x^2+ex+d\right)}{2f}-\frac{Ace\ln\left(f\,x^2+ex+d\right)}{2f^2}+\frac{Acx}{f}+\frac{Ba\ln\left(f\,x^2+ex+d\right)}{2f}-\frac{Bbe\ln\left(f\,x^2+ex+d\right)}{2f^2}+\frac{Bbx}{f}-\frac{Bcd\ln\left(f\,x^2+ex+d\right)}{2f^2}+\frac{Bc\,e^2\ln\left(f\,x^2+ex+d\right)}{2f^3}-\frac{Bcex}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(c*x^2+b*x+a)/(f*x^2+e*x+d),x)
```

```
[Out] 1/2*B*c/f*x^2+A*c/f*x+B*b/f*x-1/f^2*B*c*e*x+1/2/f*ln(f*x^2+e*x+d)*A*b-1/2/f
^2*ln(f*x^2+e*x+d)*A*c*e+1/2/f*ln(f*x^2+e*x+d)*B*a-1/2/f^2*ln(f*x^2+e*x+d)*
B*b*e-1/2/f^2*ln(f*x^2+e*x+d)*B*c*d+1/2/f^3*ln(f*x^2+e*x+d)*B*c*e^2+2/(4*d*
f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*A*a-2/f/(4*d*f-e^2)^(1/2)*
arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*A*c*d-2/f/(4*d*f-e^2)^(1/2)*arctan((2*f
*x+e)/(4*d*f-e^2)^(1/2))*B*b*d+3/f^2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*
d*f-e^2)^(1/2))*B*c*d*e-1/f/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^
(1/2))*e*A*b+1/f^2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e^
2*A*c-1/f/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e*B*a+1/f^2
/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e^2*B*b-1/f^3/(4*d*f
-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e^3*B*c
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive or negative?
```

**mupad [B]** time = 3.85, size = 273, normalized size = 1.48

$$x\left(\frac{A\,c+B\,b}{f}-\frac{B\,c\,e}{f^2}\right)-\frac{\ln\left(f\,x^2+e\,x+d\right)\,\left(B\,c\,e^4-4\,A\,b\,d\,f^3-4\,B\,a\,d\,f^3-A\,c\,e^3\,f-B\,b\,e^3\,f+A\,b\,e^2\,f^2+B\,a\,e^2\,f^2+4\,B\,c\,d^2\,f^2+4\,A\,c\,d\,e\,f^2+4\,B\,b\,d\,e\,f^2-5\,B\,c\,d\,e^2\,f\right)}{2\,\left(4\,d\,f^4-e^2\,f^3\right)}-\frac{\mathrm{atan}\left(\frac{e}{\sqrt{4\,d\,f-e^2}}+\frac{2\,f\,x}{\sqrt{4\,d\,f-e^2}}\right)\,\left(B\,c\,e^3-2\,A\,a\,f^3+A\,b\,e\,f^2+2\,A\,c\,d\,f^2+B\,a\,e\,f^2+2\,B\,b\,d\,f^2-A\,c\,e^2\,f-B\,b\,e^2\,f-3\,B\,c\,d\,e\,f\right)}{f^3\,\sqrt{4\,d\,f-e^2}}+\frac{B\,c\,x^2}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x)*(a + b*x + c*x^2))/(d + e*x + f*x^2),x)
```

```
[Out] x*((A*c + B*b)/f - (B*c*e)/f^2) - (log(d + e*x + f*x^2)*(B*c*e^4 - 4*A*b*d*
f^3 - 4*B*a*d*f^3 - A*c*e^3*f - B*b*e^3*f + A*b*e^2*f^2 + B*a*e^2*f^2 + 4*B
```

```
*c*d^2*f^2 + 4*A*c*d*e*f^2 + 4*B*b*d*e*f^2 - 5*B*c*d*e^2*f))/(2*(4*d*f^4 -
e^2*f^3)) - (atan(e/(4*d*f - e^2)^(1/2) + (2*f*x)/(4*d*f - e^2)^(1/2))*(B*c
*e^3 - 2*A*a*f^3 + A*b*e*f^2 + 2*A*c*d*f^2 + B*a*e*f^2 + 2*B*b*d*f^2 - A*c*
e^2*f - B*b*e^2*f - 3*B*c*d*e*f))/(f^3*(4*d*f - e^2)^(1/2)) + (B*c*x^2)/(2*
f)
```

**sympy [B]** time = 16.65, size = 1260, normalized size = 6.85

$$\frac{Bcx^2}{2f}+x\left(\frac{Ac}{f}+\frac{Bb}{f}-\frac{Bce}{f^2}\right)+\left(-\frac{\sqrt{-4df+e^2}\left(-2Aaf^3+Abef^2+2Acdf^2-Ace^2f+Baef^2+2Bbdf^2-Bbe^2f-3Bcdef+Bce^3\right)}{2f^3\left(4df-e^2\right)}+\frac{Abf^2-Acef+Baf^2-Bbef-Bcdf+Bce^2}{2f^3}\right)\log\left(x+\frac{-Aaef^2+2Abdf^2-Acdef+2Badf^2-Bbdef-2Bcd^2f+Bcde^2-4df^3\left(-\frac{\sqrt{-4df+e^2}\left(-2Aaf^3+Abef^2+2Acdf^2-Ace^2f+Baef^2+2Bbdf^2-Bbe^2f-3Bcdef+Bce^3\right)}{2f^3\left(4df-e^2\right)}+\frac{Abf^2-Acef+Baf^2-Bbef-Bcdf+Bce^2}{2f^3}\right)+e^2f^2\left(-\frac{\sqrt{-4df+e^2}\left(-2Aaf^3+Abef^2+2Acdf^2-Ace^2f+Baef^2+2Bbdf^2-Bbe^2f-3Bcdef+Bce^3\right)}{2f^3\left(4df-e^2\right)}+\frac{Abf^2-Acef+Baf^2-Bbef-Bcdf+Bce^2}{2f^3}\right)}{-2Aaf^3+Abef^2+2Acdf^2-Ace^2f+Baef^2+2Bbdf^2-Bbe^2f-3Bcdef+Bce^3}\right)+\left(\frac{\sqrt{-4df+e^2}\left(-2Aaf^3+Abef^2+2Acdf^2-Ace^2f+Baef^2+2Bbdf^2-Bbe^2f-3Bcdef+Bce^3\right)}{2f^3\left(4df-e^2\right)}+\frac{Abf^2-Acef+Baf^2-Bbef-Bcdf+Bce^2}{2f^3}\right)\log\left(x+\frac{-Aaef^2+2Abdf^2-Acdef+2Badf^2-Bbdef-2Bcd^2f+Bcde^2-4df^3\left(\frac{\sqrt{-4df+e^2}\left(-2Aaf^3+Abef^2+2Acdf^2-Ace^2f+Baef^2+2Bbdf^2-Bbe^2f-3Bcdef+Bce^3\right)}{2f^3\left(4df-e^2\right)}+\frac{Abf^2-Acef+Baf^2-Bbef-Bcdf+Bce^2}{2f^3}\right)+e^2f^2\left(\frac{\sqrt{-4df+e^2}\left(-2Aaf^3+Abef^2+2Acdf^2-Ace^2f+Baef^2+2Bbdf^2-Bbe^2f-3Bcdef+Bce^3\right)}{2f^3\left(4df-e^2\right)}+\frac{Abf^2-Acef+Baf^2-Bbef-Bcdf+Bce^2}{2f^3}\right)}{-2Aaf^3+Abef^2+2Acdf^2-Ace^2f+Baef^2+2Bbdf^2-Bbe^2f-3Bcdef+Bce^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x**2+b*x+a)/(f*x**2+e*x+d),x)
```

```
[Out] B*c*x**2/(2*f) + x*(A*c/f + B*b/f - B*c*e/f**2) + (-sqrt(-4*d*f + e**2)*(-2
*A*a*f**3 + A*b*e*f**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 + 2*B*b*d*f
**2 - B*b*e**2*f - 3*B*c*d*e*f + B*c*e**3)/(2*f**3*(4*d*f - e**2)) + (A*b*f
**2 - A*c*e*f + B*a*f**2 - B*b*e*f - B*c*d*f + B*c*e**2)/(2*f**3))*log(x +
(-A*a*e*f**2 + 2*A*b*d*f**2 - A*c*d*e*f + 2*B*a*d*f**2 - B*b*d*e*f - 2*B*c*
d**2*f + B*c*d*e**2 - 4*d*f**3*(-sqrt(-4*d*f + e**2)*(-2*A*a*f**3 + A*b*e*f
**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 + 2*B*b*d*f**2 - B*b*e**2*f -
3*B*c*d*e*f + B*c*e**3)/(2*f**3*(4*d*f - e**2)) + (A*b*f**2 - A*c*e*f + B*a
*f**2 - B*b*e*f - B*c*d*f + B*c*e**2)/(2*f**3)) + e**2*f**2*(-sqrt(-4*d*f +
 e**2)*(-2*A*a*f**3 + A*b*e*f**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 +
 2*B*b*d*f**2 - B*b*e**2*f - 3*B*c*d*e*f + B*c*e**3)/(2*f**3*(4*d*f - e**2)
) + (A*b*f**2 - A*c*e*f + B*a*f**2 - B*b*e*f - B*c*d*f + B*c*e**2)/(2*f**3)
))/(-2*A*a*f**3 + A*b*e*f**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 + 2*B
*b*d*f**2 - B*b*e**2*f - 3*B*c*d*e*f + B*c*e**3)) + (sqrt(-4*d*f + e**2)*(-
2*A*a*f**3 + A*b*e*f**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 + 2*B*b*d*
f**2 - B*b*e**2*f - 3*B*c*d*e*f + B*c*e**3)/(2*f**3*(4*d*f - e**2)) + (A*b*
f**2 - A*c*e*f + B*a*f**2 - B*b*e*f - B*c*d*f + B*c*e**2)/(2*f**3))*log(x +
 (-A*a*e*f**2 + 2*A*b*d*f**2 - A*c*d*e*f + 2*B*a*d*f**2 - B*b*d*e*f - 2*B*c
*d**2*f + B*c*d*e**2 - 4*d*f**3*(sqrt(-4*d*f + e**2)*(-2*A*a*f**3 + A*b*e*f
**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 + 2*B*b*d*f**2 - B*b*e**2*f -
3*B*c*d*e*f + B*c*e**3)/(2*f**3*(4*d*f - e**2)) + (A*b*f**2 - A*c*e*f + B*a
*f**2 - B*b*e*f - B*c*d*f + B*c*e**2)/(2*f**3)) + e**2*f**2*(sqrt(-4*d*f +
e**2)*(-2*A*a*f**3 + A*b*e*f**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 +
2*B*b*d*f**2 - B*b*e**2*f - 3*B*c*d*e*f + B*c*e**3)/(2*f**3*(4*d*f - e**2))
 + (A*b*f**2 - A*c*e*f + B*a*f**2 - B*b*e*f - B*c*d*f + B*c*e**2)/(2*f**3))
)/(-2*A*a*f**3 + A*b*e*f**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 + 2*B*
b*d*f**2 - B*b*e**2*f - 3*B*c*d*e*f + B*c*e**3))
```

## 3.14 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+ex+fx^2}\,dx$

**Optimal**. Leaf size=542

$$\frac{\log\left(d+ex+fx^2\right)\left(B\left(-f^2\left(-a^2f^2+2abef-\left(b^2\left(e^2-df\right)\right)\right)+2cf\left(af\left(e^2-df\right)-b\left(e^3-2def\right)\right)+c^2\left(d^2f^2-\right.\right.\right.}{2f^5}$$

**Rubi** [A] time = 1.10, antiderivative size = 542, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1011, 634, 618, 206, 628}

$$\frac{\log\left(d+ex+fx^2\right)\left(B\left(-f^2\left(-a^2f^2+2abef+b^2\left(-\left(e^2-df\right)\right)\right)+2cf\left(af\left(e^2-df\right)-b\left(e^3-2def\right)\right)+c^2\left(d^2f^2-3de^2f+e^4\right)\right)+Af(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)\right)}{2f^5}-\frac{\tanh^{-1}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)\left(Af\left(-f^2\left(-2a^2f^2+2abef+b^2\left(-\left(e^2-2df\right)\right)\right)+2cf\left(af\left(e^2-2df\right)-b\left(e^3-3def\right)\right)+c^2\left(2d^2f^2-4de^2f+e^4\right)\right)-B\left(f^2\left(a^2ef^2-2abf\left(e^2-2df\right)+b^2\left(e^3-3def\right)\right)+2cf\left(aef\left(e^2-3df\right)-b\left(2d^2f^2-4de^2f+e^4\right)\right)+c^2\left(5d^2ef^2-5de^3f+e^5\right)\right)\right)}{f^5\sqrt{e^2-4df}}-\frac{x^2\left(Acf(ce-2bf)-B\left(-2cf(be-af)+b^2f^2+c^2\left(e^2-df\right)\right)\right)}{2f^3}+\frac{x\left(Af\left(-2cf(be-af)+b^2f^2+c^2\left(e^2-df\right)\right)+B(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)\right)}{f^4}-\frac{cx^3(-Acf-2bBf+Bce)}{3f^2}+\frac{Bc^2x^4}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x + c*x^2)^2)/(d + e*x + f*x^2),x]
```

```
[Out] ((B*(c*e - b*f)*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)) + A*f*(b^2*f^2 - 2*c*f*
(b*e - a*f) + c^2*(e^2 - d*f)))*x)/f^4 - ((A*c*f*(c*e - 2*b*f) - B*(b^2*f^2
 - 2*c*f*(b*e - a*f) + c^2*(e^2 - d*f)))*x^2)/(2*f^3) - (c*(B*c*e - 2*b*B*f
 - A*c*f)*x^3)/(3*f^2) + (B*c^2*x^4)/(4*f) - ((A*f*(c^2*(e^4 - 4*d*e^2*f +
2*d^2*f^2) - f^2*(2*a*b*e*f - 2*a^2*f^2 - b^2*(e^2 - 2*d*f)) + 2*c*f*(a*f*(
e^2 - 2*d*f) - b*(e^3 - 3*d*e*f))) - B*(c^2*(e^5 - 5*d*e^3*f + 5*d^2*e*f^2)
 + f^2*(a^2*e*f^2 - 2*a*b*f*(e^2 - 2*d*f) + b^2*(e^3 - 3*d*e*f)) + 2*c*f*(a
*e*f*(e^2 - 3*d*f) - b*(e^4 - 4*d*e^2*f + 2*d^2*f^2))))*ArcTanh[(e + 2*f*x)
/Sqrt[e^2 - 4*d*f]])/(f^5*Sqrt[e^2 - 4*d*f]) + ((A*f*(c*e - b*f)*(f*(b*e -
2*a*f) - c*(e^2 - 2*d*f)) + B*(c^2*(e^4 - 3*d*e^2*f + d^2*f^2) - f^2*(2*a*b
*e*f - a^2*f^2 - b^2*(e^2 - d*f)) + 2*c*f*(a*f*(e^2 - d*f) - b*(e^3 - 2*d*e
*f))))*Log[d + e*x + f*x^2])/(2*f^5)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1011

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x +
c*x^2)^p*(d + e*x + f*x^2)^q*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && IGtQ[p, 0] && In
tegerQ[q]
```

Rubi steps

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+ex+fx^2}\,dx = \int \left(\frac{B(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+Af\left(b^2f^2-2cf(be-af)+c^2\left(e^2\right.\right.}{f^4}\right.$$

$$= \frac{\left(B(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+Af\left(b^2f^2-2cf(be-af)+c^2\left(e^2-\right.\right.\right.}{f^4}$$

$$= \frac{\left(B(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+Af\left(b^2f^2-2cf(be-af)+c^2\left(e^2-\right.\right.\right.}{f^4}$$

$$= \frac{\left(B(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+Af\left(b^2f^2-2cf(be-af)+c^2\left(e^2-\right.\right.\right.}{f^4}$$

$$= \frac{\left(B(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+Af\left(b^2f^2-2cf(be-af)+c^2\left(e^2-\right.\right.\right.}{f^4}$$

**Mathematica [A]** time = 0.60, size = 535, normalized size = 0.99

$$\frac{6\log(d+x(e+fx))\left(B\left(f^2\left(a^2f^2-2abef+b^2\left(e^2-df\right)\right)-2cf\left(af\left(df-e^2\right)+b\left(e^3-2def\right)\right)+c^2\left(d^2f^2-3de^2f+e^4\right)\right)+Af(bf-ce)\left(f(2af-be)+c\left(e^2-2df\right)\right)\right)-\frac{12\tan^{-1}\left(\frac{e+2fx}{\sqrt{4df-e^2}}\right)\left(B\left(f^2\left(a^2ef^2+2abf\left(2df-e^2\right)+b^2\left(e^3-3def\right)\right)-2cf\left(b\left(2d^2f^2-4de^2f+e^4\right)-aef\left(e^2-3df\right)\right)+c^2\left(5d^2ef^2-5de^3f+e^5\right)\right)-Af\left(f^2\left(2a^2f^2-2abef+b^2\left(e^2-2df\right)\right)+2cf\left(af\left(e^2-2df\right)-b\left(e^3-3def\right)\right)+c^2\left(2d^2f^2-4de^2f+e^4\right)\right)\right)}{\sqrt{4df-e^2}}+6f^2x^2\left(B\left(2cf(af-be)+b^2f^2+c^2\left(e^2-df\right)\right)+Acf(2bf-ce)\right)+12fx\left(Af\left(2cf(af-be)+b^2f^2+c^2\left(e^2-df\right)\right)-B(ce-bf)\left(f(2af-be)+c\left(e^2-2df\right)\right)\right)+4cf^3x^3(Acf+2bBf-Bce)+3Bc^2f^4x^4}{12f^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x + c*x^2)^2)/(d + e*x + f*x^2),x]
```

```
[Out] (12*f*(-(B*(c*e - b*f)*(f*(-(b*e) + 2*a*f) + c*(e^2 - 2*d*f))) + A*f*(b^2*f
^2 + 2*c*f*(-(b*e) + a*f) + c^2*(e^2 - d*f)))*x + 6*f^2*(A*c*f*(-(c*e) + 2*
b*f) + B*(b^2*f^2 + 2*c*f*(-(b*e) + a*f) + c^2*(e^2 - d*f)))*x^2 + 4*c*f^3*
(-(B*c*e) + 2*b*B*f + A*c*f)*x^3 + 3*B*c^2*f^4*x^4 - (12*(-(A*f*(c^2*(e^4 -
 4*d*e^2*f + 2*d^2*f^2) + f^2*(-2*a*b*e*f + 2*a^2*f^2 + b^2*(e^2 - 2*d*f))
+ 2*c*f*(a*f*(e^2 - 2*d*f) - b*(e^3 - 3*d*e*f)))) + B*(c^2*(e^5 - 5*d*e^3*f
 + 5*d^2*e*f^2) + f^2*(a^2*e*f^2 + 2*a*b*f*(-e^2 + 2*d*f) + b^2*(e^3 - 3*d*
e*f)) - 2*c*f*(-(a*e*f*(e^2 - 3*d*f)) + b*(e^4 - 4*d*e^2*f + 2*d^2*f^2))))*
ArcTan[(e + 2*f*x)/Sqrt[-e^2 + 4*d*f]])/Sqrt[-e^2 + 4*d*f] + 6*(A*f*(-(c*e)
 + b*f)*(f*(-(b*e) + 2*a*f) + c*(e^2 - 2*d*f)) + B*(c^2*(e^4 - 3*d*e^2*f +
d^2*f^2) + f^2*(-2*a*b*e*f + a^2*f^2 + b^2*(e^2 - d*f)) - 2*c*f*(a*f*(-e^2
+ d*f) + b*(e^3 - 2*d*e*f))))*Log[d + x*(e + f*x)])/(12*f^5)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+ex+fx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((A + B*x)*(a + b*x + c*x^2)^2)/(d + e*x + f*x^2),x]
```

```
[Out] IntegrateAlgebraic[((A + B*x)*(a + b*x + c*x^2)^2)/(d + e*x + f*x^2), x]
```

**fricas [A]** time = 0.61, size = 1837, normalized size = 3.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(B*c^2*e^2*f^4 - 4*B*c^2*d*f^5)*x^4 - 4*(B*c^2*e^3*f^3 + 4*(2*B*b*
c + A*c^2)*d*f^5 - (4*B*c^2*d*e + (2*B*b*c + A*c^2)*e^2)*f^4)*x^3 + 6*(B*c^
2*e^4*f^2 - 4*(B*b^2 + 2*(B*a + A*b)*c)*d*f^5 + (4*B*c^2*d^2 + 4*(2*B*b*c +
 A*c^2)*d*e + (B*b^2 + 2*(B*a + A*b)*c)*e^2)*f^4 - (5*B*c^2*d*e^2 + (2*B*b*
c + A*c^2)*e^3)*f^3)*x^2 - 6*(B*c^2*e^5 - 2*A*a^2*f^5 + (2*(2*B*a*b + A*b^2
 + 2*A*a*c)*d + (B*a^2 + 2*A*a*b)*e)*f^4 - (2*(2*B*b*c + A*c^2)*d^2 + 3*(B*
b^2 + 2*(B*a + A*b)*c)*d*e + (2*B*a*b + A*b^2 + 2*A*a*c)*e^2)*f^3 + (5*B*c^
2*d^2*e + 4*(2*B*b*c + A*c^2)*d*e^2 + (B*b^2 + 2*(B*a + A*b)*c)*e^3)*f^2 -
(5*B*c^2*d*e^3 + (2*B*b*c + A*c^2)*e^4)*f)*sqrt(e^2 - 4*d*f)*log((2*f^2*x^2
 + 2*e*f*x + e^2 - 2*d*f - sqrt(e^2 - 4*d*f)*(2*f*x + e))/(f*x^2 + e*x + d)
) - 12*(B*c^2*e^5*f + 4*(2*B*a*b + A*b^2 + 2*A*a*c)*d*f^5 - (4*(2*B*b*c + A
*c^2)*d^2 + 4*(B*b^2 + 2*(B*a + A*b)*c)*d*e + (2*B*a*b + A*b^2 + 2*A*a*c)*e
^2)*f^4 + (8*B*c^2*d^2*e + 5*(2*B*b*c + A*c^2)*d*e^2 + (B*b^2 + 2*(B*a + A*
```

```
b)*c)*e^3)*f^3 - (6*B*c^2*d*e^3 + (2*B*b*c + A*c^2)*e^4)*f^2)*x + 6*(B*c^2*
e^6 - 4*(B*a^2 + 2*A*a*b)*d*f^5 + (4*(B*b^2 + 2*(B*a + A*b)*c)*d^2 + 4*(2*B
*a*b + A*b^2 + 2*A*a*c)*d*e + (B*a^2 + 2*A*a*b)*e^2)*f^4 - (4*B*c^2*d^3 + 8
*(2*B*b*c + A*c^2)*d^2*e + 5*(B*b^2 + 2*(B*a + A*b)*c)*d*e^2 + (2*B*a*b + A
*b^2 + 2*A*a*c)*e^3)*f^3 + (13*B*c^2*d^2*e^2 + 6*(2*B*b*c + A*c^2)*d*e^3 +
(B*b^2 + 2*(B*a + A*b)*c)*e^4)*f^2 - (7*B*c^2*d*e^4 + (2*B*b*c + A*c^2)*e^5
)*f)*log(f*x^2 + e*x + d))/(e^2*f^5 - 4*d*f^6), 1/12*(3*(B*c^2*e^2*f^4 - 4*
B*c^2*d*f^5)*x^4 - 4*(B*c^2*e^3*f^3 + 4*(2*B*b*c + A*c^2)*d*f^5 - (4*B*c^2*
d*e + (2*B*b*c + A*c^2)*e^2)*f^4)*x^3 + 6*(B*c^2*e^4*f^2 - 4*(B*b^2 + 2*(B*
a + A*b)*c)*d*f^5 + (4*B*c^2*d^2 + 4*(2*B*b*c + A*c^2)*d*e + (B*b^2 + 2*(B*
a + A*b)*c)*e^2)*f^4 - (5*B*c^2*d*e^2 + (2*B*b*c + A*c^2)*e^3)*f^3)*x^2 + 1
2*(B*c^2*e^5 - 2*A*a^2*f^5 + (2*(2*B*a*b + A*b^2 + 2*A*a*c)*d + (B*a^2 + 2*
A*a*b)*e)*f^4 - (2*(2*B*b*c + A*c^2)*d^2 + 3*(B*b^2 + 2*(B*a + A*b)*c)*d*e
+ (2*B*a*b + A*b^2 + 2*A*a*c)*e^2)*f^3 + (5*B*c^2*d^2*e + 4*(2*B*b*c + A*c^
2)*d*e^2 + (B*b^2 + 2*(B*a + A*b)*c)*e^3)*f^2 - (5*B*c^2*d*e^3 + (2*B*b*c +
 A*c^2)*e^4)*f)*sqrt(-e^2 + 4*d*f)*arctan(-sqrt(-e^2 + 4*d*f)*(2*f*x + e)/(
e^2 - 4*d*f)) - 12*(B*c^2*e^5*f + 4*(2*B*a*b + A*b^2 + 2*A*a*c)*d*f^5 - (4*
(2*B*b*c + A*c^2)*d^2 + 4*(B*b^2 + 2*(B*a + A*b)*c)*d*e + (2*B*a*b + A*b^2
+ 2*A*a*c)*e^2)*f^4 + (8*B*c^2*d^2*e + 5*(2*B*b*c + A*c^2)*d*e^2 + (B*b^2 +
 2*(B*a + A*b)*c)*e^3)*f^3 - (6*B*c^2*d*e^3 + (2*B*b*c + A*c^2)*e^4)*f^2)*x
 + 6*(B*c^2*e^6 - 4*(B*a^2 + 2*A*a*b)*d*f^5 + (4*(B*b^2 + 2*(B*a + A*b)*c)*
d^2 + 4*(2*B*a*b + A*b^2 + 2*A*a*c)*d*e + (B*a^2 + 2*A*a*b)*e^2)*f^4 - (4*B
*c^2*d^3 + 8*(2*B*b*c + A*c^2)*d^2*e + 5*(B*b^2 + 2*(B*a + A*b)*c)*d*e^2 +
(2*B*a*b + A*b^2 + 2*A*a*c)*e^3)*f^3 + (13*B*c^2*d^2*e^2 + 6*(2*B*b*c + A*c
^2)*d*e^3 + (B*b^2 + 2*(B*a + A*b)*c)*e^4)*f^2 - (7*B*c^2*d*e^4 + (2*B*b*c
+ A*c^2)*e^5)*f)*log(f*x^2 + e*x + d))/(e^2*f^5 - 4*d*f^6)]
```

**giac** [**A**] time = 0.25, size = 738, normalized size = 1.36

$$\frac{3\,Bc^{2}f^{3}x^{4}+8\,Bbcf^{3}x^{3}+4\,Ac^{2}f^{3}x^{3}-4\,Bc^{2}f^{2}x^{3}e-6\,Bc^{2}df^{2}x^{2}+6\,Bb^{2}f^{3}x^{2}+12\,Bacf^{3}x^{2}+12\,Abcf^{3}x^{2}-12\,Bbcf^{2}x^{2}e-6\,Ac^{2}f^{2}x^{2}e-24\,Bbcdf^{2}x-12\,Ac^{2}df^{2}x+24\,Babf^{3}x+12\,Ab^{2}f^{3}x+24\,Aacf^{3}x+6\,Bc^{2}fx^{2}e^{2}+24\,Bc^{2}dfxe-12\,Bb^{2}f^{2}xe-24\,Bacf^{2}xe-24\,Abcf^{2}xe+24\,Bbcfxe^{2}+12\,Ac^{2}fxe^{2}-12\,Bc^{2}xe^{3}}{12\,f^{4}}+\frac{\left(Bc^{2}d^{2}f^{2}-Bb^{2}df^{3}-2\,Bacdf^{3}-2\,Abcdf^{3}+Ba^{2}f^{4}+2\,Aabf^{4}+4\,Bbcdf^{2}e+2\,Ac^{2}df^{2}e-2\,Babf^{3}e-Ab^{2}f^{3}e-2\,Aacf^{3}e-3\,Bc^{2}dfe^{2}+Bb^{2}f^{2}e^{2}+2\,Bacf^{2}e^{2}+2\,Abcf^{2}e^{2}-2\,Bbcfe^{3}-Ac^{2}fe^{3}+Bc^{2}e^{4}\right)\log\left(fx^{2}+xe+d\right)}{2\,f^{5}}+\frac{\left(4\,Bbcd^{2}f^{3}+2\,Ac^{2}d^{2}f^{3}-4\,Babdf^{4}-2\,Ab^{2}df^{4}-4\,Aacdf^{4}+2\,Aa^{2}f^{5}-5\,Bc^{2}d^{2}f^{2}e+3\,Bb^{2}df^{3}e+6\,Bacdf^{3}e+6\,Abcdf^{3}e-Ba^{2}f^{4}e-2\,Aabf^{4}e-8\,Bbcdf^{2}e^{2}-4\,Ac^{2}df^{2}e^{2}+2\,Babf^{3}e^{2}+Ab^{2}f^{3}e^{2}+2\,Aacf^{3}e^{2}+5\,Bc^{2}dfe^{3}-Bb^{2}f^{2}e^{3}-2\,Bacf^{2}e^{3}-2\,Abcf^{2}e^{3}+2\,Bbcfe^{4}+Ac^{2}fe^{4}-Bc^{2}e^{5}\right)\arctan\left(\frac{2\,fx+e}{\sqrt{4\,df-e^{2}}}\right)}{\sqrt{4\,df-e^{2}}\,f^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/12*(3*B*c^2*f^3*x^4 + 8*B*b*c*f^3*x^3 + 4*A*c^2*f^3*x^3 - 4*B*c^2*f^2*x^3
*e - 6*B*c^2*d*f^2*x^2 + 6*B*b^2*f^3*x^2 + 12*B*a*c*f^3*x^2 + 12*A*b*c*f^3*
x^2 - 12*B*b*c*f^2*x^2*e - 6*A*c^2*f^2*x^2*e - 24*B*b*c*d*f^2*x - 12*A*c^2*
d*f^2*x + 24*B*a*b*f^3*x + 12*A*b^2*f^3*x + 24*A*a*c*f^3*x + 6*B*c^2*f*x^2*
e^2 + 24*B*c^2*d*f*x*e - 12*B*b^2*f^2*x*e - 24*B*a*c*f^2*x*e - 24*A*b*c*f^2
*x*e + 24*B*b*c*f*x*e^2 + 12*A*c^2*f*x*e^2 - 12*B*c^2*x*e^3)/f^4 + 1/2*(B*c
^2*d^2*f^2 - B*b^2*d*f^3 - 2*B*a*c*d*f^3 - 2*A*b*c*d*f^3 + B*a^2*f^4 + 2*A*
a*b*f^4 + 4*B*b*c*d*f^2*e + 2*A*c^2*d*f^2*e - 2*B*a*b*f^3*e - A*b^2*f^3*e -
 2*A*a*c*f^3*e - 3*B*c^2*d*f*e^2 + B*b^2*f^2*e^2 + 2*B*a*c*f^2*e^2 + 2*A*b*
c*f^2*e^2 - 2*B*b*c*f*e^3 - A*c^2*f*e^3 + B*c^2*e^4)*log(f*x^2 + x*e + d)/f
^5 + (4*B*b*c*d^2*f^3 + 2*A*c^2*d^2*f^3 - 4*B*a*b*d*f^4 - 2*A*b^2*d*f^4 - 4
```

```
*A*a*c*d*f^4 + 2*A*a^2*f^5 - 5*B*c^2*d^2*f^2*e + 3*B*b^2*d*f^3*e + 6*B*a*c*
d*f^3*e + 6*A*b*c*d*f^3*e - B*a^2*f^4*e - 2*A*a*b*f^4*e - 8*B*b*c*d*f^2*e^2
 - 4*A*c^2*d*f^2*e^2 + 2*B*a*b*f^3*e^2 + A*b^2*f^3*e^2 + 2*A*a*c*f^3*e^2 +
5*B*c^2*d*f*e^3 - B*b^2*f^2*e^3 - 2*B*a*c*f^2*e^3 - 2*A*b*c*f^2*e^3 + 2*B*b
*c*f*e^4 + A*c^2*f*e^4 - B*c^2*e^5)*arctan((2*f*x + e)/sqrt(4*d*f - e^2))/(
sqrt(4*d*f - e^2)*f^5)
```

**maple [B]** time = 0.01, size = 1672, normalized size = 3.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x)
```

```
[Out] 1/4*B*c^2/f*x^4-2*B*b*c*d/f^2*x+1/2/f*ln(f*x^2+e*x+d)*B*a^2+2/(4*d*f-e^2)^(
1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*A*a^2+1/3*A*c^2/f*x^3+1/2*B*b^2/f*
x^2+A*b^2/f*x-A*c^2*d/f^2*x+2*B*a*b/f*x+B*a*c/f*x^2-1/2*B*c^2*d/f^2*x^2+2*A
*a*c/f*x+2/3*B*b*c/f*x^3+A*b*c/f*x^2-8/f^3/(4*d*f-e^2)^(1/2)*arctan((2*f*x+
e)/(4*d*f-e^2)^(1/2))*B*b*c*d*e^2+6/f^2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/
(4*d*f-e^2)^(1/2))*A*b*c*d*e+6/f^2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*
f-e^2)^(1/2))*B*a*c*d*e+1/f^3*A*c^2*e^2*x-1/f^2*B*b^2*e*x-1/f^4*B*c^2*e^3*x
-1/2/f^2*A*x^2*c^2*e+1/2/f^3*B*x^2*c^2*e^2-1/3/f^2*B*x^3*c^2*e-1/2/f^2*ln(f
*x^2+e*x+d)*A*b^2*e-1/2/f^4*ln(f*x^2+e*x+d)*A*c^2*e^3-1/2/f^2*ln(f*x^2+e*x+
d)*B*b^2*d+1/2/f^3*ln(f*x^2+e*x+d)*B*b^2*e^2+1/2/f^3*ln(f*x^2+e*x+d)*B*c^2*
d^2+1/2/f^5*ln(f*x^2+e*x+d)*B*c^2*e^4+1/f*ln(f*x^2+e*x+d)*A*a*b-1/f^2*B*x^2
*b*c*e-2/f^2*A*b*c*e*x-2/f^2*B*a*c*e*x+2/f^3*B*b*c*e^2*x+2/f^3*B*c^2*d*e*x+
1/f^2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e^2*A*b^2-1/f/(
4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e*B*a^2-1/f^3/(4*d*f-e
^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e^3*B*b^2-1/f^5/(4*d*f-e^2)^(
1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e^5*B*c^2-2/f/(4*d*f-e^2)^(1/2)*ar
ctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*A*b^2*d+2/f^2/(4*d*f-e^2)^(1/2)*arctan((2
*f*x+e)/(4*d*f-e^2)^(1/2))*A*c^2*d^2-1/f^2*ln(f*x^2+e*x+d)*A*a*c*e-1/f^2*ln
(f*x^2+e*x+d)*A*b*c*d+1/f^3*ln(f*x^2+e*x+d)*A*b*c*e^2+1/f^3*ln(f*x^2+e*x+d)
*A*c^2*d*e-1/f^2*ln(f*x^2+e*x+d)*B*a*b*e-1/f^2*ln(f*x^2+e*x+d)*B*a*c*d+1/f^
3*ln(f*x^2+e*x+d)*B*a*c*e^2-1/f^4*ln(f*x^2+e*x+d)*B*b*c*e^3-3/2/f^4*ln(f*x^
2+e*x+d)*B*c^2*d*e^2+1/f^4/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(
1/2))*e^4*A*c^2-2/f^3/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))
*e^3*A*b*c+2/f^2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e^2*
B*a*b-2/f^3/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e^3*B*a*c
+2/f^3*ln(f*x^2+e*x+d)*B*b*c*d*e-4/f/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*
d*f-e^2)^(1/2))*A*a*c*d-4/f^3/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2
)^(1/2))*A*c^2*d*e^2-4/f/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/
2))*B*a*b*d+3/f^2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*B*b
^2*d*e+4/f^2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*B*b*c*d^
2-5/f^3/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*B*c^2*d^2*e-2
```

```
/f/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e*A*a*b+5/f^4/(4*d
*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*B*c^2*d*e^3+2/f^4/(4*d*f-
e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e^4*B*b*c+2/f^2/(4*d*f-e^2)^
(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e^2*A*a*c
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive or negative?
```

**mupad [B]** time = 4.85, size = 893, normalized size = 1.65

$$x^3\left(\frac{Ac^2+2Bbc}{3f}-\frac{Bc^2e}{3f^2}\right)+x\left(\frac{Ab^2+2Bab+2Aac}{f}-\frac{d\left(\frac{Ac^2+2Bbc}{f}-\frac{Bc^2e}{f^2}\right)}{f}+\frac{e\left(\frac{e\left(\frac{Ac^2+2Bbc}{f}-\frac{Bc^2e}{f^2}\right)}{f}-\frac{Bb^2+2Acb+2Bac}{f}+\frac{Bc^2d}{f^2}\right)}{f}\right)-x^2\left(\frac{e\left(\frac{Ac^2+2Bbc}{f}-\frac{Bc^2e}{f^2}\right)}{2f}-\frac{Bb^2+2Acb+2Bac}{2f}+\frac{Bc^2d}{2f^2}\right)-\frac{\ln\left(fx^2+ex+d\right)\left(\text{[illegible]}\right)}{2\left(4df^6-e^2f^5\right)}\text{[illegible]}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x)*(a + b*x + c*x^2)^2)/(d + e*x + f*x^2),x)
```

```
[Out] x^3*((A*c^2 + 2*B*b*c)/(3*f) - (B*c^2*e)/(3*f^2)) + x*((A*b^2 + 2*A*a*c + 2
*B*a*b)/f - (d*((A*c^2 + 2*B*b*c)/f - (B*c^2*e)/f^2))/f + (e*((e*((A*c^2 +
2*B*b*c)/f - (B*c^2*e)/f^2))/f - (B*b^2 + 2*A*b*c + 2*B*a*c)/f + (B*c^2*d)/
f^2))/f) - x^2*((e*((A*c^2 + 2*B*b*c)/f - (B*c^2*e)/f^2))/(2*f) - (B*b^2 +
2*A*b*c + 2*B*a*c)/(2*f) + (B*c^2*d)/(2*f^2)) - (log(d + e*x + f*x^2)*(B*c^
2*e^6 - 4*B*a^2*d*f^5 - A*c^2*e^5*f - A*b^2*e^3*f^3 + B*a^2*e^2*f^4 + 4*B*b
^2*d^2*f^4 + B*b^2*e^4*f^2 - 4*B*c^2*d^3*f^3 + 6*A*c^2*d*e^3*f^2 - 8*A*c^2*
d^2*e*f^3 - 5*B*b^2*d*e^2*f^3 - 8*A*a*b*d*f^5 - 2*B*b*c*e^5*f + 13*B*c^2*d^
2*e^2*f^2 + 2*A*a*b*e^2*f^4 - 2*A*a*c*e^3*f^3 + 8*A*b*c*d^2*f^4 - 2*B*a*b*e
^3*f^3 + 8*B*a*c*d^2*f^4 + 2*A*b*c*e^4*f^2 + 2*B*a*c*e^4*f^2 + 4*A*b^2*d*e*
f^4 - 7*B*c^2*d*e^4*f - 10*A*b*c*d*e^2*f^3 - 10*B*a*c*d*e^2*f^3 + 12*B*b*c*
d*e^3*f^2 - 16*B*b*c*d^2*e*f^3 + 8*A*a*c*d*e*f^4 + 8*B*a*b*d*e*f^4))/(2*(4*
d*f^6 - e^2*f^5)) + (B*c^2*x^4)/(4*f) + (atan(e/(4*d*f - e^2)^(1/2) + (2*f*
x)/(4*d*f - e^2)^(1/2))*(2*A*a^2*f^5 - B*c^2*e^5 - 2*A*b^2*d*f^4 - B*a^2*e*
f^4 + A*c^2*e^4*f + A*b^2*e^2*f^3 + 2*A*c^2*d^2*f^3 - B*b^2*e^3*f^2 - 4*A*c
^2*d*e^2*f^2 - 5*B*c^2*d^2*e*f^2 - 2*A*a*b*e*f^4 - 4*A*a*c*d*f^4 - 4*B*a*b*
d*f^4 + 2*B*b*c*e^4*f + 2*A*a*c*e^2*f^3 + 2*B*a*b*e^2*f^3 - 2*A*b*c*e^3*f^2
 - 2*B*a*c*e^3*f^2 + 4*B*b*c*d^2*f^3 + 3*B*b^2*d*e*f^3 + 5*B*c^2*d*e^3*f -
8*B*b*c*d*e^2*f^2 + 6*A*b*c*d*e*f^3 + 6*B*a*c*d*e*f^3))/(f^5*(4*d*f - e^2)^
(1/2))
```

**sympy [B]** time = 145.64, size = 4663, normalized size = 8.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x**2+b*x+a)**2/(f*x**2+e*x+d),x)
```

```
[Out] B*c**2*x**4/(4*f) + x**3*(A*c**2/(3*f) + 2*B*b*c/(3*f) - B*c**2*e/(3*f**2))
 + x**2*(A*b*c/f - A*c**2*e/(2*f**2) + B*a*c/f + B*b**2/(2*f) - B*b*c*e/f**
2 - B*c**2*d/(2*f**2) + B*c**2*e**2/(2*f**3)) + x*(2*A*a*c/f + A*b**2/f - 2
*A*b*c*e/f**2 - A*c**2*d/f**2 + A*c**2*e**2/f**3 + 2*B*a*b/f - 2*B*a*c*e/f*
*2 - B*b**2*e/f**2 - 2*B*b*c*d/f**2 + 2*B*b*c*e**2/f**3 + 2*B*c**2*d*e/f**3
 - B*c**2*e**3/f**4) + (-sqrt(-4*d*f + e**2)*(-2*A*a**2*f**5 + 2*A*a*b*e*f*
*4 + 4*A*a*c*d*f**4 - 2*A*a*c*e**2*f**3 + 2*A*b**2*d*f**4 - A*b**2*e**2*f**
3 - 6*A*b*c*d*e*f**3 + 2*A*b*c*e**3*f**2 - 2*A*c**2*d**2*f**3 + 4*A*c**2*d*
e**2*f**2 - A*c**2*e**4*f + B*a**2*e*f**4 + 4*B*a*b*d*f**4 - 2*B*a*b*e**2*f
**3 - 6*B*a*c*d*e*f**3 + 2*B*a*c*e**3*f**2 - 3*B*b**2*d*e*f**3 + B*b**2*e**
3*f**2 - 4*B*b*c*d**2*f**3 + 8*B*b*c*d*e**2*f**2 - 2*B*b*c*e**4*f + 5*B*c**
2*d**2*e*f**2 - 5*B*c**2*d*e**3*f + B*c**2*e**5)/(2*f**5*(4*d*f - e**2)) +
(2*A*a*b*f**4 - 2*A*a*c*e*f**3 - A*b**2*e*f**3 - 2*A*b*c*d*f**3 + 2*A*b*c*e
**2*f**2 + 2*A*c**2*d*e*f**2 - A*c**2*e**3*f + B*a**2*f**4 - 2*B*a*b*e*f**3
 - 2*B*a*c*d*f**3 + 2*B*a*c*e**2*f**2 - B*b**2*d*f**3 + B*b**2*e**2*f**2 +
4*B*b*c*d*e*f**2 - 2*B*b*c*e**3*f + B*c**2*d**2*f**2 - 3*B*c**2*d*e**2*f +
B*c**2*e**4)/(2*f**5))*log(x + (-A*a**2*e*f**4 + 4*A*a*b*d*f**4 - 2*A*a*c*d
*e*f**3 - A*b**2*d*e*f**3 - 4*A*b*c*d**2*f**3 + 2*A*b*c*d*e**2*f**2 + 3*A*c
**2*d**2*e*f**2 - A*c**2*d*e**3*f + 2*B*a**2*d*f**4 - 2*B*a*b*d*e*f**3 - 4*
B*a*c*d**2*f**3 + 2*B*a*c*d*e**2*f**2 - 2*B*b**2*d**2*f**3 + B*b**2*d*e**2*
f**2 + 6*B*b*c*d**2*e*f**2 - 2*B*b*c*d*e**3*f + 2*B*c**2*d**3*f**2 - 4*B*c*
*2*d**2*e**2*f + B*c**2*d*e**4 - 4*d*f**5*(-sqrt(-4*d*f + e**2)*(-2*A*a**2*
f**5 + 2*A*a*b*e*f**4 + 4*A*a*c*d*f**4 - 2*A*a*c*e**2*f**3 + 2*A*b**2*d*f**
4 - A*b**2*e**2*f**3 - 6*A*b*c*d*e*f**3 + 2*A*b*c*e**3*f**2 - 2*A*c**2*d**2
*f**3 + 4*A*c**2*d*e**2*f**2 - A*c**2*e**4*f + B*a**2*e*f**4 + 4*B*a*b*d*f*
*4 - 2*B*a*b*e**2*f**3 - 6*B*a*c*d*e*f**3 + 2*B*a*c*e**3*f**2 - 3*B*b**2*d*
e*f**3 + B*b**2*e**3*f**2 - 4*B*b*c*d**2*f**3 + 8*B*b*c*d*e**2*f**2 - 2*B*b
*c*e**4*f + 5*B*c**2*d**2*e*f**2 - 5*B*c**2*d*e**3*f + B*c**2*e**5)/(2*f**5
*(4*d*f - e**2)) + (2*A*a*b*f**4 - 2*A*a*c*e*f**3 - A*b**2*e*f**3 - 2*A*b*c
*d*f**3 + 2*A*b*c*e**2*f**2 + 2*A*c**2*d*e*f**2 - A*c**2*e**3*f + B*a**2*f*
*4 - 2*B*a*b*e*f**3 - 2*B*a*c*d*f**3 + 2*B*a*c*e**2*f**2 - B*b**2*d*f**3 +
B*b**2*e**2*f**2 + 4*B*b*c*d*e*f**2 - 2*B*b*c*e**3*f + B*c**2*d**2*f**2 - 3
*B*c**2*d*e**2*f + B*c**2*e**4)/(2*f**5)) + e**2*f**4*(-sqrt(-4*d*f + e**2)
*(-2*A*a**2*f**5 + 2*A*a*b*e*f**4 + 4*A*a*c*d*f**4 - 2*A*a*c*e**2*f**3 + 2*
A*b**2*d*f**4 - A*b**2*e**2*f**3 - 6*A*b*c*d*e*f**3 + 2*A*b*c*e**3*f**2 - 2
*A*c**2*d**2*f**3 + 4*A*c**2*d*e**2*f**2 - A*c**2*e**4*f + B*a**2*e*f**4 +
4*B*a*b*d*f**4 - 2*B*a*b*e**2*f**3 - 6*B*a*c*d*e*f**3 + 2*B*a*c*e**3*f**2 -
 3*B*b**2*d*e*f**3 + B*b**2*e**3*f**2 - 4*B*b*c*d**2*f**3 + 8*B*b*c*d*e**2*
```

```
f**2 - 2*B*b*c*e**4*f + 5*B*c**2*d**2*e*f**2 - 5*B*c**2*d*e**3*f + B*c**2*e
**5)/(2*f**5*(4*d*f - e**2)) + (2*A*a*b*f**4 - 2*A*a*c*e*f**3 - A*b**2*e*f*
*3 - 2*A*b*c*d*f**3 + 2*A*b*c*e**2*f**2 + 2*A*c**2*d*e*f**2 - A*c**2*e**3*f
 + B*a**2*f**4 - 2*B*a*b*e*f**3 - 2*B*a*c*d*f**3 + 2*B*a*c*e**2*f**2 - B*b*
*2*d*f**3 + B*b**2*e**2*f**2 + 4*B*b*c*d*e*f**2 - 2*B*b*c*e**3*f + B*c**2*d
**2*f**2 - 3*B*c**2*d*e**2*f + B*c**2*e**4)/(2*f**5)))/(-2*A*a**2*f**5 + 2*
A*a*b*e*f**4 + 4*A*a*c*d*f**4 - 2*A*a*c*e**2*f**3 + 2*A*b**2*d*f**4 - A*b**
2*e**2*f**3 - 6*A*b*c*d*e*f**3 + 2*A*b*c*e**3*f**2 - 2*A*c**2*d**2*f**3 + 4
*A*c**2*d*e**2*f**2 - A*c**2*e**4*f + B*a**2*e*f**4 + 4*B*a*b*d*f**4 - 2*B*
a*b*e**2*f**3 - 6*B*a*c*d*e*f**3 + 2*B*a*c*e**3*f**2 - 3*B*b**2*d*e*f**3 +
B*b**2*e**3*f**2 - 4*B*b*c*d**2*f**3 + 8*B*b*c*d*e**2*f**2 - 2*B*b*c*e**4*f
 + 5*B*c**2*d**2*e*f**2 - 5*B*c**2*d*e**3*f + B*c**2*e**5)) + (sqrt(-4*d*f
+ e**2)*(-2*A*a**2*f**5 + 2*A*a*b*e*f**4 + 4*A*a*c*d*f**4 - 2*A*a*c*e**2*f*
*3 + 2*A*b**2*d*f**4 - A*b**2*e**2*f**3 - 6*A*b*c*d*e*f**3 + 2*A*b*c*e**3*f
**2 - 2*A*c**2*d**2*f**3 + 4*A*c**2*d*e**2*f**2 - A*c**2*e**4*f + B*a**2*e*
f**4 + 4*B*a*b*d*f**4 - 2*B*a*b*e**2*f**3 - 6*B*a*c*d*e*f**3 + 2*B*a*c*e**3
*f**2 - 3*B*b**2*d*e*f**3 + B*b**2*e**3*f**2 - 4*B*b*c*d**2*f**3 + 8*B*b*c*
d*e**2*f**2 - 2*B*b*c*e**4*f + 5*B*c**2*d**2*e*f**2 - 5*B*c**2*d*e**3*f + B
*c**2*e**5)/(2*f**5*(4*d*f - e**2)) + (2*A*a*b*f**4 - 2*A*a*c*e*f**3 - A*b*
*2*e*f**3 - 2*A*b*c*d*f**3 + 2*A*b*c*e**2*f**2 + 2*A*c**2*d*e*f**2 - A*c**2
*e**3*f + B*a**2*f**4 - 2*B*a*b*e*f**3 - 2*B*a*c*d*f**3 + 2*B*a*c*e**2*f**2
 - B*b**2*d*f**3 + B*b**2*e**2*f**2 + 4*B*b*c*d*e*f**2 - 2*B*b*c*e**3*f + B
*c**2*d**2*f**2 - 3*B*c**2*d*e**2*f + B*c**2*e**4)/(2*f**5))*log(x + (-A*a*
*2*e*f**4 + 4*A*a*b*d*f**4 - 2*A*a*c*d*e*f**3 - A*b**2*d*e*f**3 - 4*A*b*c*d
**2*f**3 + 2*A*b*c*d*e**2*f**2 + 3*A*c**2*d**2*e*f**2 - A*c**2*d*e**3*f + 2
*B*a**2*d*f**4 - 2*B*a*b*d*e*f**3 - 4*B*a*c*d**2*f**3 + 2*B*a*c*d*e**2*f**2
 - 2*B*b**2*d**2*f**3 + B*b**2*d*e**2*f**2 + 6*B*b*c*d**2*e*f**2 - 2*B*b*c*
d*e**3*f + 2*B*c**2*d**3*f**2 - 4*B*c**2*d**2*e**2*f + B*c**2*d*e**4 - 4*d*
f**5*(sqrt(-4*d*f + e**2)*(-2*A*a**2*f**5 + 2*A*a*b*e*f**4 + 4*A*a*c*d*f**4
 - 2*A*a*c*e**2*f**3 + 2*A*b**2*d*f**4 - A*b**2*e**2*f**3 - 6*A*b*c*d*e*f**
3 + 2*A*b*c*e**3*f**2 - 2*A*c**2*d**2*f**3 + 4*A*c**2*d*e**2*f**2 - A*c**2*
e**4*f + B*a**2*e*f**4 + 4*B*a*b*d*f**4 - 2*B*a*b*e**2*f**3 - 6*B*a*c*d*e*f
**3 + 2*B*a*c*e**3*f**2 - 3*B*b**2*d*e*f**3 + B*b**2*e**3*f**2 - 4*B*b*c*d*
*2*f**3 + 8*B*b*c*d*e**2*f**2 - 2*B*b*c*e**4*f + 5*B*c**2*d**2*e*f**2 - 5*B
*c**2*d*e**3*f + B*c**2*e**5)/(2*f**5*(4*d*f - e**2)) + (2*A*a*b*f**4 - 2*A
*a*c*e*f**3 - A*b**2*e*f**3 - 2*A*b*c*d*f**3 + 2*A*b*c*e**2*f**2 + 2*A*c**2
*d*e*f**2 - A*c**2*e**3*f + B*a**2*f**4 - 2*B*a*b*e*f**3 - 2*B*a*c*d*f**3 +
 2*B*a*c*e**2*f**2 - B*b**2*d*f**3 + B*b**2*e**2*f**2 + 4*B*b*c*d*e*f**2 -
2*B*b*c*e**3*f + B*c**2*d**2*f**2 - 3*B*c**2*d*e**2*f + B*c**2*e**4)/(2*f**
5)) + e**2*f**4*(sqrt(-4*d*f + e**2)*(-2*A*a**2*f**5 + 2*A*a*b*e*f**4 + 4*A
*a*c*d*f**4 - 2*A*a*c*e**2*f**3 + 2*A*b**2*d*f**4 - A*b**2*e**2*f**3 - 6*A*
b*c*d*e*f**3 + 2*A*b*c*e**3*f**2 - 2*A*c**2*d**2*f**3 + 4*A*c**2*d*e**2*f**
2 - A*c**2*e**4*f + B*a**2*e*f**4 + 4*B*a*b*d*f**4 - 2*B*a*b*e**2*f**3 - 6*
B*a*c*d*e*f**3 + 2*B*a*c*e**3*f**2 - 3*B*b**2*d*e*f**3 + B*b**2*e**3*f**2 -
 4*B*b*c*d**2*f**3 + 8*B*b*c*d*e**2*f**2 - 2*B*b*c*e**4*f + 5*B*c**2*d**2*e
```

```
*f**2 - 5*B*c**2*d*e**3*f + B*c**2*e**5)/(2*f**5*(4*d*f - e**2)) + (2*A*a*b
*f**4 - 2*A*a*c*e*f**3 - A*b**2*e*f**3 - 2*A*b*c*d*f**3 + 2*A*b*c*e**2*f**2
 + 2*A*c**2*d*e*f**2 - A*c**2*e**3*f + B*a**2*f**4 - 2*B*a*b*e*f**3 - 2*B*a
*c*d*f**3 + 2*B*a*c*e**2*f**2 - B*b**2*d*f**3 + B*b**2*e**2*f**2 + 4*B*b*c*
d*e*f**2 - 2*B*b*c*e**3*f + B*c**2*d**2*f**2 - 3*B*c**2*d*e**2*f + B*c**2*e
**4)/(2*f**5)))/(-2*A*a**2*f**5 + 2*A*a*b*e*f**4 + 4*A*a*c*d*f**4 - 2*A*a*c
*e**2*f**3 + 2*A*b**2*d*f**4 - A*b**2*e**2*f**3 - 6*A*b*c*d*e*f**3 + 2*A*b*
c*e**3*f**2 - 2*A*c**2*d**2*f**3 + 4*A*c**2*d*e**2*f**2 - A*c**2*e**4*f + B
*a**2*e*f**4 + 4*B*a*b*d*f**4 - 2*B*a*b*e**2*f**3 - 6*B*a*c*d*e*f**3 + 2*B*
a*c*e**3*f**2 - 3*B*b**2*d*e*f**3 + B*b**2*e**3*f**2 - 4*B*b*c*d**2*f**3 +
8*B*b*c*d*e**2*f**2 - 2*B*b*c*e**4*f + 5*B*c**2*d**2*e*f**2 - 5*B*c**2*d*e*
*3*f + B*c**2*e**5))
```

## 3.15 $\int \frac{A+Bx}{(a+bx+cx^2)(d+ex+fx^2)}\,dx$

**Optimal**. Leaf size=406

$$\frac{\tanh^{-1}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)\left(B(aef-2bdf+cde)-A\left(2af^2-bef-2cdf+ce^2\right)\right)}{\sqrt{e^2-4df}\left(f\left(a^2f-abe+b^2d\right)-c\left(bde-a\left(e^2-2df\right)\right)+c^2d^2\right)}+\frac{\log\left(a+bx+cx^2\right)(-aBf+Abf-}{2\left(f\left(a^2f-abe+b^2d\right)-c\left(bde-a\left(e^2\right.\right.\right.}$$

**Rubi [A]** time = 0.48, antiderivative size = 398, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1022, 634, 618, 206, 628}

$$\frac{\tanh^{-1}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)\left(B(aef-2bdf+cde)-A\left(2af^2-bef-2cdf+ce^2\right)\right)}{\sqrt{e^2-4df}\left(f\left(a^2f-abe+b^2d\right)+ac\left(e^2-2df\right)-bcde+c^2d^2\right)}+\frac{\log\left(a+bx+cx^2\right)(-aBf+Abf-Ace+Bcd)}{2\left(f\left(a^2f-abe+b^2d\right)+ac\left(e^2-2df\right)-bcde+c^2d^2\right)}-\frac{\log\left(d+ex+fx^2\right)(-aBf+Abf-Ace+Bcd)}{2\left(f\left(a^2f-abe+b^2d\right)+ac\left(e^2-2df\right)-bcde+c^2d^2\right)}-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-b(aBf+Ace+Bcd)+2c(-aAf+aBe+Acd)+Ab^2f\right)}{\sqrt{b^2-4ac}\left(f\left(a^2f-abe+b^2d\right)+ac\left(e^2-2df\right)-bcde+c^2d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/((a + b*x + c*x^2)*(d + e*x + f*x^2)),x]
```

```
[Out] -(((A*b^2*f + 2*c*(A*c*d + a*B*e - a*A*f) - b*(B*c*d + A*c*e + a*B*f))*ArcT
anh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c^2*d^2 - b*c*d*e +
 f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f)))) + ((B*(c*d*e - 2*b*d*f +
a*e*f) - A*(c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2))*ArcTanh[(e + 2*f*x)/Sqrt[e^
2 - 4*d*f]])/(Sqrt[e^2 - 4*d*f]*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2
*f) + a*c*(e^2 - 2*d*f))) + ((B*c*d - A*c*e + A*b*f - a*B*f)*Log[a + b*x +
c*x^2])/(2*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*
f))) - ((B*c*d - A*c*e + A*b*f - a*B*f)*Log[d + e*x + f*x^2])/(2*(c^2*d^2 -
 b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f)))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1022

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (e_.)*
(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = Simplify[c^2*d^2 - b*c*d*e +
a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2]}, Dist[1/q, Int[Simp[g*c
^2*d - g*b*c*e + a*h*c*e + g*b^2*f - a*b*h*f - a*g*c*f + c*(h*c*d - g*c*e +
 g*b*f - a*h*f)*x, x]/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[Simp[-(h*c*
d*e) + g*c*e^2 + b*h*d*f - g*c*d*f - g*b*e*f + a*g*f^2 - f*(h*c*d - g*c*e +
 g*b*f - a*h*f)*x, x]/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a,
b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{A+Bx}{\left(a+bx+cx^2\right)\left(d+ex+fx^2\right)}\,dx = \frac{\int \frac{aB(ce-bf)+A\left(c^2d+b^2f-c(be+af)\right)+c(Bcd-Ace+Abf-aBf)x}{a+bx+cx^2}\,dx}{c^2d^2-bcde+f\left(b^2d-abe+a^2f\right)+ac\left(e^2-2df\right)} + \frac{\int \frac{-Af(be-af)+Ac\left(e^2-\right.}{}}{c^2d^2-bcde+f\left(\right.}$$

$$= \frac{(Bcd-Ace+Abf-aBf)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2\left(c^2d^2-bcde+f\left(b^2d-abe+a^2f\right)+ac\left(e^2-2df\right)\right)} - \frac{(Bcd-A}{2\left(c^2d^2-bcde+\right.}$$

$$= \frac{(Bcd-Ace+Abf-aBf)\log\left(a+bx+cx^2\right)}{2\left(c^2d^2-bcde+f\left(b^2d-abe+a^2f\right)+ac\left(e^2-2df\right)\right)} - \frac{(Bcd-Ace}{2\left(c^2d^2-bcde+\right.}$$

$$= -\frac{\left(Ab^2f+2c(Acd+aBe-aAf)-b(Bcd+Ace+aBf)\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right.}{\sqrt{b^2-4ac}\left(c^2d^2-bcde+f\left(b^2d-abe+a^2f\right)+ac\left(e^2-2df\right)\right)}$$

**Mathematica [A]** time = 0.46, size = 267, normalized size = 0.66

$$\frac{\frac{2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)\left(-b(aBf+Ace+Bcd)+2c(-aAf+aBe+Acd)+Ab^2f\right)}{\sqrt{4ac-b^2}}-\frac{2\tan^{-1}\left(\frac{e+2fx}{\sqrt{4df-e^2}}\right)\left(A\left(-2af^2+bef+2cdf-ce^2\right)+B(aef-2bdf+cde)\right)}{\sqrt{4df-e^2}}+\log(a+x(b+cx))(-aBf+Abf-Ace+Bcd)+\log(d+x(e+fx))(aBf-Abf+Ace-Bcd)}{2\left(f\left(a^2f-abe+b^2d\right)+ac\left(e^2-2df\right)-bcde+c^2d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/((a + b*x + c*x^2)*(d + e*x + f*x^2)),x]
```

```
[Out] ((2*(A*b^2*f + 2*c*(A*c*d + a*B*e - a*A*f) - b*(B*c*d + A*c*e + a*B*f))*Arc
Tan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - (2*(B*(c*d*e - 2*
b*d*f + a*e*f) + A*(-(c*e^2) + 2*c*d*f + b*e*f - 2*a*f^2))*ArcTan[(e + 2*f*
x)/Sqrt[-e^2 + 4*d*f]])/Sqrt[-e^2 + 4*d*f] + (B*c*d - A*c*e + A*b*f - a*B*f
)*Log[a + x*(b + c*x)] + (-(B*c*d) + A*c*e - A*b*f + a*B*f)*Log[d + x*(e +
f*x)])/(2*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f
)))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx}{(a+bx+cx^2)(d+ex+fx^2)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x)/((a + b*x + c*x^2)*(d + e*x + f*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x)/((a + b*x + c*x^2)*(d + e*x + f*x^2)), x]
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [A]** time = 0.17, size = 416, normalized size = 1.02

$$\frac{(Bcd-Baf+Abf-Ace)\log(cx^2+bx+a)}{2(c^2d^2+b^2df-2acdf+a^2f^2-bcde-abfe+ace^2)}-\frac{(Bcd-Baf+Abf-Ace)\log(fx^2+xe+d)}{2(c^2d^2+b^2df-2acdf+a^2f^2-bcde-abfe+ace^2)}-\frac{(Bbcd-2Ac^2d+Babf-Ab^2f+2Aacf-2Bace+Abce)\arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(c^2d^2+b^2df-2acdf+a^2f^2-bcde-abfe+ace^2)\sqrt{-b^2+4ac}}+\frac{(2Bbdf-2Acdf+2Aaf^2-Bcde-Bafe-Abfe+Ace^2)\arctan\left(\frac{2fx+e}{\sqrt{4df-e^2}}\right)}{(c^2d^2+b^2df-2acdf+a^2f^2-bcde-abfe+ace^2)\sqrt{4df-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/2*(B*c*d - B*a*f + A*b*f - A*c*e)*log(c*x^2 + b*x + a)/(c^2*d^2 + b^2*d*f
 - 2*a*c*d*f + a^2*f^2 - b*c*d*e - a*b*f*e + a*c*e^2) - 1/2*(B*c*d - B*a*f
+ A*b*f - A*c*e)*log(f*x^2 + x*e + d)/(c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*
f^2 - b*c*d*e - a*b*f*e + a*c*e^2) - (B*b*c*d - 2*A*c^2*d + B*a*b*f - A*b^2
*f + 2*A*a*c*f - 2*B*a*c*e + A*b*c*e)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c)
)/((c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2 - b*c*d*e - a*b*f*e + a*c*e^2)*
```

```
sqrt(-b^2 + 4*a*c)) + (2*B*b*d*f - 2*A*c*d*f + 2*A*a*f^2 - B*c*d*e - B*a*f*
e - A*b*f*e + A*c*e^2)*arctan((2*f*x + e)/sqrt(4*d*f - e^2))/((c^2*d^2 + b^
2*d*f - 2*a*c*d*f + a^2*f^2 - b*c*d*e - a*b*f*e + a*c*e^2)*sqrt(4*d*f - e^2
))
```

**maple [B]** time = 0.01, size = 1698, normalized size = 4.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d),x)
```

```
[Out] 1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)*ln(c*x^2+b*
x+a)*A*b*f-1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)*
c*ln(c*x^2+b*x+a)*A*e-1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*
e+c^2*d^2)*ln(c*x^2+b*x+a)*B*a*f+1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2
*d*f-b*c*d*e+c^2*d^2)*c*ln(c*x^2+b*x+a)*B*d-2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*
c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^
2)^(1/2))*A*a*c*f+1/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*
d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*A*b^2*f-1/(a^2*f
^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*a*c-b^2)^(1/2)*arc
tan((2*c*x+b)/(4*a*c-b^2)^(1/2))*A*b*c*e+2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e
^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^
(1/2))*A*c^2*d-1/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2
)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*B*a*b*f+2/(a^2*f^2-
a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan
((2*c*x+b)/(4*a*c-b^2)^(1/2))*B*a*c*e-1/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+
b^2*d*f-b*c*d*e+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/
2))*B*b*c*d-1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)
*f*ln(f*x^2+e*x+d)*A*b+1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d
*e+c^2*d^2)*ln(f*x^2+e*x+d)*A*c*e+1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^
2*d*f-b*c*d*e+c^2*d^2)*f*ln(f*x^2+e*x+d)*B*a-1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f
+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)*ln(f*x^2+e*x+d)*B*c*d+2/(a^2*f^2-a*b*e*f-
2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*d*f-e^2)^(1/2)*arctan((2*f*x+
e)/(4*d*f-e^2)^(1/2))*A*a*f^2-1/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-
b*c*d*e+c^2*d^2)/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*A*b*
e*f-2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*d*f-e^
2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*A*c*d*f+1/(a^2*f^2-a*b*e*f-2*a
*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/
(4*d*f-e^2)^(1/2))*A*c*e^2-1/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c
*d*e+c^2*d^2)/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*B*a*e*f
+2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*d*f-e^2)^
(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*B*b*d*f-1/(a^2*f^2-a*b*e*f-2*a*c*
d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*
d*f-e^2)^(1/2))*B*c*d*e
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive or negative?
```

**mupad** [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/((a + b*x + c*x^2)*(d + e*x + f*x^2)),x)
```

```
[Out] \text{Hanged}
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

## 3.16 $\int \frac{A+Bx}{\left(a+bx+cx^2\right)^2\left(d+ex+fx^2\right)}\,dx$

**Optimal**. Leaf size=1075

$$-\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(fb^2+2c^2d-c(be+2af)\right)+c\left(Afb^2-(Bcd+Ace+aBf)b+2c(Acd+a\right.}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\left(cx^2+bx+a\right)}$$

**Rubi** [A] time = 4.18, antiderivative size = 1067, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1016, 1072, 634, 618, 206, 628}

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/((a + b*x + c*x^2)^2*(d + e*x + f*x^2)),x]
```

```
[Out] -((A*c*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e +
2*a*f)) + c*(A*b^2*f + 2*c*(A*c*d + a*B*e - a*A*f) - b*(B*c*d + A*c*e + a*B
*f))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(a + b*x +
 c*x^2))) - ((b^5*(B*d - A*e)*f^2 - 2*b^4*f*(B*c*d*e - a*A*f^2 - A*c*(e^2 -
 d*f)) - 4*c^2*(A*(c^3*d^3 - 3*a^3*f^3 - a^2*c*f*(e^2 - 7*d*f) + a*c^2*d*(3
*e^2 - 5*d*f)) - a*B*e*(c^2*d^2 - 3*a^2*f^2 - a*c*(e^2 - 2*d*f))) - 4*b^2*(
B*c^3*d^2*e + A*c*f*(2*c^2*d^2 + 3*a^2*f^2 + 3*a*c*(e^2 - d*f))) + 2*b*c*(B
*(c^3*d^3 + 3*a^3*f^3 + a*c^2*d*(e^2 - 7*d*f) + 3*a^2*c*f*(e^2 + d*f)) + A*
c*e*(3*c^2*d^2 + 3*a^2*f^2 + a*c*(3*e^2 + 2*d*f))) - b^3*(A*c*e*(c*e^2 - 2*
c*d*f - 4*a*f^2) + B*(4*a*c*d*f^2 + a^2*f^3 - c^2*d*(e^2 + 5*d*f))))*ArcTan
h[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*(c^2*d^2 - b*c*d*e +
 f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))^2) + ((B*(c^2*d*e*(e^2 - 3*
d*f) - 2*c*d*f*(b*e^2 - 2*b*d*f - a*e*f) + f^2*(b^2*d*e - 4*a*b*d*f + a^2*e
*f)) - A*(c^2*(e^4 - 4*d*e^2*f + 2*d^2*f^2) - f^2*(2*a*b*e*f - 2*a^2*f^2 -
b^2*(e^2 - 2*d*f)) + 2*c*f*(a*f*(e^2 - 2*d*f) - b*(e^3 - 3*d*e*f))))*ArcTan
h[(e + 2*f*x)/Sqrt[e^2 - 4*d*f]])/(Sqrt[e^2 - 4*d*f]*(c^2*d^2 - b*c*d*e + f
*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))^2) + ((A*(c*e - b*f)*(f*(b*e
- 2*a*f) - c*(e^2 - 2*d*f)) - B*(2*c*d*f*(b*e - a*f) - f^2*(b^2*d - a^2*f)
- c^2*d*(e^2 - d*f)))*Log[a + b*x + c*x^2])/(2*(c^2*d^2 - b*c*d*e + f*(b^2*
d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))^2) - ((A*(c*e - b*f)*(f*(b*e - 2*a*
f) - c*(e^2 - 2*d*f)) - B*(2*c*d*f*(b*e - a*f) - f^2*(b^2*d - a^2*f) - c^2*
d*(e^2 - d*f)))*Log[d + e*x + f*x^2])/(2*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*
b*e + a^2*f) + a*c*(e^2 - 2*d*f))^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1016

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)
*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*
c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))
 - h*(b*c*d - 2*a*c*e + a*b*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d -
a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e
))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1072

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{A+Bx}{\left(a+bx+cx^2\right)^2\left(d+ex+fx^2\right)}\,dx = -\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(2c^2d+b^2f-c(be+2af)\right)+c\left(A\ldots\right.}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce\ldots\right.}$$

$$= -\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(2c^2d+b^2f-c(be+2af)\right)+c\left(A\ldots\right.}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce\ldots\right.}$$

$$= -\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(2c^2d+b^2f-c(be+2af)\right)+c\left(A\ldots\right.}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce\ldots\right.}$$

$$= -\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(2c^2d+b^2f-c(be+2af)\right)+c\left(A\ldots\right.}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce\ldots\right.}$$

$$= -\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(2c^2d+b^2f-c(be+2af)\right)+c\left(A\ldots\right.}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce\ldots\right.}$$

**Mathematica [A]** time = 6.70, size = 952, normalized size = 0.89

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/((a + b*x + c*x^2)^2*(d + e*x + f*x^2)),x]
```

```
[Out] ((-2*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))*(A
*(b^3*f + b^2*c*(-e + f*x) + b*c*(-3*a*f + c*(d - e*x)) + 2*c^2*(c*d*x + a*
```

```
(e - f*x))) + B*(2*a^2*c*f - b*c^2*d*x - a*(b^2*f + 2*c^2*(d - e*x) + b*c*(
-e + f*x)))))/((b^2 - 4*a*c)*(a + x*(b + c*x))) - (2*(b^5*(B*d - A*e)*f^2 +
 2*b^4*f*(-(B*c*d*e) + a*A*f^2 + A*c*(e^2 - d*f)) - 4*b^2*(B*c^3*d^2*e + A*
c*f*(2*c^2*d^2 + 3*a^2*f^2 + 3*a*c*(e^2 - d*f))) + 2*b*c*(B*(c^3*d^3 + 3*a^
3*f^3 + a*c^2*d*(e^2 - 7*d*f) + 3*a^2*c*f*(e^2 + d*f)) + A*c*e*(3*c^2*d^2 +
 3*a^2*f^2 + a*c*(3*e^2 + 2*d*f))) + 4*c^2*(a*B*e*(c^2*d^2 - 3*a^2*f^2 - a*
c*(e^2 - 2*d*f)) + A*(-(c^3*d^3) + 3*a^3*f^3 + a^2*c*f*(e^2 - 7*d*f) + a*c^
2*d*(-3*e^2 + 5*d*f))) + b^3*(A*c*e*(-(c*e^2) + 2*c*d*f + 4*a*f^2) + B*(-4*
a*c*d*f^2 - a^2*f^3 + c^2*d*(e^2 + 5*d*f))))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 +
 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + (2*(B*(c^2*d*e*(-e^2 + 3*d*f) - 2*c*d*f*(-
(b*e^2) + 2*b*d*f + a*e*f) + f^2*(-(b^2*d*e) + 4*a*b*d*f - a^2*e*f)) + A*(c
^2*(e^4 - 4*d*e^2*f + 2*d^2*f^2) + f^2*(-2*a*b*e*f + 2*a^2*f^2 + b^2*(e^2 -
 2*d*f)) + 2*c*f*(a*f*(e^2 - 2*d*f) - b*(e^3 - 3*d*e*f))))*ArcTan[(e + 2*f*
x)/Sqrt[-e^2 + 4*d*f]])/Sqrt[-e^2 + 4*d*f] - (A*(c*e - b*f)*(f*(-(b*e) + 2*
a*f) + c*(e^2 - 2*d*f)) + B*(2*c*d*f*(b*e - a*f) + f^2*(-(b^2*d) + a^2*f) +
 c^2*d*(-e^2 + d*f)))*Log[a + x*(b + c*x)] + (A*(c*e - b*f)*(f*(-(b*e) + 2*
a*f) + c*(e^2 - 2*d*f)) + B*(2*c*d*f*(b*e - a*f) + f^2*(-(b^2*d) + a^2*f) +
 c^2*d*(-e^2 + d*f)))*Log[d + x*(e + f*x)])/(2*(c^2*d^2 - b*c*d*e + f*(b^2*
d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))^2)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx}{\left(a+bx+cx^2\right)^2\left(d+ex+fx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x)/((a + b*x + c*x^2)^2*(d + e*x + f*x^2)),x]
[Out] IntegrateAlgebraic[(A + B*x)/((a + b*x + c*x^2)^2*(d + e*x + f*x^2)), x]
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Timed out
```

**giac [B]** time = 0.45, size = 3226, normalized size = 3.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] -1/2*(B*c^2*d^2*f - B*b^2*d*f^2 - 2*B*a*c*d*f^2 + 2*A*b*c*d*f^2 + B*a^2*f^3
 - 2*A*a*b*f^3 + 2*B*b*c*d*f*e - 2*A*c^2*d*f*e + A*b^2*f^2*e + 2*A*a*c*f^2*
e - B*c^2*d*e^2 - 2*A*b*c*f*e^2 + A*c^2*e^3)*log(c*x^2 + b*x + a)/(c^4*d^4
+ 2*b^2*c^2*d^3*f - 4*a*c^3*d^3*f + b^4*d^2*f^2 - 4*a*b^2*c*d^2*f^2 + 6*a^2
*c^2*d^2*f^2 + 2*a^2*b^2*d*f^3 - 4*a^3*c*d*f^3 + a^4*f^4 - 2*b*c^3*d^3*e -
2*b^3*c*d^2*f*e + 2*a*b*c^2*d^2*f*e - 2*a*b^3*d*f^2*e + 2*a^2*b*c*d*f^2*e -
 2*a^3*b*f^3*e + b^2*c^2*d^2*e^2 + 2*a*c^3*d^2*e^2 + 4*a*b^2*c*d*f*e^2 - 4*
a^2*c^2*d*f*e^2 + a^2*b^2*f^2*e^2 + 2*a^3*c*f^2*e^2 - 2*a*b*c^2*d*e^3 - 2*a
^2*b*c*f*e^3 + a^2*c^2*e^4) + 1/2*(B*c^2*d^2*f - B*b^2*d*f^2 - 2*B*a*c*d*f^
2 + 2*A*b*c*d*f^2 + B*a^2*f^3 - 2*A*a*b*f^3 + 2*B*b*c*d*f*e - 2*A*c^2*d*f*e
 + A*b^2*f^2*e + 2*A*a*c*f^2*e - B*c^2*d*e^2 - 2*A*b*c*f*e^2 + A*c^2*e^3)*l
og(f*x^2 + x*e + d)/(c^4*d^4 + 2*b^2*c^2*d^3*f - 4*a*c^3*d^3*f + b^4*d^2*f^
2 - 4*a*b^2*c*d^2*f^2 + 6*a^2*c^2*d^2*f^2 + 2*a^2*b^2*d*f^3 - 4*a^3*c*d*f^3
 + a^4*f^4 - 2*b*c^3*d^3*e - 2*b^3*c*d^2*f*e + 2*a*b*c^2*d^2*f*e - 2*a*b^3*
d*f^2*e + 2*a^2*b*c*d*f^2*e - 2*a^3*b*f^3*e + b^2*c^2*d^2*e^2 + 2*a*c^3*d^2
*e^2 + 4*a*b^2*c*d*f*e^2 - 4*a^2*c^2*d*f*e^2 + a^2*b^2*f^2*e^2 + 2*a^3*c*f^
2*e^2 - 2*a*b*c^2*d*e^3 - 2*a^2*b*c*f*e^3 + a^2*c^2*e^4) + (2*B*b*c^4*d^3 -
 4*A*c^5*d^3 + 5*B*b^3*c^2*d^2*f - 14*B*a*b*c^3*d^2*f - 8*A*b^2*c^3*d^2*f +
 20*A*a*c^4*d^2*f + B*b^5*d*f^2 - 4*B*a*b^3*c*d*f^2 - 2*A*b^4*c*d*f^2 + 6*B
*a^2*b*c^2*d*f^2 + 12*A*a*b^2*c^2*d*f^2 - 28*A*a^2*c^3*d*f^2 - B*a^2*b^3*f^
3 + 2*A*a*b^4*f^3 + 6*B*a^3*b*c*f^3 - 12*A*a^2*b^2*c*f^3 + 12*A*a^3*c^2*f^3
 - 4*B*b^2*c^3*d^2*e + 4*B*a*c^4*d^2*e + 6*A*b*c^4*d^2*e - 2*B*b^4*c*d*f*e
+ 2*A*b^3*c^2*d*f*e + 8*B*a^2*c^3*d*f*e + 4*A*a*b*c^3*d*f*e - A*b^5*f^2*e +
 4*A*a*b^3*c*f^2*e - 12*B*a^3*c^2*f^2*e + 6*A*a^2*b*c^2*f^2*e + B*b^3*c^2*d
*e^2 + 2*B*a*b*c^3*d*e^2 - 12*A*a*c^4*d*e^2 + 2*A*b^4*c*f*e^2 + 6*B*a^2*b*c
^2*f*e^2 - 12*A*a*b^2*c^2*f*e^2 + 4*A*a^2*c^3*f*e^2 - A*b^3*c^2*e^3 - 4*B*a
^2*c^3*e^3 + 6*A*a*b*c^3*e^3)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*
c^4*d^4 - 4*a*c^5*d^4 + 2*b^4*c^2*d^3*f - 12*a*b^2*c^3*d^3*f + 16*a^2*c^4*d
^3*f + b^6*d^2*f^2 - 8*a*b^4*c*d^2*f^2 + 22*a^2*b^2*c^2*d^2*f^2 - 24*a^3*c^
3*d^2*f^2 + 2*a^2*b^4*d*f^3 - 12*a^3*b^2*c*d*f^3 + 16*a^4*c^2*d*f^3 + a^4*b
^2*f^4 - 4*a^5*c*f^4 - 2*b^3*c^3*d^3*e + 8*a*b*c^4*d^3*e - 2*b^5*c*d^2*f*e
+ 10*a*b^3*c^2*d^2*f*e - 8*a^2*b*c^3*d^2*f*e - 2*a*b^5*d*f^2*e + 10*a^2*b^3
*c*d*f^2*e - 8*a^3*b*c^2*d*f^2*e - 2*a^3*b^3*f^3*e + 8*a^4*b*c*f^3*e + b^4*
c^2*d^2*e^2 - 2*a*b^2*c^3*d^2*e^2 - 8*a^2*c^4*d^2*e^2 + 4*a*b^4*c*d*f*e^2 -
 20*a^2*b^2*c^2*d*f*e^2 + 16*a^3*c^3*d*f*e^2 + a^2*b^4*f^2*e^2 - 2*a^3*b^2*
c*f^2*e^2 - 8*a^4*c^2*f^2*e^2 - 2*a*b^3*c^2*d*e^3 + 8*a^2*b*c^3*d*e^3 - 2*a
^2*b^3*c*f*e^3 + 8*a^3*b*c^2*f*e^3 + a^2*b^2*c^2*e^4 - 4*a^3*c^3*e^4)*sqrt(
-b^2 + 4*a*c)) - (4*B*b*c*d^2*f^2 - 2*A*c^2*d^2*f^2 - 4*B*a*b*d*f^3 + 2*A*b
^2*d*f^3 + 4*A*a*c*d*f^3 - 2*A*a^2*f^4 - 3*B*c^2*d^2*f*e + B*b^2*d*f^2*e +
2*B*a*c*d*f^2*e - 6*A*b*c*d*f^2*e + B*a^2*f^3*e + 2*A*a*b*f^3*e - 2*B*b*c*d
*f*e^2 + 4*A*c^2*d*f*e^2 - A*b^2*f^2*e^2 - 2*A*a*c*f^2*e^2 + B*c^2*d*e^3 +
2*A*b*c*f*e^3 - A*c^2*e^4)*arctan((2*f*x + e)/sqrt(4*d*f - e^2))/((c^4*d^4
+ 2*b^2*c^2*d^3*f - 4*a*c^3*d^3*f + b^4*d^2*f^2 - 4*a*b^2*c*d^2*f^2 + 6*a^2
*c^2*d^2*f^2 + 2*a^2*b^2*d*f^3 - 4*a^3*c*d*f^3 + a^4*f^4 - 2*b*c^3*d^3*e -
2*b^3*c*d^2*f*e + 2*a*b*c^2*d^2*f*e - 2*a*b^3*d*f^2*e + 2*a^2*b*c*d*f^2*e -
```

```
 2*a^3*b*f^3*e + b^2*c^2*d^2*e^2 + 2*a*c^3*d^2*e^2 + 4*a*b^2*c*d*f*e^2 - 4*
a^2*c^2*d*f*e^2 + a^2*b^2*f^2*e^2 + 2*a^3*c*f^2*e^2 - 2*a*b*c^2*d*e^3 - 2*a
^2*b*c*f*e^3 + a^2*c^2*e^4)*sqrt(4*d*f - e^2)) + (2*B*a*c^4*d^3 - A*b*c^4*d
^3 + 3*B*a*b^2*c^2*d^2*f - 2*A*b^3*c^2*d^2*f - 6*B*a^2*c^3*d^2*f + 5*A*a*b*
c^3*d^2*f + B*a*b^4*d*f^2 - A*b^5*d*f^2 - 4*B*a^2*b^2*c*d*f^2 + 5*A*a*b^3*c
*d*f^2 + 6*B*a^3*c^2*d*f^2 - 7*A*a^2*b*c^2*d*f^2 + B*a^3*b^2*f^3 - A*a^2*b^
3*f^3 - 2*B*a^4*c*f^3 + 3*A*a^3*b*c*f^3 - 3*B*a*b*c^3*d^2*e + 2*A*b^2*c^3*d
^2*e - 2*A*a*c^4*d^2*e - 2*B*a*b^3*c*d*f*e + 2*A*b^4*c*d*f*e + 2*B*a^2*b*c^
2*d*f*e - 6*A*a*b^2*c^2*d*f*e + 4*A*a^2*c^3*d*f*e - B*a^2*b^3*f^2*e + A*a*b
^4*f^2*e + B*a^3*b*c*f^2*e - 2*A*a^2*b^2*c*f^2*e - 2*A*a^3*c^2*f^2*e + B*a*
b^2*c^2*d*e^2 - A*b^3*c^2*d*e^2 + 2*B*a^2*c^3*d*e^2 + A*a*b*c^3*d*e^2 + 2*B
*a^2*b^2*c*f*e^2 - 2*A*a*b^3*c*f*e^2 - 2*B*a^3*c^2*f*e^2 + 5*A*a^2*b*c^2*f*
e^2 - B*a^2*b*c^2*e^3 + A*a*b^2*c^2*e^3 - 2*A*a^2*c^3*e^3 + (B*b*c^4*d^3 -
2*A*c^5*d^3 + B*b^3*c^2*d^2*f - B*a*b*c^3*d^2*f - 3*A*b^2*c^3*d^2*f + 6*A*a
*c^4*d^2*f + B*a*b^3*c*d*f^2 - A*b^4*c*d*f^2 - B*a^2*b*c^2*d*f^2 + 4*A*a*b^
2*c^2*d*f^2 - 6*A*a^2*c^3*d*f^2 + B*a^3*b*c*f^3 - A*a^2*b^2*c*f^3 + 2*A*a^3
*c^2*f^3 - B*b^2*c^3*d^2*e - 2*B*a*c^4*d^2*e + 3*A*b*c^4*d^2*e - 4*B*a*b^2*
c^2*d*f*e + 2*A*b^3*c^2*d*f*e + 4*B*a^2*c^3*d*f*e - 2*A*a*b*c^3*d*f*e - B*a
^2*b^2*c*f^2*e + A*a*b^3*c*f^2*e - 2*B*a^3*c^2*f^2*e - A*a^2*b*c^2*f^2*e +
3*B*a*b*c^3*d*e^2 - A*b^2*c^3*d*e^2 - 2*A*a*c^4*d*e^2 + 3*B*a^2*b*c^2*f*e^2
 - 2*A*a*b^2*c^2*f*e^2 + 2*A*a^2*c^3*f*e^2 - 2*B*a^2*c^3*e^3 + A*a*b*c^3*e^
3)*x)/((c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2 - b*c*d*e - a*b*f*e + a*c*e
^2)^2*(c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

**maple [B]** time = 0.05, size = 51470, normalized size = 47.88

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive or negative?
```

**mupad [B]** time = 30.31, size = 118429, normalized size = 110.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/((a + b*x + c*x^2)^2*(d + e*x + f*x^2)),x)
```

```
[Out] symsum(log((x*(4*A^3*b^3*c^4*f^6 + 16*B^3*a^3*c^4*f^6 - 3*B^3*a^2*b^2*c^3*f
^6 + 4*B^3*a^2*c^5*e^2*f^4 + B^3*b^2*c^5*d^2*f^4 - 16*A^3*a*b*c^5*f^6 + 16*
A^3*a*c^6*e*f^5 + 20*A^2*B*a^2*c^5*f^6 - 3*A^2*B*b^4*c^3*f^6 + 4*A^2*B*c^7*
d^2*f^4 - 16*B^3*a^2*c^5*d*f^5 - 4*A^3*b^2*c^5*e*f^5 + 6*B^3*a*b^2*c^4*d*f^
5 - 4*B^3*a^2*b*c^4*e*f^5 + A^2*B*b^2*c^5*e^2*f^4 - 24*A^2*B*a*c^6*d*f^5 +
6*A*B^2*a*b^3*c^3*f^6 - 28*A*B^2*a^2*b*c^4*f^6 + 8*A^2*B*a*b^2*c^4*f^6 - 4*
A*B^2*b*c^6*d^2*f^4 + 8*A*B^2*a^2*c^5*e*f^5 - 6*A*B^2*b^3*c^4*d*f^5 + 8*A^2
*B*b^2*c^5*d*f^5 + 2*A^2*B*b^3*c^4*e*f^5 - 4*B^3*a*b*c^5*d*e*f^4 - 4*A*B^2*
a*b*c^5*e^2*f^4 + 2*A*B^2*a*b^2*c^4*e*f^5 + 2*A*B^2*b^2*c^5*d*e*f^4 + 16*A*
B^2*a*b*c^5*d*f^5 - 12*A^2*B*a*b*c^5*e*f^5 + 8*A*B^2*a*c^6*d*e*f^4 - 4*A^2*
B*b*c^6*d*e*f^4))/(16*a^2*c^6*d^4 + a^4*b^4*f^4 + 16*a^4*c^4*e^4 + b^4*c^4*
d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*
a^2*b^6*d*f^3 - 2*a^3*b^5*e*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 - 2*b
^5*c^3*d^3*e + 2*b^6*c^2*d^3*f + a^2*b^4*c^2*e^4 - 8*a^3*b^2*c^3*e^4 + 32*a
^3*c^5*d^2*e^2 + a^2*b^6*e^2*f^2 + 96*a^4*c^4*d^2*f^2 + b^6*c^2*d^2*e^2 + 3
2*a^5*c^3*e^2*f^2 - 2*a*b^7*d*e*f^2 - 2*b^7*c*d^2*e*f + 54*a^2*b^4*c^2*d^2*
f^2 - 112*a^3*b^2*c^3*d^2*f^2 + 16*a*b^3*c^4*d^3*e - 2*a*b^5*c^2*d*e^3 - 32
*a^2*b*c^5*d^3*e - 32*a^3*b*c^4*d*e^3 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2
*f^2 - 20*a^3*b^4*c*d*f^3 - 2*a^2*b^5*c*e^3*f - 32*a^4*b*c^3*e^3*f + 16*a^4
*b^3*c*e*f^3 - 32*a^5*b*c^2*e*f^3 - 64*a^4*c^4*d*e^2*f - 6*a*b^4*c^3*d^2*e^
2 + 16*a^2*b^3*c^3*d*e^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3 + 16
*a^3*b^3*c^2*e^3*f - 6*a^3*b^4*c*e^2*f^2 - 48*a^2*b^3*c^3*d^2*e*f - 36*a^2*
b^4*c^2*d*e^2*f + 96*a^3*b^2*c^3*d*e^2*f - 48*a^3*b^3*c^2*d*e*f^2 + 4*a*b^6
*c*d*e^2*f + 18*a*b^5*c^2*d^2*e*f + 18*a^2*b^5*c*d*e*f^2 + 32*a^3*b*c^4*d^2
*e*f + 32*a^4*b*c^3*d*e*f^2) - root(48416*a^6*b^2*c^6*d^4*e^2*f^4*z^4 - 415
44*a^5*b^4*c^5*d^4*e^2*f^4*z^4 - 31872*a^7*b^2*c^5*d^3*e^2*f^5*z^4 - 31872*
a^5*b^2*c^7*d^5*e^2*f^3*z^4 - 29184*a^6*b^2*c^6*d^3*e^4*f^3*z^4 + 28800*a^5
*b^4*c^5*d^3*e^4*f^3*z^4 + 21510*a^4*b^6*c^4*d^4*e^2*f^4*z^4 + 21408*a^6*b^
4*c^4*d^3*e^2*f^5*z^4 + 21408*a^4*b^4*c^6*d^5*e^2*f^3*z^4 - 18112*a^7*b^3*c
^4*d^2*e^3*f^5*z^4 - 18112*a^4*b^3*c^7*d^5*e^3*f^2*z^4 - 15600*a^5*b^5*c^4*
d^3*e^3*f^4*z^4 - 15600*a^4*b^5*c^5*d^4*e^3*f^3*z^4 + 15296*a^6*b^3*c^5*d^3
*e^3*f^4*z^4 + 15296*a^5*b^3*c^6*d^4*e^3*f^3*z^4 + 14016*a^7*b^2*c^5*d^2*e^
4*f^4*z^4 + 14016*a^5*b^2*c^7*d^4*e^4*f^2*z^4 - 13920*a^4*b^6*c^4*d^3*e^4*f
^3*z^4 - 11648*a^6*b^3*c^5*d^2*e^5*f^3*z^4 - 11648*a^5*b^3*c^6*d^3*e^5*f^2*
z^4 + 10432*a^6*b^2*c^6*d^2*e^6*f^2*z^4 + 9008*a^6*b^5*c^3*d^2*e^3*f^5*z^4
+ 9008*a^3*b^5*c^6*d^5*e^3*f^2*z^4 + 8544*a^5*b^5*c^4*d^2*e^5*f^3*z^4 + 854
4*a^4*b^5*c^5*d^3*e^5*f^2*z^4 - 8496*a^5*b^4*c^5*d^2*e^6*f^2*z^4 + 7488*a^8
*b^2*c^4*d^2*e^2*f^6*z^4 + 7488*a^4*b^2*c^8*d^6*e^2*f^2*z^4 + 7380*a^4*b^7*
```

c^3*d^3*e^3*f^4*z^4 + 7380*a^3*b^7*c^4*d^4*e^3*f^3*z^4 - 6720*a^3*b^8*c^3*d
^4*e^2*f^4*z^4 - 5784*a^5*b^6*c^3*d^3*e^2*f^5*z^4 - 5784*a^3*b^6*c^5*d^5*e^
2*f^3*z^4 - 3440*a^6*b^4*c^4*d^2*e^4*f^4*z^4 - 3440*a^4*b^4*c^6*d^4*e^4*f^2
*z^4 + 3360*a^3*b^8*c^3*d^3*e^4*f^3*z^4 + 3140*a^4*b^6*c^4*d^2*e^6*f^2*z^4
- 2760*a^4*b^7*c^3*d^2*e^5*f^3*z^4 - 2760*a^3*b^7*c^4*d^3*e^5*f^2*z^4 - 176
4*a^5*b^7*c^2*d^2*e^3*f^5*z^4 - 1764*a^2*b^7*c^5*d^5*e^3*f^2*z^4 - 1640*a^3
*b^9*c^2*d^3*e^3*f^4*z^4 - 1640*a^2*b^9*c^3*d^4*e^3*f^3*z^4 - 1604*a^6*b^6*
c^2*d^2*e^2*f^6*z^4 - 1604*a^2*b^6*c^6*d^6*e^2*f^2*z^4 - 1500*a^5*b^6*c^3*d
^2*e^4*f^4*z^4 - 1500*a^3*b^6*c^5*d^4*e^4*f^2*z^4 + 1140*a^2*b^10*c^2*d^4*e
^2*f^4*z^4 + 810*a^4*b^8*c^2*d^2*e^4*f^4*z^4 + 810*a^2*b^8*c^4*d^4*e^4*f^2*
z^4 - 544*a^3*b^8*c^3*d^2*e^6*f^2*z^4 + 416*a^3*b^9*c^2*d^2*e^5*f^3*z^4 + 4
16*a^2*b^9*c^3*d^3*e^5*f^2*z^4 - 384*a^2*b^10*c^2*d^3*e^4*f^3*z^4 + 180*a^4
*b^8*c^2*d^3*e^2*f^5*z^4 + 180*a^2*b^8*c^4*d^5*e^2*f^3*z^4 + 48*a^7*b^4*c^3
*d^2*e^2*f^6*z^4 + 48*a^3*b^4*c^7*d^6*e^2*f^2*z^4 + 36*a^2*b^10*c^2*d^2*e^6
*f^2*z^4 - 1024*a^10*b*c^3*d*e*f^8*z^4 - 1024*a^3*b*c^10*d^8*e*f*z^4 - 192*
a^8*b^5*c*d*e*f^8*z^4 - 192*a*b^5*c^8*d^8*e*f*z^4 + 16128*a^7*b^3*c^4*d^3*e
*f^6*z^4 + 16128*a^4*b^3*c^7*d^6*e*f^3*z^4 - 11712*a^6*b^5*c^3*d^3*e*f^6*z^
4 - 11712*a^3*b^5*c^6*d^6*e*f^3*z^4 + 11520*a^8*b*c^5*d^2*e^3*f^5*z^4 + 115
20*a^5*b*c^8*d^5*e^3*f^2*z^4 - 9984*a^6*b^3*c^5*d^4*e*f^5*z^4 - 9984*a^5*b^
3*c^6*d^5*e*f^4*z^4 + 8640*a^5*b^5*c^4*d^4*e*f^5*z^4 + 8640*a^4*b^5*c^5*d^5
*e*f^4*z^4 - 7424*a^7*b*c^6*d^3*e^3*f^4*z^4 - 7424*a^6*b*c^7*d^4*e^3*f^3*z^
4 - 6912*a^8*b^3*c^3*d^2*e*f^7*z^4 - 6912*a^3*b^3*c^8*d^7*e*f^2*z^4 + 4800*
a^7*b^3*c^4*d*e^5*f^4*z^4 + 4800*a^4*b^3*c^7*d^4*e^5*f*z^4 + 4608*a^7*b*c^6
*d^2*e^5*f^3*z^4 + 4608*a^6*b*c^7*d^3*e^5*f^2*z^4 - 4560*a^4*b^7*c^3*d^4*e*
f^5*z^4 - 4560*a^3*b^7*c^4*d^5*e*f^4*z^4 + 4176*a^5*b^7*c^2*d^3*e*f^6*z^4 +
 4176*a^2*b^7*c^5*d^6*e*f^3*z^4 + 3264*a^7*b^5*c^2*d^2*e*f^7*z^4 + 3264*a^2
*b^5*c^7*d^7*e*f^2*z^4 + 3008*a^8*b^3*c^3*d*e^3*f^6*z^4 + 3008*a^3*b^3*c^8*
d^6*e^3*f*z^4 + 2880*a^6*b^3*c^5*d*e^7*f^2*z^4 + 2880*a^5*b^3*c^6*d^2*e^7*f
*z^4 - 2240*a^7*b^4*c^3*d*e^4*f^5*z^4 - 2240*a^3*b^4*c^7*d^5*e^4*f*z^4 - 14
88*a^5*b^5*c^4*d*e^7*f^2*z^4 - 1488*a^4*b^5*c^5*d^2*e^7*f*z^4 + 1440*a^3*b^
9*c^2*d^4*e*f^5*z^4 + 1440*a^2*b^9*c^3*d^5*e*f^4*z^4 - 1328*a^6*b^5*c^3*d*e
^5*f^4*z^4 - 1328*a^3*b^5*c^6*d^4*e^5*f*z^4 - 1152*a^7*b^2*c^5*d*e^6*f^3*z^
4 - 1152*a^5*b^2*c^7*d^3*e^6*f*z^4 - 1120*a^6*b^4*c^4*d*e^6*f^3*z^4 - 1120*
a^4*b^4*c^6*d^3*e^6*f*z^4 + 912*a^6*b^6*c^2*d*e^4*f^5*z^4 + 912*a^2*b^6*c^6
*d^5*e^4*f*z^4 + 872*a^5*b^6*c^3*d*e^6*f^3*z^4 + 872*a^3*b^6*c^5*d^3*e^6*f*
z^4 + 768*a^8*b^2*c^4*d*e^4*f^5*z^4 + 768*a^4*b^2*c^8*d^5*e^4*f*z^4 - 672*a
^8*b^4*c^2*d*e^2*f^7*z^4 - 672*a^2*b^4*c^8*d^7*e^2*f*z^4 - 624*a^7*b^5*c^2*
d*e^3*f^6*z^4 - 624*a^2*b^5*c^7*d^6*e^3*f*z^4 + 480*a^5*b^8*c*d^2*e^2*f^6*z
^4 + 480*a*b^8*c^5*d^6*e^2*f^2*z^4 + 316*a^4*b^7*c^3*d*e^7*f^2*z^4 + 316*a^
3*b^7*c^4*d^2*e^7*f*z^4 - 204*a^4*b^8*c^2*d*e^6*f^3*z^4 - 204*a^2*b^8*c^4*d
^3*e^6*f*z^4 + 168*a^3*b^10*c*d^3*e^2*f^5*z^4 + 168*a*b^10*c^3*d^5*e^2*f^3*
z^4 + 156*a^2*b^11*c*d^3*e^3*f^4*z^4 + 156*a*b^11*c^2*d^4*e^3*f^3*z^4 + 128
*a^9*b^2*c^3*d*e^2*f^7*z^4 + 128*a^3*b^2*c^9*d^7*e^2*f*z^4 - 124*a^3*b^10*c
*d^2*e^4*f^4*z^4 - 124*a*b^10*c^3*d^4*e^4*f^2*z^4 + 100*a^4*b^9*c*d^2*e^3*f
^5*z^4 + 100*a*b^9*c^4*d^5*e^3*f^2*z^4 + 36*a^5*b^7*c^2*d*e^5*f^4*z^4 + 36*

a^2*b^7*c^5*d^4*e^5*f*z^4 - 24*a^3*b^9*c^2*d*e^7*f^2*z^4 - 24*a^2*b^11*c*d^2*e^5*f^3*z^4 - 24*a^2*b^9*c^3*d^2*e^7*f*z^4 - 24*a*b^11*c^2*d^3*e^5*f^2*z^4 - 9216*a^8*b*c^5*d^3*e*f^6*z^4 - 9216*a^5*b*c^8*d^6*e*f^3*z^4 - 5376*a^8*b*c^5*d*e^5*f^4*z^4 - 5376*a^5*b*c^8*d^4*e^5*f*z^4 + 5120*a^9*b*c^4*d^2*e*f^7*z^4 + 5120*a^7*b*c^6*d^4*e*f^5*z^4 + 5120*a^6*b*c^7*d^5*e*f^4*z^4 + 5120*a^4*b*c^9*d^7*e*f^2*z^4 - 4352*a^9*b*c^4*d*e^3*f^6*z^4 - 4352*a^4*b*c^9*d^6*e^3*f*z^4 - 1792*a^7*b*c^6*d*e^7*f^2*z^4 - 1792*a^6*b*c^7*d^2*e^7*f*z^4 - 1600*a^6*b^2*c^6*d*e^8*f*z^4 + 912*a^5*b^4*c^5*d*e^8*f*z^4 + 768*a^9*b^3*c^2*d*e*f^8*z^4 + 768*a^2*b^3*c^9*d^8*e*f*z^4 - 720*a^4*b^9*c*d^3*e*f^6*z^4 - 720*a*b^9*c^4*d^6*e*f^3*z^4 - 656*a^6*b^7*c*d^2*e*f^7*z^4 - 656*a*b^7*c^6*d^7*e*f^2*z^4 - 240*a^2*b^11*c*d^4*e*f^5*z^4 - 240*a*b^11*c^2*d^5*e*f^4*z^4 + 216*a^7*b^6*c*d*e^2*f^7*z^4 + 216*a*b^6*c^7*d^7*e^2*f*z^4 - 204*a^4*b^6*c^4*d*e^8*f*z^4 - 144*a^5*b^8*c*d*e^4*f^5*z^4 - 144*a*b^8*c^5*d^5*e^4*f*z^4 - 84*a*b^12*c*d^4*e^2*f^4*z^4 + 36*a^4*b^9*c*d*e^5*f^4*z^4 + 36*a*b^9*c^4*d^4*e^5*f*z^4 + 20*a^6*b^7*c*d*e^3*f^6*z^4 + 20*a*b^7*c^6*d^6*e^3*f*z^4 + 16*a^3*b^10*c*d*e^6*f^3*z^4 + 16*a^3*b^8*c^3*d*e^8*f*z^4 + 16*a*b^12*c*d^3*e^4*f^3*z^4 + 16*a*b^10*c^3*d^3*e^6*f*z^4 + 48*b^11*c^3*d^6*e*f^3*z^4 + 48*b^9*c^5*d^7*e*f^2*z^4 - 20*b^8*c^6*d^7*e^2*f*z^4 + 8*b^10*c^4*d^5*e^4*f*z^4 - 4*b^13*c*d^4*e^3*f^3*z^4 - 4*b^11*c^3*d^4*e^5*f*z^4 + 4*b^9*c^5*d^6*e^3*f*z^4 + 3072*a^9*c^5*d*e^4*f^5*z^4 + 3072*a^5*c^9*d^5*e^4*f*z^4 + 2560*a^8*c^6*d*e^6*f^3*z^4 + 2560*a^6*c^8*d^3*e^6*f*z^4 + 1536*a^10*c^4*d*e^2*f^7*z^4 + 1536*a^4*c^10*d^7*e^2*f*z^4 + 48*a^5*b^9*d^2*e*f^7*z^4 + 48*a^3*b^11*d^3*e*f^6*z^4 - 20*a^6*b^8*d*e^2*f^7*z^4 + 8*a^4*b^10*d*e^4*f^5*z^4 + 4*a^5*b^9*d*e^3*f^6*z^4 - 4*a^3*b^11*d*e^5*f^4*z^4 - 4*a*b^13*d^3*e^3*f^4*z^4 + 768*a^9*b*c^4*e^5*f^5*z^4 + 768*a^8*b*c^5*e^7*f^3*z^4 + 256*a^10*b*c^3*e^3*f^7*z^4 - 192*a^6*b^3*c^5*e^9*f*z^4 - 68*a^7*b^6*c*e^4*f^6*z^4 + 48*a^8*b^5*c*e^3*f^7*z^4 + 48*a^5*b^5*c^4*e^9*f*z^4 + 36*a^6*b^7*c*e^5*f^5*z^4 - 12*a^9*b^4*c*e^2*f^8*z^4 - 4*a^4*b^9*c*e^7*f^3*z^4 - 4*a^4*b^7*c^3*e^9*f*z^4 + 384*a^5*b^8*c*d^3*f^7*z^4 + 384*a*b^8*c^5*d^7*f^3*z^4 + 288*a^3*b^10*c*d^4*f^6*z^4 + 288*a*b^10*c^3*d^6*f^4*z^4 + 224*a^7*b^6*c*d^2*f^8*z^4 + 224*a*b^6*c^7*d^8*f^2*z^4 - 192*a^10*b^2*c^2*d*f^9*z^4 - 192*a^2*b^2*c^10*d^9*f*z^4 + 768*a^5*b*c^8*d^3*e^7*z^4 + 768*a^4*b*c^9*d^5*e^5*z^4 + 256*a^3*b*c^10*d^7*e^3*z^4 - 192*a^5*b^3*c^6*d*e^9*z^4 - 68*a*b^6*c^7*d^6*e^4*z^4 + 48*a^4*b^5*c^5*d*e^9*z^4 + 48*a*b^5*c^8*d^7*e^3*z^4 + 36*a*b^7*c^6*d^5*e^5*z^4 - 12*a*b^4*c^9*d^8*e^2*z^4 - 4*a^3*b^7*c^4*d*e^9*z^4 - 4*a*b^9*c^4*d^3*e^7*z^4 + 16*b^13*c*d^5*e*f^4*z^4 + 16*b^7*c^7*d^8*e*f*z^4 + 768*a^7*c^7*d*e^8*f*z^4 + 16*a^7*b^7*d*e*f^8*z^4 + 16*a*b^13*d^4*e*f^5*z^4 + 256*a^7*b*c^6*e^9*f*z^4 + 80*a*b^12*c*d^5*f^5*z^4 + 48*a^9*b^4*c*d*f^9*z^4 + 48*a*b^4*c^9*d^9*f*z^4 + 256*a^6*b*c^7*d*e^9*z^4 - 42*b^10*c^4*d^6*e^2*f^2*z^4 - 20*b^12*c^2*d^5*e^2*f^3*z^4 + 6*b^12*c^2*d^4*e^4*f^2*z^4 + 4*b^11*c^3*d^5*e^3*f^2*z^4 - 24960*a^7*c^7*d^4*e^2*f^4*z^4 + 18944*a^8*c^6*d^3*e^2*f^5*z^4 + 18944*a^6*c^8*d^5*e^2*f^3*z^4 + 14336*a^7*c^7*d^3*e^4*f^3*z^4 - 9984*a^8*c^6*d^2*e^4*f^4*z^4 - 9984*a^6*c^8*d^4*e^4*f^2*z^4 - 7936*a^9*c^5*d^2*e^2*f^6*z^4 - 7936*a^5*c^9*d^6*e^2*f^2*z^4 - 4352*a^7*c^7*d^2*e^6*f^2*z^4 - 42*a^4*b^10*d^2*e^2*f^6*z^4 - 20*a^2*b^12*d^3*e^2*f^5*z^4 + 6*a^2*b^12*d^2*e^4*f^4*z^4 + 4*a

^3*b^11*d^2*e^3*f^5*z^4 - 480*a^8*b^2*c^4*e^6*f^4*z^4 + 440*a^7*b^4*c^3*e^6
*f^4*z^4 - 320*a^8*b^3*c^3*e^5*f^5*z^4 - 320*a^7*b^3*c^4*e^7*f^3*z^4 + 240*
a^8*b^4*c^2*e^4*f^6*z^4 + 240*a^6*b^4*c^4*e^8*f^2*z^4 - 192*a^9*b^3*c^2*e^3
*f^7*z^4 - 192*a^9*b^2*c^3*e^4*f^6*z^4 - 192*a^7*b^2*c^5*e^8*f^2*z^4 - 90*a
^6*b^6*c^2*e^6*f^4*z^4 - 68*a^5*b^6*c^3*e^8*f^2*z^4 + 48*a^10*b^2*c^2*e^2*f
^8*z^4 - 48*a^7*b^5*c^2*e^5*f^5*z^4 - 48*a^6*b^5*c^3*e^7*f^3*z^4 + 36*a^5*b
^7*c^2*e^7*f^3*z^4 + 6*a^4*b^8*c^2*e^8*f^2*z^4 - 33920*a^6*b^2*c^6*d^5*f^5*
z^4 + 27936*a^5*b^4*c^5*d^5*f^5*z^4 + 26112*a^7*b^2*c^5*d^4*f^6*z^4 + 26112
*a^5*b^2*c^7*d^6*f^4*z^4 - 20352*a^6*b^4*c^4*d^4*f^6*z^4 - 20352*a^4*b^4*c^
6*d^6*f^4*z^4 - 13080*a^4*b^6*c^4*d^5*f^5*z^4 - 11520*a^8*b^2*c^4*d^3*f^7*z
^4 - 11520*a^4*b^2*c^8*d^7*f^3*z^4 + 8736*a^5*b^6*c^3*d^4*f^6*z^4 + 8736*a^
3*b^6*c^5*d^6*f^4*z^4 + 7488*a^7*b^4*c^3*d^3*f^7*z^4 + 7488*a^3*b^4*c^7*d^7
*f^3*z^4 + 3840*a^3*b^8*c^3*d^5*f^5*z^4 + 2560*a^9*b^2*c^3*d^2*f^8*z^4 + 25
60*a^3*b^2*c^9*d^8*f^2*z^4 - 2416*a^6*b^6*c^2*d^3*f^7*z^4 - 2416*a^2*b^6*c^
6*d^7*f^3*z^4 - 2160*a^4*b^8*c^2*d^4*f^6*z^4 - 2160*a^2*b^8*c^4*d^6*f^4*z^4
 - 1152*a^8*b^4*c^2*d^2*f^8*z^4 - 1152*a^2*b^4*c^8*d^8*f^2*z^4 - 720*a^2*b^
10*c^2*d^5*f^5*z^4 - 480*a^4*b^2*c^8*d^4*e^6*z^4 + 440*a^3*b^4*c^7*d^4*e^6*
z^4 - 320*a^4*b^3*c^7*d^3*e^7*z^4 - 320*a^3*b^3*c^8*d^5*e^5*z^4 + 240*a^4*b
^4*c^6*d^2*e^8*z^4 + 240*a^2*b^4*c^8*d^6*e^4*z^4 - 192*a^5*b^2*c^7*d^2*e^8*
z^4 - 192*a^3*b^2*c^9*d^6*e^4*z^4 - 192*a^2*b^3*c^9*d^7*e^3*z^4 - 90*a^2*b^
6*c^6*d^4*e^6*z^4 - 68*a^3*b^6*c^5*d^2*e^8*z^4 - 48*a^3*b^5*c^6*d^3*e^7*z^4
 - 48*a^2*b^5*c^7*d^5*e^5*z^4 + 48*a^2*b^2*c^10*d^8*e^2*z^4 + 36*a^2*b^7*c^
5*d^3*e^7*z^4 + 6*a^2*b^8*c^4*d^2*e^8*z^4 - 4*b^6*c^8*d^9*f*z^4 + 256*a^11*
c^3*d*f^9*z^4 + 256*a^3*c^11*d^9*f*z^4 - 4*a^8*b^6*d*f^9*z^4 - 384*a^9*c^5*
e^6*f^4*z^4 - 256*a^10*c^4*e^4*f^6*z^4 - 256*a^8*c^6*e^8*f^2*z^4 - 64*a^11*
c^3*e^2*f^8*z^4 - 24*b^10*c^4*d^7*f^3*z^4 - 16*b^12*c^2*d^6*f^4*z^4 - 16*b^
8*c^6*d^8*f^2*z^4 + 17920*a^7*c^7*d^5*f^5*z^4 - 14336*a^8*c^6*d^4*f^6*z^4 -
 14336*a^6*c^8*d^6*f^4*z^4 + 7168*a^9*c^5*d^3*f^7*z^4 + 7168*a^5*c^9*d^7*f^
3*z^4 - 2048*a^10*c^4*d^2*f^8*z^4 - 2048*a^4*c^10*d^8*f^2*z^4 + 6*b^8*c^6*d
^6*e^4*z^4 + 6*a^6*b^8*e^4*f^6*z^4 - 4*b^9*c^5*d^5*e^5*z^4 - 4*b^7*c^7*d^7*
e^3*z^4 - 4*a^7*b^7*e^3*f^7*z^4 - 4*a^5*b^9*e^5*f^5*z^4 - 384*a^5*c^9*d^4*e
^6*z^4 - 256*a^6*c^8*d^2*e^8*z^4 - 256*a^4*c^10*d^6*e^4*z^4 - 64*a^3*c^11*d
^8*e^2*z^4 - 24*a^4*b^10*d^3*f^7*z^4 - 16*a^6*b^8*d^2*f^8*z^4 - 16*a^2*b^12
*d^4*f^6*z^4 + 48*a^6*b^2*c^6*e^10*z^4 - 12*a^5*b^4*c^5*e^10*z^4 - 4*b^14*d
^5*f^5*z^4 - 64*a^7*c^7*e^10*z^4 + b^14*d^4*e^2*f^4*z^4 + b^10*c^4*d^4*e^6*
z^4 + b^6*c^8*d^8*e^2*z^4 + a^8*b^6*e^2*f^8*z^4 + a^4*b^10*e^6*f^4*z^4 + a^
4*b^6*c^4*e^10*z^4 - 4820*A*B*a^4*b*c^5*d^2*e^2*f^4*z^2 + 2976*A*B*a^3*b*c^
6*d^3*e^2*f^3*z^2 - 2328*A*B*a^3*b*c^6*d^2*e^4*f^2*z^2 + 1848*A*B*a^2*b^4*c
^4*d^3*e*f^4*z^2 - 1768*A*B*a^3*b^4*c^3*d^2*e*f^5*z^2 + 1528*A*B*a^4*b^2*c^
4*d^2*e*f^5*z^2 - 1136*A*B*a^3*b^2*c^5*d^3*e*f^4*z^2 - 974*A*B*a^4*b^3*c^3*
d*e^2*f^5*z^2 + 692*A*B*a^2*b*c^7*d^4*e^2*f^2*z^2 + 588*A*B*a*b^6*c^3*d^2*e
^3*f^3*z^2 - 580*A*B*a^3*b^3*c^4*d*e^4*f^3*z^2 + 488*A*B*a^3*b^4*c^3*d*e^3*
f^4*z^2 - 444*A*B*a^2*b^2*c^6*d^2*e^5*f*z^2 - 412*A*B*a*b^5*c^4*d^2*e^4*f^2
*z^2 + 366*A*B*a^2*b^6*c^2*d^2*e*f^5*z^2 - 352*A*B*a^2*b^2*c^6*d^4*e*f^3*z^
2 + 326*A*B*a^2*b^4*c^4*d*e^5*f^2*z^2 + 324*A*B*a*b^5*c^4*d^3*e^2*f^3*z^2 -

302*A*B*a*b^3*c^6*d^4*e^2*f^2*z^2 - 296*A*B*a*b^7*c^2*d^2*e^2*f^4*z^2 + 12
2*A*B*a^4*b^2*c^4*d*e^3*f^4*z^2 - 122*A*B*a^2*b^6*c^2*d*e^3*f^4*z^2 - 84*A*
B*a^3*b^2*c^5*d*e^5*f^2*z^2 + 72*A*B*a*b^4*c^5*d^3*e^3*f^2*z^2 - 64*A*B*a^2
*b^5*c^3*d*e^4*f^3*z^2 + 60*A*B*a^3*b^5*c^2*d*e^2*f^5*z^2 + 1312*A*B*a^5*b*
c^4*d*e^2*f^5*z^2 + 1040*A*B*a^4*b*c^5*d*e^4*f^3*z^2 - 500*A*B*a*b^6*c^3*d^
3*e*f^4*z^2 - 376*A*B*a*b^2*c^7*d^5*e*f^2*z^2 + 276*A*B*a^4*b^4*c^2*d*e*f^6
*z^2 - 262*A*B*a^2*b^3*c^5*d*e^6*f*z^2 + 238*A*B*a*b^2*c^7*d^4*e^3*f*z^2 +
232*A*B*a^5*b^2*c^3*d*e*f^6*z^2 - 176*A*B*a^2*b*c^7*d^3*e^4*f*z^2 - 120*A*B
*a*b^6*c^3*d*e^5*f^2*z^2 - 108*A*B*a*b^4*c^5*d^4*e*f^3*z^2 + 68*A*B*a*b^7*c
^2*d*e^4*f^3*z^2 + 68*A*B*a*b^4*c^5*d^2*e^5*f*z^2 + 46*A*B*a^2*b^7*c*d*e^2*
f^5*z^2 - 36*A*B*a*b^3*c^6*d^3*e^4*f*z^2 - 1932*A*B*a^2*b^3*c^5*d^3*e^2*f^3
*z^2 - 1818*A*B*a^2*b^4*c^4*d^2*e^3*f^3*z^2 + 1620*A*B*a^3*b^3*c^4*d^2*e^2*
f^4*z^2 + 1560*A*B*a^2*b^3*c^5*d^2*e^4*f^2*z^2 + 1244*A*B*a^3*b^2*c^5*d^2*e
^3*f^3*z^2 + 820*A*B*a^2*b^2*c^6*d^3*e^3*f^2*z^2 + 480*A*B*a^2*b^5*c^3*d^2*
e^2*f^4*z^2 + 352*A*B*a^3*b*c^6*d*e^6*f*z^2 - 108*A*B*a^3*b^6*c*d*e*f^6*z^2
 + 82*A*B*a*b^5*c^4*d*e^6*f*z^2 - 64*A*B*a*b*c^8*d^5*e^2*f*z^2 + 16*A*B*a*b
^8*c*d^2*e*f^5*z^2 - 4*A*B*a*b^8*c*d*e^3*f^4*z^2 + 16*B^2*a*b*c^8*d^6*e*f*z
^2 + 56*A*B*b^2*c^8*d^6*e*f*z^2 - 8*A*B*b^9*c*d*e^4*f^3*z^2 - 8*A*B*b^7*c^3
*d*e^6*f*z^2 - 800*A*B*a^6*c^4*d*e*f^6*z^2 + 10*A*B*a^2*b^8*d*e*f^6*z^2 - 6
*A*B*a*b^9*d*e^2*f^5*z^2 - 12*A*B*a^5*b^4*c*e*f^7*z^2 + 912*A*B*a^6*b*c^3*d
*f^7*z^2 + 192*A*B*a^4*b^5*c*d*f^7*z^2 + 192*A*B*a*b*c^8*d^6*f^2*z^2 - 20*A
*B*a*b^4*c^5*d*e^7*z^2 + 4*A*B*a*b*c^8*d^4*e^4*z^2 + 2144*B^2*a^4*b*c^5*d^3
*e*f^4*z^2 - 1120*B^2*a^3*b*c^6*d^4*e*f^3*z^2 - 688*B^2*a^5*b*c^4*d^2*e*f^5
*z^2 - 256*B^2*a^3*b*c^6*d^2*e^5*f*z^2 + 152*B^2*a*b^3*c^6*d^5*e*f^2*z^2 +
120*B^2*a^5*b^3*c^2*d*e*f^6*z^2 - 116*B^2*a^5*b*c^4*d*e^3*f^4*z^2 + 110*B^2
*a*b^7*c^2*d^3*e*f^4*z^2 - 80*B^2*a^2*b*c^7*d^5*e*f^2*z^2 - 72*B^2*a*b^5*c^
4*d^4*e*f^3*z^2 - 48*B^2*a^4*b*c^5*d*e^5*f^2*z^2 - 46*B^2*a*b^3*c^6*d^4*e^3
*f*z^2 - 44*B^2*a*b^4*c^5*d^3*e^4*f*z^2 - 34*B^2*a*b^5*c^4*d^2*e^5*f*z^2 +
20*B^2*a^2*b*c^7*d^4*e^3*f*z^2 - 10*B^2*a^3*b^6*c*d*e^2*f^5*z^2 - 10*B^2*a^
2*b^7*c*d^2*e*f^5*z^2 - 10*B^2*a*b^2*c^7*d^5*e^2*f*z^2 - 7*B^2*a^2*b^4*c^4*
d*e^6*f*z^2 - 6*B^2*a^3*b^2*c^5*d*e^6*f*z^2 + 4*B^2*a*b^8*c*d^2*e^2*f^4*z^2
 - 2*B^2*a^2*b^7*c*d*e^3*f^4*z^2 + 3196*A^2*a^4*b*c^5*d*e^3*f^4*z^2 - 3184*
A^2*a^4*b*c^5*d^2*e*f^5*z^2 + 1568*A^2*a^3*b*c^6*d^3*e*f^4*z^2 + 1504*A^2*a
^3*b*c^6*d*e^5*f^2*z^2 - 656*A^2*a^4*b^3*c^3*d*e*f^6*z^2 - 400*A^2*a*b^6*c^
3*d*e^4*f^3*z^2 + 314*A^2*a*b^5*c^4*d*e^5*f^2*z^2 - 264*A^2*a^3*b^5*c^2*d*e
*f^6*z^2 + 240*A^2*a^2*b^2*c^6*d*e^6*f*z^2 - 224*A^2*a^2*b*c^7*d^4*e*f^3*z^
2 + 216*A^2*a*b^5*c^4*d^3*e*f^4*z^2 - 192*A^2*a^2*b*c^7*d^2*e^5*f*z^2 + 178
*A^2*a*b^7*c^2*d*e^3*f^4*z^2 - 154*A^2*a*b^7*c^2*d^2*e*f^5*z^2 + 128*A^2*a*
b^3*c^6*d^4*e*f^3*z^2 + 106*A^2*a*b^3*c^6*d^2*e^5*f*z^2 - 12*A^2*a*b^2*c^7*
d^3*e^4*f*z^2 - 58*A*B*b^8*c^2*d^2*e^3*f^3*z^2 + 40*A*B*b^7*c^3*d^2*e^4*f^2
*z^2 - 28*A*B*b^7*c^3*d^3*e^2*f^3*z^2 - 24*A*B*b^5*c^5*d^4*e^2*f^2*z^2 - 20
*A*B*b^6*c^4*d^3*e^3*f^2*z^2 + 2768*A*B*a^4*c^6*d^2*e^3*f^3*z^2 - 1712*A*B*
a^3*c^7*d^3*e^3*f^2*z^2 - 156*A*B*a^4*b^2*c^4*e^5*f^3*z^2 + 146*A*B*a^4*b^3
*c^3*e^4*f^4*z^2 - 106*A*B*a^5*b^2*c^3*e^3*f^5*z^2 + 90*A*B*a^5*b^3*c^2*e^2
*f^6*z^2 + 38*A*B*a^3*b^3*c^4*e^6*f^2*z^2 - 36*A*B*a^3*b^5*c^2*e^4*f^4*z^2

+ 16*A*B*a^3*b^4*c^3*e^5*f^3*z^2 - 9*A*B*a^4*b^4*c^2*e^3*f^5*z^2 - 8*A*B*a^
2*b^5*c^3*e^6*f^2*z^2 + 2*A*B*a^2*b^6*c^2*e^5*f^3*z^2 + 920*A*B*a^4*b^3*c^3
*d^2*f^6*z^2 - 480*A*B*a^2*b^5*c^3*d^3*f^5*z^2 - 336*A*B*a^2*b^3*c^5*d^4*f^
4*z^2 - 272*A*B*a^3*b^3*c^4*d^3*f^5*z^2 + 240*A*B*a^3*b^5*c^2*d^2*f^6*z^2 -
32*A*B*a*c^9*d^6*e*f*z^2 - 792*B^2*a^2*b^3*c^5*d^3*e^3*f^2*z^2 + 714*B^2*a
^2*b^4*c^4*d^3*e^2*f^3*z^2 - 572*B^2*a^3*b^2*c^5*d^3*e^2*f^3*z^2 - 475*B^2*
a^2*b^2*c^6*d^4*e^2*f^2*z^2 + 265*B^2*a^4*b^2*c^4*d^2*e^2*f^4*z^2 + 260*B^2
*a^3*b^3*c^4*d^2*e^3*f^3*z^2 - 212*B^2*a^3*b^4*c^3*d^2*e^2*f^4*z^2 + 180*B^
2*a^3*b^2*c^5*d^2*e^4*f^2*z^2 - 158*B^2*a^2*b^4*c^4*d^2*e^4*f^2*z^2 + 47*B^
2*a^2*b^6*c^2*d^2*e^2*f^4*z^2 + 16*B^2*a^2*b^5*c^3*d^2*e^3*f^3*z^2 + 2752*A
^2*a^3*b^2*c^5*d^2*e^2*f^4*z^2 - 2148*A^2*a^2*b^4*c^4*d^2*e^2*f^4*z^2 + 206
4*A^2*a^2*b^3*c^5*d^2*e^3*f^3*z^2 - 424*A^2*a^2*b^2*c^6*d^3*e^2*f^3*z^2 - 1
98*A^2*a^2*b^2*c^6*d^2*e^4*f^2*z^2 - 272*B^2*a^6*b*c^3*d*e*f^6*z^2 - 24*B^2
*a^4*b^5*c*d*e*f^6*z^2 + 1808*A^2*a^5*b*c^4*d*e*f^6*z^2 - 244*A^2*a*b*c^8*d
^4*e^3*f*z^2 + 208*A^2*a*b*c^8*d^5*e*f^2*z^2 + 134*A^2*a^2*b^7*c*d*e*f^6*z^
2 - 76*A^2*a*b^4*c^5*d*e^6*f*z^2 + 4*A^2*a*b^8*c*d*e^2*f^5*z^2 + 148*A*B*b^
4*c^6*d^5*e*f^2*z^2 + 65*A*B*b^6*c^4*d^4*e*f^3*z^2 + 46*A*B*b^8*c^2*d^3*e*f
^4*z^2 - 38*A*B*b^3*c^7*d^5*e^2*f*z^2 + 34*A*B*b^9*c*d^2*e^2*f^4*z^2 - 29*A
*B*b^4*c^6*d^4*e^3*f*z^2 + 20*A*B*b^5*c^5*d^3*e^4*f*z^2 + 12*A*B*b^8*c^2*d*
e^5*f^2*z^2 - 7*A*B*b^6*c^4*d^2*e^5*f*z^2 - 2880*A*B*a^4*c^6*d^3*e*f^4*z^2
+ 2784*A*B*a^5*c^5*d^2*e*f^5*z^2 - 1112*A*B*a^5*c^5*d*e^3*f^4*z^2 + 896*A*B
*a^3*c^7*d^4*e*f^3*z^2 + 848*A*B*a^3*c^7*d^2*e^5*f*z^2 - 560*A*B*a^4*c^6*d*
e^5*f^2*z^2 + 96*A*B*a^2*c^8*d^5*e*f^2*z^2 - 88*A*B*a^2*c^8*d^4*e^3*f*z^2 -
100*A*B*a^6*b*c^3*e^2*f^6*z^2 - 76*A*B*a^5*b*c^4*e^4*f^4*z^2 + 48*A*B*a^6*
b^2*c^2*e*f^7*z^2 - 42*A*B*a^3*b^2*c^5*e^7*f*z^2 + 36*A*B*a^4*b*c^5*e^6*f^2
*z^2 - 24*A*B*a^4*b^5*c*e^2*f^6*z^2 + 10*A*B*a^3*b^6*c*e^3*f^5*z^2 + 7*A*B*
a^2*b^4*c^4*e^7*f*z^2 + 2*A*B*a^2*b^7*c*e^4*f^4*z^2 - 2496*A*B*a^5*b*c^4*d^
2*f^6*z^2 + 1872*A*B*a^4*b*c^5*d^3*f^5*z^2 - 744*A*B*a^5*b^3*c^2*d*f^7*z^2
- 720*A*B*a^2*b*c^7*d^5*f^3*z^2 + 504*A*B*a*b^3*c^6*d^5*f^3*z^2 + 256*A*B*a
^3*b*c^6*d^4*f^4*z^2 + 168*A*B*a*b^7*c^2*d^3*f^5*z^2 - 144*A*B*a^2*b^7*c*d^
2*f^6*z^2 + 144*A*B*a*b^5*c^4*d^4*f^4*z^2 + 66*A*B*a^2*b^2*c^6*d*e^7*z^2 -
36*A*B*a*b^2*c^7*d^3*e^5*z^2 + 20*A*B*a*b^3*c^6*d^2*e^6*z^2 + 12*A*B*a^2*b*
c^7*d^2*e^6*z^2 + 1208*B^2*a^3*b*c^6*d^3*e^3*f^2*z^2 - 848*B^2*a^3*b^3*c^4*
d^3*e*f^4*z^2 + 672*B^2*a^2*b^3*c^5*d^4*e*f^3*z^2 - 632*B^2*a^4*b*c^5*d^2*e
^3*f^3*z^2 + 432*B^2*a^4*b^3*c^3*d^2*e*f^5*z^2 + 276*B^2*a^2*b^2*c^6*d^3*e^
4*f*z^2 - 196*B^2*a*b^6*c^3*d^3*e^2*f^3*z^2 - 168*B^2*a^2*b^5*c^3*d^3*e*f^4
*z^2 + 154*B^2*a^2*b^3*c^5*d^2*e^5*f*z^2 + 148*B^2*a*b^5*c^4*d^3*e^3*f^2*z^
2 + 96*B^2*a*b^4*c^5*d^4*e^2*f^2*z^2 - 72*B^2*a^3*b^5*c^2*d^2*e*f^5*z^2 + 7
0*B^2*a^5*b^2*c^3*d*e^2*f^5*z^2 - 60*B^2*a^4*b^3*c^3*d*e^3*f^4*z^2 + 52*B^2
*a*b^6*c^3*d^2*e^4*f^2*z^2 + 36*B^2*a^4*b^2*c^4*d*e^4*f^3*z^2 - 32*B^2*a*b^
7*c^2*d^2*e^3*f^3*z^2 + 24*B^2*a^3*b^5*c^2*d*e^3*f^4*z^2 + 15*B^2*a^4*b^4*c
^2*d*e^2*f^5*z^2 - 8*B^2*a^3*b^4*c^3*d*e^4*f^3*z^2 + 8*B^2*a^2*b^5*c^3*d*e^
5*f^2*z^2 - 2*B^2*a^3*b^3*c^4*d*e^5*f^2*z^2 - 2*B^2*a^2*b^6*c^2*d*e^4*f^3*z
^2 - 3176*A^2*a^3*b*c^6*d^2*e^3*f^3*z^2 - 2252*A^2*a^4*b^2*c^4*d*e^2*f^5*z^
2 + 1952*A^2*a^3*b^4*c^3*d*e^2*f^5*z^2 - 1496*A^2*a^3*b^3*c^4*d*e^3*f^4*z^2

+ 1378*A^2*a^2*b^4*c^4*d*e^4*f^3*z^2 + 1184*A^2*a^3*b^3*c^4*d^2*e*f^5*z^2
- 1166*A^2*a^2*b^3*c^5*d*e^5*f^2*z^2 - 1164*A^2*a^3*b^2*c^5*d*e^4*f^3*z^2 -
1152*A^2*a^2*b^3*c^5*d^3*e*f^4*z^2 + 578*A^2*a*b^6*c^3*d^2*e^2*f^4*z^2 - 5
48*A^2*a*b^5*c^4*d^2*e^3*f^3*z^2 + 440*A^2*a*b^2*c^7*d^4*e^2*f^2*z^2 - 412*
A^2*a^2*b^6*c^2*d*e^2*f^5*z^2 - 360*A^2*a*b^3*c^6*d^3*e^3*f^2*z^2 + 312*A^2
*a*b^4*c^5*d^3*e^2*f^3*z^2 + 248*A^2*a^2*b*c^7*d^3*e^3*f^2*z^2 - 224*A^2*a^
2*b^5*c^3*d*e^3*f^4*z^2 + 216*A^2*a^2*b^5*c^3*d^2*e*f^5*z^2 + 52*A^2*a*b^4*
c^5*d^2*e^4*f^2*z^2 - 16*B^2*b^3*c^7*d^6*e*f*z^2 - 14*B^2*b^9*c*d^3*e*f^4*z
^2 + 32*B^2*a^4*c^6*d*e^6*f*z^2 - 20*A^2*b^9*c*d*e^3*f^4*z^2 + 18*A^2*b^9*c
*d^2*e*f^5*z^2 + 8*A^2*b^6*c^4*d*e^6*f*z^2 - 360*A^2*a^3*c^7*d*e^6*f*z^2 +
136*A^2*a*c^9*d^5*e^2*f*z^2 + 2*B^2*a^3*b^7*d*e*f^6*z^2 + 2*B^2*a*b^9*d^2*e
*f^5*z^2 + 12*B^2*a^4*b*c^5*e^7*f*z^2 - 204*A^2*a^3*b*c^6*e^7*f*z^2 - 128*A
^2*a^6*b*c^3*e*f^7*z^2 - 48*A^2*a*b^5*c^4*e^7*f*z^2 - 36*B^2*a^5*b^4*c*d*f^
7*z^2 - 24*A^2*a^4*b^5*c*e*f^7*z^2 - 16*B^2*a*b^8*c*d^3*f^5*z^2 - 164*A^2*a
^3*b^6*c*d*f^7*z^2 - 16*A^2*a*b^8*c*d^2*f^6*z^2 + 4*B^2*a^3*b*c^6*d*e^7*z^2
- 4*B^2*a*b*c^8*d^5*e^3*z^2 + 48*A^2*a*b*c^8*d^3*e^5*z^2 + 36*A^2*a^2*b*c^
7*d*e^7*z^2 - 6*A^2*a*b^3*c^6*d*e^7*z^2 + 136*A*B*a^6*c^4*e^3*f^5*z^2 - 96*
A*B*b^5*c^5*d^5*f^3*z^2 + 80*A*B*a^5*c^5*e^5*f^3*z^2 - 72*A*B*b^3*c^7*d^6*f
^2*z^2 - 24*A*B*b^7*c^3*d^4*f^4*z^2 + 14*A*B*b^3*c^7*d^4*e^4*z^2 - 14*A*B*b
^2*c^8*d^5*e^3*z^2 - 2*A*B*b^5*c^5*d^2*e^6*z^2 - 2*A*B*b^4*c^6*d^3*e^5*z^2
+ 2*A*B*a^3*b^7*e^2*f^6*z^2 - A*B*a^2*b^8*e^3*f^5*z^2 + 16*A*B*a^2*c^8*d^3*
e^5*z^2 - 2*A*B*a^2*b^3*c^5*e^8*z^2 + 22*B^2*b^8*c^2*d^3*e^2*f^3*z^2 - 12*B
^2*b^7*c^3*d^3*e^3*f^2*z^2 + 12*B^2*b^6*c^4*d^4*e^2*f^2*z^2 - 6*B^2*b^8*c^2
*d^2*e^4*f^2*z^2 - 864*B^2*a^4*c^6*d^3*e^2*f^3*z^2 + 496*B^2*a^3*c^7*d^4*e^
2*f^2*z^2 + 224*B^2*a^5*c^5*d^2*e^2*f^4*z^2 + 136*B^2*a^4*c^6*d^2*e^4*f^2*z
^2 - 53*A^2*b^8*c^2*d^2*e^2*f^4*z^2 + 52*A^2*b^7*c^3*d^2*e^3*f^3*z^2 + 52*A
^2*b^5*c^5*d^3*e^3*f^2*z^2 - 36*A^2*b^6*c^4*d^3*e^2*f^3*z^2 - 12*A^2*b^4*c^
6*d^4*e^2*f^2*z^2 - 9*A^2*b^6*c^4*d^2*e^4*f^2*z^2 + 836*A^2*a^4*c^6*d^2*e^2
*f^4*z^2 - 668*A^2*a^2*c^8*d^4*e^2*f^2*z^2 + 656*A^2*a^3*c^7*d^2*e^4*f^2*z^
2 + 368*A^2*a^3*c^7*d^3*e^2*f^3*z^2 - 45*B^2*a^6*b^2*c^2*e^2*f^6*z^2 - 18*B
^2*a^5*b^2*c^3*e^4*f^4*z^2 - 9*B^2*a^4*b^2*c^4*e^6*f^2*z^2 - 6*B^2*a^5*b^3*
c^2*e^3*f^5*z^2 + 3*B^2*a^4*b^4*c^2*e^4*f^4*z^2 - 2*B^2*a^4*b^3*c^3*e^5*f^3
*z^2 - 580*B^2*a^4*b^2*c^4*d^3*f^5*z^2 + 536*B^2*a^3*b^4*c^3*d^3*f^5*z^2 +
471*A^2*a^4*b^2*c^4*e^4*f^4*z^2 - 436*A^2*a^3*b^4*c^3*e^4*f^4*z^2 - 348*B^2
*a^4*b^4*c^2*d^2*f^6*z^2 + 316*B^2*a^2*b^2*c^6*d^5*f^3*z^2 + 310*A^2*a^3*b^
3*c^4*e^5*f^3*z^2 + 232*A^2*a^5*b^2*c^3*e^2*f^6*z^2 - 229*A^2*a^2*b^4*c^4*e
^6*f^2*z^2 - 216*A^2*a^4*b^4*c^2*e^2*f^6*z^2 + 204*A^2*a^4*b^3*c^3*e^3*f^5*
z^2 + 200*B^2*a^5*b^2*c^3*d^2*f^6*z^2 + 150*A^2*a^3*b^2*c^5*e^6*f^2*z^2 - 1
20*B^2*a^2*b^4*c^4*d^4*f^4*z^2 + 91*A^2*a^2*b^6*c^2*e^4*f^4*z^2 + 72*A^2*a^
3*b^5*c^2*e^3*f^5*z^2 - 66*B^2*a^2*b^6*c^2*d^3*f^5*z^2 + 44*A^2*a^2*b^5*c^3
*e^5*f^3*z^2 - 16*B^2*a^3*b^2*c^5*d^4*f^4*z^2 + 1952*A^2*a^4*b^2*c^4*d^2*f^
6*z^2 - 1792*A^2*a^3*b^2*c^5*d^3*f^5*z^2 - 1272*A^2*a^3*b^4*c^3*d^2*f^6*z^2
+ 976*A^2*a^2*b^2*c^6*d^4*f^4*z^2 + 960*A^2*a^2*b^4*c^4*d^3*f^5*z^2 + 282*
A^2*a^2*b^6*c^2*d^2*f^6*z^2 - 45*B^2*a^2*b^2*c^6*d^2*e^6*z^2 - 48*A^2*b*c^9
*d^6*e*f*z^2 - 14*A^2*a*b^9*d*e*f^6*z^2 - 7*A*B*b^10*d^2*e*f^5*z^2 + 2*A*B*

b^10*d*e^3*f^4*z^2 - 64*A*B*a^7*c^3*e*f^7*z^2 - 16*A*B*b^9*c*d^3*f^5*z^2 + 8*A*B*a^4*c^6*e^7*f*z^2 + 4*A*B*b*c^9*d^6*e^2*z^2 + 2*A*B*b^6*c^4*d*e^7*z^2 - 120*A*B*a^3*c^7*d*e^7*z^2 - 16*A*B*a^3*b^7*d*f^7*z^2 + 16*A*B*a*b^9*d^2*f^6*z^2 + 8*A*B*a*c^9*d^5*e^3*z^2 + 12*A*B*a^3*b*c^6*e^8*z^2 - 48*B^2*b^5*c^5*d^5*e*f^2*z^2 + 15*B^2*b^4*c^6*d^5*e^2*f*z^2 - 14*B^2*b^7*c^3*d^4*e*f^3*z^2 + 4*B^2*b^9*c*d^2*e^3*f^3*z^2 + 4*B^2*b^7*c^3*d^2*e^5*f*z^2 + 4*B^2*b^5*c^5*d^4*e^3*f*z^2 - B^2*b^6*c^4*d^3*e^4*f*z^2 - 336*B^2*a^3*c^7*d^3*e^4*f*z^2 + 112*B^2*a^5*c^5*d*e^4*f^3*z^2 - 112*A^2*b^3*c^7*d^5*e*f^2*z^2 + 80*B^2*a^6*c^4*d*e^2*f^5*z^2 - 48*A^2*b^5*c^5*d^4*e*f^3*z^2 + 36*A^2*b^8*c^2*d*e^4*f^3*z^2 + 36*A^2*b^3*c^7*d^4*e^3*f*z^2 - 28*A^2*b^7*c^3*d*e^5*f^2*z^2 + 20*A^2*b^2*c^8*d^5*e^2*f*z^2 + 16*B^2*a^2*c^8*d^5*e^2*f*z^2 - 14*A^2*b^7*c^3*d^3*e*f^4*z^2 - 14*A^2*b^4*c^6*d^3*e^4*f*z^2 - 10*A^2*b^5*c^5*d^2*e^5*f*z^2 - 1008*A^2*a^4*c^6*d*e^4*f^3*z^2 - 760*A^2*a^5*c^5*d*e^2*f^5*z^2 + 272*A^2*a^2*c^8*d^3*e^4*f*z^2 + 48*B^2*a^5*b*c^4*e^5*f^3*z^2 + 36*B^2*a^6*b*c^3*e^3*f^5*z^2 + 12*B^2*a^5*b^4*c*e^2*f^6*z^2 - 624*A^2*a^4*b*c^5*e^5*f^3*z^2 - 548*A^2*a^5*b*c^4*e^3*f^5*z^2 + 182*A^2*a^2*b^3*c^5*e^7*f*z^2 - 180*B^2*a*b^4*c^5*d^5*f^3*z^2 + 132*B^2*a^6*b^2*c^2*d*f^7*z^2 + 108*B^2*a^3*b^6*c*d^2*f^6*z^2 + 96*A^2*a^5*b^3*c^2*e*f^7*z^2 + 68*A^2*a*b^6*c^3*e^6*f^2*z^2 + 58*A^2*a^3*b^6*c*e^2*f^6*z^2 - 56*B^2*a*b^2*c^7*d^6*f^2*z^2 - 38*A^2*a^2*b^7*c*e^3*f^5*z^2 - 36*A^2*a*b^7*c^2*e^5*f^3*z^2 + 20*B^2*a*b^6*c^3*d^4*f^4*z^2 - 736*A^2*a^5*b^2*c^3*d*f^7*z^2 + 624*A^2*a^4*b^4*c^2*d*f^7*z^2 - 416*A^2*a*b^2*c^7*d^5*f^3*z^2 - 276*A^2*a*b^4*c^5*d^4*f^4*z^2 - 196*A^2*a*b^6*c^3*d^3*f^5*z^2 + 8*B^2*a*b^4*c^5*d^2*e^6*z^2 + 6*B^2*a*b^2*c^7*d^4*e^4*z^2 + 2*B^2*a^2*b^3*c^5*d*e^7*z^2 + 2*B^2*a*b^3*c^6*d^3*e^5*z^2 - 18*A^2*a*b^2*c^7*d^2*e^6*z^2 - 16*A*B*b*c^9*d^7*f*z^2 - B^2*b^10*d^2*e^2*f^4*z^2 + 48*B^2*a^7*c^3*e^2*f^6*z^2 - 36*B^2*a^6*c^4*e^4*f^4*z^2 + 31*B^2*b^6*c^4*d^5*f^3*z^2 - 24*B^2*a^5*c^5*e^6*f^2*z^2 + 20*B^2*b^4*c^6*d^6*f^2*z^2 - 6*A^2*b^8*c^2*e^6*f^2*z^2 + 2*B^2*b^8*c^2*d^4*f^4*z^2 - 768*B^2*a^5*c^5*d^3*f^5*z^2 + 512*B^2*a^6*c^4*d^2*f^6*z^2 + 512*B^2*a^4*c^6*d^4*f^4*z^2 + 232*A^2*a^5*c^5*e^4*f^4*z^2 + 188*A^2*a^4*c^6*e^6*f^2*z^2 - 128*B^2*a^3*c^7*d^5*f^3*z^2 + 92*A^2*a^6*c^4*e^2*f^6*z^2 + 80*A^2*b^4*c^6*d^5*f^3*z^2 + 64*A^2*b^2*c^8*d^6*f^2*z^2 + 31*A^2*b^6*c^4*d^4*f^4*z^2 + 14*A^2*b^8*c^2*d^3*f^5*z^2 - 5*B^2*b^4*c^6*d^4*e^4*z^2 + 4*B^2*b^3*c^7*d^5*e^3*z^2 + 2*B^2*b^5*c^5*d^3*e^5*z^2 - B^2*b^6*c^4*d^2*e^6*z^2 - B^2*b^2*c^8*d^6*e^2*z^2 - B^2*a^4*b^6*e^2*f^6*z^2 - 1152*A^2*a^3*c^7*d^4*f^4*z^2 + 1008*A^2*a^4*c^6*d^3*f^5*z^2 + 624*A^2*a^2*c^8*d^5*f^3*z^2 - 288*A^2*a^5*c^5*d^2*f^6*z^2 + 56*B^2*a^3*c^7*d^2*e^6*z^2 - 10*B^2*a^2*b^8*d^2*f^6*z^2 - 9*A^2*b^2*c^8*d^4*e^4*z^2 - 5*A^2*a^2*b^8*e^2*f^6*z^2 - 4*B^2*a^2*c^8*d^4*e^4*z^2 + 3*A^2*b^4*c^6*d^2*e^6*z^2 - 2*A^2*b^3*c^7*d^3*e^5*z^2 - 36*A^2*a^2*c^8*d^2*e^6*z^2 - 48*A^2*a^6*b^2*c^2*f^8*z^2 - 45*A^2*a^2*b^2*c^6*e^8*z^2 + 4*A^2*b^10*d*e^2*f^5*z^2 + 4*B^2*b^2*c^8*d^7*f*z^2 + 4*A^2*b^9*c*e^5*f^3*z^2 + 4*A^2*b^7*c^3*e^7*f*z^2 - 128*B^2*a^7*c^3*d*f^7*z^2 - 160*A^2*a*c^9*d^6*f^2*z^2 - 112*A^2*a^6*c^4*d*f^7*z^2 + 12*A^2*b*c^9*d^5*e^3*z^2 + 4*A^2*a*b^9*e^3*f^5*z^2 + 3*B^2*a^4*b^6*d*f^7*z^2 + 2*A^2*a^3*b^7*e*f^7*z^2 - 24*A^2*a*c^9*d^4*e^4*z^2 + 14*A^2*a^2*b^8*d*f^7*z^2 + 12*A^2*a^5*b^4*c*f^8*z^2 + 12*A^2*a*b^4*c^5*e^8*z^2 + A*B*a^4*b^6

*e*f^7*z^2 + B^2*a^2*b^8*d*e^2*f^5*z^2 + 16*A^2*c^10*d^7*f*z^2 + 3*B^2*b^10
*d^3*f^5*z^2 - A^2*b^10*e^4*f^4*z^2 - 4*A^2*c^10*d^6*e^2*z^2 - A^2*b^10*d^2
*f^6*z^2 + 64*A^2*a^7*c^3*f^8*z^2 - 4*B^2*a^4*c^6*e^8*z^2 - A^2*b^6*c^4*e^8
*z^2 + 48*A^2*a^3*c^7*e^8*z^2 - A^2*a^4*b^6*f^8*z^2 + 720*A^2*B*a*b^2*c^5*d
^2*e^2*f^3*z - 600*A^2*B*a^2*b^2*c^4*d*e^2*f^4*z + 576*A*B^2*a^2*b^2*c^4*d^
2*e*f^4*z + 348*A*B^2*a*b^2*c^5*d^2*e^3*f^2*z - 336*A*B^2*a^2*b*c^5*d^2*e^2
*f^3*z - 260*A*B^2*a*b^3*c^4*d^2*e^2*f^3*z - 240*A*B^2*a^2*b^2*c^4*d*e^3*f^
3*z + 196*A*B^2*a^2*b^3*c^3*d*e^2*f^4*z + 172*A^2*B*a*b*c^6*d*e^5*f*z + 20*
A*B^2*a*b^6*c*d*e*f^5*z - 912*A^2*B*a^2*b*c^5*d^2*e*f^4*z - 644*A^2*B*a*b*c
^6*d^2*e^3*f^2*z - 432*A*B^2*a*b^2*c^5*d^3*e*f^3*z + 372*A^2*B*a^2*b*c^5*d*
e^3*f^3*z - 330*A^2*B*a*b^2*c^5*d*e^4*f^2*z + 312*A*B^2*a*b*c^6*d^3*e^2*f^2
*z - 208*A*B^2*a^3*b^2*c^3*d*e*f^5*z + 192*A^2*B*a^2*b^3*c^3*d*e*f^5*z + 17
2*A^2*B*a*b^3*c^4*d*e^3*f^3*z + 108*A*B^2*a^2*b*c^5*d*e^4*f^2*z + 104*A*B^2
*a^3*b*c^4*d*e^2*f^4*z - 80*A^2*B*a*b^3*c^4*d^2*e*f^4*z + 68*A^2*B*a*b^4*c^
3*d*e^2*f^4*z - 60*A*B^2*a*b^5*c^2*d*e^2*f^4*z + 58*A*B^2*a*b^3*c^4*d*e^4*f
^2*z - 36*A*B^2*a*b^4*c^3*d^2*e*f^4*z - 24*A*B^2*a^2*b^4*c^2*d*e*f^5*z + 24
*A*B^2*a*b^4*c^3*d*e^3*f^3*z + 592*A^2*B*a*b*c^6*d^3*e*f^3*z + 240*A^2*B*a^
3*b*c^4*d*e*f^5*z - 132*A*B^2*a*b*c^6*d^2*e^4*f*z - 60*A*B^2*a*b^2*c^5*d*e^
5*f*z - 48*A^2*B*a*b^5*c^2*d*e*f^5*z + 20*B^3*a*b*c^6*d^3*e^3*f*z + 16*B^3*
a^4*b*c^3*d*e*f^5*z - 16*B^3*a*b*c^6*d^4*e*f^2*z + 12*B^3*a^2*b*c^5*d*e^5*f
*z + 320*A^3*a*b*c^6*d*e^4*f^2*z + 40*A^3*a*b^4*c^3*d*e*f^5*z - 48*A^2*B*b*
c^7*d^4*e*f^2*z - 44*A^2*B*b^3*c^5*d*e^5*f*z - 20*A*B^2*b*c^7*d^4*e^2*f*z +
14*A*B^2*b^4*c^4*d*e^5*f*z + 12*A^2*B*b*c^7*d^3*e^3*f*z + 4*A*B^2*b^7*c*d*
e^2*f^4*z + 160*A*B^2*a^4*c^4*d*e*f^5*z + 152*A^2*B*a*c^7*d^2*e^4*f*z - 40*
A*B^2*a*c^7*d^3*e^3*f*z + 32*A*B^2*a*c^7*d^4*e*f^2*z - 16*A*B^2*a^2*c^6*d*e
^5*f*z + 128*A^2*B*a^4*b*c^3*e*f^6*z + 42*A^2*B*a*b^2*c^5*e^6*f*z + 24*A^2*
B*a^2*b^5*c*e*f^6*z - 12*A*B^2*a^3*b^4*c*e*f^6*z - 12*A*B^2*a^2*b*c^5*e^6*f
*z - 10*A^2*B*a*b^6*c*e^2*f^5*z - 160*A*B^2*a*b*c^6*d^4*f^3*z + 112*A*B^2*a
^4*b*c^3*d*f^6*z - 24*A*B^2*a^2*b^5*c*d*f^6*z - 84*B^3*a*b^2*c^5*d^3*e^2*f^
2*z - 80*B^3*a^2*b^3*c^3*d^2*e*f^4*z - 60*B^3*a^2*b*c^5*d^2*e^3*f^2*z - 20*
B^3*a^3*b^2*c^3*d*e^2*f^4*z - 20*B^3*a*b^3*c^4*d^2*e^3*f^2*z - 9*B^3*a^2*b^
2*c^4*d*e^4*f^2*z - 8*B^3*a*b^4*c^3*d^2*e^2*f^3*z + 6*B^3*a^2*b^4*c^2*d*e^2
*f^4*z - 4*B^3*a^2*b^3*c^3*d*e^3*f^3*z - 216*A^2*B*b^4*c^4*d^2*e^2*f^3*z +
196*A^2*B*b^3*c^5*d^2*e^3*f^2*z - 108*A*B^2*b^3*c^5*d^3*e^2*f^2*z - 94*A*B^
2*b^4*c^4*d^2*e^3*f^2*z + 88*A^2*B*b^2*c^6*d^3*e^2*f^2*z + 80*A*B^2*b^5*c^3
*d^2*e^2*f^3*z + 360*A^2*B*a^2*c^6*d^2*e^2*f^3*z + 8*A*B^2*a^2*c^6*d^2*e^3*
f^2*z + 153*A^2*B*a^2*b^2*c^4*e^4*f^3*z - 144*A^2*B*a^2*b^3*c^3*e^3*f^4*z +
80*A^2*B*a^3*b^2*c^3*e^2*f^5*z + 36*A*B^2*a^3*b^2*c^3*e^3*f^4*z + 12*A^2*B
*a^2*b^4*c^2*e^2*f^5*z + 12*A*B^2*a^3*b^3*c^2*e^2*f^5*z + 9*A*B^2*a^2*b^2*c
^4*e^5*f^2*z - 6*A*B^2*a^2*b^4*c^2*e^3*f^4*z + 4*A*B^2*a^2*b^3*c^3*e^4*f^3*
z + 480*A^2*B*a^2*b^2*c^4*d^2*f^5*z - 176*A*B^2*a^2*b^3*c^3*d^2*f^5*z - 10*
A^2*B*a*b^6*c*d*f^6*z + 16*A*B^2*a*b*c^6*d*e^6*z + 80*B^3*a*b^3*c^4*d^3*e*f
^3*z - 48*B^3*a^3*b*c^4*d^2*e*f^4*z + 48*B^3*a^2*b*c^5*d^3*e*f^3*z + 44*B^3
*a^3*b*c^4*d*e^3*f^3*z + 24*B^3*a*b^5*c^2*d^2*e*f^4*z + 18*B^3*a*b^2*c^5*d^
2*e^4*f*z + 696*A^3*a^2*b*c^5*d*e^2*f^4*z - 504*A^3*a*b*c^6*d^2*e^2*f^3*z -

192*A^3*a*b^2*c^5*d*e^3*f^3*z - 144*A^3*a^2*b^2*c^4*d*e*f^5*z + 96*A^3*a*b
^2*c^5*d^2*e*f^4*z - 72*A^3*a*b^3*c^4*d*e^2*f^4*z - 208*A^2*B*b^3*c^5*d^3*e
*f^3*z + 152*A*B^2*b^4*c^4*d^3*e*f^3*z + 80*A^2*B*b^5*c^3*d^2*e*f^4*z + 75*
A^2*B*b^4*c^4*d*e^4*f^2*z - 59*A^2*B*b^2*c^6*d^2*e^4*f*z - 52*A^2*B*b^5*c^3
*d*e^3*f^3*z + 42*A*B^2*b^3*c^5*d^2*e^4*f*z - 21*A*B^2*b^6*c^2*d^2*e*f^4*z
- 16*A*B^2*b^5*c^3*d*e^4*f^2*z + 16*A*B^2*b^2*c^6*d^4*e*f^2*z + 16*A*B^2*b^
2*c^6*d^3*e^3*f*z + 11*A^2*B*b^6*c^2*d*e^2*f^4*z + 4*A*B^2*b^6*c^2*d*e^3*f^
3*z - 256*A^2*B*a*c^7*d^3*e^2*f^2*z - 96*A*B^2*a^3*c^5*d^2*e*f^4*z - 36*A^2
*B*a^2*c^6*d*e^4*f^2*z - 32*A^2*B*a^3*c^5*d*e^2*f^4*z - 32*A*B^2*a^2*c^6*d^
3*e*f^3*z + 8*A*B^2*a^3*c^5*d*e^3*f^3*z - 96*A^2*B*a^3*b^3*c^2*e*f^6*z + 68
*A^2*B*a^3*b*c^4*e^3*f^4*z - 60*A*B^2*a^4*b*c^3*e^2*f^5*z - 60*A*B^2*a^3*b*
c^4*e^4*f^3*z + 48*A*B^2*a^4*b^2*c^2*e*f^6*z - 38*A^2*B*a*b^3*c^4*e^5*f^2*z
 - 36*A^2*B*a^2*b*c^5*e^5*f^2*z + 36*A^2*B*a*b^5*c^2*e^3*f^4*z - 16*A^2*B*a
*b^4*c^3*e^4*f^3*z + 384*A*B^2*a^2*b*c^5*d^3*f^4*z - 352*A*B^2*a^3*b*c^4*d^
2*f^5*z - 288*A^2*B*a*b^2*c^5*d^3*f^4*z - 160*A^2*B*a^3*b^2*c^3*d*f^6*z - 1
48*A^2*B*a*b^4*c^3*d^2*f^5*z + 112*A*B^2*a*b^3*c^4*d^3*f^4*z + 72*A^2*B*a^2
*b^4*c^2*d*f^6*z + 72*A*B^2*a*b^5*c^2*d^2*f^5*z + 48*A*B^2*a^3*b^3*c^2*d*f^
6*z + 102*B^3*a^2*b^2*c^4*d^2*e^2*f^3*z - 32*B^3*b^5*c^3*d^3*e*f^3*z - 8*B^
3*b^3*c^5*d^3*e^3*f*z - 7*B^3*b^4*c^4*d^2*e^4*f*z + 5*B^3*b^2*c^6*d^4*e^2*f
*z + 80*A^3*b^2*c^6*d^3*e*f^3*z - 74*A^3*b^3*c^5*d*e^4*f^2*z - 64*A^3*b^4*c
^4*d^2*e*f^4*z + 60*A^3*b^4*c^4*d*e^3*f^3*z - 48*B^3*a^4*c^4*d*e^2*f^4*z -
24*B^3*a^3*c^5*d*e^4*f^2*z + 20*B^3*a^2*c^6*d^2*e^4*f*z - 16*A^3*b^5*c^3*d*
e^2*f^4*z + 8*A^3*b*c^7*d^3*e^2*f^2*z + 480*A^3*a^2*c^6*d^2*e*f^4*z - 392*A
^3*a^2*c^6*d*e^3*f^3*z + 280*A^3*a*c^7*d^2*e^3*f^2*z - 4*B^3*a^4*b*c^3*e^3*
f^4*z - 200*A^3*a^3*b*c^4*e^2*f^5*z - 144*A^3*a^2*b*c^5*e^4*f^3*z + 48*B^3*
a*b^2*c^5*d^4*f^3*z + 42*A^3*a*b^2*c^5*e^5*f^2*z - 36*B^3*a^4*b^2*c^2*d*f^6
*z - 32*A^3*a^3*b^2*c^3*e*f^6*z - 24*A^3*a^2*b^4*c^2*e*f^6*z - 24*A^3*a*b^5
*c^2*e^2*f^5*z + 10*A^3*a*b^3*c^4*e^4*f^3*z - 4*B^3*a*b^4*c^3*d^3*f^4*z - 4
*A^3*a*b^4*c^3*e^3*f^4*z - 480*A^3*a^2*b*c^5*d^2*f^5*z - 160*A^3*a^2*b^3*c^
3*d*f^6*z + 128*A^3*a*b^3*c^4*d^2*f^5*z + 8*A^2*B*b^5*c^3*e^5*f^2*z - 2*A^2
*B*b^6*c^2*e^4*f^3*z + 112*A^2*B*b^4*c^4*d^3*f^4*z - 92*A^2*B*a^4*c^4*e^2*f
^5*z - 64*A^2*B*a^3*c^5*e^4*f^3*z - 64*A*B^2*b^5*c^3*d^3*f^4*z + 24*A*B^2*a
^4*c^4*e^3*f^4*z + 24*A*B^2*a^3*c^5*e^5*f^2*z + 16*A^2*B*b^2*c^6*d^4*f^3*z
+ 16*A*B^2*b^3*c^5*d^4*f^3*z - A^2*B*b^6*c^2*d^2*f^5*z + 448*A^2*B*a^3*c^5*
d^2*f^5*z - 352*A^2*B*a^2*c^6*d^3*f^4*z - 5*A*B^2*b^2*c^6*d^2*e^5*z - 48*A^
2*B*a^4*b^2*c^2*f^7*z - 2*B^3*b^7*c*d^2*e*f^4*z + 34*A^3*b^2*c^6*d*e^5*f*z
+ 16*A^3*b*c^7*d^2*e^4*f*z + 2*A^3*b^6*c^2*d*e*f^5*z - 416*A^3*a^3*c^5*d*e*
f^5*z - 224*A^3*a*c^7*d^3*e*f^3*z + 12*B^3*a^3*b^4*c*d*f^6*z - 10*B^3*a*b^6
*c*d^2*f^5*z + 416*A^3*a^3*b*c^4*d*f^6*z + 224*A^3*a*b*c^6*d^3*f^4*z + 24*A
^3*a*b^5*c^2*d*f^6*z - 4*B^3*a*b*c^6*d^2*e^5*z + 20*A^2*B*c^8*d^4*e^2*f*z -
 7*A^2*B*b^4*c^4*e^6*f*z - 2*A^2*B*b^7*c*e^3*f^4*z - 64*A*B^2*a^5*c^3*e*f^6
*z + 16*A*B^2*b*c^7*d^5*f^2*z - 8*A^2*B*a^2*c^6*e^6*f*z - 2*A*B^2*b^7*c*d^2
*f^5*z - 272*A^2*B*a^4*c^4*d*f^6*z + 128*A^2*B*a*c^7*d^4*f^3*z + 9*A^2*B*b^
2*c^6*d*e^6*z - 4*A*B^2*b^3*c^5*d*e^6*z + 4*A*B^2*b*c^7*d^3*e^4*z + 8*A*B^2
*a*c^7*d^2*e^5*z + 12*A^2*B*a^3*b^4*c*f^7*z + 30*B^3*b^4*c^4*d^3*e^2*f^2*z

+ 8*B^3*b^5*c^3*d^2*e^3*f^2*z - 2*B^3*b^6*c^2*d^2*e^2*f^3*z + 152*A^3*b^3*c
^5*d^2*e^2*f^3*z - 108*A^3*b^2*c^6*d^2*e^3*f^2*z + 48*B^3*a^3*c^5*d^2*e^2*f
^3*z - 16*B^3*a^2*c^6*d^3*e^2*f^2*z - 3*B^3*a^4*b^2*c^2*e^2*f^5*z - 120*B^3
*a^2*b^2*c^4*d^3*f^4*z + 112*B^3*a^3*b^2*c^3*d^2*f^5*z + 112*A^3*a^2*b^3*c^
3*e^2*f^5*z + 12*A^3*a^2*b^2*c^4*e^3*f^4*z - 120*A^3*a*c^7*d*e^5*f*z - 52*A
^3*a*b*c^6*e^6*f*z + 10*A^3*a*b^6*c*e*f^6*z - 2*A*B^2*b^8*d*e*f^5*z - 2*A^2
*B*a*b^7*e*f^6*z - 24*A^2*B*a*c^7*d*e^6*z + 2*A*B^2*a*b^7*d*f^6*z - 12*A^2*
B*a*b*c^6*e^7*z - 2*A^3*b^7*c*d*f^6*z - 4*A^3*b*c^7*d*e^6*z + 16*B^3*a^5*c^
3*e^2*f^5*z + 11*B^3*b^6*c^2*d^3*f^4*z - 11*A^3*b^4*c^4*e^5*f^2*z - 8*B^3*b
^4*c^4*d^4*f^3*z - 4*B^3*b^2*c^6*d^5*f^2*z + 4*B^3*a^4*c^4*e^4*f^3*z + 4*A^
3*b^5*c^3*e^4*f^3*z - A^3*b^6*c^2*e^3*f^4*z + 136*A^3*a^3*c^5*e^3*f^4*z + 6
8*A^3*a^2*c^6*e^5*f^2*z - 64*A^3*b^3*c^5*d^3*f^4*z + 2*B^3*b^3*c^5*d^2*e^5*
z - B^3*b^2*c^6*d^3*e^4*z + 96*A^3*a^3*b^3*c^2*f^7*z + A*B^2*a^2*b^6*e*f^6*
z + 32*A^3*c^8*d^4*e*f^2*z - 24*A^3*c^8*d^3*e^3*f*z + 10*A^3*b^3*c^5*e^6*f*
z + 2*A^3*b^7*c*e^2*f^5*z + 128*A^3*a^4*c^4*e*f^6*z - 32*A^3*b*c^7*d^4*f^3*
z - 4*B^3*a^2*c^6*d*e^6*z - B^3*a^2*b^6*d*f^6*z - 128*A^3*a^4*b*c^3*f^7*z -
 24*A^3*a^2*b^5*c*f^7*z - 16*A^2*B*c^8*d^5*f^2*z - 4*A^2*B*c^8*d^3*e^4*z +
64*A^2*B*a^5*c^3*f^7*z + 2*A^2*B*b^3*c^5*e^7*z + 4*A*B^2*a^2*c^6*e^7*z - A^
2*B*a^2*b^6*f^7*z + 4*A^3*c^8*d^2*e^5*z - 3*A^3*b^2*c^6*e^7*z + A^2*B*b^8*d
*f^6*z - A^3*b^8*e*f^6*z + 16*A^3*a*c^7*e^7*z + 2*A^3*a*b^7*f^7*z + A^2*B*b
^8*e^2*f^5*z + B^3*b^8*d^2*f^5*z - 48*A^2*B^2*a*b*c^4*d*e*f^4 + 28*A*B^3*a*
b^2*c^3*d*e*f^4 - 16*A*B^3*a*b*c^4*d*e^2*f^3 + 16*A^3*B*a*c^5*d*e*f^4 + 32*
A^3*B*a*b*c^4*d*f^5 + 12*A^2*B^2*b^3*c^3*d*e*f^4 + 5*A*B^3*b^2*c^4*d^2*e*f^
3 + 4*A*B^3*b^3*c^3*d*e^2*f^3 + 24*A^2*B^2*a*c^5*d*e^2*f^3 + 24*A^2*B^2*a^2
*b*c^3*e*f^5 + 12*A^2*B^2*a*b*c^4*e^3*f^3 - 6*A^2*B^2*a*b^3*c^2*e*f^5 + 4*A
*B^3*a^2*b*c^3*e^2*f^4 + 3*A*B^3*a^2*b^2*c^2*e*f^5 - 18*A^2*B^2*a*b^2*c^3*d
*f^5 - 4*B^4*a^2*b*c^3*d*e*f^4 + 4*B^4*a*b*c^4*d^2*e*f^3 - 6*A*B^3*b^4*c^2*
d*e*f^4 + 4*A^3*B*b*c^5*d*e^2*f^3 - 2*A^3*B*b^2*c^4*d*e*f^4 - 8*A*B^3*a^2*c
^4*d*e*f^4 - 8*A*B^3*a*c^5*d^2*e*f^3 + 26*A^3*B*a*b^2*c^3*e*f^5 + 8*A^3*B*a
*b*c^4*e^2*f^4 + 32*A*B^3*a*b*c^4*d^2*f^4 - 28*A*B^3*a^2*b*c^3*d*f^5 + 6*A*
B^3*a*b^3*c^2*d*f^5 - 9*A^2*B^2*b^2*c^4*d*e^2*f^3 - 18*A^2*B^2*a*b^2*c^3*e^
2*f^4 - 4*A^3*B*c^6*d^2*e*f^3 - 3*A^3*B*b^4*c^2*e*f^5 - 44*A^3*B*a^2*c^4*e*
f^5 - 16*A^3*B*a*c^5*e^3*f^3 - 16*A*B^3*a^3*c^3*e*f^5 - 10*A^3*B*b^3*c^3*d*
f^5 - 4*A^3*B*b*c^5*d^2*f^4 - 4*A*B^3*b*c^5*d^3*f^3 - 28*A^3*B*a^2*b*c^3*f^
6 + 6*A^3*B*a*b^3*c^2*f^6 - 4*A^4*b*c^5*d*e*f^4 - 20*A^4*a*b*c^4*e*f^5 + 3*
A^2*B^2*b^4*c^2*e^2*f^4 - 2*A^2*B^2*b^3*c^3*e^3*f^3 + 12*A^2*B^2*a^2*c^4*e^
2*f^4 + 9*A^2*B^2*b^2*c^4*d^2*f^4 - 3*A^2*B^2*a^2*b^2*c^2*f^6 - 2*B^4*b^3*c
^3*d^2*e*f^3 + 4*B^4*a^2*c^4*d*e^2*f^3 - 10*B^4*a*b^2*c^3*d^2*f^4 - 3*B^4*a
^2*b^2*c^2*d*f^5 + 3*A^3*B*b^2*c^4*e^3*f^3 - 2*A^3*B*b^3*c^3*e^2*f^4 - 10*A
*B^3*b^3*c^3*d^2*f^4 - 4*A*B^3*a^2*c^4*e^3*f^3 + 3*A^2*B^2*b^4*c^2*d*f^5 +
36*A^2*B^2*a^2*c^4*d*f^5 - 24*A^2*B^2*a*c^5*d^2*f^4 + 4*A^2*B^2*c^6*d^3*f^3
 + 16*A^2*B^2*a^3*c^3*f^6 + 4*A^4*b^3*c^3*e*f^5 + 16*B^4*a^3*c^3*d*f^5 + 16
*A^4*a*c^5*e^2*f^4 + 8*A^4*b^2*c^4*d*f^5 - 8*A^4*a*b^2*c^3*f^6 - 24*A^4*a*c
^5*d*f^5 + 3*B^4*b^4*c^2*d^2*f^4 - 3*A^4*b^2*c^4*e^2*f^4 + 4*A^4*c^6*d^2*f^
4 + 36*A^4*a^2*c^4*f^6 + B^4*b^2*c^4*d^3*f^3, z, k)*(root(48416*a^6*b^2*c^6

*d^4*e^2*f^4*z^4 - 41544*a^5*b^4*c^5*d^4*e^2*f^4*z^4 - 31872*a^7*b^2*c^5*d^
3*e^2*f^5*z^4 - 31872*a^5*b^2*c^7*d^5*e^2*f^3*z^4 - 29184*a^6*b^2*c^6*d^3*e
^4*f^3*z^4 + 28800*a^5*b^4*c^5*d^3*e^4*f^3*z^4 + 21510*a^4*b^6*c^4*d^4*e^2*
f^4*z^4 + 21408*a^6*b^4*c^4*d^3*e^2*f^5*z^4 + 21408*a^4*b^4*c^6*d^5*e^2*f^3
*z^4 - 18112*a^7*b^3*c^4*d^2*e^3*f^5*z^4 - 18112*a^4*b^3*c^7*d^5*e^3*f^2*z^
4 - 15600*a^5*b^5*c^4*d^3*e^3*f^4*z^4 - 15600*a^4*b^5*c^5*d^4*e^3*f^3*z^4 +
 15296*a^6*b^3*c^5*d^3*e^3*f^4*z^4 + 15296*a^5*b^3*c^6*d^4*e^3*f^3*z^4 + 14
016*a^7*b^2*c^5*d^2*e^4*f^4*z^4 + 14016*a^5*b^2*c^7*d^4*e^4*f^2*z^4 - 13920
*a^4*b^6*c^4*d^3*e^4*f^3*z^4 - 11648*a^6*b^3*c^5*d^2*e^5*f^3*z^4 - 11648*a^
5*b^3*c^6*d^3*e^5*f^2*z^4 + 10432*a^6*b^2*c^6*d^2*e^6*f^2*z^4 + 9008*a^6*b^
5*c^3*d^2*e^3*f^5*z^4 + 9008*a^3*b^5*c^6*d^5*e^3*f^2*z^4 + 8544*a^5*b^5*c^4
*d^2*e^5*f^3*z^4 + 8544*a^4*b^5*c^5*d^3*e^5*f^2*z^4 - 8496*a^5*b^4*c^5*d^2*
e^6*f^2*z^4 + 7488*a^8*b^2*c^4*d^2*e^2*f^6*z^4 + 7488*a^4*b^2*c^8*d^6*e^2*f
^2*z^4 + 7380*a^4*b^7*c^3*d^3*e^3*f^4*z^4 + 7380*a^3*b^7*c^4*d^4*e^3*f^3*z^
4 - 6720*a^3*b^8*c^3*d^4*e^2*f^4*z^4 - 5784*a^5*b^6*c^3*d^3*e^2*f^5*z^4 - 5
784*a^3*b^6*c^5*d^5*e^2*f^3*z^4 - 3440*a^6*b^4*c^4*d^2*e^4*f^4*z^4 - 3440*a
^4*b^4*c^6*d^4*e^4*f^2*z^4 + 3360*a^3*b^8*c^3*d^3*e^4*f^3*z^4 + 3140*a^4*b^
6*c^4*d^2*e^6*f^2*z^4 - 2760*a^4*b^7*c^3*d^2*e^5*f^3*z^4 - 2760*a^3*b^7*c^4
*d^3*e^5*f^2*z^4 - 1764*a^5*b^7*c^2*d^2*e^3*f^5*z^4 - 1764*a^2*b^7*c^5*d^5*
e^3*f^2*z^4 - 1640*a^3*b^9*c^2*d^3*e^3*f^4*z^4 - 1640*a^2*b^9*c^3*d^4*e^3*f
^3*z^4 - 1604*a^6*b^6*c^2*d^2*e^2*f^6*z^4 - 1604*a^2*b^6*c^6*d^6*e^2*f^2*z^
4 - 1500*a^5*b^6*c^3*d^2*e^4*f^4*z^4 - 1500*a^3*b^6*c^5*d^4*e^4*f^2*z^4 + 1
140*a^2*b^10*c^2*d^4*e^2*f^4*z^4 + 810*a^4*b^8*c^2*d^2*e^4*f^4*z^4 + 810*a^
2*b^8*c^4*d^4*e^4*f^2*z^4 - 544*a^3*b^8*c^3*d^2*e^6*f^2*z^4 + 416*a^3*b^9*c
^2*d^2*e^5*f^3*z^4 + 416*a^2*b^9*c^3*d^3*e^5*f^2*z^4 - 384*a^2*b^10*c^2*d^3
*e^4*f^3*z^4 + 180*a^4*b^8*c^2*d^3*e^2*f^5*z^4 + 180*a^2*b^8*c^4*d^5*e^2*f^
3*z^4 + 48*a^7*b^4*c^3*d^2*e^2*f^6*z^4 + 48*a^3*b^4*c^7*d^6*e^2*f^2*z^4 + 3
6*a^2*b^10*c^2*d^2*e^6*f^2*z^4 - 1024*a^10*b*c^3*d*e*f^8*z^4 - 1024*a^3*b*c
^10*d^8*e*f*z^4 - 192*a^8*b^5*c*d*e*f^8*z^4 - 192*a*b^5*c^8*d^8*e*f*z^4 + 1
6128*a^7*b^3*c^4*d^3*e*f^6*z^4 + 16128*a^4*b^3*c^7*d^6*e*f^3*z^4 - 11712*a^
6*b^5*c^3*d^3*e*f^6*z^4 - 11712*a^3*b^5*c^6*d^6*e*f^3*z^4 + 11520*a^8*b*c^5
*d^2*e^3*f^5*z^4 + 11520*a^5*b*c^8*d^5*e^3*f^2*z^4 - 9984*a^6*b^3*c^5*d^4*e
*f^5*z^4 - 9984*a^5*b^3*c^6*d^5*e*f^4*z^4 + 8640*a^5*b^5*c^4*d^4*e*f^5*z^4
+ 8640*a^4*b^5*c^5*d^5*e*f^4*z^4 - 7424*a^7*b*c^6*d^3*e^3*f^4*z^4 - 7424*a^
6*b*c^7*d^4*e^3*f^3*z^4 - 6912*a^8*b^3*c^3*d^2*e*f^7*z^4 - 6912*a^3*b^3*c^8
*d^7*e*f^2*z^4 + 4800*a^7*b^3*c^4*d*e^5*f^4*z^4 + 4800*a^4*b^3*c^7*d^4*e^5*
f*z^4 + 4608*a^7*b*c^6*d^2*e^5*f^3*z^4 + 4608*a^6*b*c^7*d^3*e^5*f^2*z^4 - 4
560*a^4*b^7*c^3*d^4*e*f^5*z^4 - 4560*a^3*b^7*c^4*d^5*e*f^4*z^4 + 4176*a^5*b
^7*c^2*d^3*e*f^6*z^4 + 4176*a^2*b^7*c^5*d^6*e*f^3*z^4 + 3264*a^7*b^5*c^2*d^
2*e*f^7*z^4 + 3264*a^2*b^5*c^7*d^7*e*f^2*z^4 + 3008*a^8*b^3*c^3*d*e^3*f^6*z
^4 + 3008*a^3*b^3*c^8*d^6*e^3*f*z^4 + 2880*a^6*b^3*c^5*d*e^7*f^2*z^4 + 2880
*a^5*b^3*c^6*d^2*e^7*f*z^4 - 2240*a^7*b^4*c^3*d*e^4*f^5*z^4 - 2240*a^3*b^4*
c^7*d^5*e^4*f*z^4 - 1488*a^5*b^5*c^4*d*e^7*f^2*z^4 - 1488*a^4*b^5*c^5*d^2*e
^7*f*z^4 + 1440*a^3*b^9*c^2*d^4*e*f^5*z^4 + 1440*a^2*b^9*c^3*d^5*e*f^4*z^4
- 1328*a^6*b^5*c^3*d*e^5*f^4*z^4 - 1328*a^3*b^5*c^6*d^4*e^5*f*z^4 - 1152*a^

7*b^2*c^5*d*e^6*f^3*z^4 - 1152*a^5*b^2*c^7*d^3*e^6*f*z^4 - 1120*a^6*b^4*c^4
*d*e^6*f^3*z^4 - 1120*a^4*b^4*c^6*d^3*e^6*f*z^4 + 912*a^6*b^6*c^2*d*e^4*f^5
*z^4 + 912*a^2*b^6*c^6*d^5*e^4*f*z^4 + 872*a^5*b^6*c^3*d*e^6*f^3*z^4 + 872*
a^3*b^6*c^5*d^3*e^6*f*z^4 + 768*a^8*b^2*c^4*d*e^4*f^5*z^4 + 768*a^4*b^2*c^8
*d^5*e^4*f*z^4 - 672*a^8*b^4*c^2*d*e^2*f^7*z^4 - 672*a^2*b^4*c^8*d^7*e^2*f*
z^4 - 624*a^7*b^5*c^2*d*e^3*f^6*z^4 - 624*a^2*b^5*c^7*d^6*e^3*f*z^4 + 480*a
^5*b^8*c*d^2*e^2*f^6*z^4 + 480*a*b^8*c^5*d^6*e^2*f^2*z^4 + 316*a^4*b^7*c^3*
d*e^7*f^2*z^4 + 316*a^3*b^7*c^4*d^2*e^7*f*z^4 - 204*a^4*b^8*c^2*d*e^6*f^3*z
^4 - 204*a^2*b^8*c^4*d^3*e^6*f*z^4 + 168*a^3*b^10*c*d^3*e^2*f^5*z^4 + 168*a
*b^10*c^3*d^5*e^2*f^3*z^4 + 156*a^2*b^11*c*d^3*e^3*f^4*z^4 + 156*a*b^11*c^2
*d^4*e^3*f^3*z^4 + 128*a^9*b^2*c^3*d*e^2*f^7*z^4 + 128*a^3*b^2*c^9*d^7*e^2*
f*z^4 - 124*a^3*b^10*c*d^2*e^4*f^4*z^4 - 124*a*b^10*c^3*d^4*e^4*f^2*z^4 + 1
00*a^4*b^9*c*d^2*e^3*f^5*z^4 + 100*a*b^9*c^4*d^5*e^3*f^2*z^4 + 36*a^5*b^7*c
^2*d*e^5*f^4*z^4 + 36*a^2*b^7*c^5*d^4*e^5*f*z^4 - 24*a^3*b^9*c^2*d*e^7*f^2*
z^4 - 24*a^2*b^11*c*d^2*e^5*f^3*z^4 - 24*a^2*b^9*c^3*d^2*e^7*f*z^4 - 24*a*b
^11*c^2*d^3*e^5*f^2*z^4 - 9216*a^8*b*c^5*d^3*e*f^6*z^4 - 9216*a^5*b*c^8*d^6
*e*f^3*z^4 - 5376*a^8*b*c^5*d*e^5*f^4*z^4 - 5376*a^5*b*c^8*d^4*e^5*f*z^4 +
5120*a^9*b*c^4*d^2*e*f^7*z^4 + 5120*a^7*b*c^6*d^4*e*f^5*z^4 + 5120*a^6*b*c^
7*d^5*e*f^4*z^4 + 5120*a^4*b*c^9*d^7*e*f^2*z^4 - 4352*a^9*b*c^4*d*e^3*f^6*z
^4 - 4352*a^4*b*c^9*d^6*e^3*f*z^4 - 1792*a^7*b*c^6*d*e^7*f^2*z^4 - 1792*a^6
*b*c^7*d^2*e^7*f*z^4 - 1600*a^6*b^2*c^6*d*e^8*f*z^4 + 912*a^5*b^4*c^5*d*e^8
*f*z^4 + 768*a^9*b^3*c^2*d*e*f^8*z^4 + 768*a^2*b^3*c^9*d^8*e*f*z^4 - 720*a^
4*b^9*c*d^3*e*f^6*z^4 - 720*a*b^9*c^4*d^6*e*f^3*z^4 - 656*a^6*b^7*c*d^2*e*f
^7*z^4 - 656*a*b^7*c^6*d^7*e*f^2*z^4 - 240*a^2*b^11*c*d^4*e*f^5*z^4 - 240*a
*b^11*c^2*d^5*e*f^4*z^4 + 216*a^7*b^6*c*d*e^2*f^7*z^4 + 216*a*b^6*c^7*d^7*e
^2*f*z^4 - 204*a^4*b^6*c^4*d*e^8*f*z^4 - 144*a^5*b^8*c*d*e^4*f^5*z^4 - 144*
a*b^8*c^5*d^5*e^4*f*z^4 - 84*a*b^12*c*d^4*e^2*f^4*z^4 + 36*a^4*b^9*c*d*e^5*
f^4*z^4 + 36*a*b^9*c^4*d^4*e^5*f*z^4 + 20*a^6*b^7*c*d*e^3*f^6*z^4 + 20*a*b^
7*c^6*d^6*e^3*f*z^4 + 16*a^3*b^10*c*d*e^6*f^3*z^4 + 16*a^3*b^8*c^3*d*e^8*f*
z^4 + 16*a*b^12*c*d^3*e^4*f^3*z^4 + 16*a*b^10*c^3*d^3*e^6*f*z^4 + 48*b^11*c
^3*d^6*e*f^3*z^4 + 48*b^9*c^5*d^7*e*f^2*z^4 - 20*b^8*c^6*d^7*e^2*f*z^4 + 8*
b^10*c^4*d^5*e^4*f*z^4 - 4*b^13*c*d^4*e^3*f^3*z^4 - 4*b^11*c^3*d^4*e^5*f*z^
4 + 4*b^9*c^5*d^6*e^3*f*z^4 + 3072*a^9*c^5*d*e^4*f^5*z^4 + 3072*a^5*c^9*d^5
*e^4*f*z^4 + 2560*a^8*c^6*d*e^6*f^3*z^4 + 2560*a^6*c^8*d^3*e^6*f*z^4 + 1536
*a^10*c^4*d*e^2*f^7*z^4 + 1536*a^4*c^10*d^7*e^2*f*z^4 + 48*a^5*b^9*d^2*e*f^
7*z^4 + 48*a^3*b^11*d^3*e*f^6*z^4 - 20*a^6*b^8*d*e^2*f^7*z^4 + 8*a^4*b^10*d
*e^4*f^5*z^4 + 4*a^5*b^9*d*e^3*f^6*z^4 - 4*a^3*b^11*d*e^5*f^4*z^4 - 4*a*b^1
3*d^3*e^3*f^4*z^4 + 768*a^9*b*c^4*e^5*f^5*z^4 + 768*a^8*b*c^5*e^7*f^3*z^4 +
256*a^10*b*c^3*e^3*f^7*z^4 - 192*a^6*b^3*c^5*e^9*f*z^4 - 68*a^7*b^6*c*e^4*
f^6*z^4 + 48*a^8*b^5*c*e^3*f^7*z^4 + 48*a^5*b^5*c^4*e^9*f*z^4 + 36*a^6*b^7*
c*e^5*f^5*z^4 - 12*a^9*b^4*c*e^2*f^8*z^4 - 4*a^4*b^9*c*e^7*f^3*z^4 - 4*a^4*
b^7*c^3*e^9*f*z^4 + 384*a^5*b^8*c*d^3*f^7*z^4 + 384*a*b^8*c^5*d^7*f^3*z^4 +
288*a^3*b^10*c*d^4*f^6*z^4 + 288*a*b^10*c^3*d^6*f^4*z^4 + 224*a^7*b^6*c*d^
2*f^8*z^4 + 224*a*b^6*c^7*d^8*f^2*z^4 - 192*a^10*b^2*c^2*d*f^9*z^4 - 192*a^
2*b^2*c^10*d^9*f*z^4 + 768*a^5*b*c^8*d^3*e^7*z^4 + 768*a^4*b*c^9*d^5*e^5*z^

4 + 256*a^3*b*c^10*d^7*e^3*z^4 - 192*a^5*b^3*c^6*d*e^9*z^4 - 68*a*b^6*c^7*d
^6*e^4*z^4 + 48*a^4*b^5*c^5*d*e^9*z^4 + 48*a*b^5*c^8*d^7*e^3*z^4 + 36*a*b^7
*c^6*d^5*e^5*z^4 - 12*a*b^4*c^9*d^8*e^2*z^4 - 4*a^3*b^7*c^4*d*e^9*z^4 - 4*a
*b^9*c^4*d^3*e^7*z^4 + 16*b^13*c*d^5*e*f^4*z^4 + 16*b^7*c^7*d^8*e*f*z^4 + 7
68*a^7*c^7*d*e^8*f*z^4 + 16*a^7*b^7*d*e*f^8*z^4 + 16*a*b^13*d^4*e*f^5*z^4 +
256*a^7*b*c^6*e^9*f*z^4 + 80*a*b^12*c*d^5*f^5*z^4 + 48*a^9*b^4*c*d*f^9*z^4
+ 48*a*b^4*c^9*d^9*f*z^4 + 256*a^6*b*c^7*d*e^9*z^4 - 42*b^10*c^4*d^6*e^2*f
^2*z^4 - 20*b^12*c^2*d^5*e^2*f^3*z^4 + 6*b^12*c^2*d^4*e^4*f^2*z^4 + 4*b^11*
c^3*d^5*e^3*f^2*z^4 - 24960*a^7*c^7*d^4*e^2*f^4*z^4 + 18944*a^8*c^6*d^3*e^2
*f^5*z^4 + 18944*a^6*c^8*d^5*e^2*f^3*z^4 + 14336*a^7*c^7*d^3*e^4*f^3*z^4 -
9984*a^8*c^6*d^2*e^4*f^4*z^4 - 9984*a^6*c^8*d^4*e^4*f^2*z^4 - 7936*a^9*c^5*
d^2*e^2*f^6*z^4 - 7936*a^5*c^9*d^6*e^2*f^2*z^4 - 4352*a^7*c^7*d^2*e^6*f^2*z
^4 - 42*a^4*b^10*d^2*e^2*f^6*z^4 - 20*a^2*b^12*d^3*e^2*f^5*z^4 + 6*a^2*b^12
*d^2*e^4*f^4*z^4 + 4*a^3*b^11*d^2*e^3*f^5*z^4 - 480*a^8*b^2*c^4*e^6*f^4*z^4
+ 440*a^7*b^4*c^3*e^6*f^4*z^4 - 320*a^8*b^3*c^3*e^5*f^5*z^4 - 320*a^7*b^3*
c^4*e^7*f^3*z^4 + 240*a^8*b^4*c^2*e^4*f^6*z^4 + 240*a^6*b^4*c^4*e^8*f^2*z^4
- 192*a^9*b^3*c^2*e^3*f^7*z^4 - 192*a^9*b^2*c^3*e^4*f^6*z^4 - 192*a^7*b^2*
c^5*e^8*f^2*z^4 - 90*a^6*b^6*c^2*e^6*f^4*z^4 - 68*a^5*b^6*c^3*e^8*f^2*z^4 +
48*a^10*b^2*c^2*e^2*f^8*z^4 - 48*a^7*b^5*c^2*e^5*f^5*z^4 - 48*a^6*b^5*c^3*
e^7*f^3*z^4 + 36*a^5*b^7*c^2*e^7*f^3*z^4 + 6*a^4*b^8*c^2*e^8*f^2*z^4 - 3392
0*a^6*b^2*c^6*d^5*f^5*z^4 + 27936*a^5*b^4*c^5*d^5*f^5*z^4 + 26112*a^7*b^2*c
^5*d^4*f^6*z^4 + 26112*a^5*b^2*c^7*d^6*f^4*z^4 - 20352*a^6*b^4*c^4*d^4*f^6*
z^4 - 20352*a^4*b^4*c^6*d^6*f^4*z^4 - 13080*a^4*b^6*c^4*d^5*f^5*z^4 - 11520
*a^8*b^2*c^4*d^3*f^7*z^4 - 11520*a^4*b^2*c^8*d^7*f^3*z^4 + 8736*a^5*b^6*c^3
*d^4*f^6*z^4 + 8736*a^3*b^6*c^5*d^6*f^4*z^4 + 7488*a^7*b^4*c^3*d^3*f^7*z^4
+ 7488*a^3*b^4*c^7*d^7*f^3*z^4 + 3840*a^3*b^8*c^3*d^5*f^5*z^4 + 2560*a^9*b^
2*c^3*d^2*f^8*z^4 + 2560*a^3*b^2*c^9*d^8*f^2*z^4 - 2416*a^6*b^6*c^2*d^3*f^7
*z^4 - 2416*a^2*b^6*c^6*d^7*f^3*z^4 - 2160*a^4*b^8*c^2*d^4*f^6*z^4 - 2160*a
^2*b^8*c^4*d^6*f^4*z^4 - 1152*a^8*b^4*c^2*d^2*f^8*z^4 - 1152*a^2*b^4*c^8*d^
8*f^2*z^4 - 720*a^2*b^10*c^2*d^5*f^5*z^4 - 480*a^4*b^2*c^8*d^4*e^6*z^4 + 44
0*a^3*b^4*c^7*d^4*e^6*z^4 - 320*a^4*b^3*c^7*d^3*e^7*z^4 - 320*a^3*b^3*c^8*d
^5*e^5*z^4 + 240*a^4*b^4*c^6*d^2*e^8*z^4 + 240*a^2*b^4*c^8*d^6*e^4*z^4 - 19
2*a^5*b^2*c^7*d^2*e^8*z^4 - 192*a^3*b^2*c^9*d^6*e^4*z^4 - 192*a^2*b^3*c^9*d
^7*e^3*z^4 - 90*a^2*b^6*c^6*d^4*e^6*z^4 - 68*a^3*b^6*c^5*d^2*e^8*z^4 - 48*a
^3*b^5*c^6*d^3*e^7*z^4 - 48*a^2*b^5*c^7*d^5*e^5*z^4 + 48*a^2*b^2*c^10*d^8*e
^2*z^4 + 36*a^2*b^7*c^5*d^3*e^7*z^4 + 6*a^2*b^8*c^4*d^2*e^8*z^4 - 4*b^6*c^8
*d^9*f*z^4 + 256*a^11*c^3*d*f^9*z^4 + 256*a^3*c^11*d^9*f*z^4 - 4*a^8*b^6*d*
f^9*z^4 - 384*a^9*c^5*e^6*f^4*z^4 - 256*a^10*c^4*e^4*f^6*z^4 - 256*a^8*c^6*
e^8*f^2*z^4 - 64*a^11*c^3*e^2*f^8*z^4 - 24*b^10*c^4*d^7*f^3*z^4 - 16*b^12*c
^2*d^6*f^4*z^4 - 16*b^8*c^6*d^8*f^2*z^4 + 17920*a^7*c^7*d^5*f^5*z^4 - 14336
*a^8*c^6*d^4*f^6*z^4 - 14336*a^6*c^8*d^6*f^4*z^4 + 7168*a^9*c^5*d^3*f^7*z^4
+ 7168*a^5*c^9*d^7*f^3*z^4 - 2048*a^10*c^4*d^2*f^8*z^4 - 2048*a^4*c^10*d^8
*f^2*z^4 + 6*b^8*c^6*d^6*e^4*z^4 + 6*a^6*b^8*e^4*f^6*z^4 - 4*b^9*c^5*d^5*e^
5*z^4 - 4*b^7*c^7*d^7*e^3*z^4 - 4*a^7*b^7*e^3*f^7*z^4 - 4*a^5*b^9*e^5*f^5*z
^4 - 384*a^5*c^9*d^4*e^6*z^4 - 256*a^6*c^8*d^2*e^8*z^4 - 256*a^4*c^10*d^6*e

^4*z^4 - 64*a^3*c^11*d^8*e^2*z^4 - 24*a^4*b^10*d^3*f^7*z^4 - 16*a^6*b^8*d^2
*f^8*z^4 - 16*a^2*b^12*d^4*f^6*z^4 + 48*a^6*b^2*c^6*e^10*z^4 - 12*a^5*b^4*c
^5*e^10*z^4 - 4*b^14*d^5*f^5*z^4 - 64*a^7*c^7*e^10*z^4 + b^14*d^4*e^2*f^4*z
^4 + b^10*c^4*d^4*e^6*z^4 + b^6*c^8*d^8*e^2*z^4 + a^8*b^6*e^2*f^8*z^4 + a^4
*b^10*e^6*f^4*z^4 + a^4*b^6*c^4*e^10*z^4 - 4820*A*B*a^4*b*c^5*d^2*e^2*f^4*z
^2 + 2976*A*B*a^3*b*c^6*d^3*e^2*f^3*z^2 - 2328*A*B*a^3*b*c^6*d^2*e^4*f^2*z^
2 + 1848*A*B*a^2*b^4*c^4*d^3*e*f^4*z^2 - 1768*A*B*a^3*b^4*c^3*d^2*e*f^5*z^2
 + 1528*A*B*a^4*b^2*c^4*d^2*e*f^5*z^2 - 1136*A*B*a^3*b^2*c^5*d^3*e*f^4*z^2
 - 974*A*B*a^4*b^3*c^3*d*e^2*f^5*z^2 + 692*A*B*a^2*b*c^7*d^4*e^2*f^2*z^2 + 5
88*A*B*a*b^6*c^3*d^2*e^3*f^3*z^2 - 580*A*B*a^3*b^3*c^4*d*e^4*f^3*z^2 + 488*
A*B*a^3*b^4*c^3*d*e^3*f^4*z^2 - 444*A*B*a^2*b^2*c^6*d^2*e^5*f*z^2 - 412*A*B
*a*b^5*c^4*d^2*e^4*f^2*z^2 + 366*A*B*a^2*b^6*c^2*d^2*e*f^5*z^2 - 352*A*B*a^
2*b^2*c^6*d^4*e*f^3*z^2 + 326*A*B*a^2*b^4*c^4*d*e^5*f^2*z^2 + 324*A*B*a*b^5
*c^4*d^3*e^2*f^3*z^2 - 302*A*B*a*b^3*c^6*d^4*e^2*f^2*z^2 - 296*A*B*a*b^7*c^
2*d^2*e^2*f^4*z^2 + 122*A*B*a^4*b^2*c^4*d*e^3*f^4*z^2 - 122*A*B*a^2*b^6*c^2
*d*e^3*f^4*z^2 - 84*A*B*a^3*b^2*c^5*d*e^5*f^2*z^2 + 72*A*B*a*b^4*c^5*d^3*e^
3*f^2*z^2 - 64*A*B*a^2*b^5*c^3*d*e^4*f^3*z^2 + 60*A*B*a^3*b^5*c^2*d*e^2*f^5
*z^2 + 1312*A*B*a^5*b*c^4*d*e^2*f^5*z^2 + 1040*A*B*a^4*b*c^5*d*e^4*f^3*z^2
 - 500*A*B*a*b^6*c^3*d^3*e*f^4*z^2 - 376*A*B*a*b^2*c^7*d^5*e*f^2*z^2 + 276*A
*B*a^4*b^4*c^2*d*e*f^6*z^2 - 262*A*B*a^2*b^3*c^5*d*e^6*f*z^2 + 238*A*B*a*b^
2*c^7*d^4*e^3*f*z^2 + 232*A*B*a^5*b^2*c^3*d*e*f^6*z^2 - 176*A*B*a^2*b*c^7*d
^3*e^4*f*z^2 - 120*A*B*a*b^6*c^3*d*e^5*f^2*z^2 - 108*A*B*a*b^4*c^5*d^4*e*f^
3*z^2 + 68*A*B*a*b^7*c^2*d*e^4*f^3*z^2 + 68*A*B*a*b^4*c^5*d^2*e^5*f*z^2 + 4
6*A*B*a^2*b^7*c*d*e^2*f^5*z^2 - 36*A*B*a*b^3*c^6*d^3*e^4*f*z^2 - 1932*A*B*a
^2*b^3*c^5*d^3*e^2*f^3*z^2 - 1818*A*B*a^2*b^4*c^4*d^2*e^3*f^3*z^2 + 1620*A*
B*a^3*b^3*c^4*d^2*e^2*f^4*z^2 + 1560*A*B*a^2*b^3*c^5*d^2*e^4*f^2*z^2 + 1244
*A*B*a^3*b^2*c^5*d^2*e^3*f^3*z^2 + 820*A*B*a^2*b^2*c^6*d^3*e^3*f^2*z^2 + 48
0*A*B*a^2*b^5*c^3*d^2*e^2*f^4*z^2 + 352*A*B*a^3*b*c^6*d*e^6*f*z^2 - 108*A*B
*a^3*b^6*c*d*e*f^6*z^2 + 82*A*B*a*b^5*c^4*d*e^6*f*z^2 - 64*A*B*a*b*c^8*d^5*
e^2*f*z^2 + 16*A*B*a*b^8*c*d^2*e*f^5*z^2 - 4*A*B*a*b^8*c*d*e^3*f^4*z^2 + 16
*B^2*a*b*c^8*d^6*e*f*z^2 + 56*A*B*b^2*c^8*d^6*e*f*z^2 - 8*A*B*b^9*c*d*e^4*f
^3*z^2 - 8*A*B*b^7*c^3*d*e^6*f*z^2 - 800*A*B*a^6*c^4*d*e*f^6*z^2 + 10*A*B*a
^2*b^8*d*e*f^6*z^2 - 6*A*B*a*b^9*d*e^2*f^5*z^2 - 12*A*B*a^5*b^4*c*e*f^7*z^2
 + 912*A*B*a^6*b*c^3*d*f^7*z^2 + 192*A*B*a^4*b^5*c*d*f^7*z^2 + 192*A*B*a*b*
c^8*d^6*f^2*z^2 - 20*A*B*a*b^4*c^5*d*e^7*z^2 + 4*A*B*a*b*c^8*d^4*e^4*z^2 +
2144*B^2*a^4*b*c^5*d^3*e*f^4*z^2 - 1120*B^2*a^3*b*c^6*d^4*e*f^3*z^2 - 688*B
^2*a^5*b*c^4*d^2*e*f^5*z^2 - 256*B^2*a^3*b*c^6*d^2*e^5*f*z^2 + 152*B^2*a*b^
3*c^6*d^5*e*f^2*z^2 + 120*B^2*a^5*b^3*c^2*d*e*f^6*z^2 - 116*B^2*a^5*b*c^4*d
*e^3*f^4*z^2 + 110*B^2*a*b^7*c^2*d^3*e*f^4*z^2 - 80*B^2*a^2*b*c^7*d^5*e*f^2
*z^2 - 72*B^2*a*b^5*c^4*d^4*e*f^3*z^2 - 48*B^2*a^4*b*c^5*d*e^5*f^2*z^2 - 46
*B^2*a*b^3*c^6*d^4*e^3*f*z^2 - 44*B^2*a*b^4*c^5*d^3*e^4*f*z^2 - 34*B^2*a*b^
5*c^4*d^2*e^5*f*z^2 + 20*B^2*a^2*b*c^7*d^4*e^3*f*z^2 - 10*B^2*a^3*b^6*c*d*e
^2*f^5*z^2 - 10*B^2*a^2*b^7*c*d^2*e*f^5*z^2 - 10*B^2*a*b^2*c^7*d^5*e^2*f*z^
2 - 7*B^2*a^2*b^4*c^4*d*e^6*f*z^2 - 6*B^2*a^3*b^2*c^5*d*e^6*f*z^2 + 4*B^2*a
*b^8*c*d^2*e^2*f^4*z^2 - 2*B^2*a^2*b^7*c*d*e^3*f^4*z^2 + 3196*A^2*a^4*b*c^5

*d*e^3*f^4*z^2 - 3184*A^2*a^4*b*c^5*d^2*e*f^5*z^2 + 1568*A^2*a^3*b*c^6*d^3*
e*f^4*z^2 + 1504*A^2*a^3*b*c^6*d*e^5*f^2*z^2 - 656*A^2*a^4*b^3*c^3*d*e*f^6*
z^2 - 400*A^2*a*b^6*c^3*d*e^4*f^3*z^2 + 314*A^2*a*b^5*c^4*d*e^5*f^2*z^2 - 2
64*A^2*a^3*b^5*c^2*d*e*f^6*z^2 + 240*A^2*a^2*b^2*c^6*d*e^6*f*z^2 - 224*A^2*
a^2*b*c^7*d^4*e*f^3*z^2 + 216*A^2*a*b^5*c^4*d^3*e*f^4*z^2 - 192*A^2*a^2*b*c
^7*d^2*e^5*f*z^2 + 178*A^2*a*b^7*c^2*d*e^3*f^4*z^2 - 154*A^2*a*b^7*c^2*d^2*
e*f^5*z^2 + 128*A^2*a*b^3*c^6*d^4*e*f^3*z^2 + 106*A^2*a*b^3*c^6*d^2*e^5*f*z
^2 - 12*A^2*a*b^2*c^7*d^3*e^4*f*z^2 - 58*A*B*b^8*c^2*d^2*e^3*f^3*z^2 + 40*A
*B*b^7*c^3*d^2*e^4*f^2*z^2 - 28*A*B*b^7*c^3*d^3*e^2*f^3*z^2 - 24*A*B*b^5*c^
5*d^4*e^2*f^2*z^2 - 20*A*B*b^6*c^4*d^3*e^3*f^2*z^2 + 2768*A*B*a^4*c^6*d^2*e
^3*f^3*z^2 - 1712*A*B*a^3*c^7*d^3*e^3*f^2*z^2 - 156*A*B*a^4*b^2*c^4*e^5*f^3
*z^2 + 146*A*B*a^4*b^3*c^3*e^4*f^4*z^2 - 106*A*B*a^5*b^2*c^3*e^3*f^5*z^2 +
90*A*B*a^5*b^3*c^2*e^2*f^6*z^2 + 38*A*B*a^3*b^3*c^4*e^6*f^2*z^2 - 36*A*B*a^
3*b^5*c^2*e^4*f^4*z^2 + 16*A*B*a^3*b^4*c^3*e^5*f^3*z^2 - 9*A*B*a^4*b^4*c^2*
e^3*f^5*z^2 - 8*A*B*a^2*b^5*c^3*e^6*f^2*z^2 + 2*A*B*a^2*b^6*c^2*e^5*f^3*z^2
 + 920*A*B*a^4*b^3*c^3*d^2*f^6*z^2 - 480*A*B*a^2*b^5*c^3*d^3*f^5*z^2 - 336*
A*B*a^2*b^3*c^5*d^4*f^4*z^2 - 272*A*B*a^3*b^3*c^4*d^3*f^5*z^2 + 240*A*B*a^3
*b^5*c^2*d^2*f^6*z^2 - 32*A*B*a*c^9*d^6*e*f*z^2 - 792*B^2*a^2*b^3*c^5*d^3*e
^3*f^2*z^2 + 714*B^2*a^2*b^4*c^4*d^3*e^2*f^3*z^2 - 572*B^2*a^3*b^2*c^5*d^3*
e^2*f^3*z^2 - 475*B^2*a^2*b^2*c^6*d^4*e^2*f^2*z^2 + 265*B^2*a^4*b^2*c^4*d^2
*e^2*f^4*z^2 + 260*B^2*a^3*b^3*c^4*d^2*e^3*f^3*z^2 - 212*B^2*a^3*b^4*c^3*d^
2*e^2*f^4*z^2 + 180*B^2*a^3*b^2*c^5*d^2*e^4*f^2*z^2 - 158*B^2*a^2*b^4*c^4*d
^2*e^4*f^2*z^2 + 47*B^2*a^2*b^6*c^2*d^2*e^2*f^4*z^2 + 16*B^2*a^2*b^5*c^3*d^
2*e^3*f^3*z^2 + 2752*A^2*a^3*b^2*c^5*d^2*e^2*f^4*z^2 - 2148*A^2*a^2*b^4*c^4
*d^2*e^2*f^4*z^2 + 2064*A^2*a^2*b^3*c^5*d^2*e^3*f^3*z^2 - 424*A^2*a^2*b^2*c
^6*d^3*e^2*f^3*z^2 - 198*A^2*a^2*b^2*c^6*d^2*e^4*f^2*z^2 - 272*B^2*a^6*b*c^
3*d*e*f^6*z^2 - 24*B^2*a^4*b^5*c*d*e*f^6*z^2 + 1808*A^2*a^5*b*c^4*d*e*f^6*z
^2 - 244*A^2*a*b*c^8*d^4*e^3*f*z^2 + 208*A^2*a*b*c^8*d^5*e*f^2*z^2 + 134*A^
2*a^2*b^7*c*d*e*f^6*z^2 - 76*A^2*a*b^4*c^5*d*e^6*f*z^2 + 4*A^2*a*b^8*c*d*e^
2*f^5*z^2 + 148*A*B*b^4*c^6*d^5*e*f^2*z^2 + 65*A*B*b^6*c^4*d^4*e*f^3*z^2 +
46*A*B*b^8*c^2*d^3*e*f^4*z^2 - 38*A*B*b^3*c^7*d^5*e^2*f*z^2 + 34*A*B*b^9*c*
d^2*e^2*f^4*z^2 - 29*A*B*b^4*c^6*d^4*e^3*f*z^2 + 20*A*B*b^5*c^5*d^3*e^4*f*z
^2 + 12*A*B*b^8*c^2*d*e^5*f^2*z^2 - 7*A*B*b^6*c^4*d^2*e^5*f*z^2 - 2880*A*B*
a^4*c^6*d^3*e*f^4*z^2 + 2784*A*B*a^5*c^5*d^2*e*f^5*z^2 - 1112*A*B*a^5*c^5*d
*e^3*f^4*z^2 + 896*A*B*a^3*c^7*d^4*e*f^3*z^2 + 848*A*B*a^3*c^7*d^2*e^5*f*z^
2 - 560*A*B*a^4*c^6*d*e^5*f^2*z^2 + 96*A*B*a^2*c^8*d^5*e*f^2*z^2 - 88*A*B*a
^2*c^8*d^4*e^3*f*z^2 - 100*A*B*a^6*b*c^3*e^2*f^6*z^2 - 76*A*B*a^5*b*c^4*e^4
*f^4*z^2 + 48*A*B*a^6*b^2*c^2*e*f^7*z^2 - 42*A*B*a^3*b^2*c^5*e^7*f*z^2 + 36
*A*B*a^4*b*c^5*e^6*f^2*z^2 - 24*A*B*a^4*b^5*c*e^2*f^6*z^2 + 10*A*B*a^3*b^6*
c*e^3*f^5*z^2 + 7*A*B*a^2*b^4*c^4*e^7*f*z^2 + 2*A*B*a^2*b^7*c*e^4*f^4*z^2 -
 2496*A*B*a^5*b*c^4*d^2*f^6*z^2 + 1872*A*B*a^4*b*c^5*d^3*f^5*z^2 - 744*A*B*
a^5*b^3*c^2*d*f^7*z^2 - 720*A*B*a^2*b*c^7*d^5*f^3*z^2 + 504*A*B*a*b^3*c^6*d
^5*f^3*z^2 + 256*A*B*a^3*b*c^6*d^4*f^4*z^2 + 168*A*B*a*b^7*c^2*d^3*f^5*z^2
- 144*A*B*a^2*b^7*c*d^2*f^6*z^2 + 144*A*B*a*b^5*c^4*d^4*f^4*z^2 + 66*A*B*a^
2*b^2*c^6*d*e^7*z^2 - 36*A*B*a*b^2*c^7*d^3*e^5*z^2 + 20*A*B*a*b^3*c^6*d^2*e

^6*z^2 + 12*A*B*a^2*b*c^7*d^2*e^6*z^2 + 1208*B^2*a^3*b*c^6*d^3*e^3*f^2*z^2
- 848*B^2*a^3*b^3*c^4*d^3*e*f^4*z^2 + 672*B^2*a^2*b^3*c^5*d^4*e*f^3*z^2 - 6
32*B^2*a^4*b*c^5*d^2*e^3*f^3*z^2 + 432*B^2*a^4*b^3*c^3*d^2*e*f^5*z^2 + 276*
B^2*a^2*b^2*c^6*d^3*e^4*f*z^2 - 196*B^2*a*b^6*c^3*d^3*e^2*f^3*z^2 - 168*B^2
*a^2*b^5*c^3*d^3*e*f^4*z^2 + 154*B^2*a^2*b^3*c^5*d^2*e^5*f*z^2 + 148*B^2*a*
b^5*c^4*d^3*e^3*f^2*z^2 + 96*B^2*a*b^4*c^5*d^4*e^2*f^2*z^2 - 72*B^2*a^3*b^5
*c^2*d^2*e*f^5*z^2 + 70*B^2*a^5*b^2*c^3*d*e^2*f^5*z^2 - 60*B^2*a^4*b^3*c^3*
d*e^3*f^4*z^2 + 52*B^2*a*b^6*c^3*d^2*e^4*f^2*z^2 + 36*B^2*a^4*b^2*c^4*d*e^4
*f^3*z^2 - 32*B^2*a*b^7*c^2*d^2*e^3*f^3*z^2 + 24*B^2*a^3*b^5*c^2*d*e^3*f^4*
z^2 + 15*B^2*a^4*b^4*c^2*d*e^2*f^5*z^2 - 8*B^2*a^3*b^4*c^3*d*e^4*f^3*z^2 +
8*B^2*a^2*b^5*c^3*d*e^5*f^2*z^2 - 2*B^2*a^3*b^3*c^4*d*e^5*f^2*z^2 - 2*B^2*a
^2*b^6*c^2*d*e^4*f^3*z^2 - 3176*A^2*a^3*b*c^6*d^2*e^3*f^3*z^2 - 2252*A^2*a^
4*b^2*c^4*d*e^2*f^5*z^2 + 1952*A^2*a^3*b^4*c^3*d*e^2*f^5*z^2 - 1496*A^2*a^3
*b^3*c^4*d*e^3*f^4*z^2 + 1378*A^2*a^2*b^4*c^4*d*e^4*f^3*z^2 + 1184*A^2*a^3*
b^3*c^4*d^2*e*f^5*z^2 - 1166*A^2*a^2*b^3*c^5*d*e^5*f^2*z^2 - 1164*A^2*a^3*b
^2*c^5*d*e^4*f^3*z^2 - 1152*A^2*a^2*b^3*c^5*d^3*e*f^4*z^2 + 578*A^2*a*b^6*c
^3*d^2*e^2*f^4*z^2 - 548*A^2*a*b^5*c^4*d^2*e^3*f^3*z^2 + 440*A^2*a*b^2*c^7*
d^4*e^2*f^2*z^2 - 412*A^2*a^2*b^6*c^2*d*e^2*f^5*z^2 - 360*A^2*a*b^3*c^6*d^3
*e^3*f^2*z^2 + 312*A^2*a*b^4*c^5*d^3*e^2*f^3*z^2 + 248*A^2*a^2*b*c^7*d^3*e^
3*f^2*z^2 - 224*A^2*a^2*b^5*c^3*d*e^3*f^4*z^2 + 216*A^2*a^2*b^5*c^3*d^2*e*f
^5*z^2 + 52*A^2*a*b^4*c^5*d^2*e^4*f^2*z^2 - 16*B^2*b^3*c^7*d^6*e*f*z^2 - 14
*B^2*b^9*c*d^3*e*f^4*z^2 + 32*B^2*a^4*c^6*d*e^6*f*z^2 - 20*A^2*b^9*c*d*e^3*
f^4*z^2 + 18*A^2*b^9*c*d^2*e*f^5*z^2 + 8*A^2*b^6*c^4*d*e^6*f*z^2 - 360*A^2*
a^3*c^7*d*e^6*f*z^2 + 136*A^2*a*c^9*d^5*e^2*f*z^2 + 2*B^2*a^3*b^7*d*e*f^6*z
^2 + 2*B^2*a*b^9*d^2*e*f^5*z^2 + 12*B^2*a^4*b*c^5*e^7*f*z^2 - 204*A^2*a^3*b
*c^6*e^7*f*z^2 - 128*A^2*a^6*b*c^3*e*f^7*z^2 - 48*A^2*a*b^5*c^4*e^7*f*z^2 -
36*B^2*a^5*b^4*c*d*f^7*z^2 - 24*A^2*a^4*b^5*c*e*f^7*z^2 - 16*B^2*a*b^8*c*d
^3*f^5*z^2 - 164*A^2*a^3*b^6*c*d*f^7*z^2 - 16*A^2*a*b^8*c*d^2*f^6*z^2 + 4*B
^2*a^3*b*c^6*d*e^7*z^2 - 4*B^2*a*b*c^8*d^5*e^3*z^2 + 48*A^2*a*b*c^8*d^3*e^5
*z^2 + 36*A^2*a^2*b*c^7*d*e^7*z^2 - 6*A^2*a*b^3*c^6*d*e^7*z^2 + 136*A*B*a^6
*c^4*e^3*f^5*z^2 - 96*A*B*b^5*c^5*d^5*f^3*z^2 + 80*A*B*a^5*c^5*e^5*f^3*z^2
- 72*A*B*b^3*c^7*d^6*f^2*z^2 - 24*A*B*b^7*c^3*d^4*f^4*z^2 + 14*A*B*b^3*c^7*
d^4*e^4*z^2 - 14*A*B*b^2*c^8*d^5*e^3*z^2 - 2*A*B*b^5*c^5*d^2*e^6*z^2 - 2*A*
B*b^4*c^6*d^3*e^5*z^2 + 2*A*B*a^3*b^7*e^2*f^6*z^2 - A*B*a^2*b^8*e^3*f^5*z^2
+ 16*A*B*a^2*c^8*d^3*e^5*z^2 - 2*A*B*a^2*b^3*c^5*e^8*z^2 + 22*B^2*b^8*c^2*
d^3*e^2*f^3*z^2 - 12*B^2*b^7*c^3*d^3*e^3*f^2*z^2 + 12*B^2*b^6*c^4*d^4*e^2*f
^2*z^2 - 6*B^2*b^8*c^2*d^2*e^4*f^2*z^2 - 864*B^2*a^4*c^6*d^3*e^2*f^3*z^2 +
496*B^2*a^3*c^7*d^4*e^2*f^2*z^2 + 224*B^2*a^5*c^5*d^2*e^2*f^4*z^2 + 136*B^2
*a^4*c^6*d^2*e^4*f^2*z^2 - 53*A^2*b^8*c^2*d^2*e^2*f^4*z^2 + 52*A^2*b^7*c^3*
d^2*e^3*f^3*z^2 + 52*A^2*b^5*c^5*d^3*e^3*f^2*z^2 - 36*A^2*b^6*c^4*d^3*e^2*f
^3*z^2 - 12*A^2*b^4*c^6*d^4*e^2*f^2*z^2 - 9*A^2*b^6*c^4*d^2*e^4*f^2*z^2 + 8
36*A^2*a^4*c^6*d^2*e^2*f^4*z^2 - 668*A^2*a^2*c^8*d^4*e^2*f^2*z^2 + 656*A^2*
a^3*c^7*d^2*e^4*f^2*z^2 + 368*A^2*a^3*c^7*d^3*e^2*f^3*z^2 - 45*B^2*a^6*b^2*
c^2*e^2*f^6*z^2 - 18*B^2*a^5*b^2*c^3*e^4*f^4*z^2 - 9*B^2*a^4*b^2*c^4*e^6*f^
2*z^2 - 6*B^2*a^5*b^3*c^2*e^3*f^5*z^2 + 3*B^2*a^4*b^4*c^2*e^4*f^4*z^2 - 2*B

^2*a^4*b^3*c^3*e^5*f^3*z^2 - 580*B^2*a^4*b^2*c^4*d^3*f^5*z^2 + 536*B^2*a^3*
b^4*c^3*d^3*f^5*z^2 + 471*A^2*a^4*b^2*c^4*e^4*f^4*z^2 - 436*A^2*a^3*b^4*c^3
*e^4*f^4*z^2 - 348*B^2*a^4*b^4*c^2*d^2*f^6*z^2 + 316*B^2*a^2*b^2*c^6*d^5*f^
3*z^2 + 310*A^2*a^3*b^3*c^4*e^5*f^3*z^2 + 232*A^2*a^5*b^2*c^3*e^2*f^6*z^2 -
229*A^2*a^2*b^4*c^4*e^6*f^2*z^2 - 216*A^2*a^4*b^4*c^2*e^2*f^6*z^2 + 204*A^
2*a^4*b^3*c^3*e^3*f^5*z^2 + 200*B^2*a^5*b^2*c^3*d^2*f^6*z^2 + 150*A^2*a^3*b
^2*c^5*e^6*f^2*z^2 - 120*B^2*a^2*b^4*c^4*d^4*f^4*z^2 + 91*A^2*a^2*b^6*c^2*e
^4*f^4*z^2 + 72*A^2*a^3*b^5*c^2*e^3*f^5*z^2 - 66*B^2*a^2*b^6*c^2*d^3*f^5*z^
2 + 44*A^2*a^2*b^5*c^3*e^5*f^3*z^2 - 16*B^2*a^3*b^2*c^5*d^4*f^4*z^2 + 1952*
A^2*a^4*b^2*c^4*d^2*f^6*z^2 - 1792*A^2*a^3*b^2*c^5*d^3*f^5*z^2 - 1272*A^2*a
^3*b^4*c^3*d^2*f^6*z^2 + 976*A^2*a^2*b^2*c^6*d^4*f^4*z^2 + 960*A^2*a^2*b^4*
c^4*d^3*f^5*z^2 + 282*A^2*a^2*b^6*c^2*d^2*f^6*z^2 - 45*B^2*a^2*b^2*c^6*d^2*
e^6*z^2 - 48*A^2*b*c^9*d^6*e*f*z^2 - 14*A^2*a*b^9*d*e*f^6*z^2 - 7*A*B*b^10*
d^2*e*f^5*z^2 + 2*A*B*b^10*d*e^3*f^4*z^2 - 64*A*B*a^7*c^3*e*f^7*z^2 - 16*A*
B*b^9*c*d^3*f^5*z^2 + 8*A*B*a^4*c^6*e^7*f*z^2 + 4*A*B*b*c^9*d^6*e^2*z^2 + 2
*A*B*b^6*c^4*d*e^7*z^2 - 120*A*B*a^3*c^7*d*e^7*z^2 - 16*A*B*a^3*b^7*d*f^7*z
^2 + 16*A*B*a*b^9*d^2*f^6*z^2 + 8*A*B*a*c^9*d^5*e^3*z^2 + 12*A*B*a^3*b*c^6*
e^8*z^2 - 48*B^2*b^5*c^5*d^5*e*f^2*z^2 + 15*B^2*b^4*c^6*d^5*e^2*f*z^2 - 14*
B^2*b^7*c^3*d^4*e*f^3*z^2 + 4*B^2*b^9*c*d^2*e^3*f^3*z^2 + 4*B^2*b^7*c^3*d^2
*e^5*f*z^2 + 4*B^2*b^5*c^5*d^4*e^3*f*z^2 - B^2*b^6*c^4*d^3*e^4*f*z^2 - 336*
B^2*a^3*c^7*d^3*e^4*f*z^2 + 112*B^2*a^5*c^5*d*e^4*f^3*z^2 - 112*A^2*b^3*c^7
*d^5*e*f^2*z^2 + 80*B^2*a^6*c^4*d*e^2*f^5*z^2 - 48*A^2*b^5*c^5*d^4*e*f^3*z^
2 + 36*A^2*b^8*c^2*d*e^4*f^3*z^2 + 36*A^2*b^3*c^7*d^4*e^3*f*z^2 - 28*A^2*b^
7*c^3*d*e^5*f^2*z^2 + 20*A^2*b^2*c^8*d^5*e^2*f*z^2 + 16*B^2*a^2*c^8*d^5*e^2
*f*z^2 - 14*A^2*b^7*c^3*d^3*e*f^4*z^2 - 14*A^2*b^4*c^6*d^3*e^4*f*z^2 - 10*A
^2*b^5*c^5*d^2*e^5*f*z^2 - 1008*A^2*a^4*c^6*d*e^4*f^3*z^2 - 760*A^2*a^5*c^5
*d*e^2*f^5*z^2 + 272*A^2*a^2*c^8*d^3*e^4*f*z^2 + 48*B^2*a^5*b*c^4*e^5*f^3*z
^2 + 36*B^2*a^6*b*c^3*e^3*f^5*z^2 + 12*B^2*a^5*b^4*c*e^2*f^6*z^2 - 624*A^2*
a^4*b*c^5*e^5*f^3*z^2 - 548*A^2*a^5*b*c^4*e^3*f^5*z^2 + 182*A^2*a^2*b^3*c^5
*e^7*f*z^2 - 180*B^2*a*b^4*c^5*d^5*f^3*z^2 + 132*B^2*a^6*b^2*c^2*d*f^7*z^2
+ 108*B^2*a^3*b^6*c*d^2*f^6*z^2 + 96*A^2*a^5*b^3*c^2*e*f^7*z^2 + 68*A^2*a*b
^6*c^3*e^6*f^2*z^2 + 58*A^2*a^3*b^6*c*e^2*f^6*z^2 - 56*B^2*a*b^2*c^7*d^6*f^
2*z^2 - 38*A^2*a^2*b^7*c*e^3*f^5*z^2 - 36*A^2*a*b^7*c^2*e^5*f^3*z^2 + 20*B^
2*a*b^6*c^3*d^4*f^4*z^2 - 736*A^2*a^5*b^2*c^3*d*f^7*z^2 + 624*A^2*a^4*b^4*c
^2*d*f^7*z^2 - 416*A^2*a*b^2*c^7*d^5*f^3*z^2 - 276*A^2*a*b^4*c^5*d^4*f^4*z^
2 - 196*A^2*a*b^6*c^3*d^3*f^5*z^2 + 8*B^2*a*b^4*c^5*d^2*e^6*z^2 + 6*B^2*a*b
^2*c^7*d^4*e^4*z^2 + 2*B^2*a^2*b^3*c^5*d*e^7*z^2 + 2*B^2*a*b^3*c^6*d^3*e^5*
z^2 - 18*A^2*a*b^2*c^7*d^2*e^6*z^2 - 16*A*B*b*c^9*d^7*f*z^2 - B^2*b^10*d^2*
e^2*f^4*z^2 + 48*B^2*a^7*c^3*e^2*f^6*z^2 - 36*B^2*a^6*c^4*e^4*f^4*z^2 + 31*
B^2*b^6*c^4*d^5*f^3*z^2 - 24*B^2*a^5*c^5*e^6*f^2*z^2 + 20*B^2*b^4*c^6*d^6*f
^2*z^2 - 6*A^2*b^8*c^2*e^6*f^2*z^2 + 2*B^2*b^8*c^2*d^4*f^4*z^2 - 768*B^2*a^
5*c^5*d^3*f^5*z^2 + 512*B^2*a^6*c^4*d^2*f^6*z^2 + 512*B^2*a^4*c^6*d^4*f^4*z
^2 + 232*A^2*a^5*c^5*e^4*f^4*z^2 + 188*A^2*a^4*c^6*e^6*f^2*z^2 - 128*B^2*a^
3*c^7*d^5*f^3*z^2 + 92*A^2*a^6*c^4*e^2*f^6*z^2 + 80*A^2*b^4*c^6*d^5*f^3*z^2
+ 64*A^2*b^2*c^8*d^6*f^2*z^2 + 31*A^2*b^6*c^4*d^4*f^4*z^2 + 14*A^2*b^8*c^2

*d^3*f^5*z^2 - 5*B^2*b^4*c^6*d^4*e^4*z^2 + 4*B^2*b^3*c^7*d^5*e^3*z^2 + 2*B^
2*b^5*c^5*d^3*e^5*z^2 - B^2*b^6*c^4*d^2*e^6*z^2 - B^2*b^2*c^8*d^6*e^2*z^2 -
B^2*a^4*b^6*e^2*f^6*z^2 - 1152*A^2*a^3*c^7*d^4*f^4*z^2 + 1008*A^2*a^4*c^6*
d^3*f^5*z^2 + 624*A^2*a^2*c^8*d^5*f^3*z^2 - 288*A^2*a^5*c^5*d^2*f^6*z^2 + 5
6*B^2*a^3*c^7*d^2*e^6*z^2 - 10*B^2*a^2*b^8*d^2*f^6*z^2 - 9*A^2*b^2*c^8*d^4*
e^4*z^2 - 5*A^2*a^2*b^8*e^2*f^6*z^2 - 4*B^2*a^2*c^8*d^4*e^4*z^2 + 3*A^2*b^4
*c^6*d^2*e^6*z^2 - 2*A^2*b^3*c^7*d^3*e^5*z^2 - 36*A^2*a^2*c^8*d^2*e^6*z^2 -
48*A^2*a^6*b^2*c^2*f^8*z^2 - 45*A^2*a^2*b^2*c^6*e^8*z^2 + 4*A^2*b^10*d*e^2
*f^5*z^2 + 4*B^2*b^2*c^8*d^7*f*z^2 + 4*A^2*b^9*c*e^5*f^3*z^2 + 4*A^2*b^7*c^
3*e^7*f*z^2 - 128*B^2*a^7*c^3*d*f^7*z^2 - 160*A^2*a*c^9*d^6*f^2*z^2 - 112*A
^2*a^6*c^4*d*f^7*z^2 + 12*A^2*b*c^9*d^5*e^3*z^2 + 4*A^2*a*b^9*e^3*f^5*z^2 +
3*B^2*a^4*b^6*d*f^7*z^2 + 2*A^2*a^3*b^7*e*f^7*z^2 - 24*A^2*a*c^9*d^4*e^4*z
^2 + 14*A^2*a^2*b^8*d*f^7*z^2 + 12*A^2*a^5*b^4*c*f^8*z^2 + 12*A^2*a*b^4*c^5
*e^8*z^2 + A*B*a^4*b^6*e*f^7*z^2 + B^2*a^2*b^8*d*e^2*f^5*z^2 + 16*A^2*c^10*
d^7*f*z^2 + 3*B^2*b^10*d^3*f^5*z^2 - A^2*b^10*e^4*f^4*z^2 - 4*A^2*c^10*d^6*
e^2*z^2 - A^2*b^10*d^2*f^6*z^2 + 64*A^2*a^7*c^3*f^8*z^2 - 4*B^2*a^4*c^6*e^8
*z^2 - A^2*b^6*c^4*e^8*z^2 + 48*A^2*a^3*c^7*e^8*z^2 - A^2*a^4*b^6*f^8*z^2 +
720*A^2*B*a*b^2*c^5*d^2*e^2*f^3*z - 600*A^2*B*a^2*b^2*c^4*d*e^2*f^4*z + 57
6*A*B^2*a^2*b^2*c^4*d^2*e*f^4*z + 348*A*B^2*a*b^2*c^5*d^2*e^3*f^2*z - 336*A
*B^2*a^2*b*c^5*d^2*e^2*f^3*z - 260*A*B^2*a*b^3*c^4*d^2*e^2*f^3*z - 240*A*B^
2*a^2*b^2*c^4*d*e^3*f^3*z + 196*A*B^2*a^2*b^3*c^3*d*e^2*f^4*z + 172*A^2*B*a
*b*c^6*d*e^5*f*z + 20*A*B^2*a*b^6*c*d*e*f^5*z - 912*A^2*B*a^2*b*c^5*d^2*e*f
^4*z - 644*A^2*B*a*b*c^6*d^2*e^3*f^2*z - 432*A*B^2*a*b^2*c^5*d^3*e*f^3*z +
372*A^2*B*a^2*b*c^5*d*e^3*f^3*z - 330*A^2*B*a*b^2*c^5*d*e^4*f^2*z + 312*A*B
^2*a*b*c^6*d^3*e^2*f^2*z - 208*A*B^2*a^3*b^2*c^3*d*e*f^5*z + 192*A^2*B*a^2*
b^3*c^3*d*e*f^5*z + 172*A^2*B*a*b^3*c^4*d*e^3*f^3*z + 108*A*B^2*a^2*b*c^5*d
*e^4*f^2*z + 104*A*B^2*a^3*b*c^4*d*e^2*f^4*z - 80*A^2*B*a*b^3*c^4*d^2*e*f^4
*z + 68*A^2*B*a*b^4*c^3*d*e^2*f^4*z - 60*A*B^2*a*b^5*c^2*d*e^2*f^4*z + 58*A
*B^2*a*b^3*c^4*d*e^4*f^2*z - 36*A*B^2*a*b^4*c^3*d^2*e*f^4*z - 24*A*B^2*a^2*
b^4*c^2*d*e*f^5*z + 24*A*B^2*a*b^4*c^3*d*e^3*f^3*z + 592*A^2*B*a*b*c^6*d^3*
e*f^3*z + 240*A^2*B*a^3*b*c^4*d*e*f^5*z - 132*A*B^2*a*b*c^6*d^2*e^4*f*z - 6
0*A*B^2*a*b^2*c^5*d*e^5*f*z - 48*A^2*B*a*b^5*c^2*d*e*f^5*z + 20*B^3*a*b*c^6
*d^3*e^3*f*z + 16*B^3*a^4*b*c^3*d*e*f^5*z - 16*B^3*a*b*c^6*d^4*e*f^2*z + 12
*B^3*a^2*b*c^5*d*e^5*f*z + 320*A^3*a*b*c^6*d*e^4*f^2*z + 40*A^3*a*b^4*c^3*d
*e*f^5*z - 48*A^2*B*b*c^7*d^4*e*f^2*z - 44*A^2*B*b^3*c^5*d*e^5*f*z - 20*A*B
^2*b*c^7*d^4*e^2*f*z + 14*A*B^2*b^4*c^4*d*e^5*f*z + 12*A^2*B*b*c^7*d^3*e^3*
f*z + 4*A*B^2*b^7*c*d*e^2*f^4*z + 160*A*B^2*a^4*c^4*d*e*f^5*z + 152*A^2*B*a
*c^7*d^2*e^4*f*z - 40*A*B^2*a*c^7*d^3*e^3*f*z + 32*A*B^2*a*c^7*d^4*e*f^2*z
- 16*A*B^2*a^2*c^6*d*e^5*f*z + 128*A^2*B*a^4*b*c^3*e*f^6*z + 42*A^2*B*a*b^2
*c^5*e^6*f*z + 24*A^2*B*a^2*b^5*c*e*f^6*z - 12*A*B^2*a^3*b^4*c*e*f^6*z - 12
*A*B^2*a^2*b*c^5*e^6*f*z - 10*A^2*B*a*b^6*c*e^2*f^5*z - 160*A*B^2*a*b*c^6*d
^4*f^3*z + 112*A*B^2*a^4*b*c^3*d*f^6*z - 24*A*B^2*a^2*b^5*c*d*f^6*z - 84*B^
3*a*b^2*c^5*d^3*e^2*f^2*z - 80*B^3*a^2*b^3*c^3*d^2*e*f^4*z - 60*B^3*a^2*b*c
^5*d^2*e^3*f^2*z - 20*B^3*a^3*b^2*c^3*d*e^2*f^4*z - 20*B^3*a*b^3*c^4*d^2*e^
3*f^2*z - 9*B^3*a^2*b^2*c^4*d*e^4*f^2*z - 8*B^3*a*b^4*c^3*d^2*e^2*f^3*z + 6

*B^3*a^2*b^4*c^2*d*e^2*f^4*z - 4*B^3*a^2*b^3*c^3*d*e^3*f^3*z - 216*A^2*B*b^
4*c^4*d^2*e^2*f^3*z + 196*A^2*B*b^3*c^5*d^2*e^3*f^2*z - 108*A*B^2*b^3*c^5*d
^3*e^2*f^2*z - 94*A*B^2*b^4*c^4*d^2*e^3*f^2*z + 88*A^2*B*b^2*c^6*d^3*e^2*f^
2*z + 80*A*B^2*b^5*c^3*d^2*e^2*f^3*z + 360*A^2*B*a^2*c^6*d^2*e^2*f^3*z + 8*
A*B^2*a^2*c^6*d^2*e^3*f^2*z + 153*A^2*B*a^2*b^2*c^4*e^4*f^3*z - 144*A^2*B*a
^2*b^3*c^3*e^3*f^4*z + 80*A^2*B*a^3*b^2*c^3*e^2*f^5*z + 36*A*B^2*a^3*b^2*c^
3*e^3*f^4*z + 12*A^2*B*a^2*b^4*c^2*e^2*f^5*z + 12*A*B^2*a^3*b^3*c^2*e^2*f^5
*z + 9*A*B^2*a^2*b^2*c^4*e^5*f^2*z - 6*A*B^2*a^2*b^4*c^2*e^3*f^4*z + 4*A*B^
2*a^2*b^3*c^3*e^4*f^3*z + 480*A^2*B*a^2*b^2*c^4*d^2*f^5*z - 176*A*B^2*a^2*b
^3*c^3*d^2*f^5*z - 10*A^2*B*a*b^6*c*d*f^6*z + 16*A*B^2*a*b*c^6*d*e^6*z + 80
*B^3*a*b^3*c^4*d^3*e*f^3*z - 48*B^3*a^3*b*c^4*d^2*e*f^4*z + 48*B^3*a^2*b*c^
5*d^3*e*f^3*z + 44*B^3*a^3*b*c^4*d*e^3*f^3*z + 24*B^3*a*b^5*c^2*d^2*e*f^4*z
 + 18*B^3*a*b^2*c^5*d^2*e^4*f*z + 696*A^3*a^2*b*c^5*d*e^2*f^4*z - 504*A^3*a
*b*c^6*d^2*e^2*f^3*z - 192*A^3*a*b^2*c^5*d*e^3*f^3*z - 144*A^3*a^2*b^2*c^4*
d*e*f^5*z + 96*A^3*a*b^2*c^5*d^2*e*f^4*z - 72*A^3*a*b^3*c^4*d*e^2*f^4*z - 2
08*A^2*B*b^3*c^5*d^3*e*f^3*z + 152*A*B^2*b^4*c^4*d^3*e*f^3*z + 80*A^2*B*b^5
*c^3*d^2*e*f^4*z + 75*A^2*B*b^4*c^4*d*e^4*f^2*z - 59*A^2*B*b^2*c^6*d^2*e^4*
f*z - 52*A^2*B*b^5*c^3*d*e^3*f^3*z + 42*A*B^2*b^3*c^5*d^2*e^4*f*z - 21*A*B^
2*b^6*c^2*d^2*e*f^4*z - 16*A*B^2*b^5*c^3*d*e^4*f^2*z + 16*A*B^2*b^2*c^6*d^4
*e*f^2*z + 16*A*B^2*b^2*c^6*d^3*e^3*f*z + 11*A^2*B*b^6*c^2*d*e^2*f^4*z + 4*
A*B^2*b^6*c^2*d*e^3*f^3*z - 256*A^2*B*a*c^7*d^3*e^2*f^2*z - 96*A*B^2*a^3*c^
5*d^2*e*f^4*z - 36*A^2*B*a^2*c^6*d*e^4*f^2*z - 32*A^2*B*a^3*c^5*d*e^2*f^4*z
 - 32*A*B^2*a^2*c^6*d^3*e*f^3*z + 8*A*B^2*a^3*c^5*d*e^3*f^3*z - 96*A^2*B*a^
3*b^3*c^2*e*f^6*z + 68*A^2*B*a^3*b*c^4*e^3*f^4*z - 60*A*B^2*a^4*b*c^3*e^2*f
^5*z - 60*A*B^2*a^3*b*c^4*e^4*f^3*z + 48*A*B^2*a^4*b^2*c^2*e*f^6*z - 38*A^2
*B*a*b^3*c^4*e^5*f^2*z - 36*A^2*B*a^2*b*c^5*e^5*f^2*z + 36*A^2*B*a*b^5*c^2*
e^3*f^4*z - 16*A^2*B*a*b^4*c^3*e^4*f^3*z + 384*A*B^2*a^2*b*c^5*d^3*f^4*z -
352*A*B^2*a^3*b*c^4*d^2*f^5*z - 288*A^2*B*a*b^2*c^5*d^3*f^4*z - 160*A^2*B*a
^3*b^2*c^3*d*f^6*z - 148*A^2*B*a*b^4*c^3*d^2*f^5*z + 112*A*B^2*a*b^3*c^4*d^
3*f^4*z + 72*A^2*B*a^2*b^4*c^2*d*f^6*z + 72*A*B^2*a*b^5*c^2*d^2*f^5*z + 48*
A*B^2*a^3*b^3*c^2*d*f^6*z + 102*B^3*a^2*b^2*c^4*d^2*e^2*f^3*z - 32*B^3*b^5*
c^3*d^3*e*f^3*z - 8*B^3*b^3*c^5*d^3*e^3*f*z - 7*B^3*b^4*c^4*d^2*e^4*f*z + 5
*B^3*b^2*c^6*d^4*e^2*f*z + 80*A^3*b^2*c^6*d^3*e*f^3*z - 74*A^3*b^3*c^5*d*e^
4*f^2*z - 64*A^3*b^4*c^4*d^2*e*f^4*z + 60*A^3*b^4*c^4*d*e^3*f^3*z - 48*B^3*
a^4*c^4*d*e^2*f^4*z - 24*B^3*a^3*c^5*d*e^4*f^2*z + 20*B^3*a^2*c^6*d^2*e^4*f
*z - 16*A^3*b^5*c^3*d*e^2*f^4*z + 8*A^3*b*c^7*d^3*e^2*f^2*z + 480*A^3*a^2*c
^6*d^2*e*f^4*z - 392*A^3*a^2*c^6*d*e^3*f^3*z + 280*A^3*a*c^7*d^2*e^3*f^2*z
- 4*B^3*a^4*b*c^3*e^3*f^4*z - 200*A^3*a^3*b*c^4*e^2*f^5*z - 144*A^3*a^2*b*c
^5*e^4*f^3*z + 48*B^3*a*b^2*c^5*d^4*f^3*z + 42*A^3*a*b^2*c^5*e^5*f^2*z - 36
*B^3*a^4*b^2*c^2*d*f^6*z - 32*A^3*a^3*b^2*c^3*e*f^6*z - 24*A^3*a^2*b^4*c^2*
e*f^6*z - 24*A^3*a*b^5*c^2*e^2*f^5*z + 10*A^3*a*b^3*c^4*e^4*f^3*z - 4*B^3*a
*b^4*c^3*d^3*f^4*z - 4*A^3*a*b^4*c^3*e^3*f^4*z - 480*A^3*a^2*b*c^5*d^2*f^5*
z - 160*A^3*a^2*b^3*c^3*d*f^6*z + 128*A^3*a*b^3*c^4*d^2*f^5*z + 8*A^2*B*b^5
*c^3*e^5*f^2*z - 2*A^2*B*b^6*c^2*e^4*f^3*z + 112*A^2*B*b^4*c^4*d^3*f^4*z -
92*A^2*B*a^4*c^4*e^2*f^5*z - 64*A^2*B*a^3*c^5*e^4*f^3*z - 64*A*B^2*b^5*c^3*

d^3*f^4*z + 24*A*B^2*a^4*c^4*e^3*f^4*z + 24*A*B^2*a^3*c^5*e^5*f^2*z + 16*A^
2*B*b^2*c^6*d^4*f^3*z + 16*A*B^2*b^3*c^5*d^4*f^3*z - A^2*B*b^6*c^2*d^2*f^5*
z + 448*A^2*B*a^3*c^5*d^2*f^5*z - 352*A^2*B*a^2*c^6*d^3*f^4*z - 5*A*B^2*b^2
*c^6*d^2*e^5*z - 48*A^2*B*a^4*b^2*c^2*f^7*z - 2*B^3*b^7*c*d^2*e*f^4*z + 34*
A^3*b^2*c^6*d*e^5*f*z + 16*A^3*b*c^7*d^2*e^4*f*z + 2*A^3*b^6*c^2*d*e*f^5*z
- 416*A^3*a^3*c^5*d*e*f^5*z - 224*A^3*a*c^7*d^3*e*f^3*z + 12*B^3*a^3*b^4*c*
d*f^6*z - 10*B^3*a*b^6*c*d^2*f^5*z + 416*A^3*a^3*b*c^4*d*f^6*z + 224*A^3*a*
b*c^6*d^3*f^4*z + 24*A^3*a*b^5*c^2*d*f^6*z - 4*B^3*a*b*c^6*d^2*e^5*z + 20*A
^2*B*c^8*d^4*e^2*f*z - 7*A^2*B*b^4*c^4*e^6*f*z - 2*A^2*B*b^7*c*e^3*f^4*z -
64*A*B^2*a^5*c^3*e*f^6*z + 16*A*B^2*b*c^7*d^5*f^2*z - 8*A^2*B*a^2*c^6*e^6*f
*z - 2*A*B^2*b^7*c*d^2*f^5*z - 272*A^2*B*a^4*c^4*d*f^6*z + 128*A^2*B*a*c^7*
d^4*f^3*z + 9*A^2*B*b^2*c^6*d*e^6*z - 4*A*B^2*b^3*c^5*d*e^6*z + 4*A*B^2*b*c
^7*d^3*e^4*z + 8*A*B^2*a*c^7*d^2*e^5*z + 12*A^2*B*a^3*b^4*c*f^7*z + 30*B^3*
b^4*c^4*d^3*e^2*f^2*z + 8*B^3*b^5*c^3*d^2*e^3*f^2*z - 2*B^3*b^6*c^2*d^2*e^2
*f^3*z + 152*A^3*b^3*c^5*d^2*e^2*f^3*z - 108*A^3*b^2*c^6*d^2*e^3*f^2*z + 48
*B^3*a^3*c^5*d^2*e^2*f^3*z - 16*B^3*a^2*c^6*d^3*e^2*f^2*z - 3*B^3*a^4*b^2*c
^2*e^2*f^5*z - 120*B^3*a^2*b^2*c^4*d^3*f^4*z + 112*B^3*a^3*b^2*c^3*d^2*f^5*
z + 112*A^3*a^2*b^3*c^3*e^2*f^5*z + 12*A^3*a^2*b^2*c^4*e^3*f^4*z - 120*A^3*
a*c^7*d*e^5*f*z - 52*A^3*a*b*c^6*e^6*f*z + 10*A^3*a*b^6*c*e*f^6*z - 2*A*B^2
*b^8*d*e*f^5*z - 2*A^2*B*a*b^7*e*f^6*z - 24*A^2*B*a*c^7*d*e^6*z + 2*A*B^2*a
*b^7*d*f^6*z - 12*A^2*B*a*b*c^6*e^7*z - 2*A^3*b^7*c*d*f^6*z - 4*A^3*b*c^7*d
*e^6*z + 16*B^3*a^5*c^3*e^2*f^5*z + 11*B^3*b^6*c^2*d^3*f^4*z - 11*A^3*b^4*c
^4*e^5*f^2*z - 8*B^3*b^4*c^4*d^4*f^3*z - 4*B^3*b^2*c^6*d^5*f^2*z + 4*B^3*a^
4*c^4*e^4*f^3*z + 4*A^3*b^5*c^3*e^4*f^3*z - A^3*b^6*c^2*e^3*f^4*z + 136*A^3
*a^3*c^5*e^3*f^4*z + 68*A^3*a^2*c^6*e^5*f^2*z - 64*A^3*b^3*c^5*d^3*f^4*z +
2*B^3*b^3*c^5*d^2*e^5*z - B^3*b^2*c^6*d^3*e^4*z + 96*A^3*a^3*b^3*c^2*f^7*z
+ A*B^2*a^2*b^6*e*f^6*z + 32*A^3*c^8*d^4*e*f^2*z - 24*A^3*c^8*d^3*e^3*f*z +
10*A^3*b^3*c^5*e^6*f*z + 2*A^3*b^7*c*e^2*f^5*z + 128*A^3*a^4*c^4*e*f^6*z -
32*A^3*b*c^7*d^4*f^3*z - 4*B^3*a^2*c^6*d*e^6*z - B^3*a^2*b^6*d*f^6*z - 128
*A^3*a^4*b*c^3*f^7*z - 24*A^3*a^2*b^5*c*f^7*z - 16*A^2*B*c^8*d^5*f^2*z - 4*
A^2*B*c^8*d^3*e^4*z + 64*A^2*B*a^5*c^3*f^7*z + 2*A^2*B*b^3*c^5*e^7*z + 4*A*
B^2*a^2*c^6*e^7*z - A^2*B*a^2*b^6*f^7*z + 4*A^3*c^8*d^2*e^5*z - 3*A^3*b^2*c
^6*e^7*z + A^2*B*b^8*d*f^6*z - A^3*b^8*e*f^6*z + 16*A^3*a*c^7*e^7*z + 2*A^3
*a*b^7*f^7*z + A^2*B*b^8*e^2*f^5*z + B^3*b^8*d^2*f^5*z - 48*A^2*B^2*a*b*c^4
*d*e*f^4 + 28*A*B^3*a*b^2*c^3*d*e*f^4 - 16*A*B^3*a*b*c^4*d*e^2*f^3 + 16*A^3
*B*a*c^5*d*e*f^4 + 32*A^3*B*a*b*c^4*d*f^5 + 12*A^2*B^2*b^3*c^3*d*e*f^4 + 5*
A*B^3*b^2*c^4*d^2*e*f^3 + 4*A*B^3*b^3*c^3*d*e^2*f^3 + 24*A^2*B^2*a*c^5*d*e^
2*f^3 + 24*A^2*B^2*a^2*b*c^3*e*f^5 + 12*A^2*B^2*a*b*c^4*e^3*f^3 - 6*A^2*B^2
*a*b^3*c^2*e*f^5 + 4*A*B^3*a^2*b*c^3*e^2*f^4 + 3*A*B^3*a^2*b^2*c^2*e*f^5 -
18*A^2*B^2*a*b^2*c^3*d*f^5 - 4*B^4*a^2*b*c^3*d*e*f^4 + 4*B^4*a*b*c^4*d^2*e*
f^3 - 6*A*B^3*b^4*c^2*d*e*f^4 + 4*A^3*B*b*c^5*d*e^2*f^3 - 2*A^3*B*b^2*c^4*d
*e*f^4 - 8*A*B^3*a^2*c^4*d*e*f^4 - 8*A*B^3*a*c^5*d^2*e*f^3 + 26*A^3*B*a*b^2
*c^3*e*f^5 + 8*A^3*B*a*b*c^4*e^2*f^4 + 32*A*B^3*a*b*c^4*d^2*f^4 - 28*A*B^3*
a^2*b*c^3*d*f^5 + 6*A*B^3*a*b^3*c^2*d*f^5 - 9*A^2*B^2*b^2*c^4*d*e^2*f^3 - 1
8*A^2*B^2*a*b^2*c^3*e^2*f^4 - 4*A^3*B*c^6*d^2*e*f^3 - 3*A^3*B*b^4*c^2*e*f^5

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 - 44*A^3*B*a^2*c^4*e*f^5 - 16*A^3*B*a*c^5*e^3*f^3 - 16*A*B^3*a^3*c^3*e*f^5
 - 10*A^3*B*b^3*c^3*d*f^5 - 4*A^3*B*b*c^5*d^2*f^4 - 4*A*B^3*b*c^5*d^3*f^3 -
 28*A^3*B*a^2*b*c^3*f^6 + 6*A^3*B*a*b^3*c^2*f^6 - 4*A^4*b*c^5*d*e*f^4 - 20*
A^4*a*b*c^4*e*f^5 + 3*A^2*B^2*b^4*c^2*e^2*f^4 - 2*A^2*B^2*b^3*c^3*e^3*f^3 +
 12*A^2*B^2*a^2*c^4*e^2*f^4 + 9*A^2*B^2*b^2*c^4*d^2*f^4 - 3*A^2*B^2*a^2*b^2
*c^2*f^6 - 2*B^4*b^3*c^3*d^2*e*f^3 + 4*B^4*a^2*c^4*d*e^2*f^3 - 10*B^4*a*b^2
*c^3*d^2*f^4 - 3*B^4*a^2*b^2*c^2*d*f^5 + 3*A^3*B*b^2*c^4*e^3*f^3 - 2*A^3*B*
b^3*c^3*e^2*f^4 - 10*A*B^3*b^3*c^3*d^2*f^4 - 4*A*B^3*a^2*c^4*e^3*f^3 + 3*A^
2*B^2*b^4*c^2*d*f^5 + 36*A^2*B^2*a^2*c^4*d*f^5 - 24*A^2*B^2*a*c^5*d^2*f^4 +
 4*A^2*B^2*c^6*d^3*f^3 + 16*A^2*B^2*a^3*c^3*f^6 + 4*A^4*b^3*c^3*e*f^5 + 16*
B^4*a^3*c^3*d*f^5 + 16*A^4*a*c^5*e^2*f^4 + 8*A^4*b^2*c^4*d*f^5 - 8*A^4*a*b^
2*c^3*f^6 - 24*A^4*a*c^5*d*f^5 + 3*B^4*b^4*c^2*d^2*f^4 - 3*A^4*b^2*c^4*e^2*
f^4 + 4*A^4*c^6*d^2*f^4 + 36*A^4*a^2*c^4*f^6 + B^4*b^2*c^4*d^3*f^3, z, k)*(
root(48416*a^6*b^2*c^6*d^4*e^2*f^4*z^4 - 41544*a^5*b^4*c^5*d^4*e^2*f^4*z^4
 - 31872*a^7*b^2*c^5*d^3*e^2*f^5*z^4 - 31872*a^5*b^2*c^7*d^5*e^2*f^3*z^4 - 2
9184*a^6*b^2*c^6*d^3*e^4*f^3*z^4 + 28800*a^5*b^4*c^5*d^3*e^4*f^3*z^4 + 2151
0*a^4*b^6*c^4*d^4*e^2*f^4*z^4 + 21408*a^6*b^4*c^4*d^3*e^2*f^5*z^4 + 21408*a
^4*b^4*c^6*d^5*e^2*f^3*z^4 - 18112*a^7*b^3*c^4*d^2*e^3*f^5*z^4 - 18112*a^4*
b^3*c^7*d^5*e^3*f^2*z^4 - 15600*a^5*b^5*c^4*d^3*e^3*f^4*z^4 - 15600*a^4*b^5
*c^5*d^4*e^3*f^3*z^4 + 15296*a^6*b^3*c^5*d^3*e^3*f^4*z^4 + 15296*a^5*b^3*c^
6*d^4*e^3*f^3*z^4 + 14016*a^7*b^2*c^5*d^2*e^4*f^4*z^4 + 14016*a^5*b^2*c^7*d
^4*e^4*f^2*z^4 - 13920*a^4*b^6*c^4*d^3*e^4*f^3*z^4 - 11648*a^6*b^3*c^5*d^2*
e^5*f^3*z^4 - 11648*a^5*b^3*c^6*d^3*e^5*f^2*z^4 + 10432*a^6*b^2*c^6*d^2*e^6
*f^2*z^4 + 9008*a^6*b^5*c^3*d^2*e^3*f^5*z^4 + 9008*a^3*b^5*c^6*d^5*e^3*f^2*
z^4 + 8544*a^5*b^5*c^4*d^2*e^5*f^3*z^4 + 8544*a^4*b^5*c^5*d^3*e^5*f^2*z^4 -
 8496*a^5*b^4*c^5*d^2*e^6*f^2*z^4 + 7488*a^8*b^2*c^4*d^2*e^2*f^6*z^4 + 7488
*a^4*b^2*c^8*d^6*e^2*f^2*z^4 + 7380*a^4*b^7*c^3*d^3*e^3*f^4*z^4 + 7380*a^3*
b^7*c^4*d^4*e^3*f^3*z^4 - 6720*a^3*b^8*c^3*d^4*e^2*f^4*z^4 - 5784*a^5*b^6*c
^3*d^3*e^2*f^5*z^4 - 5784*a^3*b^6*c^5*d^5*e^2*f^3*z^4 - 3440*a^6*b^4*c^4*d^
2*e^4*f^4*z^4 - 3440*a^4*b^4*c^6*d^4*e^4*f^2*z^4 + 3360*a^3*b^8*c^3*d^3*e^4
*f^3*z^4 + 3140*a^4*b^6*c^4*d^2*e^6*f^2*z^4 - 2760*a^4*b^7*c^3*d^2*e^5*f^3*
z^4 - 2760*a^3*b^7*c^4*d^3*e^5*f^2*z^4 - 1764*a^5*b^7*c^2*d^2*e^3*f^5*z^4 -
 1764*a^2*b^7*c^5*d^5*e^3*f^2*z^4 - 1640*a^3*b^9*c^2*d^3*e^3*f^4*z^4 - 1640
*a^2*b^9*c^3*d^4*e^3*f^3*z^4 - 1604*a^6*b^6*c^2*d^2*e^2*f^6*z^4 - 1604*a^2*
b^6*c^6*d^6*e^2*f^2*z^4 - 1500*a^5*b^6*c^3*d^2*e^4*f^4*z^4 - 1500*a^3*b^6*c
^5*d^4*e^4*f^2*z^4 + 1140*a^2*b^10*c^2*d^4*e^2*f^4*z^4 + 810*a^4*b^8*c^2*d^
2*e^4*f^4*z^4 + 810*a^2*b^8*c^4*d^4*e^4*f^2*z^4 - 544*a^3*b^8*c^3*d^2*e^6*f
^2*z^4 + 416*a^3*b^9*c^2*d^2*e^5*f^3*z^4 + 416*a^2*b^9*c^3*d^3*e^5*f^2*z^4
 - 384*a^2*b^10*c^2*d^3*e^4*f^3*z^4 + 180*a^4*b^8*c^2*d^3*e^2*f^5*z^4 + 180*
a^2*b^8*c^4*d^5*e^2*f^3*z^4 + 48*a^7*b^4*c^3*d^2*e^2*f^6*z^4 + 48*a^3*b^4*c
^7*d^6*e^2*f^2*z^4 + 36*a^2*b^10*c^2*d^2*e^6*f^2*z^4 - 1024*a^10*b*c^3*d*e*
f^8*z^4 - 1024*a^3*b*c^10*d^8*e*f*z^4 - 192*a^8*b^5*c*d*e*f^8*z^4 - 192*a*b
^5*c^8*d^8*e*f*z^4 + 16128*a^7*b^3*c^4*d^3*e*f^6*z^4 + 16128*a^4*b^3*c^7*d^
6*e*f^3*z^4 - 11712*a^6*b^5*c^3*d^3*e*f^6*z^4 - 11712*a^3*b^5*c^6*d^6*e*f^3
*z^4 + 11520*a^8*b*c^5*d^2*e^3*f^5*z^4 + 11520*a^5*b*c^8*d^5*e^3*f^2*z^4 -
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9984*a^6*b^3*c^5*d^4*e*f^5*z^4 - 9984*a^5*b^3*c^6*d^5*e*f^4*z^4 + 8640*a^5*
b^5*c^4*d^4*e*f^5*z^4 + 8640*a^4*b^5*c^5*d^5*e*f^4*z^4 - 7424*a^7*b*c^6*d^3
*e^3*f^4*z^4 - 7424*a^6*b*c^7*d^4*e^3*f^3*z^4 - 6912*a^8*b^3*c^3*d^2*e*f^7*
z^4 - 6912*a^3*b^3*c^8*d^7*e*f^2*z^4 + 4800*a^7*b^3*c^4*d*e^5*f^4*z^4 + 480
0*a^4*b^3*c^7*d^4*e^5*f*z^4 + 4608*a^7*b*c^6*d^2*e^5*f^3*z^4 + 4608*a^6*b*c
^7*d^3*e^5*f^2*z^4 - 4560*a^4*b^7*c^3*d^4*e*f^5*z^4 - 4560*a^3*b^7*c^4*d^5*
e*f^4*z^4 + 4176*a^5*b^7*c^2*d^3*e*f^6*z^4 + 4176*a^2*b^7*c^5*d^6*e*f^3*z^4
 + 3264*a^7*b^5*c^2*d^2*e*f^7*z^4 + 3264*a^2*b^5*c^7*d^7*e*f^2*z^4 + 3008*a
^8*b^3*c^3*d*e^3*f^6*z^4 + 3008*a^3*b^3*c^8*d^6*e^3*f*z^4 + 2880*a^6*b^3*c^
5*d*e^7*f^2*z^4 + 2880*a^5*b^3*c^6*d^2*e^7*f*z^4 - 2240*a^7*b^4*c^3*d*e^4*f
^5*z^4 - 2240*a^3*b^4*c^7*d^5*e^4*f*z^4 - 1488*a^5*b^5*c^4*d*e^7*f^2*z^4 -
1488*a^4*b^5*c^5*d^2*e^7*f*z^4 + 1440*a^3*b^9*c^2*d^4*e*f^5*z^4 + 1440*a^2*
b^9*c^3*d^5*e*f^4*z^4 - 1328*a^6*b^5*c^3*d*e^5*f^4*z^4 - 1328*a^3*b^5*c^6*d
^4*e^5*f*z^4 - 1152*a^7*b^2*c^5*d*e^6*f^3*z^4 - 1152*a^5*b^2*c^7*d^3*e^6*f*
z^4 - 1120*a^6*b^4*c^4*d*e^6*f^3*z^4 - 1120*a^4*b^4*c^6*d^3*e^6*f*z^4 + 912
*a^6*b^6*c^2*d*e^4*f^5*z^4 + 912*a^2*b^6*c^6*d^5*e^4*f*z^4 + 872*a^5*b^6*c^
3*d*e^6*f^3*z^4 + 872*a^3*b^6*c^5*d^3*e^6*f*z^4 + 768*a^8*b^2*c^4*d*e^4*f^5
*z^4 + 768*a^4*b^2*c^8*d^5*e^4*f*z^4 - 672*a^8*b^4*c^2*d*e^2*f^7*z^4 - 672*
a^2*b^4*c^8*d^7*e^2*f*z^4 - 624*a^7*b^5*c^2*d*e^3*f^6*z^4 - 624*a^2*b^5*c^7
*d^6*e^3*f*z^4 + 480*a^5*b^8*c*d^2*e^2*f^6*z^4 + 480*a*b^8*c^5*d^6*e^2*f^2*
z^4 + 316*a^4*b^7*c^3*d*e^7*f^2*z^4 + 316*a^3*b^7*c^4*d^2*e^7*f*z^4 - 204*a
^4*b^8*c^2*d*e^6*f^3*z^4 - 204*a^2*b^8*c^4*d^3*e^6*f*z^4 + 168*a^3*b^10*c*d
^3*e^2*f^5*z^4 + 168*a*b^10*c^3*d^5*e^2*f^3*z^4 + 156*a^2*b^11*c*d^3*e^3*f^
4*z^4 + 156*a*b^11*c^2*d^4*e^3*f^3*z^4 + 128*a^9*b^2*c^3*d*e^2*f^7*z^4 + 12
8*a^3*b^2*c^9*d^7*e^2*f*z^4 - 124*a^3*b^10*c*d^2*e^4*f^4*z^4 - 124*a*b^10*c
^3*d^4*e^4*f^2*z^4 + 100*a^4*b^9*c*d^2*e^3*f^5*z^4 + 100*a*b^9*c^4*d^5*e^3*
f^2*z^4 + 36*a^5*b^7*c^2*d*e^5*f^4*z^4 + 36*a^2*b^7*c^5*d^4*e^5*f*z^4 - 24*
a^3*b^9*c^2*d*e^7*f^2*z^4 - 24*a^2*b^11*c*d^2*e^5*f^3*z^4 - 24*a^2*b^9*c^3*
d^2*e^7*f*z^4 - 24*a*b^11*c^2*d^3*e^5*f^2*z^4 - 9216*a^8*b*c^5*d^3*e*f^6*z^
4 - 9216*a^5*b*c^8*d^6*e*f^3*z^4 - 5376*a^8*b*c^5*d*e^5*f^4*z^4 - 5376*a^5*
b*c^8*d^4*e^5*f*z^4 + 5120*a^9*b*c^4*d^2*e*f^7*z^4 + 5120*a^7*b*c^6*d^4*e*f
^5*z^4 + 5120*a^6*b*c^7*d^5*e*f^4*z^4 + 5120*a^4*b*c^9*d^7*e*f^2*z^4 - 4352
*a^9*b*c^4*d*e^3*f^6*z^4 - 4352*a^4*b*c^9*d^6*e^3*f*z^4 - 1792*a^7*b*c^6*d*
e^7*f^2*z^4 - 1792*a^6*b*c^7*d^2*e^7*f*z^4 - 1600*a^6*b^2*c^6*d*e^8*f*z^4 +
 912*a^5*b^4*c^5*d*e^8*f*z^4 + 768*a^9*b^3*c^2*d*e*f^8*z^4 + 768*a^2*b^3*c^
9*d^8*e*f*z^4 - 720*a^4*b^9*c*d^3*e*f^6*z^4 - 720*a*b^9*c^4*d^6*e*f^3*z^4 -
 656*a^6*b^7*c*d^2*e*f^7*z^4 - 656*a*b^7*c^6*d^7*e*f^2*z^4 - 240*a^2*b^11*c
*d^4*e*f^5*z^4 - 240*a*b^11*c^2*d^5*e*f^4*z^4 + 216*a^7*b^6*c*d*e^2*f^7*z^4
 + 216*a*b^6*c^7*d^7*e^2*f*z^4 - 204*a^4*b^6*c^4*d*e^8*f*z^4 - 144*a^5*b^8*
c*d*e^4*f^5*z^4 - 144*a*b^8*c^5*d^5*e^4*f*z^4 - 84*a*b^12*c*d^4*e^2*f^4*z^4
 + 36*a^4*b^9*c*d*e^5*f^4*z^4 + 36*a*b^9*c^4*d^4*e^5*f*z^4 + 20*a^6*b^7*c*d
*e^3*f^6*z^4 + 20*a*b^7*c^6*d^6*e^3*f*z^4 + 16*a^3*b^10*c*d*e^6*f^3*z^4 + 1
6*a^3*b^8*c^3*d*e^8*f*z^4 + 16*a*b^12*c*d^3*e^4*f^3*z^4 + 16*a*b^10*c^3*d^3
*e^6*f*z^4 + 48*b^11*c^3*d^6*e*f^3*z^4 + 48*b^9*c^5*d^7*e*f^2*z^4 - 20*b^8*
c^6*d^7*e^2*f*z^4 + 8*b^10*c^4*d^5*e^4*f*z^4 - 4*b^13*c*d^4*e^3*f^3*z^4 - 4

*b^11*c^3*d^4*e^5*f*z^4 + 4*b^9*c^5*d^6*e^3*f*z^4 + 3072*a^9*c^5*d*e^4*f^5*
z^4 + 3072*a^5*c^9*d^5*e^4*f*z^4 + 2560*a^8*c^6*d*e^6*f^3*z^4 + 2560*a^6*c^
8*d^3*e^6*f*z^4 + 1536*a^10*c^4*d*e^2*f^7*z^4 + 1536*a^4*c^10*d^7*e^2*f*z^4
 + 48*a^5*b^9*d^2*e*f^7*z^4 + 48*a^3*b^11*d^3*e*f^6*z^4 - 20*a^6*b^8*d*e^2*
f^7*z^4 + 8*a^4*b^10*d*e^4*f^5*z^4 + 4*a^5*b^9*d*e^3*f^6*z^4 - 4*a^3*b^11*d
*e^5*f^4*z^4 - 4*a*b^13*d^3*e^3*f^4*z^4 + 768*a^9*b*c^4*e^5*f^5*z^4 + 768*a
^8*b*c^5*e^7*f^3*z^4 + 256*a^10*b*c^3*e^3*f^7*z^4 - 192*a^6*b^3*c^5*e^9*f*z
^4 - 68*a^7*b^6*c*e^4*f^6*z^4 + 48*a^8*b^5*c*e^3*f^7*z^4 + 48*a^5*b^5*c^4*e
^9*f*z^4 + 36*a^6*b^7*c*e^5*f^5*z^4 - 12*a^9*b^4*c*e^2*f^8*z^4 - 4*a^4*b^9*
c*e^7*f^3*z^4 - 4*a^4*b^7*c^3*e^9*f*z^4 + 384*a^5*b^8*c*d^3*f^7*z^4 + 384*a
*b^8*c^5*d^7*f^3*z^4 + 288*a^3*b^10*c*d^4*f^6*z^4 + 288*a*b^10*c^3*d^6*f^4*
z^4 + 224*a^7*b^6*c*d^2*f^8*z^4 + 224*a*b^6*c^7*d^8*f^2*z^4 - 192*a^10*b^2*
c^2*d*f^9*z^4 - 192*a^2*b^2*c^10*d^9*f*z^4 + 768*a^5*b*c^8*d^3*e^7*z^4 + 76
8*a^4*b*c^9*d^5*e^5*z^4 + 256*a^3*b*c^10*d^7*e^3*z^4 - 192*a^5*b^3*c^6*d*e^
9*z^4 - 68*a*b^6*c^7*d^6*e^4*z^4 + 48*a^4*b^5*c^5*d*e^9*z^4 + 48*a*b^5*c^8*
d^7*e^3*z^4 + 36*a*b^7*c^6*d^5*e^5*z^4 - 12*a*b^4*c^9*d^8*e^2*z^4 - 4*a^3*b
^7*c^4*d*e^9*z^4 - 4*a*b^9*c^4*d^3*e^7*z^4 + 16*b^13*c*d^5*e*f^4*z^4 + 16*b
^7*c^7*d^8*e*f*z^4 + 768*a^7*c^7*d*e^8*f*z^4 + 16*a^7*b^7*d*e*f^8*z^4 + 16*
a*b^13*d^4*e*f^5*z^4 + 256*a^7*b*c^6*e^9*f*z^4 + 80*a*b^12*c*d^5*f^5*z^4 +
48*a^9*b^4*c*d*f^9*z^4 + 48*a*b^4*c^9*d^9*f*z^4 + 256*a^6*b*c^7*d*e^9*z^4 -
 42*b^10*c^4*d^6*e^2*f^2*z^4 - 20*b^12*c^2*d^5*e^2*f^3*z^4 + 6*b^12*c^2*d^4
*e^4*f^2*z^4 + 4*b^11*c^3*d^5*e^3*f^2*z^4 - 24960*a^7*c^7*d^4*e^2*f^4*z^4 +
 18944*a^8*c^6*d^3*e^2*f^5*z^4 + 18944*a^6*c^8*d^5*e^2*f^3*z^4 + 14336*a^7*
c^7*d^3*e^4*f^3*z^4 - 9984*a^8*c^6*d^2*e^4*f^4*z^4 - 9984*a^6*c^8*d^4*e^4*f
^2*z^4 - 7936*a^9*c^5*d^2*e^2*f^6*z^4 - 7936*a^5*c^9*d^6*e^2*f^2*z^4 - 4352
*a^7*c^7*d^2*e^6*f^2*z^4 - 42*a^4*b^10*d^2*e^2*f^6*z^4 - 20*a^2*b^12*d^3*e^
2*f^5*z^4 + 6*a^2*b^12*d^2*e^4*f^4*z^4 + 4*a^3*b^11*d^2*e^3*f^5*z^4 - 480*a
^8*b^2*c^4*e^6*f^4*z^4 + 440*a^7*b^4*c^3*e^6*f^4*z^4 - 320*a^8*b^3*c^3*e^5*
f^5*z^4 - 320*a^7*b^3*c^4*e^7*f^3*z^4 + 240*a^8*b^4*c^2*e^4*f^6*z^4 + 240*a
^6*b^4*c^4*e^8*f^2*z^4 - 192*a^9*b^3*c^2*e^3*f^7*z^4 - 192*a^9*b^2*c^3*e^4*
f^6*z^4 - 192*a^7*b^2*c^5*e^8*f^2*z^4 - 90*a^6*b^6*c^2*e^6*f^4*z^4 - 68*a^5
*b^6*c^3*e^8*f^2*z^4 + 48*a^10*b^2*c^2*e^2*f^8*z^4 - 48*a^7*b^5*c^2*e^5*f^5
*z^4 - 48*a^6*b^5*c^3*e^7*f^3*z^4 + 36*a^5*b^7*c^2*e^7*f^3*z^4 + 6*a^4*b^8*
c^2*e^8*f^2*z^4 - 33920*a^6*b^2*c^6*d^5*f^5*z^4 + 27936*a^5*b^4*c^5*d^5*f^5
*z^4 + 26112*a^7*b^2*c^5*d^4*f^6*z^4 + 26112*a^5*b^2*c^7*d^6*f^4*z^4 - 2035
2*a^6*b^4*c^4*d^4*f^6*z^4 - 20352*a^4*b^4*c^6*d^6*f^4*z^4 - 13080*a^4*b^6*c
^4*d^5*f^5*z^4 - 11520*a^8*b^2*c^4*d^3*f^7*z^4 - 11520*a^4*b^2*c^8*d^7*f^3*
z^4 + 8736*a^5*b^6*c^3*d^4*f^6*z^4 + 8736*a^3*b^6*c^5*d^6*f^4*z^4 + 7488*a^
7*b^4*c^3*d^3*f^7*z^4 + 7488*a^3*b^4*c^7*d^7*f^3*z^4 + 3840*a^3*b^8*c^3*d^5
*f^5*z^4 + 2560*a^9*b^2*c^3*d^2*f^8*z^4 + 2560*a^3*b^2*c^9*d^8*f^2*z^4 - 24
16*a^6*b^6*c^2*d^3*f^7*z^4 - 2416*a^2*b^6*c^6*d^7*f^3*z^4 - 2160*a^4*b^8*c^
2*d^4*f^6*z^4 - 2160*a^2*b^8*c^4*d^6*f^4*z^4 - 1152*a^8*b^4*c^2*d^2*f^8*z^4
 - 1152*a^2*b^4*c^8*d^8*f^2*z^4 - 720*a^2*b^10*c^2*d^5*f^5*z^4 - 480*a^4*b^
2*c^8*d^4*e^6*z^4 + 440*a^3*b^4*c^7*d^4*e^6*z^4 - 320*a^4*b^3*c^7*d^3*e^7*z
^4 - 320*a^3*b^3*c^8*d^5*e^5*z^4 + 240*a^4*b^4*c^6*d^2*e^8*z^4 + 240*a^2*b^

4*c^8*d^6*e^4*z^4 - 192*a^5*b^2*c^7*d^2*e^8*z^4 - 192*a^3*b^2*c^9*d^6*e^4*z
^4 - 192*a^2*b^3*c^9*d^7*e^3*z^4 - 90*a^2*b^6*c^6*d^4*e^6*z^4 - 68*a^3*b^6*
c^5*d^2*e^8*z^4 - 48*a^3*b^5*c^6*d^3*e^7*z^4 - 48*a^2*b^5*c^7*d^5*e^5*z^4 +
48*a^2*b^2*c^10*d^8*e^2*z^4 + 36*a^2*b^7*c^5*d^3*e^7*z^4 + 6*a^2*b^8*c^4*d
^2*e^8*z^4 - 4*b^6*c^8*d^9*f*z^4 + 256*a^11*c^3*d*f^9*z^4 + 256*a^3*c^11*d^
9*f*z^4 - 4*a^8*b^6*d*f^9*z^4 - 384*a^9*c^5*e^6*f^4*z^4 - 256*a^10*c^4*e^4*
f^6*z^4 - 256*a^8*c^6*e^8*f^2*z^4 - 64*a^11*c^3*e^2*f^8*z^4 - 24*b^10*c^4*d
^7*f^3*z^4 - 16*b^12*c^2*d^6*f^4*z^4 - 16*b^8*c^6*d^8*f^2*z^4 + 17920*a^7*c
^7*d^5*f^5*z^4 - 14336*a^8*c^6*d^4*f^6*z^4 - 14336*a^6*c^8*d^6*f^4*z^4 + 71
68*a^9*c^5*d^3*f^7*z^4 + 7168*a^5*c^9*d^7*f^3*z^4 - 2048*a^10*c^4*d^2*f^8*z
^4 - 2048*a^4*c^10*d^8*f^2*z^4 + 6*b^8*c^6*d^6*e^4*z^4 + 6*a^6*b^8*e^4*f^6*
z^4 - 4*b^9*c^5*d^5*e^5*z^4 - 4*b^7*c^7*d^7*e^3*z^4 - 4*a^7*b^7*e^3*f^7*z^4
 - 4*a^5*b^9*e^5*f^5*z^4 - 384*a^5*c^9*d^4*e^6*z^4 - 256*a^6*c^8*d^2*e^8*z^
4 - 256*a^4*c^10*d^6*e^4*z^4 - 64*a^3*c^11*d^8*e^2*z^4 - 24*a^4*b^10*d^3*f^
7*z^4 - 16*a^6*b^8*d^2*f^8*z^4 - 16*a^2*b^12*d^4*f^6*z^4 + 48*a^6*b^2*c^6*e
^10*z^4 - 12*a^5*b^4*c^5*e^10*z^4 - 4*b^14*d^5*f^5*z^4 - 64*a^7*c^7*e^10*z^
4 + b^14*d^4*e^2*f^4*z^4 + b^10*c^4*d^4*e^6*z^4 + b^6*c^8*d^8*e^2*z^4 + a^8
*b^6*e^2*f^8*z^4 + a^4*b^10*e^6*f^4*z^4 + a^4*b^6*c^4*e^10*z^4 - 4820*A*B*a
^4*b*c^5*d^2*e^2*f^4*z^2 + 2976*A*B*a^3*b*c^6*d^3*e^2*f^3*z^2 - 2328*A*B*a^
3*b*c^6*d^2*e^4*f^2*z^2 + 1848*A*B*a^2*b^4*c^4*d^3*e*f^4*z^2 - 1768*A*B*a^3
*b^4*c^3*d^2*e*f^5*z^2 + 1528*A*B*a^4*b^2*c^4*d^2*e*f^5*z^2 - 1136*A*B*a^3*
b^2*c^5*d^3*e*f^4*z^2 - 974*A*B*a^4*b^3*c^3*d*e^2*f^5*z^2 + 692*A*B*a^2*b*c
^7*d^4*e^2*f^2*z^2 + 588*A*B*a*b^6*c^3*d^2*e^3*f^3*z^2 - 580*A*B*a^3*b^3*c^
4*d*e^4*f^3*z^2 + 488*A*B*a^3*b^4*c^3*d*e^3*f^4*z^2 - 444*A*B*a^2*b^2*c^6*d
^2*e^5*f*z^2 - 412*A*B*a*b^5*c^4*d^2*e^4*f^2*z^2 + 366*A*B*a^2*b^6*c^2*d^2*
e*f^5*z^2 - 352*A*B*a^2*b^2*c^6*d^4*e*f^3*z^2 + 326*A*B*a^2*b^4*c^4*d*e^5*f
^2*z^2 + 324*A*B*a*b^5*c^4*d^3*e^2*f^3*z^2 - 302*A*B*a*b^3*c^6*d^4*e^2*f^2*
z^2 - 296*A*B*a*b^7*c^2*d^2*e^2*f^4*z^2 + 122*A*B*a^4*b^2*c^4*d*e^3*f^4*z^2
 - 122*A*B*a^2*b^6*c^2*d*e^3*f^4*z^2 - 84*A*B*a^3*b^2*c^5*d*e^5*f^2*z^2 + 7
2*A*B*a*b^4*c^5*d^3*e^3*f^2*z^2 - 64*A*B*a^2*b^5*c^3*d*e^4*f^3*z^2 + 60*A*B
*a^3*b^5*c^2*d*e^2*f^5*z^2 + 1312*A*B*a^5*b*c^4*d*e^2*f^5*z^2 + 1040*A*B*a^
4*b*c^5*d*e^4*f^3*z^2 - 500*A*B*a*b^6*c^3*d^3*e*f^4*z^2 - 376*A*B*a*b^2*c^7
*d^5*e*f^2*z^2 + 276*A*B*a^4*b^4*c^2*d*e*f^6*z^2 - 262*A*B*a^2*b^3*c^5*d*e^
6*f*z^2 + 238*A*B*a*b^2*c^7*d^4*e^3*f*z^2 + 232*A*B*a^5*b^2*c^3*d*e*f^6*z^2
 - 176*A*B*a^2*b*c^7*d^3*e^4*f*z^2 - 120*A*B*a*b^6*c^3*d*e^5*f^2*z^2 - 108*
A*B*a*b^4*c^5*d^4*e*f^3*z^2 + 68*A*B*a*b^7*c^2*d*e^4*f^3*z^2 + 68*A*B*a*b^4
*c^5*d^2*e^5*f*z^2 + 46*A*B*a^2*b^7*c*d*e^2*f^5*z^2 - 36*A*B*a*b^3*c^6*d^3*
e^4*f*z^2 - 1932*A*B*a^2*b^3*c^5*d^3*e^2*f^3*z^2 - 1818*A*B*a^2*b^4*c^4*d^2
*e^3*f^3*z^2 + 1620*A*B*a^3*b^3*c^4*d^2*e^2*f^4*z^2 + 1560*A*B*a^2*b^3*c^5*
d^2*e^4*f^2*z^2 + 1244*A*B*a^3*b^2*c^5*d^2*e^3*f^3*z^2 + 820*A*B*a^2*b^2*c^
6*d^3*e^3*f^2*z^2 + 480*A*B*a^2*b^5*c^3*d^2*e^2*f^4*z^2 + 352*A*B*a^3*b*c^6
*d*e^6*f*z^2 - 108*A*B*a^3*b^6*c*d*e*f^6*z^2 + 82*A*B*a*b^5*c^4*d*e^6*f*z^2
 - 64*A*B*a*b*c^8*d^5*e^2*f*z^2 + 16*A*B*a*b^8*c*d^2*e*f^5*z^2 - 4*A*B*a*b^
8*c*d*e^3*f^4*z^2 + 16*B^2*a*b*c^8*d^6*e*f*z^2 + 56*A*B*b^2*c^8*d^6*e*f*z^2
 - 8*A*B*b^9*c*d*e^4*f^3*z^2 - 8*A*B*b^7*c^3*d*e^6*f*z^2 - 800*A*B*a^6*c^4*

d*e*f^6*z^2 + 10*A*B*a^2*b^8*d*e*f^6*z^2 - 6*A*B*a*b^9*d*e^2*f^5*z^2 - 12*A
*B*a^5*b^4*c*e*f^7*z^2 + 912*A*B*a^6*b*c^3*d*f^7*z^2 + 192*A*B*a^4*b^5*c*d*
f^7*z^2 + 192*A*B*a*b*c^8*d^6*f^2*z^2 - 20*A*B*a*b^4*c^5*d*e^7*z^2 + 4*A*B*
a*b*c^8*d^4*e^4*z^2 + 2144*B^2*a^4*b*c^5*d^3*e*f^4*z^2 - 1120*B^2*a^3*b*c^6
*d^4*e*f^3*z^2 - 688*B^2*a^5*b*c^4*d^2*e*f^5*z^2 - 256*B^2*a^3*b*c^6*d^2*e^
5*f*z^2 + 152*B^2*a*b^3*c^6*d^5*e*f^2*z^2 + 120*B^2*a^5*b^3*c^2*d*e*f^6*z^2
 - 116*B^2*a^5*b*c^4*d*e^3*f^4*z^2 + 110*B^2*a*b^7*c^2*d^3*e*f^4*z^2 - 80*B
^2*a^2*b*c^7*d^5*e*f^2*z^2 - 72*B^2*a*b^5*c^4*d^4*e*f^3*z^2 - 48*B^2*a^4*b*
c^5*d*e^5*f^2*z^2 - 46*B^2*a*b^3*c^6*d^4*e^3*f*z^2 - 44*B^2*a*b^4*c^5*d^3*e
^4*f*z^2 - 34*B^2*a*b^5*c^4*d^2*e^5*f*z^2 + 20*B^2*a^2*b*c^7*d^4*e^3*f*z^2
- 10*B^2*a^3*b^6*c*d*e^2*f^5*z^2 - 10*B^2*a^2*b^7*c*d^2*e*f^5*z^2 - 10*B^2*
a*b^2*c^7*d^5*e^2*f*z^2 - 7*B^2*a^2*b^4*c^4*d*e^6*f*z^2 - 6*B^2*a^3*b^2*c^5
*d*e^6*f*z^2 + 4*B^2*a*b^8*c*d^2*e^2*f^4*z^2 - 2*B^2*a^2*b^7*c*d*e^3*f^4*z^
2 + 3196*A^2*a^4*b*c^5*d*e^3*f^4*z^2 - 3184*A^2*a^4*b*c^5*d^2*e*f^5*z^2 + 1
568*A^2*a^3*b*c^6*d^3*e*f^4*z^2 + 1504*A^2*a^3*b*c^6*d*e^5*f^2*z^2 - 656*A^
2*a^4*b^3*c^3*d*e*f^6*z^2 - 400*A^2*a*b^6*c^3*d*e^4*f^3*z^2 + 314*A^2*a*b^5
*c^4*d*e^5*f^2*z^2 - 264*A^2*a^3*b^5*c^2*d*e*f^6*z^2 + 240*A^2*a^2*b^2*c^6*
d*e^6*f*z^2 - 224*A^2*a^2*b*c^7*d^4*e*f^3*z^2 + 216*A^2*a*b^5*c^4*d^3*e*f^4
*z^2 - 192*A^2*a^2*b*c^7*d^2*e^5*f*z^2 + 178*A^2*a*b^7*c^2*d*e^3*f^4*z^2 -
154*A^2*a*b^7*c^2*d^2*e*f^5*z^2 + 128*A^2*a*b^3*c^6*d^4*e*f^3*z^2 + 106*A^2
*a*b^3*c^6*d^2*e^5*f*z^2 - 12*A^2*a*b^2*c^7*d^3*e^4*f*z^2 - 58*A*B*b^8*c^2*
d^2*e^3*f^3*z^2 + 40*A*B*b^7*c^3*d^2*e^4*f^2*z^2 - 28*A*B*b^7*c^3*d^3*e^2*f
^3*z^2 - 24*A*B*b^5*c^5*d^4*e^2*f^2*z^2 - 20*A*B*b^6*c^4*d^3*e^3*f^2*z^2 +
2768*A*B*a^4*c^6*d^2*e^3*f^3*z^2 - 1712*A*B*a^3*c^7*d^3*e^3*f^2*z^2 - 156*A
*B*a^4*b^2*c^4*e^5*f^3*z^2 + 146*A*B*a^4*b^3*c^3*e^4*f^4*z^2 - 106*A*B*a^5*
b^2*c^3*e^3*f^5*z^2 + 90*A*B*a^5*b^3*c^2*e^2*f^6*z^2 + 38*A*B*a^3*b^3*c^4*e
^6*f^2*z^2 - 36*A*B*a^3*b^5*c^2*e^4*f^4*z^2 + 16*A*B*a^3*b^4*c^3*e^5*f^3*z^
2 - 9*A*B*a^4*b^4*c^2*e^3*f^5*z^2 - 8*A*B*a^2*b^5*c^3*e^6*f^2*z^2 + 2*A*B*a
^2*b^6*c^2*e^5*f^3*z^2 + 920*A*B*a^4*b^3*c^3*d^2*f^6*z^2 - 480*A*B*a^2*b^5*
c^3*d^3*f^5*z^2 - 336*A*B*a^2*b^3*c^5*d^4*f^4*z^2 - 272*A*B*a^3*b^3*c^4*d^3
*f^5*z^2 + 240*A*B*a^3*b^5*c^2*d^2*f^6*z^2 - 32*A*B*a*c^9*d^6*e*f*z^2 - 792
*B^2*a^2*b^3*c^5*d^3*e^3*f^2*z^2 + 714*B^2*a^2*b^4*c^4*d^3*e^2*f^3*z^2 - 57
2*B^2*a^3*b^2*c^5*d^3*e^2*f^3*z^2 - 475*B^2*a^2*b^2*c^6*d^4*e^2*f^2*z^2 + 2
65*B^2*a^4*b^2*c^4*d^2*e^2*f^4*z^2 + 260*B^2*a^3*b^3*c^4*d^2*e^3*f^3*z^2 -
212*B^2*a^3*b^4*c^3*d^2*e^2*f^4*z^2 + 180*B^2*a^3*b^2*c^5*d^2*e^4*f^2*z^2 -
 158*B^2*a^2*b^4*c^4*d^2*e^4*f^2*z^2 + 47*B^2*a^2*b^6*c^2*d^2*e^2*f^4*z^2 +
 16*B^2*a^2*b^5*c^3*d^2*e^3*f^3*z^2 + 2752*A^2*a^3*b^2*c^5*d^2*e^2*f^4*z^2
- 2148*A^2*a^2*b^4*c^4*d^2*e^2*f^4*z^2 + 2064*A^2*a^2*b^3*c^5*d^2*e^3*f^3*z
^2 - 424*A^2*a^2*b^2*c^6*d^3*e^2*f^3*z^2 - 198*A^2*a^2*b^2*c^6*d^2*e^4*f^2*
z^2 - 272*B^2*a^6*b*c^3*d*e*f^6*z^2 - 24*B^2*a^4*b^5*c*d*e*f^6*z^2 + 1808*A
^2*a^5*b*c^4*d*e*f^6*z^2 - 244*A^2*a*b*c^8*d^4*e^3*f*z^2 + 208*A^2*a*b*c^8*
d^5*e*f^2*z^2 + 134*A^2*a^2*b^7*c*d*e*f^6*z^2 - 76*A^2*a*b^4*c^5*d*e^6*f*z^
2 + 4*A^2*a*b^8*c*d*e^2*f^5*z^2 + 148*A*B*b^4*c^6*d^5*e*f^2*z^2 + 65*A*B*b^
6*c^4*d^4*e*f^3*z^2 + 46*A*B*b^8*c^2*d^3*e*f^4*z^2 - 38*A*B*b^3*c^7*d^5*e^2
*f*z^2 + 34*A*B*b^9*c*d^2*e^2*f^4*z^2 - 29*A*B*b^4*c^6*d^4*e^3*f*z^2 + 20*A

```
*B*b^5*c^5*d^3*e^4*f*z^2 + 12*A*B*b^8*c^2*d*e^5*f^2*z^2 - 7*A*B*b^6*c^4*d^2
*e^5*f*z^2 - 2880*A*B*a^4*c^6*d^3*e*f^4*z^2 + 2784*A*B*a^5*c^5*d^2*e*f^5*z^
2 - 1112*A*B*a^5*c^5*d*e^3*f^4*z^2 + 896*A*B*a^3*c^7*d^4*e*f^3*z^2 + 848*A*
B*a^3*c^7*d^2*e^5*f*z^2 - 560*A*B*a^4*c^6*d*e^5*f^2*z^2 + 96*A*B*a^2*c^8*d^
5*e*f^2*z^2 - 88*A*B*a^2*c^8*d^4*e^3*f*z^2 - 100*A*B*a^6*b*c^3*e^2*f^6*z^2
- 76*A*B*a^5*b*c^4*e^4*f^4*z^2 + 48*A*B*a^6*b^2*c^2*e*f^7*z^2 - 42*A*B*a^3*
b^2*c^5*e^7*f*z^2 + 36*A*B*a^4*b*c^5*e^6*f^2*z^2 - 24*A*B*a^4*b^5*c*e^2*f^6
*z^2 + 10*A*B*a^3*b^6*c*e^3*f^5*z^2 + 7*A*B*a^2*b^4*c^4*e^7*f*z^2 + 2*A*B*a
^2*b^7*c*e^4*f^4*z^2 - 2496*A*B*a^5*b*c^4*d^2*f^6*z^2 + 1872*A*B*a^4*b*c^5*
d^3*f^5*z^2 - 744*A*B*a^5*b^3*c^2*d*f^7*z^2 - 720*A*B*a^2*b*c^7*d^5*f^3*z^2
 + 504*A*B*a*b^3*c^6*d^5*f^3*z^2 + 256*A*B*a^3*b*c^6*d^4*f^4*z^2 + 168*A*B*
a*b^7*c^2*d^3*f^5*z^2 - 144*A*B*a^2*b^7*c*d^2*f^6*z^2 + 144*A*B*a*b^5*c^4*d
^4*f^4*z^2 + 66*A*B*a^2*b^2*c^6*d*e^7*z^2 - 36*A*B*a*b^2*c^7*d^3*e^5*z^2 +
20*A*B*a*b^3*c^6*d^2*e^6*z^2 + 12*A*B*a^2*b*c^7*d^2*e^6*z^2 + 1208*B^2*a^3*
b*c^6*d^3*e^3*f^2*z^2 - 848*B^2*a^3*b^3*c^4*d^3*e*f^4*z^2 + 672*B^2*a^2*b^3
*c^5*d^4*e*f^3*z^2 - 632*B^2*a^4*b*c^5*d^2*e^3*f^3*z^2 + 432*B^2*a^4*b^3*c^
3*d^2*e*f^5*z^2 + 276*B^2*a^2*b^2*c^6*d^3*e^4*f*z^2 - 196*B^2*a*b^6*c^3*d^3
*e^2*f^3*z^2 - 168*B^2*a^2*b^5*c^3*d^3*e*f^4*z^2 + 154*B^2*a^2*b^3*c^5*d^2*
e^5*f*z^2 + 148*B^2*a*b^5*c^4*d^3*e^3*f^2*z^2 + 96*B^2*a*b^4*c^5*d^4*e^2*f^
2*z^2 - 72*B^2*a^3*b^5*c^2*d^2*e*f^5*z^2 + 70*B^2*a^5*b^2*c^3*d*e^2*f^5*z^2
 - 60*B^2*a^4*b^3*c^3*d*e^3*f^4*z^2 + 52*B^2*a*b^6*c^3*d^2*e^4*f^2*z^2 + 36
*B^2*a^4*b^2*c^4*d*e^4*f^3*z^2 - 32*B^2*a*b^7*c^2*d^2*e^3*f^3*z^2 + 24*B^2*
a^3*b^5*c^2*d*e^3*f^4*z^2 + 15*B^2*a^4*b^4*c^2*d*e^2*f^5*z^2 - 8*B^2*a^3*b^
4*c^3*d*e^4*f^3*z^2 + 8*B^2*a^2*b^5*c^3*d*e^5*f^2*z^2 - 2*B^2*a^3*b^3*c^4*d
*e^5*f^2*z^2 - 2*B^2*a^2*b^6*c^2*d*e^4*f^3*z^2 - 3176*A^2*a^3*b*c^6*d^2*e^3
*f^3*z^2 - 2252*A^2*a^4*b^2*c^4*d*e^2*f^5*z^2 + 1952*A^2*a^3*b^4*c^3*d*e^2*
f^5*z^2 - 1496*A^2*a^3*b^3*c^4*d*e^3*f^4*z^2 + 1378*A^2*a^2*b^4*c^4*d*e^4*f
^3*z^2 + 1184*A^2*a^3*b^3*c^4*d^2*e*f^5*z^2 - 1166*A^2*a^2*b^3*c^5*d*e^5*f^
2*z^2 - 1164*A^2*a^3*b^2*c^5*d*e^4*f^3*z^2 - 1152*A^2*a^2*b^3*c^5*d^3*e*f^4
*z^2 + 578*A^2*a*b^6*c^3*d^2*e^2*f^4*z^2 - 548*A^2*a*b^5*c^4*d^2*e^3*f^3*z^
2 + 440*A^2*a*b^2*c^7*d^4*e^2*f^2*z^2 - 412*A^2*a^2*b^6*c^2*d*e^2*f^5*z^2 -
 360*A^2*a*b^3*c^6*d^3*e^3*f^2*z^2 + 312*A^2*a*b^4*c^5*d^3*e^2*f^3*z^2 + 24
8*A^2*a^2*b*c^7*d^3*e^3*f^2*z^2 - 224*A^2*a^2*b^5*c^3*d*e^3*f^4*z^2 + 216*A
^2*a^2*b^5*c^3*d^2*e*f^5*z^2 + 52*A^2*a*b^4*c^5*d^2*e^4*f^2*z^2 - 16*B^2*b^
3*c^7*d^6*e*f*z^2 - 14*B^2*b^9*c*d^3*e*f^4*z^2 + 32*B^2*a^4*c^6*d*e^6*f*z^2
 - 20*A^2*b^9*c*d*e^3*f^4*z^2 + 18*A^2*b^9*c*d^2*e*f^5*z^2 + 8*A^2*b^6*c^4*
d*e^6*f*z^2 - 360*A^2*a^3*c^7*d*e^6*f*z^2 + 136*A^2*a*c^9*d^5*e^2*f*z^2 + 2
*B^2*a^3*b^7*d*e*f^6*z^2 + 2*B^2*a*b^9*d^2*e*f^5*z^2 + 12*B^2*a^4*b*c^5*e^7
*f*z^2 - 204*A^2*a^3*b*c^6*e^7*f*z^2 - 128*A^2*a^6*b*c^3*e*f^7*z^2 - 48*A^2
*a*b^5*c^4*e^7*f*z^2 - 36*B^2*a^5*b^4*c*d*f^7*z^2 - 24*A^2*a^4*b^5*c*e*f^7*
z^2 - 16*B^2*a*b^8*c*d^3*f^5*z^2 - 164*A^2*a^3*b^6*c*d*f^7*z^2 - 16*A^2*a*b
^8*c*d^2*f^6*z^2 + 4*B^2*a^3*b*c^6*d*e^7*z^2 - 4*B^2*a*b*c^8*d^5*e^3*z^2 +
48*A^2*a*b*c^8*d^3*e^5*z^2 + 36*A^2*a^2*b*c^7*d*e^7*z^2 - 6*A^2*a*b^3*c^6*d
*e^7*z^2 + 136*A*B*a^6*c^4*e^3*f^5*z^2 - 96*A*B*b^5*c^5*d^5*f^3*z^2 + 80*A*
B*a^5*c^5*e^5*f^3*z^2 - 72*A*B*b^3*c^7*d^6*f^2*z^2 - 24*A*B*b^7*c^3*d^4*f^4
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*z^2 + 14*A*B*b^3*c^7*d^4*e^4*z^2 - 14*A*B*b^2*c^8*d^5*e^3*z^2 - 2*A*B*b^5*
c^5*d^2*e^6*z^2 - 2*A*B*b^4*c^6*d^3*e^5*z^2 + 2*A*B*a^3*b^7*e^2*f^6*z^2 - A
*B*a^2*b^8*e^3*f^5*z^2 + 16*A*B*a^2*c^8*d^3*e^5*z^2 - 2*A*B*a^2*b^3*c^5*e^8
*z^2 + 22*B^2*b^8*c^2*d^3*e^2*f^3*z^2 - 12*B^2*b^7*c^3*d^3*e^3*f^2*z^2 + 12
*B^2*b^6*c^4*d^4*e^2*f^2*z^2 - 6*B^2*b^8*c^2*d^2*e^4*f^2*z^2 - 864*B^2*a^4*
c^6*d^3*e^2*f^3*z^2 + 496*B^2*a^3*c^7*d^4*e^2*f^2*z^2 + 224*B^2*a^5*c^5*d^2
*e^2*f^4*z^2 + 136*B^2*a^4*c^6*d^2*e^4*f^2*z^2 - 53*A^2*b^8*c^2*d^2*e^2*f^4
*z^2 + 52*A^2*b^7*c^3*d^2*e^3*f^3*z^2 + 52*A^2*b^5*c^5*d^3*e^3*f^2*z^2 - 36
*A^2*b^6*c^4*d^3*e^2*f^3*z^2 - 12*A^2*b^4*c^6*d^4*e^2*f^2*z^2 - 9*A^2*b^6*c
^4*d^2*e^4*f^2*z^2 + 836*A^2*a^4*c^6*d^2*e^2*f^4*z^2 - 668*A^2*a^2*c^8*d^4*
e^2*f^2*z^2 + 656*A^2*a^3*c^7*d^2*e^4*f^2*z^2 + 368*A^2*a^3*c^7*d^3*e^2*f^3
*z^2 - 45*B^2*a^6*b^2*c^2*e^2*f^6*z^2 - 18*B^2*a^5*b^2*c^3*e^4*f^4*z^2 - 9*
B^2*a^4*b^2*c^4*e^6*f^2*z^2 - 6*B^2*a^5*b^3*c^2*e^3*f^5*z^2 + 3*B^2*a^4*b^4
*c^2*e^4*f^4*z^2 - 2*B^2*a^4*b^3*c^3*e^5*f^3*z^2 - 580*B^2*a^4*b^2*c^4*d^3*
f^5*z^2 + 536*B^2*a^3*b^4*c^3*d^3*f^5*z^2 + 471*A^2*a^4*b^2*c^4*e^4*f^4*z^2
 - 436*A^2*a^3*b^4*c^3*e^4*f^4*z^2 - 348*B^2*a^4*b^4*c^2*d^2*f^6*z^2 + 316*
B^2*a^2*b^2*c^6*d^5*f^3*z^2 + 310*A^2*a^3*b^3*c^4*e^5*f^3*z^2 + 232*A^2*a^5
*b^2*c^3*e^2*f^6*z^2 - 229*A^2*a^2*b^4*c^4*e^6*f^2*z^2 - 216*A^2*a^4*b^4*c^
2*e^2*f^6*z^2 + 204*A^2*a^4*b^3*c^3*e^3*f^5*z^2 + 200*B^2*a^5*b^2*c^3*d^2*f
^6*z^2 + 150*A^2*a^3*b^2*c^5*e^6*f^2*z^2 - 120*B^2*a^2*b^4*c^4*d^4*f^4*z^2
 + 91*A^2*a^2*b^6*c^2*e^4*f^4*z^2 + 72*A^2*a^3*b^5*c^2*e^3*f^5*z^2 - 66*B^2*
a^2*b^6*c^2*d^3*f^5*z^2 + 44*A^2*a^2*b^5*c^3*e^5*f^3*z^2 - 16*B^2*a^3*b^2*c
^5*d^4*f^4*z^2 + 1952*A^2*a^4*b^2*c^4*d^2*f^6*z^2 - 1792*A^2*a^3*b^2*c^5*d^
3*f^5*z^2 - 1272*A^2*a^3*b^4*c^3*d^2*f^6*z^2 + 976*A^2*a^2*b^2*c^6*d^4*f^4*
z^2 + 960*A^2*a^2*b^4*c^4*d^3*f^5*z^2 + 282*A^2*a^2*b^6*c^2*d^2*f^6*z^2 - 4
5*B^2*a^2*b^2*c^6*d^2*e^6*z^2 - 48*A^2*b*c^9*d^6*e*f*z^2 - 14*A^2*a*b^9*d*e
*f^6*z^2 - 7*A*B*b^10*d^2*e*f^5*z^2 + 2*A*B*b^10*d*e^3*f^4*z^2 - 64*A*B*a^7
*c^3*e*f^7*z^2 - 16*A*B*b^9*c*d^3*f^5*z^2 + 8*A*B*a^4*c^6*e^7*f*z^2 + 4*A*B
*b*c^9*d^6*e^2*z^2 + 2*A*B*b^6*c^4*d*e^7*z^2 - 120*A*B*a^3*c^7*d*e^7*z^2 -
16*A*B*a^3*b^7*d*f^7*z^2 + 16*A*B*a*b^9*d^2*f^6*z^2 + 8*A*B*a*c^9*d^5*e^3*z
^2 + 12*A*B*a^3*b*c^6*e^8*z^2 - 48*B^2*b^5*c^5*d^5*e*f^2*z^2 + 15*B^2*b^4*c
^6*d^5*e^2*f*z^2 - 14*B^2*b^7*c^3*d^4*e*f^3*z^2 + 4*B^2*b^9*c*d^2*e^3*f^3*z
^2 + 4*B^2*b^7*c^3*d^2*e^5*f*z^2 + 4*B^2*b^5*c^5*d^4*e^3*f*z^2 - B^2*b^6*c^
4*d^3*e^4*f*z^2 - 336*B^2*a^3*c^7*d^3*e^4*f*z^2 + 112*B^2*a^5*c^5*d*e^4*f^3
*z^2 - 112*A^2*b^3*c^7*d^5*e*f^2*z^2 + 80*B^2*a^6*c^4*d*e^2*f^5*z^2 - 48*A^
2*b^5*c^5*d^4*e*f^3*z^2 + 36*A^2*b^8*c^2*d*e^4*f^3*z^2 + 36*A^2*b^3*c^7*d^4
*e^3*f*z^2 - 28*A^2*b^7*c^3*d*e^5*f^2*z^2 + 20*A^2*b^2*c^8*d^5*e^2*f*z^2 +
16*B^2*a^2*c^8*d^5*e^2*f*z^2 - 14*A^2*b^7*c^3*d^3*e*f^4*z^2 - 14*A^2*b^4*c^
6*d^3*e^4*f*z^2 - 10*A^2*b^5*c^5*d^2*e^5*f*z^2 - 1008*A^2*a^4*c^6*d*e^4*f^3
*z^2 - 760*A^2*a^5*c^5*d*e^2*f^5*z^2 + 272*A^2*a^2*c^8*d^3*e^4*f*z^2 + 48*B
^2*a^5*b*c^4*e^5*f^3*z^2 + 36*B^2*a^6*b*c^3*e^3*f^5*z^2 + 12*B^2*a^5*b^4*c*
e^2*f^6*z^2 - 624*A^2*a^4*b*c^5*e^5*f^3*z^2 - 548*A^2*a^5*b*c^4*e^3*f^5*z^2
 + 182*A^2*a^2*b^3*c^5*e^7*f*z^2 - 180*B^2*a*b^4*c^5*d^5*f^3*z^2 + 132*B^2*
a^6*b^2*c^2*d*f^7*z^2 + 108*B^2*a^3*b^6*c*d^2*f^6*z^2 + 96*A^2*a^5*b^3*c^2*
e*f^7*z^2 + 68*A^2*a*b^6*c^3*e^6*f^2*z^2 + 58*A^2*a^3*b^6*c*e^2*f^6*z^2 - 5

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6*B^2*a*b^2*c^7*d^6*f^2*z^2 - 38*A^2*a^2*b^7*c*e^3*f^5*z^2 - 36*A^2*a*b^7*c
^2*e^5*f^3*z^2 + 20*B^2*a*b^6*c^3*d^4*f^4*z^2 - 736*A^2*a^5*b^2*c^3*d*f^7*z
^2 + 624*A^2*a^4*b^4*c^2*d*f^7*z^2 - 416*A^2*a*b^2*c^7*d^5*f^3*z^2 - 276*A^
2*a*b^4*c^5*d^4*f^4*z^2 - 196*A^2*a*b^6*c^3*d^3*f^5*z^2 + 8*B^2*a*b^4*c^5*d
^2*e^6*z^2 + 6*B^2*a*b^2*c^7*d^4*e^4*z^2 + 2*B^2*a^2*b^3*c^5*d*e^7*z^2 + 2*
B^2*a*b^3*c^6*d^3*e^5*z^2 - 18*A^2*a*b^2*c^7*d^2*e^6*z^2 - 16*A*B*b*c^9*d^7
*f*z^2 - B^2*b^10*d^2*e^2*f^4*z^2 + 48*B^2*a^7*c^3*e^2*f^6*z^2 - 36*B^2*a^6
*c^4*e^4*f^4*z^2 + 31*B^2*b^6*c^4*d^5*f^3*z^2 - 24*B^2*a^5*c^5*e^6*f^2*z^2
+ 20*B^2*b^4*c^6*d^6*f^2*z^2 - 6*A^2*b^8*c^2*e^6*f^2*z^2 + 2*B^2*b^8*c^2*d^
4*f^4*z^2 - 768*B^2*a^5*c^5*d^3*f^5*z^2 + 512*B^2*a^6*c^4*d^2*f^6*z^2 + 512
*B^2*a^4*c^6*d^4*f^4*z^2 + 232*A^2*a^5*c^5*e^4*f^4*z^2 + 188*A^2*a^4*c^6*e^
6*f^2*z^2 - 128*B^2*a^3*c^7*d^5*f^3*z^2 + 92*A^2*a^6*c^4*e^2*f^6*z^2 + 80*A
^2*b^4*c^6*d^5*f^3*z^2 + 64*A^2*b^2*c^8*d^6*f^2*z^2 + 31*A^2*b^6*c^4*d^4*f^
4*z^2 + 14*A^2*b^8*c^2*d^3*f^5*z^2 - 5*B^2*b^4*c^6*d^4*e^4*z^2 + 4*B^2*b^3*
c^7*d^5*e^3*z^2 + 2*B^2*b^5*c^5*d^3*e^5*z^2 - B^2*b^6*c^4*d^2*e^6*z^2 - B^2
*b^2*c^8*d^6*e^2*z^2 - B^2*a^4*b^6*e^2*f^6*z^2 - 1152*A^2*a^3*c^7*d^4*f^4*z
^2 + 1008*A^2*a^4*c^6*d^3*f^5*z^2 + 624*A^2*a^2*c^8*d^5*f^3*z^2 - 288*A^2*a
^5*c^5*d^2*f^6*z^2 + 56*B^2*a^3*c^7*d^2*e^6*z^2 - 10*B^2*a^2*b^8*d^2*f^6*z^
2 - 9*A^2*b^2*c^8*d^4*e^4*z^2 - 5*A^2*a^2*b^8*e^2*f^6*z^2 - 4*B^2*a^2*c^8*d
^4*e^4*z^2 + 3*A^2*b^4*c^6*d^2*e^6*z^2 - 2*A^2*b^3*c^7*d^3*e^5*z^2 - 36*A^2
*a^2*c^8*d^2*e^6*z^2 - 48*A^2*a^6*b^2*c^2*f^8*z^2 - 45*A^2*a^2*b^2*c^6*e^8*
z^2 + 4*A^2*b^10*d*e^2*f^5*z^2 + 4*B^2*b^2*c^8*d^7*f*z^2 + 4*A^2*b^9*c*e^5*
f^3*z^2 + 4*A^2*b^7*c^3*e^7*f*z^2 - 128*B^2*a^7*c^3*d*f^7*z^2 - 160*A^2*a*c
^9*d^6*f^2*z^2 - 112*A^2*a^6*c^4*d*f^7*z^2 + 12*A^2*b*c^9*d^5*e^3*z^2 + 4*A
^2*a*b^9*e^3*f^5*z^2 + 3*B^2*a^4*b^6*d*f^7*z^2 + 2*A^2*a^3*b^7*e*f^7*z^2 -
24*A^2*a*c^9*d^4*e^4*z^2 + 14*A^2*a^2*b^8*d*f^7*z^2 + 12*A^2*a^5*b^4*c*f^8*
z^2 + 12*A^2*a*b^4*c^5*e^8*z^2 + A*B*a^4*b^6*e*f^7*z^2 + B^2*a^2*b^8*d*e^2*
f^5*z^2 + 16*A^2*c^10*d^7*f*z^2 + 3*B^2*b^10*d^3*f^5*z^2 - A^2*b^10*e^4*f^4
*z^2 - 4*A^2*c^10*d^6*e^2*z^2 - A^2*b^10*d^2*f^6*z^2 + 64*A^2*a^7*c^3*f^8*z
^2 - 4*B^2*a^4*c^6*e^8*z^2 - A^2*b^6*c^4*e^8*z^2 + 48*A^2*a^3*c^7*e^8*z^2 -
 A^2*a^4*b^6*f^8*z^2 + 720*A^2*B*a*b^2*c^5*d^2*e^2*f^3*z - 600*A^2*B*a^2*b^
2*c^4*d*e^2*f^4*z + 576*A*B^2*a^2*b^2*c^4*d^2*e*f^4*z + 348*A*B^2*a*b^2*c^5
*d^2*e^3*f^2*z - 336*A*B^2*a^2*b*c^5*d^2*e^2*f^3*z - 260*A*B^2*a*b^3*c^4*d^
2*e^2*f^3*z - 240*A*B^2*a^2*b^2*c^4*d*e^3*f^3*z + 196*A*B^2*a^2*b^3*c^3*d*e
^2*f^4*z + 172*A^2*B*a*b*c^6*d*e^5*f*z + 20*A*B^2*a*b^6*c*d*e*f^5*z - 912*A
^2*B*a^2*b*c^5*d^2*e*f^4*z - 644*A^2*B*a*b*c^6*d^2*e^3*f^2*z - 432*A*B^2*a*
b^2*c^5*d^3*e*f^3*z + 372*A^2*B*a^2*b*c^5*d*e^3*f^3*z - 330*A^2*B*a*b^2*c^5
*d*e^4*f^2*z + 312*A*B^2*a*b*c^6*d^3*e^2*f^2*z - 208*A*B^2*a^3*b^2*c^3*d*e*
f^5*z + 192*A^2*B*a^2*b^3*c^3*d*e*f^5*z + 172*A^2*B*a*b^3*c^4*d*e^3*f^3*z +
 108*A*B^2*a^2*b*c^5*d*e^4*f^2*z + 104*A*B^2*a^3*b*c^4*d*e^2*f^4*z - 80*A^2
*B*a*b^3*c^4*d^2*e*f^4*z + 68*A^2*B*a*b^4*c^3*d*e^2*f^4*z - 60*A*B^2*a*b^5*
c^2*d*e^2*f^4*z + 58*A*B^2*a*b^3*c^4*d*e^4*f^2*z - 36*A*B^2*a*b^4*c^3*d^2*e
*f^4*z - 24*A*B^2*a^2*b^4*c^2*d*e*f^5*z + 24*A*B^2*a*b^4*c^3*d*e^3*f^3*z +
592*A^2*B*a*b*c^6*d^3*e*f^3*z + 240*A^2*B*a^3*b*c^4*d*e*f^5*z - 132*A*B^2*a
*b*c^6*d^2*e^4*f*z - 60*A*B^2*a*b^2*c^5*d*e^5*f*z - 48*A^2*B*a*b^5*c^2*d*e*
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f^5*z + 20*B^3*a*b*c^6*d^3*e^3*f*z + 16*B^3*a^4*b*c^3*d*e*f^5*z - 16*B^3*a*
b*c^6*d^4*e*f^2*z + 12*B^3*a^2*b*c^5*d*e^5*f*z + 320*A^3*a*b*c^6*d*e^4*f^2*
z + 40*A^3*a*b^4*c^3*d*e*f^5*z - 48*A^2*B*b*c^7*d^4*e*f^2*z - 44*A^2*B*b^3*
c^5*d*e^5*f*z - 20*A*B^2*b*c^7*d^4*e^2*f*z + 14*A*B^2*b^4*c^4*d*e^5*f*z + 1
2*A^2*B*b*c^7*d^3*e^3*f*z + 4*A*B^2*b^7*c*d*e^2*f^4*z + 160*A*B^2*a^4*c^4*d
*e*f^5*z + 152*A^2*B*a*c^7*d^2*e^4*f*z - 40*A*B^2*a*c^7*d^3*e^3*f*z + 32*A*
B^2*a*c^7*d^4*e*f^2*z - 16*A*B^2*a^2*c^6*d*e^5*f*z + 128*A^2*B*a^4*b*c^3*e*
f^6*z + 42*A^2*B*a*b^2*c^5*e^6*f*z + 24*A^2*B*a^2*b^5*c*e*f^6*z - 12*A*B^2*
a^3*b^4*c*e*f^6*z - 12*A*B^2*a^2*b*c^5*e^6*f*z - 10*A^2*B*a*b^6*c*e^2*f^5*z
 - 160*A*B^2*a*b*c^6*d^4*f^3*z + 112*A*B^2*a^4*b*c^3*d*f^6*z - 24*A*B^2*a^2
*b^5*c*d*f^6*z - 84*B^3*a*b^2*c^5*d^3*e^2*f^2*z - 80*B^3*a^2*b^3*c^3*d^2*e*
f^4*z - 60*B^3*a^2*b*c^5*d^2*e^3*f^2*z - 20*B^3*a^3*b^2*c^3*d*e^2*f^4*z - 2
0*B^3*a*b^3*c^4*d^2*e^3*f^2*z - 9*B^3*a^2*b^2*c^4*d*e^4*f^2*z - 8*B^3*a*b^4
*c^3*d^2*e^2*f^3*z + 6*B^3*a^2*b^4*c^2*d*e^2*f^4*z - 4*B^3*a^2*b^3*c^3*d*e^
3*f^3*z - 216*A^2*B*b^4*c^4*d^2*e^2*f^3*z + 196*A^2*B*b^3*c^5*d^2*e^3*f^2*z
 - 108*A*B^2*b^3*c^5*d^3*e^2*f^2*z - 94*A*B^2*b^4*c^4*d^2*e^3*f^2*z + 88*A^
2*B*b^2*c^6*d^3*e^2*f^2*z + 80*A*B^2*b^5*c^3*d^2*e^2*f^3*z + 360*A^2*B*a^2*
c^6*d^2*e^2*f^3*z + 8*A*B^2*a^2*c^6*d^2*e^3*f^2*z + 153*A^2*B*a^2*b^2*c^4*e
^4*f^3*z - 144*A^2*B*a^2*b^3*c^3*e^3*f^4*z + 80*A^2*B*a^3*b^2*c^3*e^2*f^5*z
 + 36*A*B^2*a^3*b^2*c^3*e^3*f^4*z + 12*A^2*B*a^2*b^4*c^2*e^2*f^5*z + 12*A*B
^2*a^3*b^3*c^2*e^2*f^5*z + 9*A*B^2*a^2*b^2*c^4*e^5*f^2*z - 6*A*B^2*a^2*b^4*
c^2*e^3*f^4*z + 4*A*B^2*a^2*b^3*c^3*e^4*f^3*z + 480*A^2*B*a^2*b^2*c^4*d^2*f
^5*z - 176*A*B^2*a^2*b^3*c^3*d^2*f^5*z - 10*A^2*B*a*b^6*c*d*f^6*z + 16*A*B^
2*a*b*c^6*d*e^6*z + 80*B^3*a*b^3*c^4*d^3*e*f^3*z - 48*B^3*a^3*b*c^4*d^2*e*f
^4*z + 48*B^3*a^2*b*c^5*d^3*e*f^3*z + 44*B^3*a^3*b*c^4*d*e^3*f^3*z + 24*B^3
*a*b^5*c^2*d^2*e*f^4*z + 18*B^3*a*b^2*c^5*d^2*e^4*f*z + 696*A^3*a^2*b*c^5*d
*e^2*f^4*z - 504*A^3*a*b*c^6*d^2*e^2*f^3*z - 192*A^3*a*b^2*c^5*d*e^3*f^3*z
- 144*A^3*a^2*b^2*c^4*d*e*f^5*z + 96*A^3*a*b^2*c^5*d^2*e*f^4*z - 72*A^3*a*b
^3*c^4*d*e^2*f^4*z - 208*A^2*B*b^3*c^5*d^3*e*f^3*z + 152*A*B^2*b^4*c^4*d^3*
e*f^3*z + 80*A^2*B*b^5*c^3*d^2*e*f^4*z + 75*A^2*B*b^4*c^4*d*e^4*f^2*z - 59*
A^2*B*b^2*c^6*d^2*e^4*f*z - 52*A^2*B*b^5*c^3*d*e^3*f^3*z + 42*A*B^2*b^3*c^5
*d^2*e^4*f*z - 21*A*B^2*b^6*c^2*d^2*e*f^4*z - 16*A*B^2*b^5*c^3*d*e^4*f^2*z
+ 16*A*B^2*b^2*c^6*d^4*e*f^2*z + 16*A*B^2*b^2*c^6*d^3*e^3*f*z + 11*A^2*B*b^
6*c^2*d*e^2*f^4*z + 4*A*B^2*b^6*c^2*d*e^3*f^3*z - 256*A^2*B*a*c^7*d^3*e^2*f
^2*z - 96*A*B^2*a^3*c^5*d^2*e*f^4*z - 36*A^2*B*a^2*c^6*d*e^4*f^2*z - 32*A^2
*B*a^3*c^5*d*e^2*f^4*z - 32*A*B^2*a^2*c^6*d^3*e*f^3*z + 8*A*B^2*a^3*c^5*d*e
^3*f^3*z - 96*A^2*B*a^3*b^3*c^2*e*f^6*z + 68*A^2*B*a^3*b*c^4*e^3*f^4*z - 60
*A*B^2*a^4*b*c^3*e^2*f^5*z - 60*A*B^2*a^3*b*c^4*e^4*f^3*z + 48*A*B^2*a^4*b^
2*c^2*e*f^6*z - 38*A^2*B*a*b^3*c^4*e^5*f^2*z - 36*A^2*B*a^2*b*c^5*e^5*f^2*z
 + 36*A^2*B*a*b^5*c^2*e^3*f^4*z - 16*A^2*B*a*b^4*c^3*e^4*f^3*z + 384*A*B^2*
a^2*b*c^5*d^3*f^4*z - 352*A*B^2*a^3*b*c^4*d^2*f^5*z - 288*A^2*B*a*b^2*c^5*d
^3*f^4*z - 160*A^2*B*a^3*b^2*c^3*d*f^6*z - 148*A^2*B*a*b^4*c^3*d^2*f^5*z +
112*A*B^2*a*b^3*c^4*d^3*f^4*z + 72*A^2*B*a^2*b^4*c^2*d*f^6*z + 72*A*B^2*a*b
^5*c^2*d^2*f^5*z + 48*A*B^2*a^3*b^3*c^2*d*f^6*z + 102*B^3*a^2*b^2*c^4*d^2*e
^2*f^3*z - 32*B^3*b^5*c^3*d^3*e*f^3*z - 8*B^3*b^3*c^5*d^3*e^3*f*z - 7*B^3*b

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^4*c^4*d^2*e^4*f*z + 5*B^3*b^2*c^6*d^4*e^2*f*z + 80*A^3*b^2*c^6*d^3*e*f^3*z
 - 74*A^3*b^3*c^5*d*e^4*f^2*z - 64*A^3*b^4*c^4*d^2*e*f^4*z + 60*A^3*b^4*c^4
*d*e^3*f^3*z - 48*B^3*a^4*c^4*d*e^2*f^4*z - 24*B^3*a^3*c^5*d*e^4*f^2*z + 20
*B^3*a^2*c^6*d^2*e^4*f*z - 16*A^3*b^5*c^3*d*e^2*f^4*z + 8*A^3*b*c^7*d^3*e^2
*f^2*z + 480*A^3*a^2*c^6*d^2*e*f^4*z - 392*A^3*a^2*c^6*d*e^3*f^3*z + 280*A^
3*a*c^7*d^2*e^3*f^2*z - 4*B^3*a^4*b*c^3*e^3*f^4*z - 200*A^3*a^3*b*c^4*e^2*f
^5*z - 144*A^3*a^2*b*c^5*e^4*f^3*z + 48*B^3*a*b^2*c^5*d^4*f^3*z + 42*A^3*a*
b^2*c^5*e^5*f^2*z - 36*B^3*a^4*b^2*c^2*d*f^6*z - 32*A^3*a^3*b^2*c^3*e*f^6*z
 - 24*A^3*a^2*b^4*c^2*e*f^6*z - 24*A^3*a*b^5*c^2*e^2*f^5*z + 10*A^3*a*b^3*c
^4*e^4*f^3*z - 4*B^3*a*b^4*c^3*d^3*f^4*z - 4*A^3*a*b^4*c^3*e^3*f^4*z - 480*
A^3*a^2*b*c^5*d^2*f^5*z - 160*A^3*a^2*b^3*c^3*d*f^6*z + 128*A^3*a*b^3*c^4*d
^2*f^5*z + 8*A^2*B*b^5*c^3*e^5*f^2*z - 2*A^2*B*b^6*c^2*e^4*f^3*z + 112*A^2*
B*b^4*c^4*d^3*f^4*z - 92*A^2*B*a^4*c^4*e^2*f^5*z - 64*A^2*B*a^3*c^5*e^4*f^3
*z - 64*A*B^2*b^5*c^3*d^3*f^4*z + 24*A*B^2*a^4*c^4*e^3*f^4*z + 24*A*B^2*a^3
*c^5*e^5*f^2*z + 16*A^2*B*b^2*c^6*d^4*f^3*z + 16*A*B^2*b^3*c^5*d^4*f^3*z -
A^2*B*b^6*c^2*d^2*f^5*z + 448*A^2*B*a^3*c^5*d^2*f^5*z - 352*A^2*B*a^2*c^6*d
^3*f^4*z - 5*A*B^2*b^2*c^6*d^2*e^5*z - 48*A^2*B*a^4*b^2*c^2*f^7*z - 2*B^3*b
^7*c*d^2*e*f^4*z + 34*A^3*b^2*c^6*d*e^5*f*z + 16*A^3*b*c^7*d^2*e^4*f*z + 2*
A^3*b^6*c^2*d*e*f^5*z - 416*A^3*a^3*c^5*d*e*f^5*z - 224*A^3*a*c^7*d^3*e*f^3
*z + 12*B^3*a^3*b^4*c*d*f^6*z - 10*B^3*a*b^6*c*d^2*f^5*z + 416*A^3*a^3*b*c^
4*d*f^6*z + 224*A^3*a*b*c^6*d^3*f^4*z + 24*A^3*a*b^5*c^2*d*f^6*z - 4*B^3*a*
b*c^6*d^2*e^5*z + 20*A^2*B*c^8*d^4*e^2*f*z - 7*A^2*B*b^4*c^4*e^6*f*z - 2*A^
2*B*b^7*c*e^3*f^4*z - 64*A*B^2*a^5*c^3*e*f^6*z + 16*A*B^2*b*c^7*d^5*f^2*z -
 8*A^2*B*a^2*c^6*e^6*f*z - 2*A*B^2*b^7*c*d^2*f^5*z - 272*A^2*B*a^4*c^4*d*f^
6*z + 128*A^2*B*a*c^7*d^4*f^3*z + 9*A^2*B*b^2*c^6*d*e^6*z - 4*A*B^2*b^3*c^5
*d*e^6*z + 4*A*B^2*b*c^7*d^3*e^4*z + 8*A*B^2*a*c^7*d^2*e^5*z + 12*A^2*B*a^3
*b^4*c*f^7*z + 30*B^3*b^4*c^4*d^3*e^2*f^2*z + 8*B^3*b^5*c^3*d^2*e^3*f^2*z -
 2*B^3*b^6*c^2*d^2*e^2*f^3*z + 152*A^3*b^3*c^5*d^2*e^2*f^3*z - 108*A^3*b^2*
c^6*d^2*e^3*f^2*z + 48*B^3*a^3*c^5*d^2*e^2*f^3*z - 16*B^3*a^2*c^6*d^3*e^2*f
^2*z - 3*B^3*a^4*b^2*c^2*e^2*f^5*z - 120*B^3*a^2*b^2*c^4*d^3*f^4*z + 112*B^
3*a^3*b^2*c^3*d^2*f^5*z + 112*A^3*a^2*b^3*c^3*e^2*f^5*z + 12*A^3*a^2*b^2*c^
4*e^3*f^4*z - 120*A^3*a*c^7*d*e^5*f*z - 52*A^3*a*b*c^6*e^6*f*z + 10*A^3*a*b
^6*c*e*f^6*z - 2*A*B^2*b^8*d*e*f^5*z - 2*A^2*B*a*b^7*e*f^6*z - 24*A^2*B*a*c
^7*d*e^6*z + 2*A*B^2*a*b^7*d*f^6*z - 12*A^2*B*a*b*c^6*e^7*z - 2*A^3*b^7*c*d
*f^6*z - 4*A^3*b*c^7*d*e^6*z + 16*B^3*a^5*c^3*e^2*f^5*z + 11*B^3*b^6*c^2*d^
3*f^4*z - 11*A^3*b^4*c^4*e^5*f^2*z - 8*B^3*b^4*c^4*d^4*f^3*z - 4*B^3*b^2*c^
6*d^5*f^2*z + 4*B^3*a^4*c^4*e^4*f^3*z + 4*A^3*b^5*c^3*e^4*f^3*z - A^3*b^6*c
^2*e^3*f^4*z + 136*A^3*a^3*c^5*e^3*f^4*z + 68*A^3*a^2*c^6*e^5*f^2*z - 64*A^
3*b^3*c^5*d^3*f^4*z + 2*B^3*b^3*c^5*d^2*e^5*z - B^3*b^2*c^6*d^3*e^4*z + 96*
A^3*a^3*b^3*c^2*f^7*z + A*B^2*a^2*b^6*e*f^6*z + 32*A^3*c^8*d^4*e*f^2*z - 24
*A^3*c^8*d^3*e^3*f*z + 10*A^3*b^3*c^5*e^6*f*z + 2*A^3*b^7*c*e^2*f^5*z + 128
*A^3*a^4*c^4*e*f^6*z - 32*A^3*b*c^7*d^4*f^3*z - 4*B^3*a^2*c^6*d*e^6*z - B^3
*a^2*b^6*d*f^6*z - 128*A^3*a^4*b*c^3*f^7*z - 24*A^3*a^2*b^5*c*f^7*z - 16*A^
2*B*c^8*d^5*f^2*z - 4*A^2*B*c^8*d^3*e^4*z + 64*A^2*B*a^5*c^3*f^7*z + 2*A^2*
B*b^3*c^5*e^7*z + 4*A*B^2*a^2*c^6*e^7*z - A^2*B*a^2*b^6*f^7*z + 4*A^3*c^8*d
```

^2*e^5*z - 3*A^3*b^2*c^6*e^7*z + A^2*B*b^8*d*f^6*z - A^3*b^8*e*f^6*z + 16*A
^3*a*c^7*e^7*z + 2*A^3*a*b^7*f^7*z + A^2*B*b^8*e^2*f^5*z + B^3*b^8*d^2*f^5*
z - 48*A^2*B^2*a*b*c^4*d*e*f^4 + 28*A*B^3*a*b^2*c^3*d*e*f^4 - 16*A*B^3*a*b*
c^4*d*e^2*f^3 + 16*A^3*B*a*c^5*d*e*f^4 + 32*A^3*B*a*b*c^4*d*f^5 + 12*A^2*B^
2*b^3*c^3*d*e*f^4 + 5*A*B^3*b^2*c^4*d^2*e*f^3 + 4*A*B^3*b^3*c^3*d*e^2*f^3 +
 24*A^2*B^2*a*c^5*d*e^2*f^3 + 24*A^2*B^2*a^2*b*c^3*e*f^5 + 12*A^2*B^2*a*b*c
^4*e^3*f^3 - 6*A^2*B^2*a*b^3*c^2*e*f^5 + 4*A*B^3*a^2*b*c^3*e^2*f^4 + 3*A*B^
3*a^2*b^2*c^2*e*f^5 - 18*A^2*B^2*a*b^2*c^3*d*f^5 - 4*B^4*a^2*b*c^3*d*e*f^4
+ 4*B^4*a*b*c^4*d^2*e*f^3 - 6*A*B^3*b^4*c^2*d*e*f^4 + 4*A^3*B*b*c^5*d*e^2*f
^3 - 2*A^3*B*b^2*c^4*d*e*f^4 - 8*A*B^3*a^2*c^4*d*e*f^4 - 8*A*B^3*a*c^5*d^2*
e*f^3 + 26*A^3*B*a*b^2*c^3*e*f^5 + 8*A^3*B*a*b*c^4*e^2*f^4 + 32*A*B^3*a*b*c
^4*d^2*f^4 - 28*A*B^3*a^2*b*c^3*d*f^5 + 6*A*B^3*a*b^3*c^2*d*f^5 - 9*A^2*B^2
*b^2*c^4*d*e^2*f^3 - 18*A^2*B^2*a*b^2*c^3*e^2*f^4 - 4*A^3*B*c^6*d^2*e*f^3 -
 3*A^3*B*b^4*c^2*e*f^5 - 44*A^3*B*a^2*c^4*e*f^5 - 16*A^3*B*a*c^5*e^3*f^3 -
16*A*B^3*a^3*c^3*e*f^5 - 10*A^3*B*b^3*c^3*d*f^5 - 4*A^3*B*b*c^5*d^2*f^4 - 4
*A*B^3*b*c^5*d^3*f^3 - 28*A^3*B*a^2*b*c^3*f^6 + 6*A^3*B*a*b^3*c^2*f^6 - 4*A
^4*b*c^5*d*e*f^4 - 20*A^4*a*b*c^4*e*f^5 + 3*A^2*B^2*b^4*c^2*e^2*f^4 - 2*A^2
*B^2*b^3*c^3*e^3*f^3 + 12*A^2*B^2*a^2*c^4*e^2*f^4 + 9*A^2*B^2*b^2*c^4*d^2*f
^4 - 3*A^2*B^2*a^2*b^2*c^2*f^6 - 2*B^4*b^3*c^3*d^2*e*f^3 + 4*B^4*a^2*c^4*d*
e^2*f^3 - 10*B^4*a*b^2*c^3*d^2*f^4 - 3*B^4*a^2*b^2*c^2*d*f^5 + 3*A^3*B*b^2*
c^4*e^3*f^3 - 2*A^3*B*b^3*c^3*e^2*f^4 - 10*A*B^3*b^3*c^3*d^2*f^4 - 4*A*B^3*
a^2*c^4*e^3*f^3 + 3*A^2*B^2*b^4*c^2*d*f^5 + 36*A^2*B^2*a^2*c^4*d*f^5 - 24*A
^2*B^2*a*c^5*d^2*f^4 + 4*A^2*B^2*c^6*d^3*f^3 + 16*A^2*B^2*a^3*c^3*f^6 + 4*A
^4*b^3*c^3*e*f^5 + 16*B^4*a^3*c^3*d*f^5 + 16*A^4*a*c^5*e^2*f^4 + 8*A^4*b^2*
c^4*d*f^5 - 8*A^4*a*b^2*c^3*f^6 - 24*A^4*a*c^5*d*f^5 + 3*B^4*b^4*c^2*d^2*f^
4 - 3*A^4*b^2*c^4*e^2*f^4 + 4*A^4*c^6*d^2*f^4 + 36*A^4*a^2*c^4*f^6 + B^4*b^
2*c^4*d^3*f^3, z, k)*((64*a^9*c^4*e*f^8 - 64*a^6*c^7*e^7*f^2 - 64*a^7*c^6*e
^5*f^4 + 64*a^8*c^5*e^3*f^6 + 4*b^5*c^8*d^7*f^2 + 4*b^7*c^6*d^6*f^3 - 4*b^9
*c^4*d^5*f^4 - 4*b^11*c^2*d^4*f^5 - 612*a^2*b^5*c^6*d^5*f^4 - 712*a^2*b^7*c
^4*d^4*f^5 - 132*a^2*b^9*c^2*d^3*f^6 + 1696*a^3*b^3*c^7*d^5*f^4 + 2736*a^3*
b^5*c^5*d^4*f^5 + 896*a^3*b^7*c^3*d^3*f^6 - 5120*a^4*b^3*c^6*d^4*f^5 - 3140
*a^4*b^5*c^4*d^3*f^6 - 220*a^4*b^7*c^2*d^2*f^7 + 5664*a^5*b^3*c^5*d^3*f^6 +
 1128*a^5*b^5*c^3*d^2*f^7 - 2560*a^6*b^3*c^4*d^2*f^7 + 4*a^3*b^6*c^4*e^7*f^
2 - 6*a^3*b^7*c^3*e^6*f^3 + 4*a^3*b^8*c^2*e^5*f^4 - 36*a^4*b^4*c^5*e^7*f^2
+ 57*a^4*b^5*c^4*e^6*f^3 - 37*a^4*b^6*c^3*e^5*f^4 + 7*a^4*b^7*c^2*e^4*f^5 +
 96*a^5*b^2*c^6*e^7*f^2 - 168*a^5*b^3*c^5*e^6*f^3 + 100*a^5*b^4*c^4*e^5*f^4
 - 3*a^5*b^5*c^3*e^4*f^5 - 10*a^5*b^6*c^2*e^3*f^6 - 48*a^6*b^2*c^5*e^5*f^4
- 56*a^6*b^3*c^4*e^4*f^5 + 36*a^6*b^4*c^3*e^3*f^6 - 13*a^6*b^5*c^2*e^2*f^7
- 64*a^7*b^2*c^4*e^3*f^6 + 56*a^7*b^3*c^3*e^2*f^7 - 1472*a^4*c^9*d^4*e^3*f^
2 - 1088*a^5*c^8*d^2*e^5*f^2 + 3584*a^5*c^8*d^3*e^3*f^3 - 3200*a^6*c^7*d^2*
e^3*f^4 - 2*b^7*c^6*d^5*e^2*f^2 - b^8*c^5*d^4*e^3*f^2 + 4*b^9*c^4*d^3*e^4*f
^2 - 5*b^9*c^4*d^4*e^2*f^3 - 6*b^10*c^3*d^3*e^3*f^3 + 4*b^11*c^2*d^3*e^2*f^
4 + 8*a*b^11*c*d^3*f^6 + 8*a^5*b^7*c*d*f^8 - 448*a^8*b*c^4*d*f^8 - 16*a^5*b
*c^7*e^8*f - a^6*b^6*c*e*f^8 + 128*a^5*c^8*d*e^7*f + 128*a^8*c^5*d*e*f^7 -
b^12*c*d^3*e*f^5 - 32*a*b^3*c^9*d^7*f^2 - 24*a*b^5*c^7*d^6*f^3 + 88*a*b^7*c

^5*d^5*f^4 + 88*a*b^9*c^3*d^4*f^5 + 64*a^2*b*c^10*d^7*f^2 + 128*a^3*b*c^9*d
^6*f^3 + 16*a^3*b^9*c*d^2*f^7 - 1600*a^4*b*c^8*d^5*f^4 + 3840*a^5*b*c^7*d^4
*f^5 - 4160*a^6*b*c^6*d^3*f^6 - 92*a^6*b^5*c^2*d*f^8 + 2176*a^7*b*c^5*d^2*f
^7 + 352*a^7*b^3*c^3*d*f^8 - a^3*b^5*c^5*e^8*f - a^3*b^9*c*e^4*f^5 + 8*a^4*
b^3*c^6*e^8*f + a^4*b^8*c*e^3*f^6 + a^5*b^7*c*e^2*f^7 + 144*a^6*b*c^6*e^6*f
^3 + 80*a^7*b*c^5*e^4*f^5 + 12*a^7*b^4*c^2*e*f^8 - 80*a^8*b*c^4*e^2*f^7 - 4
8*a^8*b^2*c^3*e*f^8 + 128*a^3*c^10*d^5*e^3*f - 448*a^3*c^10*d^6*e*f^2 + 256
*a^4*c^9*d^3*e^5*f + 2176*a^4*c^9*d^5*e*f^3 - 4160*a^5*c^8*d^4*e*f^4 + 896*
a^6*c^7*d*e^5*f^3 + 3840*a^6*c^7*d^3*e*f^5 + 896*a^7*c^6*d*e^3*f^5 - 1600*a
^7*c^6*d^2*e*f^6 - b^5*c^8*d^6*e^2*f + b^6*c^7*d^5*e^3*f - 5*b^6*c^7*d^6*e*
f^2 + b^7*c^6*d^4*e^4*f - b^8*c^5*d^3*e^5*f + 5*b^8*c^5*d^5*e*f^3 + 9*b^10*
c^3*d^4*e*f^4 + 8*a*b^3*c^9*d^6*e^2*f + 12*a*b^4*c^8*d^6*e*f^2 - 27*a*b^5*c
^7*d^4*e^4*f + 18*a*b^6*c^6*d^3*e^5*f - 154*a*b^6*c^6*d^5*e*f^3 + a*b^7*c^5
*d^2*e^6*f - 200*a*b^8*c^4*d^4*e*f^4 - 2*a*b^10*c^2*d^3*e*f^5 + a*b^11*c*d^
2*e^2*f^5 - 16*a^2*b*c^10*d^6*e^2*f + a^2*b^6*c^5*d*e^7*f + a^2*b^10*c*d*e^
3*f^5 - 19*a^2*b^10*c*d^2*e*f^6 - 304*a^3*b*c^9*d^4*e^4*f + 10*a^3*b^9*c*d*
e^2*f^6 - 304*a^4*b*c^8*d^2*e^6*f - 48*a^4*b^2*c^7*d*e^7*f - 160*a^5*b*c^7*
d*e^6*f^2 + 214*a^5*b^6*c^2*d*e*f^7 - 2048*a^6*b*c^6*d*e^4*f^4 - 792*a^6*b^
4*c^3*d*e*f^7 - 1824*a^7*b*c^5*d*e^2*f^6 + 928*a^7*b^2*c^4*d*e*f^7 + 78*a*b
^5*c^7*d^5*e^2*f^2 + 10*a*b^6*c^6*d^4*e^3*f^2 - 68*a*b^7*c^5*d^3*e^4*f^2 +
129*a*b^7*c^5*d^4*e^2*f^3 - 4*a*b^8*c^4*d^2*e^5*f^2 + 96*a*b^8*c^4*d^3*e^3*
f^3 + 6*a*b^9*c^3*d^2*e^4*f^3 - 52*a*b^9*c^3*d^3*e^2*f^4 - 4*a*b^10*c^2*d^2
*e^3*f^4 - 48*a^2*b^2*c^9*d^5*e^3*f + 144*a^2*b^2*c^9*d^6*e*f^2 + 168*a^2*b
^3*c^8*d^4*e^4*f - 80*a^2*b^4*c^7*d^3*e^5*f + 1128*a^2*b^4*c^7*d^5*e*f^3 -
27*a^2*b^5*c^6*d^2*e^6*f + 1481*a^2*b^6*c^5*d^4*e*f^4 - 4*a^2*b^7*c^4*d*e^6
*f^2 + 6*a^2*b^8*c^3*d*e^5*f^3 + 126*a^2*b^8*c^3*d^3*e*f^5 - 4*a^2*b^9*c^2*
d*e^4*f^4 + 992*a^3*b*c^9*d^5*e^2*f^2 + 32*a^3*b^2*c^8*d^3*e^5*f - 2912*a^3
*b^2*c^8*d^5*e*f^3 + 168*a^3*b^3*c^7*d^2*e^6*f - 4812*a^3*b^4*c^6*d^4*e*f^4
 + 22*a^3*b^5*c^5*d*e^6*f^2 - 68*a^3*b^6*c^4*d*e^5*f^3 - 860*a^3*b^6*c^4*d^
3*e*f^5 + 80*a^3*b^7*c^3*d*e^4*f^4 - 44*a^3*b^8*c^2*d*e^3*f^5 + 232*a^3*b^8
*c^2*d^2*e*f^6 + 1280*a^4*b*c^8*d^3*e^4*f^2 - 816*a^4*b*c^8*d^4*e^2*f^3 + 7
088*a^4*b^2*c^7*d^4*e*f^4 + 16*a^4*b^3*c^6*d*e^6*f^2 + 312*a^4*b^4*c^5*d*e^
5*f^3 + 2864*a^4*b^4*c^5*d^3*e*f^5 - 576*a^4*b^5*c^4*d*e^4*f^4 + 393*a^4*b^
6*c^3*d*e^3*f^5 - 995*a^4*b^6*c^3*d^2*e*f^6 - 78*a^4*b^7*c^2*d*e^2*f^6 + 99
2*a^5*b*c^7*d^2*e^4*f^3 - 3264*a^5*b*c^7*d^3*e^2*f^4 - 768*a^5*b^2*c^6*d*e^
5*f^3 - 5184*a^5*b^2*c^6*d^3*e*f^5 + 1792*a^5*b^3*c^5*d*e^4*f^4 - 1232*a^5*
b^4*c^4*d*e^3*f^5 + 1588*a^5*b^4*c^4*d^2*e*f^6 + 30*a^5*b^5*c^3*d*e^2*f^6 +
 5008*a^6*b*c^6*d^2*e^2*f^5 + 976*a^6*b^2*c^5*d*e^3*f^5 - 16*a^6*b^2*c^5*d^
2*e*f^6 + 944*a^6*b^3*c^4*d*e^2*f^6 - 19*a^4*b^8*c*d*e*f^7 - 528*a^2*b^3*c^
8*d^5*e^2*f^2 - 124*a^2*b^4*c^7*d^4*e^3*f^2 + 432*a^2*b^5*c^6*d^3*e^4*f^2 -
 843*a^2*b^5*c^6*d^4*e^2*f^3 + 83*a^2*b^6*c^5*d^2*e^5*f^2 - 722*a^2*b^6*c^5
*d^3*e^3*f^3 - 80*a^2*b^7*c^4*d^2*e^4*f^3 + 376*a^2*b^7*c^4*d^3*e^2*f^4 + 4
3*a^2*b^9*c^2*d^2*e^2*f^5 + 768*a^3*b^2*c^8*d^4*e^3*f^2 - 1216*a^3*b^3*c^7*
d^3*e^4*f^2 + 1832*a^3*b^3*c^7*d^4*e^2*f^3 - 540*a^3*b^4*c^6*d^2*e^5*f^2 +
2928*a^3*b^4*c^6*d^3*e^3*f^3 + 414*a^3*b^5*c^5*d^2*e^4*f^3 - 1740*a^3*b^5*c

^5*d^3*e^2*f^4 + 344*a^3*b^6*c^4*d^2*e^3*f^4 - 634*a^3*b^7*c^3*d^2*e^2*f^5 + 1360*a^4*b^2*c^7*d^2*e^5*f^2 - 5664*a^4*b^2*c^7*d^3*e^3*f^3 - 1008*a^4*b^3*c^6*d^2*e^4*f^3 + 4064*a^4*b^3*c^6*d^3*e^2*f^4 - 1928*a^4*b^4*c^5*d^2*e^3*f^4 + 3065*a^4*b^5*c^4*d^2*e^2*f^5 + 4032*a^5*b^2*c^6*d^2*e^3*f^4 - 6376*a^5*b^3*c^5*d^2*e^2*f^5)/(16*a^2*c^6*d^4 + a^4*b^4*f^4 + 16*a^4*c^4*e^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 2*a^3*b^5*e*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 - 2*b^5*c^3*d^3*e + 2*b^6*c^2*d^3*f + a^2*b^4*c^2*e^4 - 8*a^3*b^2*c^3*e^4 + 32*a^3*c^5*d^2*e^2 + a^2*b^6*e^2*f^2 + 96*a^4*c^4*d^2*f^2 + b^6*c^2*d^2*e^2 + 32*a^5*c^3*e^2*f^2 - 2*a*b^7*d*e*f^2 - 2*b^7*c*d^2*e*f + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 + 16*a*b^3*c^4*d^3*e - 2*a*b^5*c^2*d*e^3 - 32*a^2*b*c^5*d^3*e - 32*a^3*b*c^4*d*e^3 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 - 2*a^2*b^5*c*e^3*f - 32*a^4*b*c^3*e^3*f + 16*a^4*b^3*c*e*f^3 - 32*a^5*b*c^2*e*f^3 - 64*a^4*c^4*d*e^2*f - 6*a*b^4*c^3*d^2*e^2 + 16*a^2*b^3*c^3*d*e^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3 + 16*a^3*b^3*c^2*e^3*f - 6*a^3*b^4*c*e^2*f^2 - 48*a^2*b^3*c^3*d^2*e*f - 36*a^2*b^4*c^2*d*e^2*f + 96*a^3*b^2*c^3*d*e^2*f - 48*a^3*b^3*c^2*d*e*f^2 + 4*a*b^6*c*d*e^2*f + 18*a*b^5*c^2*d^2*e*f + 18*a^2*b^5*c*d*e*f^2 + 32*a^3*b*c^4*d^2*e*f + 32*a^4*b*c^3*d*e*f^2) + (x*(128*a^9*c^4*f^9 - 2*a^6*b^6*c*f^9 - 640*a^8*c^5*d*f^8 + 96*a^5*c^8*e^8*f + 6*b^12*c*d^3*f^6 + 24*a^7*b^4*c^2*f^9 - 96*a^8*b^2*c^3*f^9 + 128*a^2*c^11*d^7*f^2 - 640*a^3*c^10*d^6*f^3 + 1152*a^4*c^9*d^5*f^4 - 640*a^5*c^8*d^4*f^5 - 640*a^6*c^7*d^3*f^6 + 1152*a^7*c^6*d^2*f^7 + 288*a^6*c^7*e^6*f^3 + 416*a^7*c^6*e^4*f^5 + 352*a^8*c^5*e^2*f^7 + 8*b^4*c^9*d^7*f^2 + 22*b^6*c^7*d^6*f^3 + 26*b^8*c^5*d^5*f^4 + 18*b^10*c^3*d^4*f^5 + 672*a^2*b^2*c^9*d^6*f^3 + 1224*a^2*b^4*c^7*d^5*f^4 + 1202*a^2*b^6*c^5*d^4*f^5 + 564*a^2*b^8*c^3*d^3*f^6 - 2048*a^3*b^2*c^8*d^5*f^4 - 2744*a^3*b^4*c^6*d^4*f^5 - 1736*a^3*b^6*c^4*d^3*f^6 - 128*a^3*b^8*c^2*d^2*f^7 + 2656*a^4*b^2*c^7*d^4*f^5 + 2648*a^4*b^4*c^5*d^3*f^6 + 570*a^4*b^6*c^3*d^2*f^7 - 1344*a^5*b^2*c^6*d^3*f^6 - 904*a^5*b^4*c^4*d^2*f^7 - 160*a^6*b^2*c^5*d^2*f^7 + 8*a^2*b^7*c^4*e^7*f^2 - 12*a^2*b^8*c^3*e^6*f^3 + 8*a^2*b^9*c^2*e^5*f^4 - 90*a^3*b^5*c^5*e^7*f^2 + 132*a^3*b^6*c^4*e^6*f^3 - 76*a^3*b^7*c^3*e^5*f^4 + 6*a^3*b^8*c^2*e^4*f^5 + 336*a^4*b^3*c^6*e^7*f^2 - 462*a^4*b^4*c^5*e^6*f^3 + 164*a^4*b^5*c^4*e^5*f^4 + 106*a^4*b^6*c^3*e^4*f^5 - 56*a^4*b^7*c^2*e^3*f^6 + 432*a^5*b^2*c^6*e^6*f^3 + 288*a^5*b^3*c^5*e^5*f^4 - 598*a^5*b^4*c^4*e^4*f^5 + 102*a^5*b^5*c^3*e^3*f^6 + 90*a^5*b^6*c^2*e^2*f^7 + 720*a^6*b^2*c^5*e^4*f^5 + 336*a^6*b^3*c^4*e^3*f^6 - 314*a^6*b^4*c^3*e^2*f^7 + 240*a^7*b^2*c^4*e^2*f^7 + 64*a^3*c^10*d^5*e^2*f^2 - 768*a^4*c^9*d^3*e^4*f^2 + 416*a^4*c^9*d^4*e^2*f^3 + 1856*a^5*c^8*d^2*e^4*f^3 - 1664*a^5*c^8*d^3*e^2*f^4 + 2336*a^6*c^7*d^2*e^2*f^5 + 26*b^6*c^7*d^5*e^2*f^2 - 8*b^7*c^6*d^4*e^3*f^2 - 10*b^8*c^5*d^3*e^4*f^2 + 58*b^8*c^5*d^4*e^2*f^3 + 8*b^9*c^4*d^2*e^5*f^2 - 12*b^9*c^4*d^3*e^3*f^3 - 12*b^10*c^3*d^2*e^4*f^3 + 36*b^10*c^3*d^3*e^2*f^4 + 8*b^11*c^2*d^2*e^3*f^4 + 2*a^4*b^8*c*d*f^8 + 6*a^5*b^7*c*e*f^8 - 384*a^8*b*c^4*e*f^8 - 64*a*b^2*c^10*d^7*f^2 - 216*a*b^4*c^8*d^6*f^3 - 300*a*b^6*c^6*d^5*f^4 - 240*a*b^8*c^4*d^4*f^5 - 92*a*b^10*c^2*d^3*f^6 + 10*a^2*b^10*c*d^2*f^7 - 12*a^5*b^6*c^2*d*f^8 - 40*a^6*b^4*c^3*d*f^8 + 384*a^7*b^2*c^4*d*f

^8 - 2*a^2*b^6*c^5*e^8*f - 2*a^2*b^10*c*e^4*f^5 + 22*a^3*b^4*c^6*e^8*f + 6*
a^3*b^9*c*e^3*f^6 - 80*a^4*b^2*c^7*e^8*f - 8*a^4*b^8*c*e^2*f^7 - 416*a^5*b*
c^7*e^7*f^2 - 960*a^6*b*c^6*e^5*f^4 - 72*a^6*b^5*c^2*e*f^8 - 928*a^7*b*c^5*
e^3*f^6 + 288*a^7*b^3*c^3*e*f^8 - 32*a^2*c^11*d^6*e^2*f + 32*a^3*c^10*d^4*e
^4*f + 160*a^4*c^9*d^2*e^6*f - 704*a^5*c^8*d*e^6*f^2 - 1536*a^6*c^7*d*e^4*f
^4 - 1472*a^7*c^6*d*e^2*f^6 - 2*b^4*c^9*d^6*e^2*f + 6*b^5*c^8*d^5*e^3*f - 2
4*b^5*c^8*d^6*e*f^2 - 8*b^6*c^7*d^4*e^4*f + 6*b^7*c^6*d^3*e^5*f - 58*b^7*c^
6*d^5*e*f^3 - 2*b^8*c^5*d^2*e^6*f - 60*b^9*c^4*d^4*e*f^4 - 26*b^11*c^2*d^3*
e*f^5 - 2*b^12*c*d^2*e^2*f^5 + 16*a*b^2*c^10*d^6*e^2*f - 48*a*b^3*c^9*d^5*e
^3*f + 192*a*b^3*c^9*d^6*e*f^2 + 66*a*b^4*c^8*d^4*e^4*f - 52*a*b^5*c^7*d^3*
e^5*f + 576*a*b^5*c^7*d^5*e*f^3 + 14*a*b^6*c^6*d^2*e^6*f + 718*a*b^7*c^5*d^
4*e*f^4 - 16*a*b^8*c^4*d*e^6*f^2 + 24*a*b^9*c^3*d*e^5*f^3 + 356*a*b^9*c^3*d
^3*e*f^5 - 16*a*b^10*c^2*d*e^4*f^4 + 96*a^2*b*c^10*d^5*e^3*f - 384*a^2*b*c^
10*d^6*e*f^2 - 42*a^2*b^5*c^6*d*e^7*f - 2*a^2*b^10*c*d*e^2*f^6 - 64*a^3*b*c
^9*d^3*e^5*f + 1792*a^3*b*c^9*d^5*e*f^3 + 144*a^3*b^3*c^7*d*e^7*f - 3712*a^
4*b*c^8*d^4*e*f^4 + 14*a^4*b^7*c^2*d*e*f^7 + 2368*a^5*b*c^7*d*e^5*f^3 + 460
8*a^5*b*c^7*d^3*e*f^5 + 192*a^5*b^5*c^3*d*e*f^7 + 3808*a^6*b*c^6*d*e^3*f^5
- 3712*a^6*b*c^6*d^2*e*f^6 - 1184*a^6*b^3*c^4*d*e*f^7 - 204*a*b^4*c^8*d^5*e
^2*f^2 + 46*a*b^5*c^7*d^4*e^3*f^2 + 132*a*b^6*c^6*d^3*e^4*f^2 - 590*a*b^6*c
^6*d^4*e^2*f^3 - 90*a*b^7*c^5*d^2*e^5*f^2 + 64*a*b^7*c^5*d^3*e^3*f^3 + 196*
a*b^8*c^4*d^2*e^4*f^3 - 408*a*b^8*c^4*d^3*e^2*f^4 - 188*a*b^9*c^3*d^2*e^3*f
^4 + 78*a*b^10*c^2*d^2*e^2*f^5 - 144*a^2*b^2*c^9*d^4*e^4*f + 128*a^2*b^3*c^
8*d^3*e^5*f - 1824*a^2*b^3*c^8*d^5*e*f^3 - 6*a^2*b^4*c^7*d^2*e^6*f - 3096*a
^2*b^5*c^6*d^4*e*f^4 + 190*a^2*b^6*c^5*d*e^6*f^2 - 316*a^2*b^7*c^4*d*e^5*f^
3 - 1908*a^2*b^7*c^4*d^3*e*f^5 + 228*a^2*b^8*c^3*d*e^4*f^4 - 58*a^2*b^9*c^2
*d*e^3*f^5 + 92*a^2*b^9*c^2*d^2*e*f^6 - 288*a^3*b*c^9*d^4*e^3*f^2 - 112*a^3
*b^2*c^8*d^2*e^6*f + 5664*a^3*b^3*c^7*d^4*e*f^4 - 796*a^3*b^4*c^6*d*e^6*f^2
 + 1524*a^3*b^5*c^5*d*e^5*f^3 + 5120*a^3*b^5*c^5*d^3*e*f^5 - 1176*a^3*b^6*c
^4*d*e^4*f^4 + 240*a^3*b^7*c^3*d*e^3*f^5 - 116*a^3*b^7*c^3*d^2*e*f^6 + 68*a
^3*b^8*c^2*d*e^2*f^6 + 192*a^4*b*c^8*d^2*e^5*f^2 + 2240*a^4*b*c^8*d^3*e^3*f
^3 + 1344*a^4*b^2*c^7*d*e^6*f^2 - 3168*a^4*b^3*c^6*d*e^5*f^3 - 7232*a^4*b^3
*c^6*d^3*e*f^5 + 2464*a^4*b^4*c^5*d*e^4*f^4 + 78*a^4*b^5*c^4*d*e^3*f^5 - 11
60*a^4*b^5*c^4*d^2*e*f^6 - 574*a^4*b^6*c^3*d*e^2*f^6 - 4928*a^5*b*c^7*d^2*e
^3*f^4 - 1152*a^5*b^2*c^6*d*e^4*f^4 - 2416*a^5*b^3*c^5*d*e^3*f^5 + 4096*a^5
*b^3*c^5*d^2*e*f^6 + 1748*a^5*b^4*c^4*d*e^2*f^6 - 1280*a^6*b^2*c^5*d*e^2*f^
6 + 4*a*b^7*c^5*d*e^7*f + 4*a*b^11*c*d*e^3*f^5 - 10*a*b^11*c*d^2*e*f^6 - 4*
a^3*b^9*c*d*e*f^7 - 160*a^4*b*c^8*d*e^7*f + 1792*a^7*b*c^5*d*e*f^7 + 384*a^
2*b^2*c^9*d^5*e^2*f^2 + 16*a^2*b^3*c^8*d^4*e^3*f^2 - 624*a^2*b^4*c^7*d^3*e^
4*f^2 + 1962*a^2*b^4*c^7*d^4*e^2*f^3 + 348*a^2*b^5*c^6*d^2*e^5*f^2 + 204*a^
2*b^5*c^6*d^3*e^3*f^3 - 1214*a^2*b^6*c^5*d^2*e^4*f^3 + 1636*a^2*b^6*c^5*d^3
*e^2*f^4 + 1520*a^2*b^7*c^4*d^2*e^3*f^4 - 750*a^2*b^8*c^3*d^2*e^2*f^5 + 121
6*a^3*b^2*c^8*d^3*e^4*f^2 - 2224*a^3*b^2*c^8*d^4*e^2*f^3 - 512*a^3*b^3*c^7*
d^2*e^5*f^2 - 1632*a^3*b^3*c^7*d^3*e^3*f^3 + 3492*a^3*b^4*c^6*d^2*e^4*f^3 -
 2824*a^3*b^4*c^6*d^3*e^2*f^4 - 5492*a^3*b^5*c^5*d^2*e^3*f^4 + 2868*a^3*b^6
*c^4*d^2*e^2*f^5 - 4480*a^4*b^2*c^7*d^2*e^4*f^3 + 2432*a^4*b^2*c^7*d^3*e^2*

f^4 + 8864*a^4*b^3*c^6*d^2*e^3*f^4 - 4206*a^4*b^4*c^5*d^2*e^2*f^5 + 432*a^5
*b^2*c^6*d^2*e^2*f^5))/(16*a^2*c^6*d^4 + a^4*b^4*f^4 + 16*a^4*c^4*e^4 + b^4
*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4
 + 2*a^2*b^6*d*f^3 - 2*a^3*b^5*e*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3
- 2*b^5*c^3*d^3*e + 2*b^6*c^2*d^3*f + a^2*b^4*c^2*e^4 - 8*a^3*b^2*c^3*e^4 +
 32*a^3*c^5*d^2*e^2 + a^2*b^6*e^2*f^2 + 96*a^4*c^4*d^2*f^2 + b^6*c^2*d^2*e^
2 + 32*a^5*c^3*e^2*f^2 - 2*a*b^7*d*e*f^2 - 2*b^7*c*d^2*e*f + 54*a^2*b^4*c^2
*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 + 16*a*b^3*c^4*d^3*e - 2*a*b^5*c^2*d*e^3
 - 32*a^2*b*c^5*d^3*e - 32*a^3*b*c^4*d*e^3 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*
c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 - 2*a^2*b^5*c*e^3*f - 32*a^4*b*c^3*e^3*f + 1
6*a^4*b^3*c*e*f^3 - 32*a^5*b*c^2*e*f^3 - 64*a^4*c^4*d*e^2*f - 6*a*b^4*c^3*d
^2*e^2 + 16*a^2*b^3*c^3*d*e^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3
 + 16*a^3*b^3*c^2*e^3*f - 6*a^3*b^4*c*e^2*f^2 - 48*a^2*b^3*c^3*d^2*e*f - 36
*a^2*b^4*c^2*d*e^2*f + 96*a^3*b^2*c^3*d*e^2*f - 48*a^3*b^3*c^2*d*e*f^2 + 4*
a*b^6*c*d*e^2*f + 18*a*b^5*c^2*d^2*e*f + 18*a^2*b^5*c*d*e*f^2 + 32*a^3*b*c^
4*d^2*e*f + 32*a^4*b*c^3*d*e*f^2)) - (64*A*a^7*c^4*f^8 - A*a^4*b^6*c*f^8 +
32*A*a*c^10*d^6*f^2 - 352*A*a^6*c^5*d*f^7 - A*b^10*c*d^2*f^6 + 8*B*a^4*c^7*
e^7*f - 64*B*a^7*c^4*e*f^7 + 12*A*a^5*b^4*c^2*f^8 - 48*A*a^6*b^2*c^3*f^8 -
224*A*a^2*c^9*d^5*f^3 + 640*A*a^3*c^8*d^4*f^4 - 960*A*a^4*c^7*d^3*f^5 + 800
*A*a^5*c^6*d^2*f^6 - 40*A*a^4*c^7*e^6*f^2 - 80*A*a^5*c^6*e^4*f^4 + 24*A*a^6
*c^5*e^2*f^6 - 8*A*b^2*c^9*d^6*f^2 - 16*A*b^4*c^7*d^5*f^3 - A*b^6*c^5*d^4*f
^4 + 6*A*b^8*c^3*d^3*f^5 + 16*B*a^5*c^6*e^5*f^3 - 56*B*a^6*c^5*e^3*f^5 + 4*
B*b^3*c^8*d^6*f^2 + 12*B*b^5*c^6*d^5*f^3 + 4*B*b^7*c^4*d^4*f^4 - 4*B*b^9*c^
2*d^3*f^5 + 120*A*a*b^2*c^8*d^5*f^3 + 60*A*a*b^4*c^6*d^4*f^4 - 36*A*a*b^6*c
^4*d^3*f^5 + 8*A*a*b^8*c^2*d^2*f^6 + 20*A*a^3*b^6*c^2*d*f^7 - 80*A*a^4*b^4*
c^3*d*f^7 + 216*A*a^5*b^2*c^4*d*f^7 + 4*A*a*b^6*c^4*e^6*f^2 - 6*A*a*b^7*c^3
*e^5*f^3 + 4*A*a*b^8*c^2*e^4*f^4 + 9*A*a^2*b^3*c^6*e^7*f + 2*A*a^2*b^8*c*e^
2*f^6 + 88*A*a^4*b*c^6*e^5*f^3 + 172*A*a^5*b*c^5*e^3*f^5 - 92*B*a*b^3*c^7*d
^5*f^3 - 72*B*a*b^5*c^5*d^4*f^4 + 20*B*a*b^7*c^3*d^3*f^5 + 176*B*a^2*b*c^8*
d^5*f^3 - 544*B*a^3*b*c^7*d^4*f^4 + 736*B*a^4*b*c^6*d^3*f^5 + 4*B*a^4*b^5*c
^2*d*f^7 - 464*B*a^5*b*c^5*d^2*f^6 - 44*B*a^5*b^3*c^3*d*f^7 - 2*B*a^3*b^2*c
^6*e^7*f - B*a^3*b^7*c*e^2*f^6 - 28*B*a^4*b*c^6*e^6*f^2 - 56*B*a^5*b*c^5*e^
4*f^4 - 12*B*a^5*b^4*c^2*e*f^7 + 36*B*a^6*b*c^4*e^2*f^6 + 48*B*a^6*b^2*c^3*
e*f^7 - 16*A*a^2*c^9*d^3*e^4*f + 48*A*a^4*c^7*d*e^4*f^3 - 168*A*a^5*c^6*d*e
^2*f^5 + 2*A*b^2*c^9*d^5*e^2*f - 3*A*b^3*c^8*d^4*e^3*f + 12*A*b^3*c^8*d^5*e
*f^2 - 4*A*b^7*c^4*d*e^5*f^2 - 12*A*b^7*c^4*d^3*e*f^4 + 6*A*b^8*c^3*d*e^4*f
^3 - 4*A*b^9*c^2*d*e^3*f^4 + 8*A*b^9*c^2*d^2*e*f^5 + 8*B*a^2*c^9*d^4*e^3*f
- 32*B*a^2*c^9*d^5*e*f^2 + 16*B*a^3*c^8*d^2*e^5*f + 64*B*a^3*c^8*d^4*e*f^3
+ 64*B*a^4*c^7*d^3*e*f^4 + 96*B*a^5*c^6*d*e^3*f^4 - 256*B*a^5*c^6*d^2*e*f^5
 - B*b^3*c^8*d^5*e^2*f + 2*B*b^4*c^7*d^4*e^3*f - 8*B*b^4*c^7*d^5*e*f^2 - B*
b^6*c^5*d^2*e^5*f - 3*B*b^6*c^5*d^4*e*f^3 + 12*B*b^8*c^3*d^3*e*f^4 - 384*A*
a^2*b^2*c^7*d^4*f^4 - 32*A*a^2*b^4*c^5*d^3*f^5 - 14*A*a^2*b^6*c^3*d^2*f^6 +
 560*A*a^3*b^2*c^6*d^3*f^5 + 56*A*a^3*b^4*c^4*d^2*f^6 - 456*A*a^4*b^2*c^5*d
^2*f^6 - 38*A*a^2*b^4*c^5*e^6*f^2 + 58*A*a^2*b^5*c^4*e^5*f^3 - 36*A*a^2*b^6
*c^3*e^4*f^4 + 5*A*a^2*b^7*c^2*e^3*f^5 + 98*A*a^3*b^2*c^6*e^6*f^2 - 158*A*a

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^3*b^3*c^5*e^5*f^3 + 80*A*a^3*b^4*c^4*e^4*f^4 + 22*A*a^3*b^5*c^3*e^3*f^5 -
22*A*a^3*b^6*c^2*e^2*f^6 + 20*A*a^4*b^2*c^5*e^4*f^4 - 147*A*a^4*b^3*c^4*e^3
*f^5 + 80*A*a^4*b^4*c^3*e^2*f^6 - 102*A*a^5*b^2*c^4*e^2*f^6 + 360*B*a^2*b^3
*c^6*d^4*f^4 + 64*B*a^2*b^5*c^4*d^3*f^5 - 504*B*a^3*b^3*c^5*d^3*f^5 - 40*B*
a^3*b^5*c^3*d^2*f^6 + 276*B*a^4*b^3*c^4*d^2*f^6 + 7*B*a^3*b^3*c^5*e^6*f^2 -
 8*B*a^3*b^4*c^4*e^5*f^3 + 2*B*a^3*b^5*c^3*e^4*f^4 + 2*B*a^3*b^6*c^2*e^3*f^
5 + 28*B*a^4*b^2*c^5*e^5*f^3 + 6*B*a^4*b^3*c^4*e^4*f^4 - 26*B*a^4*b^4*c^3*e
^3*f^5 + 11*B*a^4*b^5*c^2*e^2*f^6 + 86*B*a^5*b^2*c^4*e^3*f^5 - 37*B*a^5*b^3
*c^3*e^2*f^6 + 120*A*a^2*c^9*d^4*e^2*f^2 + 48*A*a^3*c^8*d^2*e^4*f^2 - 336*A
*a^3*c^8*d^3*e^2*f^3 + 368*A*a^4*c^7*d^2*e^2*f^4 + 4*A*b^4*c^7*d^4*e^2*f^2
- 5*A*b^6*c^5*d^2*e^4*f^2 + 6*A*b^6*c^5*d^3*e^2*f^3 + 16*A*b^7*c^4*d^2*e^3*
f^3 - 18*A*b^8*c^3*d^2*e^2*f^4 - 32*B*a^3*c^8*d^3*e^3*f^2 - 16*B*a^4*c^7*d^
2*e^3*f^3 - 3*B*b^5*c^6*d^4*e^2*f^2 + 4*B*b^6*c^5*d^3*e^3*f^2 + 4*B*b^7*c^4
*d^2*e^4*f^2 - 12*B*b^7*c^4*d^3*e^2*f^3 - 6*B*b^8*c^3*d^2*e^3*f^3 + 4*B*b^9
*c^2*d^2*e^2*f^4 - 2*A*a^2*b^8*c*d*f^7 - A*a*b^5*c^5*e^7*f - A*a*b^9*c*e^3*
f^5 - 20*A*a^3*b*c^7*e^7*f - 16*B*a*b*c^9*d^6*f^2 + 112*B*a^6*b*c^4*d*f^7 +
 B*a^4*b^6*c*e*f^7 - 8*A*a*c^10*d^5*e^2*f - 8*A*a^3*c^8*d*e^6*f + A*b^6*c^5
*d*e^6*f + A*b^10*c*d*e^2*f^5 + 224*B*a^6*c^5*d*e*f^6 - B*b^10*c*d^2*e*f^5
+ 12*A*a*b*c^9*d^4*e^3*f - 48*A*a*b*c^9*d^5*e*f^2 - 10*A*a*b^4*c^6*d*e^6*f
+ 80*A*a^5*b*c^5*d*e*f^6 + 4*B*a*b*c^9*d^5*e^2*f + B*a*b^5*c^5*d*e^6*f + B*
a*b^9*c*d*e^2*f^5 + 4*B*a^3*b*c^7*d*e^6*f - 92*A*a^2*b^2*c^7*d^2*e^4*f^2 +
132*A*a^2*b^2*c^7*d^3*e^2*f^3 + 446*A*a^2*b^3*c^6*d^2*e^3*f^3 - 604*A*a^2*b
^4*c^5*d^2*e^2*f^4 + 340*A*a^3*b^2*c^6*d^2*e^2*f^4 + 72*B*a^2*b^2*c^7*d^3*e
^3*f^2 + 134*B*a^2*b^3*c^6*d^2*e^4*f^2 - 306*B*a^2*b^3*c^6*d^3*e^2*f^3 - 26
4*B*a^2*b^4*c^5*d^2*e^3*f^3 + 188*B*a^2*b^5*c^4*d^2*e^2*f^4 + 292*B*a^3*b^2
*c^6*d^2*e^3*f^3 + 6*B*a^3*b^3*c^5*d^2*e^2*f^4 + 4*A*a*b^2*c^8*d^3*e^4*f +
2*A*a*b^3*c^7*d^2*e^5*f - 16*A*a*b^3*c^7*d^4*e*f^3 + 48*A*a*b^5*c^5*d*e^5*f
^2 + 72*A*a*b^5*c^5*d^3*e*f^4 - 84*A*a*b^6*c^4*d*e^4*f^3 + 64*A*a*b^7*c^3*d
*e^3*f^4 - 88*A*a*b^7*c^3*d^2*e*f^5 - 18*A*a*b^8*c^2*d*e^2*f^5 - 8*A*a^2*b*
c^8*d^2*e^5*f + 64*A*a^2*b*c^8*d^4*e*f^3 + 26*A*a^2*b^2*c^7*d*e^6*f + 16*A*
a^2*b^7*c^2*d*e*f^6 + 192*A*a^3*b*c^7*d*e^5*f^2 + 96*A*a^3*b*c^7*d^3*e*f^4
- 136*A*a^3*b^5*c^3*d*e*f^6 + 80*A*a^4*b*c^6*d*e^3*f^4 - 192*A*a^4*b*c^6*d^
2*e*f^5 + 268*A*a^4*b^3*c^4*d*e*f^6 - 10*B*a*b^2*c^8*d^4*e^3*f + 40*B*a*b^2
*c^8*d^5*e*f^2 - 2*B*a*b^3*c^7*d^3*e^4*f + 8*B*a*b^4*c^6*d^2*e^5*f + 36*B*a
*b^4*c^6*d^4*e*f^3 - 4*B*a*b^6*c^4*d*e^5*f^2 - 88*B*a*b^6*c^4*d^3*e*f^4 + 6
*B*a*b^7*c^3*d*e^4*f^3 - 4*B*a*b^8*c^2*d*e^3*f^4 + 16*B*a*b^8*c^2*d^2*e*f^5
 + 8*B*a^2*b*c^8*d^3*e^4*f - 5*B*a^2*b^3*c^6*d*e^6*f - 8*B*a^3*b^6*c^2*d*e*
f^6 + 40*B*a^4*b*c^6*d*e^4*f^3 + 104*B*a^4*b^4*c^3*d*e*f^6 + 148*B*a^5*b*c^
5*d*e^2*f^5 - 344*B*a^5*b^2*c^4*d*e*f^6 - 46*A*a*b^2*c^8*d^4*e^2*f^2 - 4*A*
a*b^3*c^7*d^3*e^3*f^2 + 40*A*a*b^4*c^6*d^2*e^4*f^2 - 36*A*a*b^4*c^6*d^3*e^2
*f^3 - 158*A*a*b^5*c^5*d^2*e^3*f^3 + 196*A*a*b^6*c^4*d^2*e^2*f^4 + 16*A*a^2
*b*c^8*d^3*e^3*f^2 - 176*A*a^2*b^3*c^6*d*e^5*f^2 - 120*A*a^2*b^3*c^6*d^3*e*
f^4 + 380*A*a^2*b^4*c^5*d*e^4*f^3 - 324*A*a^2*b^5*c^4*d*e^3*f^4 + 272*A*a^2
*b^5*c^4*d^2*e*f^5 + 80*A*a^2*b^6*c^3*d*e^2*f^5 - 280*A*a^3*b*c^7*d^2*e^3*f
^3 - 572*A*a^3*b^2*c^6*d*e^4*f^3 + 508*A*a^3*b^3*c^5*d*e^3*f^4 - 144*A*a^3*
```

b^3*c^5*d^2*e*f^5 - 4*A*a^3*b^4*c^4*d*e^2*f^5 - 326*A*a^4*b^2*c^5*d*e^2*f^5
 + 31*B*a*b^3*c^7*d^4*e^2*f^2 - 32*B*a*b^4*c^6*d^3*e^3*f^2 - 40*B*a*b^5*c^5
*d^2*e^4*f^2 + 102*B*a*b^5*c^5*d^3*e^2*f^3 + 72*B*a*b^6*c^4*d^2*e^3*f^3 - 5
6*B*a*b^7*c^3*d^2*e^2*f^4 - 76*B*a^2*b*c^8*d^4*e^2*f^2 - 20*B*a^2*b^2*c^7*d
^2*e^5*f - 112*B*a^2*b^2*c^7*d^4*e*f^3 + 24*B*a^2*b^4*c^5*d*e^5*f^2 + 192*B
*a^2*b^4*c^5*d^3*e*f^4 - 42*B*a^2*b^5*c^4*d*e^4*f^3 + 32*B*a^2*b^6*c^3*d*e^
3*f^4 - 38*B*a^2*b^6*c^3*d^2*e*f^5 - 9*B*a^2*b^7*c^2*d*e^2*f^5 - 152*B*a^3*
b*c^7*d^2*e^4*f^2 + 360*B*a^3*b*c^7*d^3*e^2*f^3 - 32*B*a^3*b^2*c^6*d*e^5*f^
2 - 144*B*a^3*b^2*c^6*d^3*e*f^4 + 62*B*a^3*b^3*c^5*d*e^4*f^3 - 40*B*a^3*b^4
*c^4*d*e^3*f^4 - 152*B*a^3*b^4*c^4*d^2*e*f^5 + 14*B*a^3*b^5*c^3*d*e^2*f^5 -
 472*B*a^4*b*c^6*d^2*e^2*f^4 - 120*B*a^4*b^2*c^5*d*e^3*f^4 + 512*B*a^4*b^2*
c^5*d^2*e*f^5 - 13*B*a^4*b^3*c^4*d*e^2*f^5)/(16*a^2*c^6*d^4 + a^4*b^4*f^4 +
 16*a^4*c^4*e^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*
d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 2*a^3*b^5*e*f^3 - 64*a^3*c^5*d^3*
f - 64*a^5*c^3*d*f^3 - 2*b^5*c^3*d^3*e + 2*b^6*c^2*d^3*f + a^2*b^4*c^2*e^4
- 8*a^3*b^2*c^3*e^4 + 32*a^3*c^5*d^2*e^2 + a^2*b^6*e^2*f^2 + 96*a^4*c^4*d^2
*f^2 + b^6*c^2*d^2*e^2 + 32*a^5*c^3*e^2*f^2 - 2*a*b^7*d*e*f^2 - 2*b^7*c*d^2
*e*f + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 + 16*a*b^3*c^4*d^3*
e - 2*a*b^5*c^2*d*e^3 - 32*a^2*b*c^5*d^3*e - 32*a^3*b*c^4*d*e^3 - 20*a*b^4*
c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 - 2*a^2*b^5*c*e^3*f - 3
2*a^4*b*c^3*e^3*f + 16*a^4*b^3*c*e*f^3 - 32*a^5*b*c^2*e*f^3 - 64*a^4*c^4*d*
e^2*f - 6*a*b^4*c^3*d^2*e^2 + 16*a^2*b^3*c^3*d*e^3 + 64*a^2*b^2*c^4*d^3*f +
 64*a^4*b^2*c^2*d*f^3 + 16*a^3*b^3*c^2*e^3*f - 6*a^3*b^4*c*e^2*f^2 - 48*a^2
*b^3*c^3*d^2*e*f - 36*a^2*b^4*c^2*d*e^2*f + 96*a^3*b^2*c^3*d*e^2*f - 48*a^3
*b^3*c^2*d*e*f^2 + 4*a*b^6*c*d*e^2*f + 18*a*b^5*c^2*d^2*e*f + 18*a^2*b^5*c*
d*e*f^2 + 32*a^3*b*c^4*d^2*e*f + 32*a^4*b*c^3*d*e*f^2) + (x*(64*B*a^7*c^4*f
^8 + 4*A*a^3*b^7*c*f^8 - 256*A*a^6*b*c^4*f^8 - B*a^4*b^6*c*f^8 + 48*A*a^3*c
^8*e^7*f + 256*A*a^6*c^5*e*f^7 - 320*B*a^6*c^5*d*f^7 + 3*B*b^10*c*d^2*f^6 -
 48*A*a^4*b^5*c^2*f^8 + 192*A*a^5*b^3*c^3*f^8 + 12*B*a^5*b^4*c^2*f^8 - 48*B
*a^6*b^2*c^3*f^8 + 256*A*a^4*c^7*e^5*f^3 + 464*A*a^5*c^6*e^3*f^5 - 16*A*b^3
*c^8*d^5*f^3 - 48*A*b^5*c^6*d^4*f^4 - 36*A*b^7*c^4*d^3*f^5 - 4*A*b^9*c^2*d^
2*f^6 - 64*B*a^2*c^9*d^5*f^3 + 320*B*a^3*c^8*d^4*f^4 - 640*B*a^4*c^7*d^3*f^
5 + 640*B*a^5*c^6*d^2*f^6 - 16*B*a^4*c^7*e^6*f^2 - 64*B*a^5*c^6*e^4*f^4 + 1
6*B*a^6*c^5*e^2*f^6 + 4*B*b^4*c^7*d^5*f^3 + 23*B*b^6*c^5*d^4*f^4 + 22*B*b^8
*c^3*d^3*f^5 + 320*A*a*b^3*c^7*d^4*f^4 + 352*A*a*b^5*c^5*d^3*f^5 + 76*A*a*b
^7*c^3*d^2*f^6 - 512*A*a^2*b*c^8*d^4*f^4 - 60*A*a^2*b^7*c^2*d*f^7 + 1408*A*
a^3*b*c^7*d^3*f^5 + 352*A*a^3*b^5*c^3*d*f^7 - 1792*A*a^4*b*c^6*d^2*f^6 - 97
6*A*a^4*b^3*c^4*d*f^7 - 6*A*a*b^5*c^5*e^6*f^2 + 4*A*a*b^6*c^4*e^5*f^3 + 4*A
*a*b^7*c^3*e^4*f^4 - 6*A*a*b^8*c^2*e^3*f^5 - 20*A*a^2*b^2*c^7*e^7*f - 144*A
*a^3*b*c^7*e^6*f^2 + 68*A*a^3*b^6*c^2*e*f^7 - 640*A*a^4*b*c^6*e^4*f^4 - 240
*A*a^4*b^4*c^3*e*f^7 - 848*A*a^5*b*c^5*e^2*f^6 + 192*A*a^5*b^2*c^4*e*f^7 -
132*B*a*b^4*c^6*d^4*f^4 - 196*B*a*b^6*c^4*d^3*f^5 - 40*B*a*b^8*c^2*d^2*f^6
- 20*B*a^3*b^6*c^2*d*f^7 + 52*B*a^4*b^4*c^3*d*f^7 + 64*B*a^5*b^2*c^4*d*f^7
+ 2*B*a^2*b^3*c^6*e^7*f + B*a^2*b^8*c*e^2*f^6 + 16*B*a^4*b*c^6*e^5*f^3 + 12
0*B*a^5*b*c^5*e^3*f^5 + 64*A*a^2*c^9*d^2*e^5*f + 512*A*a^2*c^9*d^4*e*f^3 -

384*A*a^3*c^8*d*e^5*f^2 - 1408*A*a^3*c^8*d^3*e*f^4 - 1280*A*a^4*c^7*d*e^3*f
^4 + 1792*A*a^4*c^7*d^2*e*f^5 - 4*A*b^2*c^9*d^4*e^3*f + 16*A*b^2*c^9*d^5*e*
f^2 + 8*A*b^3*c^8*d^3*e^4*f - 2*A*b^4*c^7*d^2*e^5*f + 80*A*b^4*c^7*d^4*e*f^
3 + 6*A*b^6*c^5*d*e^5*f^2 + 76*A*b^6*c^5*d^3*e*f^4 - 4*A*b^7*c^4*d*e^4*f^3
- 4*A*b^8*c^3*d*e^3*f^4 - 2*A*b^8*c^3*d^2*e*f^5 + 6*A*b^9*c^2*d*e^2*f^5 - 3
2*B*a^2*c^9*d^3*e^4*f - 96*B*a^4*c^7*d*e^4*f^3 - 192*B*a^5*c^6*d*e^2*f^5 +
2*B*b^3*c^8*d^4*e^3*f - 8*B*b^3*c^8*d^5*e*f^2 - 6*B*b^4*c^7*d^3*e^4*f + 4*B
*b^5*c^6*d^2*e^5*f - 48*B*b^5*c^6*d^4*e*f^3 - 60*B*b^7*c^4*d^3*e*f^4 - 8*B*
b^9*c^2*d^2*e*f^5 + 4*A*a*b^9*c*d*f^7 - 2*A*b^10*c*d*e*f^6 - 1184*A*a^2*b^3
*c^6*d^3*f^5 - 544*A*a^2*b^5*c^4*d^2*f^6 + 1664*A*a^3*b^3*c^5*d^2*f^6 + 60*
A*a^2*b^3*c^6*e^6*f^2 - 30*A*a^2*b^4*c^5*e^5*f^3 - 64*A*a^2*b^5*c^4*e^4*f^4
 + 72*A*a^2*b^6*c^3*e^3*f^5 - 12*A*a^2*b^7*c^2*e^2*f^6 - 8*A*a^3*b^2*c^6*e^
5*f^3 + 352*A*a^3*b^3*c^5*e^4*f^4 - 268*A*a^3*b^4*c^4*e^3*f^5 - 52*A*a^3*b^
5*c^3*e^2*f^6 + 188*A*a^4*b^2*c^5*e^3*f^5 + 484*A*a^4*b^3*c^4*e^2*f^6 + 80*
B*a^2*b^2*c^7*d^4*f^4 + 520*B*a^2*b^4*c^5*d^3*f^5 + 210*B*a^2*b^6*c^3*d^2*f
^6 - 192*B*a^3*b^2*c^6*d^3*f^5 - 456*B*a^3*b^4*c^4*d^2*f^6 + 96*B*a^4*b^2*c
^5*d^2*f^6 - 7*B*a^2*b^4*c^5*e^6*f^2 + 8*B*a^2*b^5*c^4*e^5*f^3 - 2*B*a^2*b^
6*c^3*e^4*f^4 - 2*B*a^2*b^7*c^2*e^3*f^5 + 32*B*a^3*b^2*c^6*e^6*f^2 - 36*B*a
^3*b^3*c^5*e^5*f^3 - 4*B*a^3*b^4*c^4*e^4*f^4 + 28*B*a^3*b^5*c^3*e^3*f^5 - 1
2*B*a^3*b^6*c^2*e^2*f^6 + 64*B*a^4*b^2*c^5*e^4*f^4 - 110*B*a^4*b^3*c^4*e^3*
f^5 + 47*B*a^4*b^4*c^3*e^2*f^6 - 64*B*a^5*b^2*c^4*e^2*f^6 - 384*A*a^2*c^9*d
^3*e^3*f^2 + 1184*A*a^3*c^8*d^2*e^3*f^3 - 28*A*b^3*c^8*d^4*e^2*f^2 - 12*A*b
^4*c^7*d^3*e^3*f^2 + 20*A*b^5*c^6*d^2*e^4*f^2 - 36*A*b^5*c^6*d^3*e^2*f^3 -
44*A*b^6*c^5*d^2*e^3*f^3 + 32*A*b^7*c^4*d^2*e^2*f^4 + 144*B*a^2*c^9*d^4*e^2
*f^2 + 192*B*a^3*c^8*d^2*e^4*f^2 - 448*B*a^3*c^8*d^3*e^2*f^3 + 480*B*a^4*c^
7*d^2*e^2*f^4 + 23*B*b^4*c^7*d^4*e^2*f^2 - 4*B*b^5*c^6*d^3*e^3*f^2 - 13*B*b
^6*c^5*d^2*e^4*f^2 + 48*B*b^6*c^5*d^3*e^2*f^3 + 12*B*b^7*c^4*d^2*e^3*f^3 +
2*B*b^8*c^3*d^2*e^2*f^4 + 64*A*a*b*c^9*d^5*f^3 + 1088*A*a^5*b*c^5*d*f^7 + 2
*A*a*b^4*c^6*e^7*f + 2*A*a*b^9*c*e^2*f^6 - 6*A*a^2*b^8*c*e*f^7 + 2*B*a^2*b^
8*c*d*f^7 - 8*B*a^3*b*c^7*e^7*f + 16*A*a*c^10*d^4*e^3*f - 64*A*a*c^10*d^5*e
*f^2 - 1088*A*a^5*c^6*d*e*f^6 - 2*A*b^5*c^6*d*e^6*f - 32*B*a^3*c^8*d*e^6*f
- 32*A*a*b*c^9*d^3*e^4*f + 24*A*a*b^3*c^7*d*e^6*f + 24*A*a*b^8*c^2*d*e*f^6
- 64*A*a^2*b*c^8*d*e^6*f - 8*B*a*b*c^9*d^4*e^3*f + 32*B*a*b*c^9*d^5*e*f^2 -
 6*B*a*b^4*c^6*d*e^6*f + 288*B*a^5*b*c^5*d*e*f^6 - 1496*A*a^2*b^2*c^7*d^2*e
^3*f^3 + 1496*A*a^2*b^3*c^6*d^2*e^2*f^4 - 272*B*a^2*b^2*c^7*d^2*e^4*f^2 + 6
40*B*a^2*b^2*c^7*d^3*e^2*f^3 + 636*B*a^2*b^3*c^6*d^2*e^3*f^3 - 286*B*a^2*b^
4*c^5*d^2*e^2*f^4 + 192*B*a^3*b^2*c^6*d^2*e^2*f^4 + 112*A*a*b*c^9*d^4*e^2*f
^2 - 8*A*a*b^2*c^8*d^2*e^5*f - 448*A*a*b^2*c^8*d^4*e*f^3 - 88*A*a*b^4*c^6*d
*e^5*f^2 - 576*A*a*b^4*c^6*d^3*e*f^4 + 84*A*a*b^5*c^5*d*e^4*f^3 + 32*A*a*b^
6*c^4*d*e^3*f^4 + 12*A*a*b^6*c^4*d^2*e*f^5 - 80*A*a*b^7*c^3*d*e^2*f^5 - 108
*A*a^2*b^6*c^3*d*e*f^6 + 736*A*a^3*b*c^7*d*e^4*f^3 + 192*A*a^3*b^4*c^4*d*e*
f^6 + 2048*A*a^4*b*c^6*d*e^2*f^5 + 208*A*a^4*b^2*c^5*d*e*f^6 + 32*B*a*b^2*c
^8*d^3*e^4*f - 20*B*a*b^3*c^7*d^2*e^5*f + 288*B*a*b^3*c^7*d^4*e*f^3 + 20*B*
a*b^5*c^5*d*e^5*f^2 + 480*B*a*b^5*c^5*d^3*e*f^4 - 20*B*a*b^6*c^4*d*e^4*f^3
+ 72*B*a*b^7*c^3*d^2*e*f^5 + 10*B*a*b^8*c^2*d*e^2*f^5 + 16*B*a^2*b*c^8*d^2*

e^5*f - 384*B*a^2*b*c^8*d^4*e*f^3 + 32*B*a^2*b^2*c^7*d*e^6*f + 44*B*a^2*b^7*c^2*d*e*f^6 + 192*B*a^3*b*c^7*d*e^5*f^2 + 960*B*a^3*b*c^7*d^3*e*f^4 - 160*B*a^3*b^5*c^3*d*e*f^6 + 544*B*a^4*b*c^6*d*e^3*f^4 - 896*B*a^4*b*c^6*d^2*e*f^5 + 120*B*a^4*b^3*c^4*d*e*f^6 - 4*B*a*b^9*c*d*e*f^6 + 144*A*a*b^2*c^8*d^3*e^3*f^2 - 144*A*a*b^3*c^7*d^2*e^4*f^2 + 112*A*a*b^3*c^7*d^3*e^2*f^3 + 476*A*a*b^4*c^6*d^2*e^3*f^3 - 412*A*a*b^5*c^5*d^2*e^2*f^4 + 256*A*a^2*b*c^8*d^2*e^4*f^2 + 128*A*a^2*b*c^8*d^3*e^2*f^3 + 352*A*a^2*b^2*c^7*d*e^5*f^2 + 1440*A*a^2*b^2*c^7*d^3*e*f^4 - 456*A*a^2*b^3*c^6*d*e^4*f^3 - 116*A*a^2*b^4*c^5*d*e^3*f^4 + 224*A*a^2*b^4*c^5*d^2*e*f^5 + 452*A*a^2*b^5*c^4*d*e^2*f^5 - 1440*A*a^3*b*c^7*d^2*e^2*f^4 + 528*A*a^3*b^2*c^6*d*e^3*f^4 - 1408*A*a^3*b^2*c^6*d^2*e*f^5 - 1424*A*a^3*b^3*c^5*d*e^2*f^5 - 128*B*a*b^2*c^8*d^4*e^2*f^2 + 8*B*a*b^3*c^7*d^3*e^3*f^2 + 108*B*a*b^4*c^6*d^2*e^4*f^2 - 324*B*a*b^4*c^6*d^3*e^2*f^3 - 164*B*a*b^5*c^5*d^2*e^3*f^3 + 44*B*a*b^6*c^4*d^2*e^2*f^4 + 32*B*a^2*b*c^8*d^3*e^3*f^2 - 128*B*a^2*b^3*c^6*d*e^5*f^2 - 1200*B*a^2*b^3*c^6*d^3*e*f^4 + 142*B*a^2*b^4*c^5*d*e^4*f^3 + 20*B*a^2*b^5*c^4*d*e^3*f^4 - 304*B*a^2*b^5*c^4*d^2*e*f^5 - 112*B*a^2*b^6*c^3*d*e^2*f^5 - 688*B*a^3*b*c^7*d^2*e^3*f^3 - 224*B*a^3*b^2*c^6*d*e^4*f^3 - 216*B*a^3*b^3*c^5*d*e^3*f^4 + 800*B*a^3*b^3*c^5*d^2*e*f^5 + 460*B*a^3*b^4*c^4*d*e^2*f^5 - 640*B*a^4*b^2*c^5*d*e^2*f^5))/(16*a^2*c^6*d^4 + a^4*b^4*f^4 + 16*a^4*c^4*e^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 2*a^3*b^5*e*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 - 2*b^5*c^3*d^3*e + 2*b^6*c^2*d^3*f + a^2*b^4*c^2*e^4 - 8*a^3*b^2*c^3*e^4 + 32*a^3*c^5*d^2*e^2 + a^2*b^6*e^2*f^2 + 96*a^4*c^4*d^2*f^2 + b^6*c^2*d^2*e^2 + 32*a^5*c^3*e^2*f^2 - 2*a*b^7*d*e*f^2 - 2*b^7*c*d^2*e*f + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 + 16*a*b^3*c^4*d^3*e - 2*a*b^5*c^2*d*e^3 - 32*a^2*b*c^5*d^3*e - 32*a^3*b*c^4*d*e^3 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 - 2*a^2*b^5*c*e^3*f - 32*a^4*b*c^3*e^3*f + 16*a^4*b^3*c*e*f^3 - 32*a^5*b*c^2*e*f^3 - 64*a^4*c^4*d*e^2*f - 6*a*b^4*c^3*d^2*e^2 + 16*a^2*b^3*c^3*d*e^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3 + 16*a^3*b^3*c^2*e^3*f - 6*a^3*b^4*c*e^2*f^2 - 48*a^2*b^3*c^3*d^2*e*f - 36*a^2*b^4*c^2*d*e^2*f + 96*a^3*b^2*c^3*d*e^2*f - 48*a^3*b^3*c^2*d*e*f^2 + 4*a*b^6*c*d*e^2*f + 18*a*b^5*c^2*d^2*e*f + 18*a^2*b^5*c*d*e*f^2 + 32*a^3*b*c^4*d^2*e*f + 32*a^4*b*c^3*d*e*f^2)) - (56*A^2*a^3*b^3*c^3*f^7 - 13*A^2*a^2*b^5*c^2*f^7 - 24*A^2*a^2*c^7*e^5*f^2 - 16*A^2*b^3*c^6*d^3*f^4 - A^2*b^5*c^4*d^2*f^5 + 2*A^2*b^4*c^5*e^5*f^2 - A^2*b^5*c^4*e^4*f^3 - A^2*b^6*c^3*e^3*f^4 + 2*A^2*b^7*c^2*e^2*f^5 - 12*B^2*a^4*c^5*e^3*f^4 - 8*B^2*b^3*c^6*d^4*f^3 - 9*B^2*b^5*c^4*d^3*f^4 + 64*A*B*a^5*c^4*f^7 + A^2*a*b^7*c*f^7 - A^2*b^8*c*e*f^6 - 80*A^2*a^4*b*c^4*f^7 - 16*A^2*b*c^8*d^4*f^3 + 28*A^2*a^4*c^5*e*f^6 - 2*A^2*b^7*c^2*d*f^6 - A^2*b^3*c^6*e^6*f - 16*B^2*a^5*c^4*e*f^6 - 4*A^2*c^9*d^3*e^3*f + 12*A^2*c^9*d^4*e*f^2 + 48*A^2*a*b*c^7*d^3*f^4 + 22*A^2*a*b^5*c^3*d*f^6 + 48*A^2*a^3*b*c^5*d*f^6 + 12*A^2*a*b^6*c^2*e*f^6 + 16*B^2*a*b*c^7*d^4*f^3 + 64*B^2*a^4*b*c^4*d*f^6 - 16*A^2*a*c^8*d^3*e*f^3 + 80*A^2*a^3*c^6*d*e*f^5 + 4*A^2*b*c^8*d^2*e^4*f + A^2*b^2*c^7*d*e^5*f + 2*A^2*b^6*c^3*d*e*f^5 + 4*B^2*a^2*c^7*d*e^5*f - 64*B^2*a^4*c^5*d*e*f^5 + A*B*b^8*c*d*f^6 + 8*A^2*a*b^3*c^5*d^2*f^5 - 64*A^2*a^2*b^3*c^4*d*f^6 - 2*A^2*a*b^2*c^6*e^5*f^2 - 10*A^2*a*b^

3*c^5*e^4*f^3 + 20*A^2*a*b^4*c^4*e^3*f^4 - 25*A^2*a*b^5*c^3*e^2*f^5 + 56*A^
2*a^2*b*c^6*e^4*f^3 - 44*A^2*a^2*b^4*c^3*e*f^6 - 76*A^2*a^3*b*c^5*e^2*f^5 +
 40*A^2*a^3*b^2*c^4*e*f^6 + 56*B^2*a*b^3*c^5*d^3*f^4 - 2*B^2*a*b^5*c^3*d^2*
f^5 - 96*B^2*a^2*b*c^6*d^3*f^4 + 11*B^2*a^2*b^5*c^2*d*f^6 + 16*B^2*a^3*b*c^
5*d^2*f^5 - 40*B^2*a^3*b^3*c^3*d*f^6 + 16*B^2*a^4*b*c^4*e^2*f^5 + 3*B^2*a^4
*b^2*c^3*e*f^6 + 24*A^2*a*c^8*d^2*e^3*f^2 + 92*A^2*a^2*c^7*d*e^3*f^3 - 104*
A^2*a^2*c^7*d^2*e*f^4 - 4*A^2*b*c^8*d^3*e^2*f^2 + 20*A^2*b^2*c^7*d^3*e*f^3
 - A^2*b^4*c^5*d*e^3*f^3 - 8*A^2*b^4*c^5*d^2*e*f^4 + 32*B^2*a^2*c^7*d^3*e*f^
3 - 8*B^2*a^3*c^6*d*e^3*f^3 + 48*B^2*a^3*c^6*d^2*e*f^4 - B^2*b^2*c^7*d^3*e^
3*f + 3*B^2*b^2*c^7*d^4*e*f^2 + B^2*b^3*c^6*d^2*e^4*f + 8*B^2*b^4*c^5*d^3*e
*f^3 - 3*B^2*b^6*c^3*d^2*e*f^4 - A*B*a^2*b^6*c*f^7 - 32*A*B*a*c^8*d^4*f^3 -
 32*A*B*a^4*c^5*d*f^6 - 8*A*B*a^2*c^7*e^6*f + 4*A^2*a*b*c^7*e^6*f - B^2*a*b
^7*c*d*f^6 - 8*A^2*a*c^8*d*e^5*f - 65*A^2*a^2*b^2*c^5*e^3*f^4 + 88*A^2*a^2*
b^3*c^4*e^2*f^5 + 8*B^2*a^2*b^3*c^4*d^2*f^5 + 2*B^2*a^3*b^2*c^4*e^3*f^4 - 3
*B^2*a^3*b^3*c^3*e^2*f^5 - 13*A^2*b^2*c^7*d^2*e^3*f^2 + 16*A^2*b^3*c^6*d^2*
e^2*f^3 - 28*B^2*a^2*c^7*d^2*e^3*f^2 + B^2*b^3*c^6*d^3*e^2*f^2 - 5*B^2*b^4*
c^5*d^2*e^3*f^2 + 7*B^2*b^5*c^4*d^2*e^2*f^3 + 12*A*B*a^3*b^4*c^2*f^7 - 48*A
*B*a^4*b^2*c^3*f^7 + 160*A*B*a^2*c^7*d^3*f^4 - 160*A*B*a^3*c^6*d^2*f^5 - 16
*A*B*a^3*c^6*e^4*f^3 + 48*A*B*a^4*c^5*e^2*f^5 + 24*A*B*b^2*c^7*d^4*f^3 + 24
*A*B*b^4*c^5*d^3*f^4 - A*B*b^6*c^3*d^2*f^5 + 20*B^2*a*b^2*c^6*d^2*e^3*f^2 -
 49*B^2*a*b^3*c^5*d^2*e^2*f^3 + 96*B^2*a^2*b*c^6*d^2*e^2*f^3 + 19*B^2*a^2*b
^2*c^5*d*e^3*f^3 - 102*B^2*a^2*b^2*c^5*d^2*e*f^4 + 3*B^2*a^2*b^3*c^4*d*e^2*
f^4 - 120*A*B*a*b^2*c^6*d^3*f^4 + 4*A*B*a*b^4*c^4*d^2*f^5 + 24*A*B*a^2*b^4*
c^3*d*f^6 - 8*A*B*a^3*b^2*c^4*d*f^6 - 7*A*B*a*b^3*c^5*e^5*f^2 + 8*A*B*a*b^4
*c^4*e^4*f^3 - 2*A*B*a*b^5*c^3*e^3*f^4 - 2*A*B*a*b^6*c^2*e^2*f^5 + 28*A*B*a
^2*b*c^6*e^5*f^2 - 11*A*B*a^2*b^5*c^2*e*f^6 + 72*A*B*a^3*b*c^5*e^3*f^4 + 34
*A*B*a^3*b^3*c^3*e*f^6 + 16*A*B*a*c^8*d^3*e^2*f^2 + 80*A*B*a^2*c^7*d*e^4*f^
2 + 16*A*B*a^3*c^6*d*e^2*f^4 - 4*A*B*b^2*c^7*d^2*e^4*f - 22*A*B*b^3*c^6*d^3
*e*f^3 + 3*A*B*b^4*c^5*d*e^4*f^2 - 6*A*B*b^5*c^4*d*e^3*f^3 + 15*A*B*b^5*c^4
*d^2*e*f^4 + 4*A*B*b^6*c^3*d*e^2*f^4 + 12*A^2*a*b*c^7*d*e^4*f^2 - 28*A^2*a*
b^4*c^4*d*e*f^5 - 2*B^2*a*b^2*c^6*d*e^5*f + 2*B^2*a*b^6*c^2*d*e*f^5 + A*B*a
*b^7*c*e*f^6 + 24*A*B*a^2*b^2*c^5*d^2*f^5 - 28*A*B*a^2*b^2*c^5*e^4*f^3 - 9*
A*B*a^2*b^3*c^4*e^3*f^4 + 29*A*B*a^2*b^4*c^3*e^2*f^5 - 100*A*B*a^3*b^2*c^4*
e^2*f^5 - 208*A*B*a^2*c^7*d^2*e^2*f^3 + 13*A*B*b^3*c^6*d^2*e^3*f^2 - 23*A*B
*b^4*c^5*d^2*e^2*f^3 - 52*A^2*a*b*c^7*d^2*e^2*f^3 - 34*A^2*a*b^2*c^6*d*e^3*
f^3 + 48*A^2*a*b^2*c^6*d^2*e*f^4 + 40*A^2*a*b^3*c^5*d*e^2*f^4 - 108*A^2*a^2
*b*c^6*d*e^2*f^4 + 36*A^2*a^2*b^2*c^5*d*e*f^5 - 8*B^2*a*b*c^7*d^3*e^2*f^2 -
 24*B^2*a*b^2*c^6*d^3*e*f^3 + 7*B^2*a*b^3*c^5*d*e^4*f^2 - 8*B^2*a*b^4*c^4*d
*e^3*f^3 + 32*B^2*a*b^4*c^4*d^2*e*f^4 + 2*B^2*a*b^5*c^3*d*e^2*f^4 - 16*B^2*
a^2*b*c^6*d*e^4*f^2 - 20*B^2*a^2*b^4*c^3*d*e*f^5 + 8*B^2*a^3*b*c^5*d*e^2*f^
4 + 40*B^2*a^3*b^2*c^4*d*e*f^5 - 10*A*B*a*b^6*c^2*d*f^6 + 2*A*B*a*b^2*c^6*e
^6*f - 20*A*B*a^4*b*c^4*e*f^6 + 4*A*B*b*c^8*d^3*e^3*f - 12*A*B*b*c^8*d^4*e*
f^2 - 2*A*B*b^7*c^2*d*e*f^5 + 132*A*B*a*b^2*c^6*d^2*e^2*f^3 + 96*A*B*a^2*b^
2*c^5*d*e^2*f^4 + 24*A*B*a*b*c^7*d^3*e*f^3 + 20*A*B*a*b^5*c^3*d*e*f^5 - 24*
A*B*a^3*b*c^5*d*e*f^5 - 24*A*B*a*b*c^7*d^2*e^3*f^2 - 44*A*B*a*b^2*c^6*d*e^4

*f^2 + 84*A*B*a*b^3*c^5*d*e^3*f^3 - 122*A*B*a*b^3*c^5*d^2*e*f^4 - 54*A*B*a*
b^4*c^4*d*e^2*f^4 - 180*A*B*a^2*b*c^6*d*e^3*f^3 + 288*A*B*a^2*b*c^6*d^2*e*f
^4 - 18*A*B*a^2*b^3*c^4*d*e*f^5 + 4*A*B*a*b*c^7*d*e^5*f)/(16*a^2*c^6*d^4 +
a^4*b^4*f^4 + 16*a^4*c^4*e^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 -
8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 2*a^3*b^5*e*f^3 - 64
*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 - 2*b^5*c^3*d^3*e + 2*b^6*c^2*d^3*f + a^2
*b^4*c^2*e^4 - 8*a^3*b^2*c^3*e^4 + 32*a^3*c^5*d^2*e^2 + a^2*b^6*e^2*f^2 + 9
6*a^4*c^4*d^2*f^2 + b^6*c^2*d^2*e^2 + 32*a^5*c^3*e^2*f^2 - 2*a*b^7*d*e*f^2
- 2*b^7*c*d^2*e*f + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 + 16*a
*b^3*c^4*d^3*e - 2*a*b^5*c^2*d*e^3 - 32*a^2*b*c^5*d^3*e - 32*a^3*b*c^4*d*e^
3 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 - 2*a^2*b^
5*c*e^3*f - 32*a^4*b*c^3*e^3*f + 16*a^4*b^3*c*e*f^3 - 32*a^5*b*c^2*e*f^3 -
64*a^4*c^4*d*e^2*f - 6*a*b^4*c^3*d^2*e^2 + 16*a^2*b^3*c^3*d*e^3 + 64*a^2*b^
2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3 + 16*a^3*b^3*c^2*e^3*f - 6*a^3*b^4*c*e^2
*f^2 - 48*a^2*b^3*c^3*d^2*e*f - 36*a^2*b^4*c^2*d*e^2*f + 96*a^3*b^2*c^3*d*e
^2*f - 48*a^3*b^3*c^2*d*e*f^2 + 4*a*b^6*c*d*e^2*f + 18*a*b^5*c^2*d^2*e*f +
18*a^2*b^5*c*d*e*f^2 + 32*a^3*b*c^4*d^2*e*f + 32*a^4*b*c^3*d*e*f^2) + (x*(1
04*A^2*a^4*c^5*f^7 - 32*B^2*a^5*c^4*f^7 + 8*A^2*c^9*d^4*f^3 + A^2*b^8*c*f^7
+ 50*A^2*a^2*b^4*c^3*f^7 - 96*A^2*a^3*b^2*c^4*f^7 - 12*B^2*a^3*b^4*c^2*f^7
+ 42*B^2*a^4*b^2*c^3*f^7 + 208*A^2*a^2*c^7*d^2*f^5 + 36*A^2*a^2*c^7*e^4*f^
3 + 72*A^2*a^3*c^6*e^2*f^5 + 8*A^2*b^2*c^7*d^3*f^4 + 18*A^2*b^4*c^5*d^2*f^5
- 32*B^2*a^2*c^7*d^3*f^4 + 32*B^2*a^3*c^6*d^2*f^5 - 2*A^2*b^3*c^6*e^5*f^2
+ A^2*b^4*c^5*e^4*f^3 + A^2*b^6*c^3*e^2*f^5 + 24*B^2*a^3*c^6*e^4*f^3 + 56*B
^2*a^4*c^5*e^2*f^5 + 2*B^2*b^2*c^7*d^4*f^3 - 6*B^2*b^4*c^5*d^3*f^4 + 9*B^2*
b^6*c^3*d^2*f^5 - 16*A^2*c^9*d^3*e^2*f^2 - 12*A^2*a*b^6*c^2*f^7 + B^2*a^2*b
^6*c*f^7 - 64*A^2*a*c^8*d^3*f^4 - 256*A^2*a^3*c^6*d*f^6 + 2*A^2*b^6*c^3*d*f
^6 + 32*B^2*a^4*c^5*d*f^6 + A^2*b^2*c^7*e^6*f - 2*A^2*b^7*c^2*e*f^6 + 4*B^2
*a^2*c^7*e^6*f + 4*A^2*c^9*d^2*e^4*f - 36*A^2*a*b^4*c^4*d*f^6 - 4*A^2*a*b*c
^7*e^5*f^2 + 22*A^2*a*b^5*c^3*e*f^6 - 16*A^2*a^3*b*c^5*e*f^6 - 2*B^2*a*b^6*
c^2*d*f^6 - 40*B^2*a^4*b*c^4*e*f^6 + 8*A^2*a*c^8*d*e^4*f^2 + 16*A^2*b*c^8*d
^3*e*f^3 + 6*A^2*b^5*c^4*d*e*f^5 - 2*A*B*a*b^7*c*f^7 - 144*A^2*a*b^2*c^6*d^
2*f^5 + 168*A^2*a^2*b^2*c^5*d*f^6 + 2*A^2*a*b^2*c^6*e^4*f^3 + 10*A^2*a*b^3*
c^5*e^3*f^4 - 18*A^2*a*b^4*c^4*e^2*f^5 - 80*A^2*a^2*b*c^6*e^3*f^4 - 56*A^2*
a^2*b^3*c^4*e*f^6 + 24*B^2*a*b^2*c^6*d^3*f^4 - 64*B^2*a*b^4*c^4*d^2*f^5 + 2
6*B^2*a^2*b^4*c^3*d*f^6 - 88*B^2*a^3*b^2*c^4*d*f^6 - 12*B^2*a^2*b*c^6*e^5*f
^2 - 40*B^2*a^3*b*c^5*e^3*f^4 + 6*B^2*a^3*b^3*c^3*e*f^6 + 8*A^2*a*c^8*d^2*e
^2*f^3 - 128*A^2*a^2*c^7*d*e^2*f^4 + 8*A^2*b*c^8*d^2*e^3*f^2 + 4*A^2*b^2*c^
7*d*e^4*f^2 + 10*A^2*b^3*c^6*d*e^3*f^3 - 18*A^2*b^4*c^5*d*e^2*f^4 - 32*B^2*
a^2*c^7*d*e^4*f^2 - 80*B^2*a^3*c^6*d*e^2*f^4 + B^2*b^2*c^7*d^2*e^4*f + 10*B
^2*b^3*c^6*d^3*e*f^3 - 12*B^2*b^5*c^4*d^2*e*f^4 + 72*A*B*a^4*b*c^4*f^7 - 8*
A*B*b*c^8*d^4*f^3 - 176*A*B*a^4*c^5*e*f^6 + 2*A*B*b^7*c^2*d*f^6 - 4*A^2*b*c
^8*d*e^5*f + 54*A^2*a^2*b^2*c^5*e^2*f^5 + 84*B^2*a^2*b^2*c^5*d^2*f^5 + 9*B^
2*a^2*b^2*c^5*e^4*f^3 + 2*B^2*a^3*b^2*c^4*e^2*f^5 - 26*A^2*b^2*c^7*d^2*e^2*
f^3 + 88*B^2*a^2*c^7*d^2*e^2*f^3 - 4*B^2*b^2*c^7*d^3*e^2*f^2 - 4*B^2*b^3*c^
6*d^2*e^3*f^2 + 8*B^2*b^4*c^5*d^2*e^2*f^3 + 24*A*B*a^2*b^5*c^2*f^7 - 84*A*B

*a^3*b^3*c^3*f^7 + 8*A*B*a^2*c^7*e^5*f^2 - 32*A*B*a^3*c^6*e^3*f^4 + 4*A*B*b
^3*c^6*d^3*f^4 - 20*A*B*b^5*c^4*d^2*f^5 - 4*B^2*a*b*c^7*d*e^5*f - 78*B^2*a*
b^2*c^6*d^2*e^2*f^3 - 48*B^2*a^2*b^2*c^5*d*e^2*f^4 + 148*A*B*a*b^3*c^5*d^2*
f^5 - 192*A*B*a^2*b*c^6*d^2*f^5 - 4*A*B*a^2*b^3*c^4*d*f^6 + 10*A*B*a*b^2*c^
6*e^5*f^2 - 2*A*B*a*b^3*c^5*e^4*f^3 - 12*A*B*a*b^4*c^4*e^3*f^4 + 8*A*B*a*b^
5*c^3*e^2*f^5 - 52*A*B*a^2*b*c^6*e^4*f^3 - 44*A*B*a^2*b^4*c^3*e*f^6 - 48*A*
B*a^3*b*c^5*e^2*f^5 + 204*A*B*a^3*b^2*c^4*e*f^6 - 48*A*B*a*c^8*d^2*e^3*f^2
- 48*A*B*a^2*c^7*d*e^3*f^3 - 16*A*B*a^2*c^7*d^2*e*f^4 + 16*A*B*b*c^8*d^3*e^
2*f^2 - 28*A*B*b^2*c^7*d^3*e*f^3 - 6*A*B*b^3*c^6*d*e^4*f^2 + 8*A*B*b^4*c^5*
d^2*e*f^4 + 12*A*B*b^5*c^4*d*e^2*f^4 - 40*A^2*a*b*c^7*d*e^3*f^3 + 80*A^2*a*
b*c^7*d^2*e*f^4 - 24*A^2*a*b^3*c^5*d*e*f^5 + 48*A^2*a^2*b*c^6*d*e*f^5 - 24*
B^2*a*b*c^7*d^3*e*f^3 - 8*B^2*a*b^5*c^3*d*e*f^5 + 56*B^2*a^3*b*c^5*d*e*f^5
- 4*A*B*a*b*c^7*e^6*f + 8*A*B*a*c^8*d*e^5*f + 96*A*B*a^2*b^2*c^5*e^3*f^4 -
36*A*B*a^2*b^3*c^4*e^2*f^5 + 4*A*B*b^2*c^7*d^2*e^3*f^2 + 8*A*B*b^3*c^6*d^2*
e^2*f^3 + 84*A^2*a*b^2*c^6*d*e^2*f^4 + 24*B^2*a*b*c^7*d^2*e^3*f^2 + 14*B^2*
a*b^2*c^6*d*e^4*f^2 - 16*B^2*a*b^3*c^5*d*e^3*f^3 + 98*B^2*a*b^3*c^5*d^2*e*f
^4 + 12*B^2*a*b^4*c^4*d*e^2*f^4 + 64*B^2*a^2*b*c^6*d*e^3*f^3 - 120*B^2*a^2*
b*c^6*d^2*e*f^4 + 30*B^2*a^2*b^3*c^4*d*e*f^5 + 16*A*B*a*b*c^7*d^3*f^4 - 12*
A*B*a*b^5*c^3*d*f^6 + 112*A*B*a^3*b*c^5*d*f^6 + 2*A*B*a*b^6*c^2*e*f^6 + 48*
A*B*a*c^8*d^3*e*f^3 + 144*A*B*a^3*c^6*d*e*f^5 - 4*A*B*b*c^8*d^2*e^4*f + 2*A
*B*b^2*c^7*d*e^5*f - 10*A*B*b^6*c^3*d*e*f^5 + 100*A*B*a*b^4*c^4*d*e*f^5 + 6
4*A*B*a*b*c^7*d^2*e^2*f^3 + 12*A*B*a*b^2*c^6*d*e^3*f^3 - 108*A*B*a*b^2*c^6*
d^2*e*f^4 - 100*A*B*a*b^3*c^5*d*e^2*f^4 + 288*A*B*a^2*b*c^6*d*e^2*f^4 - 324
*A*B*a^2*b^2*c^5*d*e*f^5))/(16*a^2*c^6*d^4 + a^4*b^4*f^4 + 16*a^4*c^4*e^4 +
b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c
*f^4 + 2*a^2*b^6*d*f^3 - 2*a^3*b^5*e*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*
f^3 - 2*b^5*c^3*d^3*e + 2*b^6*c^2*d^3*f + a^2*b^4*c^2*e^4 - 8*a^3*b^2*c^3*e
^4 + 32*a^3*c^5*d^2*e^2 + a^2*b^6*e^2*f^2 + 96*a^4*c^4*d^2*f^2 + b^6*c^2*d^
2*e^2 + 32*a^5*c^3*e^2*f^2 - 2*a*b^7*d*e*f^2 - 2*b^7*c*d^2*e*f + 54*a^2*b^4
*c^2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 + 16*a*b^3*c^4*d^3*e - 2*a*b^5*c^2*d
*e^3 - 32*a^2*b*c^5*d^3*e - 32*a^3*b*c^4*d*e^3 - 20*a*b^4*c^3*d^3*f - 12*a*
b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 - 2*a^2*b^5*c*e^3*f - 32*a^4*b*c^3*e^3*f
+ 16*a^4*b^3*c*e*f^3 - 32*a^5*b*c^2*e*f^3 - 64*a^4*c^4*d*e^2*f - 6*a*b^4*c
^3*d^2*e^2 + 16*a^2*b^3*c^3*d*e^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d
*f^3 + 16*a^3*b^3*c^2*e^3*f - 6*a^3*b^4*c*e^2*f^2 - 48*a^2*b^3*c^3*d^2*e*f
- 36*a^2*b^4*c^2*d*e^2*f + 96*a^3*b^2*c^3*d*e^2*f - 48*a^3*b^3*c^2*d*e*f^2
+ 4*a*b^6*c*d*e^2*f + 18*a*b^5*c^2*d^2*e*f + 18*a^2*b^5*c*d*e*f^2 + 32*a^3*
b*c^4*d^2*e*f + 32*a^4*b*c^3*d*e*f^2)) - (3*A^3*b^2*c^5*e^2*f^4 - 4*A^3*c^7
*d^2*f^4 - 12*A^3*a^2*c^5*f^6 - B^3*b^3*c^4*d^2*f^4 + 16*A^3*a*c^6*d*f^5 -
16*A*B^2*a^3*c^4*f^6 + 2*A^3*a*b^2*c^4*f^6 - 16*A^3*a*c^6*e^2*f^4 - 6*A^3*b
^2*c^5*d*f^5 - 3*A^3*b^3*c^4*e*f^5 + 4*B^3*a*b*c^5*d^2*f^4 + 3*B^3*a*b^3*c^
3*d*f^5 - 12*B^3*a^2*b*c^4*d*f^5 + 8*B^3*a^2*c^5*d*e*f^4 + 3*A*B^2*a^2*b^2*
c^3*f^6 - 12*A*B^2*a^2*c^5*e^2*f^4 + A*B^2*b^2*c^5*d^2*f^4 - 3*A^2*B*b^3*c^
4*e^2*f^4 + 16*A^3*a*b*c^5*e*f^5 + 4*A^3*b*c^6*d*e*f^4 - 3*A^2*B*a*b^3*c^3*
f^6 + 16*A^2*B*a^2*b*c^4*f^6 - 8*A*B^2*a*c^6*d^2*f^4 + 24*A*B^2*a^2*c^5*d*f

^5 - 3*A*B^2*b^4*c^3*d*f^5 + 4*A^2*B*b*c^6*d^2*f^4 - 8*A^2*B*a^2*c^5*e*f^5
+ 9*A^2*B*b^3*c^4*d*f^5 + 3*A^2*B*b^4*c^3*e*f^5 + 4*A*B^2*a*b^2*c^4*d*f^5 -
3*A*B^2*a*b^3*c^3*e*f^5 + 16*A*B^2*a^2*b*c^4*e*f^5 + 16*A^2*B*a*b*c^5*e^2*
f^4 - 14*A^2*B*a*b^2*c^4*e*f^5 + 4*A*B^2*b^3*c^4*d*e*f^4 - 8*A^2*B*b^2*c^5*
d*e*f^4 - 2*B^3*a*b^2*c^4*d*e*f^4 + 2*A*B^2*a*b^2*c^4*e^2*f^4 - 28*A^2*B*a*
b*c^5*d*f^5 + 16*A^2*B*a*c^6*d*e*f^4 - 12*A*B^2*a*b*c^5*d*e*f^4)/(16*a^2*c^
6*d^4 + a^4*b^4*f^4 + 16*a^4*c^4*e^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d
^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 2*a^3*b^5*e*
f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 - 2*b^5*c^3*d^3*e + 2*b^6*c^2*d^3
*f + a^2*b^4*c^2*e^4 - 8*a^3*b^2*c^3*e^4 + 32*a^3*c^5*d^2*e^2 + a^2*b^6*e^2
*f^2 + 96*a^4*c^4*d^2*f^2 + b^6*c^2*d^2*e^2 + 32*a^5*c^3*e^2*f^2 - 2*a*b^7*
d*e*f^2 - 2*b^7*c*d^2*e*f + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^
2 + 16*a*b^3*c^4*d^3*e - 2*a*b^5*c^2*d*e^3 - 32*a^2*b*c^5*d^3*e - 32*a^3*b*
c^4*d*e^3 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 -
2*a^2*b^5*c*e^3*f - 32*a^4*b*c^3*e^3*f + 16*a^4*b^3*c*e*f^3 - 32*a^5*b*c^2*
e*f^3 - 64*a^4*c^4*d*e^2*f - 6*a*b^4*c^3*d^2*e^2 + 16*a^2*b^3*c^3*d*e^3 + 6
4*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3 + 16*a^3*b^3*c^2*e^3*f - 6*a^3*b
^4*c*e^2*f^2 - 48*a^2*b^3*c^3*d^2*e*f - 36*a^2*b^4*c^2*d*e^2*f + 96*a^3*b^2
*c^3*d*e^2*f - 48*a^3*b^3*c^2*d*e*f^2 + 4*a*b^6*c*d*e^2*f + 18*a*b^5*c^2*d^
2*e*f + 18*a^2*b^5*c*d*e*f^2 + 32*a^3*b*c^4*d^2*e*f + 32*a^4*b*c^3*d*e*f^2)
)*root(48416*a^6*b^2*c^6*d^4*e^2*f^4*z^4 - 41544*a^5*b^4*c^5*d^4*e^2*f^4*z^
4 - 31872*a^7*b^2*c^5*d^3*e^2*f^5*z^4 - 31872*a^5*b^2*c^7*d^5*e^2*f^3*z^4 -
29184*a^6*b^2*c^6*d^3*e^4*f^3*z^4 + 28800*a^5*b^4*c^5*d^3*e^4*f^3*z^4 + 21
510*a^4*b^6*c^4*d^4*e^2*f^4*z^4 + 21408*a^6*b^4*c^4*d^3*e^2*f^5*z^4 + 21408
*a^4*b^4*c^6*d^5*e^2*f^3*z^4 - 18112*a^7*b^3*c^4*d^2*e^3*f^5*z^4 - 18112*a^
4*b^3*c^7*d^5*e^3*f^2*z^4 - 15600*a^5*b^5*c^4*d^3*e^3*f^4*z^4 - 15600*a^4*b
^5*c^5*d^4*e^3*f^3*z^4 + 15296*a^6*b^3*c^5*d^3*e^3*f^4*z^4 + 15296*a^5*b^3*
c^6*d^4*e^3*f^3*z^4 + 14016*a^7*b^2*c^5*d^2*e^4*f^4*z^4 + 14016*a^5*b^2*c^7
*d^4*e^4*f^2*z^4 - 13920*a^4*b^6*c^4*d^3*e^4*f^3*z^4 - 11648*a^6*b^3*c^5*d^
2*e^5*f^3*z^4 - 11648*a^5*b^3*c^6*d^3*e^5*f^2*z^4 + 10432*a^6*b^2*c^6*d^2*e
^6*f^2*z^4 + 9008*a^6*b^5*c^3*d^2*e^3*f^5*z^4 + 9008*a^3*b^5*c^6*d^5*e^3*f^
2*z^4 + 8544*a^5*b^5*c^4*d^2*e^5*f^3*z^4 + 8544*a^4*b^5*c^5*d^3*e^5*f^2*z^4
- 8496*a^5*b^4*c^5*d^2*e^6*f^2*z^4 + 7488*a^8*b^2*c^4*d^2*e^2*f^6*z^4 + 74
88*a^4*b^2*c^8*d^6*e^2*f^2*z^4 + 7380*a^4*b^7*c^3*d^3*e^3*f^4*z^4 + 7380*a^
3*b^7*c^4*d^4*e^3*f^3*z^4 - 6720*a^3*b^8*c^3*d^4*e^2*f^4*z^4 - 5784*a^5*b^6
*c^3*d^3*e^2*f^5*z^4 - 5784*a^3*b^6*c^5*d^5*e^2*f^3*z^4 - 3440*a^6*b^4*c^4*
d^2*e^4*f^4*z^4 - 3440*a^4*b^4*c^6*d^4*e^4*f^2*z^4 + 3360*a^3*b^8*c^3*d^3*e
^4*f^3*z^4 + 3140*a^4*b^6*c^4*d^2*e^6*f^2*z^4 - 2760*a^4*b^7*c^3*d^2*e^5*f^
3*z^4 - 2760*a^3*b^7*c^4*d^3*e^5*f^2*z^4 - 1764*a^5*b^7*c^2*d^2*e^3*f^5*z^4
- 1764*a^2*b^7*c^5*d^5*e^3*f^2*z^4 - 1640*a^3*b^9*c^2*d^3*e^3*f^4*z^4 - 16
40*a^2*b^9*c^3*d^4*e^3*f^3*z^4 - 1604*a^6*b^6*c^2*d^2*e^2*f^6*z^4 - 1604*a^
2*b^6*c^6*d^6*e^2*f^2*z^4 - 1500*a^5*b^6*c^3*d^2*e^4*f^4*z^4 - 1500*a^3*b^6
*c^5*d^4*e^4*f^2*z^4 + 1140*a^2*b^10*c^2*d^4*e^2*f^4*z^4 + 810*a^4*b^8*c^2*
d^2*e^4*f^4*z^4 + 810*a^2*b^8*c^4*d^4*e^4*f^2*z^4 - 544*a^3*b^8*c^3*d^2*e^6
*f^2*z^4 + 416*a^3*b^9*c^2*d^2*e^5*f^3*z^4 + 416*a^2*b^9*c^3*d^3*e^5*f^2*z^

4 - 384*a^2*b^10*c^2*d^3*e^4*f^3*z^4 + 180*a^4*b^8*c^2*d^3*e^2*f^5*z^4 + 18
0*a^2*b^8*c^4*d^5*e^2*f^3*z^4 + 48*a^7*b^4*c^3*d^2*e^2*f^6*z^4 + 48*a^3*b^4
*c^7*d^6*e^2*f^2*z^4 + 36*a^2*b^10*c^2*d^2*e^6*f^2*z^4 - 1024*a^10*b*c^3*d*
e*f^8*z^4 - 1024*a^3*b*c^10*d^8*e*f*z^4 - 192*a^8*b^5*c*d*e*f^8*z^4 - 192*a
*b^5*c^8*d^8*e*f*z^4 + 16128*a^7*b^3*c^4*d^3*e*f^6*z^4 + 16128*a^4*b^3*c^7*
d^6*e*f^3*z^4 - 11712*a^6*b^5*c^3*d^3*e*f^6*z^4 - 11712*a^3*b^5*c^6*d^6*e*f
^3*z^4 + 11520*a^8*b*c^5*d^2*e^3*f^5*z^4 + 11520*a^5*b*c^8*d^5*e^3*f^2*z^4
- 9984*a^6*b^3*c^5*d^4*e*f^5*z^4 - 9984*a^5*b^3*c^6*d^5*e*f^4*z^4 + 8640*a^
5*b^5*c^4*d^4*e*f^5*z^4 + 8640*a^4*b^5*c^5*d^5*e*f^4*z^4 - 7424*a^7*b*c^6*d
^3*e^3*f^4*z^4 - 7424*a^6*b*c^7*d^4*e^3*f^3*z^4 - 6912*a^8*b^3*c^3*d^2*e*f^
7*z^4 - 6912*a^3*b^3*c^8*d^7*e*f^2*z^4 + 4800*a^7*b^3*c^4*d*e^5*f^4*z^4 + 4
800*a^4*b^3*c^7*d^4*e^5*f*z^4 + 4608*a^7*b*c^6*d^2*e^5*f^3*z^4 + 4608*a^6*b
*c^7*d^3*e^5*f^2*z^4 - 4560*a^4*b^7*c^3*d^4*e*f^5*z^4 - 4560*a^3*b^7*c^4*d^
5*e*f^4*z^4 + 4176*a^5*b^7*c^2*d^3*e*f^6*z^4 + 4176*a^2*b^7*c^5*d^6*e*f^3*z
^4 + 3264*a^7*b^5*c^2*d^2*e*f^7*z^4 + 3264*a^2*b^5*c^7*d^7*e*f^2*z^4 + 3008
*a^8*b^3*c^3*d*e^3*f^6*z^4 + 3008*a^3*b^3*c^8*d^6*e^3*f*z^4 + 2880*a^6*b^3*
c^5*d*e^7*f^2*z^4 + 2880*a^5*b^3*c^6*d^2*e^7*f*z^4 - 2240*a^7*b^4*c^3*d*e^4
*f^5*z^4 - 2240*a^3*b^4*c^7*d^5*e^4*f*z^4 - 1488*a^5*b^5*c^4*d*e^7*f^2*z^4
- 1488*a^4*b^5*c^5*d^2*e^7*f*z^4 + 1440*a^3*b^9*c^2*d^4*e*f^5*z^4 + 1440*a^
2*b^9*c^3*d^5*e*f^4*z^4 - 1328*a^6*b^5*c^3*d*e^5*f^4*z^4 - 1328*a^3*b^5*c^6
*d^4*e^5*f*z^4 - 1152*a^7*b^2*c^5*d*e^6*f^3*z^4 - 1152*a^5*b^2*c^7*d^3*e^6*
f*z^4 - 1120*a^6*b^4*c^4*d*e^6*f^3*z^4 - 1120*a^4*b^4*c^6*d^3*e^6*f*z^4 + 9
12*a^6*b^6*c^2*d*e^4*f^5*z^4 + 912*a^2*b^6*c^6*d^5*e^4*f*z^4 + 872*a^5*b^6*
c^3*d*e^6*f^3*z^4 + 872*a^3*b^6*c^5*d^3*e^6*f*z^4 + 768*a^8*b^2*c^4*d*e^4*f
^5*z^4 + 768*a^4*b^2*c^8*d^5*e^4*f*z^4 - 672*a^8*b^4*c^2*d*e^2*f^7*z^4 - 67
2*a^2*b^4*c^8*d^7*e^2*f*z^4 - 624*a^7*b^5*c^2*d*e^3*f^6*z^4 - 624*a^2*b^5*c
^7*d^6*e^3*f*z^4 + 480*a^5*b^8*c*d^2*e^2*f^6*z^4 + 480*a*b^8*c^5*d^6*e^2*f^
2*z^4 + 316*a^4*b^7*c^3*d*e^7*f^2*z^4 + 316*a^3*b^7*c^4*d^2*e^7*f*z^4 - 204
*a^4*b^8*c^2*d*e^6*f^3*z^4 - 204*a^2*b^8*c^4*d^3*e^6*f*z^4 + 168*a^3*b^10*c
*d^3*e^2*f^5*z^4 + 168*a*b^10*c^3*d^5*e^2*f^3*z^4 + 156*a^2*b^11*c*d^3*e^3*
f^4*z^4 + 156*a*b^11*c^2*d^4*e^3*f^3*z^4 + 128*a^9*b^2*c^3*d*e^2*f^7*z^4 +
128*a^3*b^2*c^9*d^7*e^2*f*z^4 - 124*a^3*b^10*c*d^2*e^4*f^4*z^4 - 124*a*b^10
*c^3*d^4*e^4*f^2*z^4 + 100*a^4*b^9*c*d^2*e^3*f^5*z^4 + 100*a*b^9*c^4*d^5*e^
3*f^2*z^4 + 36*a^5*b^7*c^2*d*e^5*f^4*z^4 + 36*a^2*b^7*c^5*d^4*e^5*f*z^4 - 2
4*a^3*b^9*c^2*d*e^7*f^2*z^4 - 24*a^2*b^11*c*d^2*e^5*f^3*z^4 - 24*a^2*b^9*c^
3*d^2*e^7*f*z^4 - 24*a*b^11*c^2*d^3*e^5*f^2*z^4 - 9216*a^8*b*c^5*d^3*e*f^6*
z^4 - 9216*a^5*b*c^8*d^6*e*f^3*z^4 - 5376*a^8*b*c^5*d*e^5*f^4*z^4 - 5376*a^
5*b*c^8*d^4*e^5*f*z^4 + 5120*a^9*b*c^4*d^2*e*f^7*z^4 + 5120*a^7*b*c^6*d^4*e
*f^5*z^4 + 5120*a^6*b*c^7*d^5*e*f^4*z^4 + 5120*a^4*b*c^9*d^7*e*f^2*z^4 - 43
52*a^9*b*c^4*d*e^3*f^6*z^4 - 4352*a^4*b*c^9*d^6*e^3*f*z^4 - 1792*a^7*b*c^6*
d*e^7*f^2*z^4 - 1792*a^6*b*c^7*d^2*e^7*f*z^4 - 1600*a^6*b^2*c^6*d*e^8*f*z^4
+ 912*a^5*b^4*c^5*d*e^8*f*z^4 + 768*a^9*b^3*c^2*d*e*f^8*z^4 + 768*a^2*b^3*
c^9*d^8*e*f*z^4 - 720*a^4*b^9*c*d^3*e*f^6*z^4 - 720*a*b^9*c^4*d^6*e*f^3*z^4
- 656*a^6*b^7*c*d^2*e*f^7*z^4 - 656*a*b^7*c^6*d^7*e*f^2*z^4 - 240*a^2*b^11
*c*d^4*e*f^5*z^4 - 240*a*b^11*c^2*d^5*e*f^4*z^4 + 216*a^7*b^6*c*d*e^2*f^7*z

^4 + 216*a*b^6*c^7*d^7*e^2*f*z^4 - 204*a^4*b^6*c^4*d*e^8*f*z^4 - 144*a^5*b^
8*c*d*e^4*f^5*z^4 - 144*a*b^8*c^5*d^5*e^4*f*z^4 - 84*a*b^12*c*d^4*e^2*f^4*z
^4 + 36*a^4*b^9*c*d*e^5*f^4*z^4 + 36*a*b^9*c^4*d^4*e^5*f*z^4 + 20*a^6*b^7*c
*d*e^3*f^6*z^4 + 20*a*b^7*c^6*d^6*e^3*f*z^4 + 16*a^3*b^10*c*d*e^6*f^3*z^4 +
 16*a^3*b^8*c^3*d*e^8*f*z^4 + 16*a*b^12*c*d^3*e^4*f^3*z^4 + 16*a*b^10*c^3*d
^3*e^6*f*z^4 + 48*b^11*c^3*d^6*e*f^3*z^4 + 48*b^9*c^5*d^7*e*f^2*z^4 - 20*b^
8*c^6*d^7*e^2*f*z^4 + 8*b^10*c^4*d^5*e^4*f*z^4 - 4*b^13*c*d^4*e^3*f^3*z^4 -
 4*b^11*c^3*d^4*e^5*f*z^4 + 4*b^9*c^5*d^6*e^3*f*z^4 + 3072*a^9*c^5*d*e^4*f^
5*z^4 + 3072*a^5*c^9*d^5*e^4*f*z^4 + 2560*a^8*c^6*d*e^6*f^3*z^4 + 2560*a^6*
c^8*d^3*e^6*f*z^4 + 1536*a^10*c^4*d*e^2*f^7*z^4 + 1536*a^4*c^10*d^7*e^2*f*z
^4 + 48*a^5*b^9*d^2*e*f^7*z^4 + 48*a^3*b^11*d^3*e*f^6*z^4 - 20*a^6*b^8*d*e^
2*f^7*z^4 + 8*a^4*b^10*d*e^4*f^5*z^4 + 4*a^5*b^9*d*e^3*f^6*z^4 - 4*a^3*b^11
*d*e^5*f^4*z^4 - 4*a*b^13*d^3*e^3*f^4*z^4 + 768*a^9*b*c^4*e^5*f^5*z^4 + 768
*a^8*b*c^5*e^7*f^3*z^4 + 256*a^10*b*c^3*e^3*f^7*z^4 - 192*a^6*b^3*c^5*e^9*f
*z^4 - 68*a^7*b^6*c*e^4*f^6*z^4 + 48*a^8*b^5*c*e^3*f^7*z^4 + 48*a^5*b^5*c^4
*e^9*f*z^4 + 36*a^6*b^7*c*e^5*f^5*z^4 - 12*a^9*b^4*c*e^2*f^8*z^4 - 4*a^4*b^
9*c*e^7*f^3*z^4 - 4*a^4*b^7*c^3*e^9*f*z^4 + 384*a^5*b^8*c*d^3*f^7*z^4 + 384
*a*b^8*c^5*d^7*f^3*z^4 + 288*a^3*b^10*c*d^4*f^6*z^4 + 288*a*b^10*c^3*d^6*f^
4*z^4 + 224*a^7*b^6*c*d^2*f^8*z^4 + 224*a*b^6*c^7*d^8*f^2*z^4 - 192*a^10*b^
2*c^2*d*f^9*z^4 - 192*a^2*b^2*c^10*d^9*f*z^4 + 768*a^5*b*c^8*d^3*e^7*z^4 +
768*a^4*b*c^9*d^5*e^5*z^4 + 256*a^3*b*c^10*d^7*e^3*z^4 - 192*a^5*b^3*c^6*d*
e^9*z^4 - 68*a*b^6*c^7*d^6*e^4*z^4 + 48*a^4*b^5*c^5*d*e^9*z^4 + 48*a*b^5*c^
8*d^7*e^3*z^4 + 36*a*b^7*c^6*d^5*e^5*z^4 - 12*a*b^4*c^9*d^8*e^2*z^4 - 4*a^3
*b^7*c^4*d*e^9*z^4 - 4*a*b^9*c^4*d^3*e^7*z^4 + 16*b^13*c*d^5*e*f^4*z^4 + 16
*b^7*c^7*d^8*e*f*z^4 + 768*a^7*c^7*d*e^8*f*z^4 + 16*a^7*b^7*d*e*f^8*z^4 + 1
6*a*b^13*d^4*e*f^5*z^4 + 256*a^7*b*c^6*e^9*f*z^4 + 80*a*b^12*c*d^5*f^5*z^4
+ 48*a^9*b^4*c*d*f^9*z^4 + 48*a*b^4*c^9*d^9*f*z^4 + 256*a^6*b*c^7*d*e^9*z^4
 - 42*b^10*c^4*d^6*e^2*f^2*z^4 - 20*b^12*c^2*d^5*e^2*f^3*z^4 + 6*b^12*c^2*d
^4*e^4*f^2*z^4 + 4*b^11*c^3*d^5*e^3*f^2*z^4 - 24960*a^7*c^7*d^4*e^2*f^4*z^4
 + 18944*a^8*c^6*d^3*e^2*f^5*z^4 + 18944*a^6*c^8*d^5*e^2*f^3*z^4 + 14336*a^
7*c^7*d^3*e^4*f^3*z^4 - 9984*a^8*c^6*d^2*e^4*f^4*z^4 - 9984*a^6*c^8*d^4*e^4
*f^2*z^4 - 7936*a^9*c^5*d^2*e^2*f^6*z^4 - 7936*a^5*c^9*d^6*e^2*f^2*z^4 - 43
52*a^7*c^7*d^2*e^6*f^2*z^4 - 42*a^4*b^10*d^2*e^2*f^6*z^4 - 20*a^2*b^12*d^3*
e^2*f^5*z^4 + 6*a^2*b^12*d^2*e^4*f^4*z^4 + 4*a^3*b^11*d^2*e^3*f^5*z^4 - 480
*a^8*b^2*c^4*e^6*f^4*z^4 + 440*a^7*b^4*c^3*e^6*f^4*z^4 - 320*a^8*b^3*c^3*e^
5*f^5*z^4 - 320*a^7*b^3*c^4*e^7*f^3*z^4 + 240*a^8*b^4*c^2*e^4*f^6*z^4 + 240
*a^6*b^4*c^4*e^8*f^2*z^4 - 192*a^9*b^3*c^2*e^3*f^7*z^4 - 192*a^9*b^2*c^3*e^
4*f^6*z^4 - 192*a^7*b^2*c^5*e^8*f^2*z^4 - 90*a^6*b^6*c^2*e^6*f^4*z^4 - 68*a
^5*b^6*c^3*e^8*f^2*z^4 + 48*a^10*b^2*c^2*e^2*f^8*z^4 - 48*a^7*b^5*c^2*e^5*f
^5*z^4 - 48*a^6*b^5*c^3*e^7*f^3*z^4 + 36*a^5*b^7*c^2*e^7*f^3*z^4 + 6*a^4*b^
8*c^2*e^8*f^2*z^4 - 33920*a^6*b^2*c^6*d^5*f^5*z^4 + 27936*a^5*b^4*c^5*d^5*f
^5*z^4 + 26112*a^7*b^2*c^5*d^4*f^6*z^4 + 26112*a^5*b^2*c^7*d^6*f^4*z^4 - 20
352*a^6*b^4*c^4*d^4*f^6*z^4 - 20352*a^4*b^4*c^6*d^6*f^4*z^4 - 13080*a^4*b^6
*c^4*d^5*f^5*z^4 - 11520*a^8*b^2*c^4*d^3*f^7*z^4 - 11520*a^4*b^2*c^8*d^7*f^
3*z^4 + 8736*a^5*b^6*c^3*d^4*f^6*z^4 + 8736*a^3*b^6*c^5*d^6*f^4*z^4 + 7488*

a^7*b^4*c^3*d^3*f^7*z^4 + 7488*a^3*b^4*c^7*d^7*f^3*z^4 + 3840*a^3*b^8*c^3*d
^5*f^5*z^4 + 2560*a^9*b^2*c^3*d^2*f^8*z^4 + 2560*a^3*b^2*c^9*d^8*f^2*z^4 -
2416*a^6*b^6*c^2*d^3*f^7*z^4 - 2416*a^2*b^6*c^6*d^7*f^3*z^4 - 2160*a^4*b^8*
c^2*d^4*f^6*z^4 - 2160*a^2*b^8*c^4*d^6*f^4*z^4 - 1152*a^8*b^4*c^2*d^2*f^8*z
^4 - 1152*a^2*b^4*c^8*d^8*f^2*z^4 - 720*a^2*b^10*c^2*d^5*f^5*z^4 - 480*a^4*
b^2*c^8*d^4*e^6*z^4 + 440*a^3*b^4*c^7*d^4*e^6*z^4 - 320*a^4*b^3*c^7*d^3*e^7
*z^4 - 320*a^3*b^3*c^8*d^5*e^5*z^4 + 240*a^4*b^4*c^6*d^2*e^8*z^4 + 240*a^2*
b^4*c^8*d^6*e^4*z^4 - 192*a^5*b^2*c^7*d^2*e^8*z^4 - 192*a^3*b^2*c^9*d^6*e^4
*z^4 - 192*a^2*b^3*c^9*d^7*e^3*z^4 - 90*a^2*b^6*c^6*d^4*e^6*z^4 - 68*a^3*b^
6*c^5*d^2*e^8*z^4 - 48*a^3*b^5*c^6*d^3*e^7*z^4 - 48*a^2*b^5*c^7*d^5*e^5*z^4
 + 48*a^2*b^2*c^10*d^8*e^2*z^4 + 36*a^2*b^7*c^5*d^3*e^7*z^4 + 6*a^2*b^8*c^4
*d^2*e^8*z^4 - 4*b^6*c^8*d^9*f*z^4 + 256*a^11*c^3*d*f^9*z^4 + 256*a^3*c^11*
d^9*f*z^4 - 4*a^8*b^6*d*f^9*z^4 - 384*a^9*c^5*e^6*f^4*z^4 - 256*a^10*c^4*e^
4*f^6*z^4 - 256*a^8*c^6*e^8*f^2*z^4 - 64*a^11*c^3*e^2*f^8*z^4 - 24*b^10*c^4
*d^7*f^3*z^4 - 16*b^12*c^2*d^6*f^4*z^4 - 16*b^8*c^6*d^8*f^2*z^4 + 17920*a^7
*c^7*d^5*f^5*z^4 - 14336*a^8*c^6*d^4*f^6*z^4 - 14336*a^6*c^8*d^6*f^4*z^4 +
7168*a^9*c^5*d^3*f^7*z^4 + 7168*a^5*c^9*d^7*f^3*z^4 - 2048*a^10*c^4*d^2*f^8
*z^4 - 2048*a^4*c^10*d^8*f^2*z^4 + 6*b^8*c^6*d^6*e^4*z^4 + 6*a^6*b^8*e^4*f^
6*z^4 - 4*b^9*c^5*d^5*e^5*z^4 - 4*b^7*c^7*d^7*e^3*z^4 - 4*a^7*b^7*e^3*f^7*z
^4 - 4*a^5*b^9*e^5*f^5*z^4 - 384*a^5*c^9*d^4*e^6*z^4 - 256*a^6*c^8*d^2*e^8*
z^4 - 256*a^4*c^10*d^6*e^4*z^4 - 64*a^3*c^11*d^8*e^2*z^4 - 24*a^4*b^10*d^3*
f^7*z^4 - 16*a^6*b^8*d^2*f^8*z^4 - 16*a^2*b^12*d^4*f^6*z^4 + 48*a^6*b^2*c^6
*e^10*z^4 - 12*a^5*b^4*c^5*e^10*z^4 - 4*b^14*d^5*f^5*z^4 - 64*a^7*c^7*e^10*
z^4 + b^14*d^4*e^2*f^4*z^4 + b^10*c^4*d^4*e^6*z^4 + b^6*c^8*d^8*e^2*z^4 + a
^8*b^6*e^2*f^8*z^4 + a^4*b^10*e^6*f^4*z^4 + a^4*b^6*c^4*e^10*z^4 - 4820*A*B
*a^4*b*c^5*d^2*e^2*f^4*z^2 + 2976*A*B*a^3*b*c^6*d^3*e^2*f^3*z^2 - 2328*A*B*
a^3*b*c^6*d^2*e^4*f^2*z^2 + 1848*A*B*a^2*b^4*c^4*d^3*e*f^4*z^2 - 1768*A*B*a
^3*b^4*c^3*d^2*e*f^5*z^2 + 1528*A*B*a^4*b^2*c^4*d^2*e*f^5*z^2 - 1136*A*B*a^
3*b^2*c^5*d^3*e*f^4*z^2 - 974*A*B*a^4*b^3*c^3*d*e^2*f^5*z^2 + 692*A*B*a^2*b
*c^7*d^4*e^2*f^2*z^2 + 588*A*B*a*b^6*c^3*d^2*e^3*f^3*z^2 - 580*A*B*a^3*b^3*
c^4*d*e^4*f^3*z^2 + 488*A*B*a^3*b^4*c^3*d*e^3*f^4*z^2 - 444*A*B*a^2*b^2*c^6
*d^2*e^5*f*z^2 - 412*A*B*a*b^5*c^4*d^2*e^4*f^2*z^2 + 366*A*B*a^2*b^6*c^2*d^
2*e*f^5*z^2 - 352*A*B*a^2*b^2*c^6*d^4*e*f^3*z^2 + 326*A*B*a^2*b^4*c^4*d*e^5
*f^2*z^2 + 324*A*B*a*b^5*c^4*d^3*e^2*f^3*z^2 - 302*A*B*a*b^3*c^6*d^4*e^2*f^
2*z^2 - 296*A*B*a*b^7*c^2*d^2*e^2*f^4*z^2 + 122*A*B*a^4*b^2*c^4*d*e^3*f^4*z
^2 - 122*A*B*a^2*b^6*c^2*d*e^3*f^4*z^2 - 84*A*B*a^3*b^2*c^5*d*e^5*f^2*z^2 +
 72*A*B*a*b^4*c^5*d^3*e^3*f^2*z^2 - 64*A*B*a^2*b^5*c^3*d*e^4*f^3*z^2 + 60*A
*B*a^3*b^5*c^2*d*e^2*f^5*z^2 + 1312*A*B*a^5*b*c^4*d*e^2*f^5*z^2 + 1040*A*B*
a^4*b*c^5*d*e^4*f^3*z^2 - 500*A*B*a*b^6*c^3*d^3*e*f^4*z^2 - 376*A*B*a*b^2*c
^7*d^5*e*f^2*z^2 + 276*A*B*a^4*b^4*c^2*d*e*f^6*z^2 - 262*A*B*a^2*b^3*c^5*d*
e^6*f*z^2 + 238*A*B*a*b^2*c^7*d^4*e^3*f*z^2 + 232*A*B*a^5*b^2*c^3*d*e*f^6*z
^2 - 176*A*B*a^2*b*c^7*d^3*e^4*f*z^2 - 120*A*B*a*b^6*c^3*d*e^5*f^2*z^2 - 10
8*A*B*a*b^4*c^5*d^4*e*f^3*z^2 + 68*A*B*a*b^7*c^2*d*e^4*f^3*z^2 + 68*A*B*a*b
^4*c^5*d^2*e^5*f*z^2 + 46*A*B*a^2*b^7*c*d*e^2*f^5*z^2 - 36*A*B*a*b^3*c^6*d^
3*e^4*f*z^2 - 1932*A*B*a^2*b^3*c^5*d^3*e^2*f^3*z^2 - 1818*A*B*a^2*b^4*c^4*d

^2*e^3*f^3*z^2 + 1620*A*B*a^3*b^3*c^4*d^2*e^2*f^4*z^2 + 1560*A*B*a^2*b^3*c^5*d^2*e^4*f^2*z^2 + 1244*A*B*a^3*b^2*c^5*d^2*e^3*f^3*z^2 + 820*A*B*a^2*b^2*c^6*d^3*e^3*f^2*z^2 + 480*A*B*a^2*b^5*c^3*d^2*e^2*f^4*z^2 + 352*A*B*a^3*b*c^6*d*e^6*f*z^2 - 108*A*B*a^3*b^6*c*d*e*f^6*z^2 + 82*A*B*a*b^5*c^4*d*e^6*f*z^2 - 64*A*B*a*b*c^8*d^5*e^2*f*z^2 + 16*A*B*a*b^8*c*d^2*e*f^5*z^2 - 4*A*B*a*b^8*c*d*e^3*f^4*z^2 + 16*B^2*a*b*c^8*d^6*e*f*z^2 + 56*A*B*b^2*c^8*d^6*e*f*z^2 - 8*A*B*b^9*c*d*e^4*f^3*z^2 - 8*A*B*b^7*c^3*d*e^6*f*z^2 - 800*A*B*a^6*c^4*d*e*f^6*z^2 + 10*A*B*a^2*b^8*d*e*f^6*z^2 - 6*A*B*a*b^9*d*e^2*f^5*z^2 - 12*A*B*a^5*b^4*c*e*f^7*z^2 + 912*A*B*a^6*b*c^3*d*f^7*z^2 + 192*A*B*a^4*b^5*c*d*f^7*z^2 + 192*A*B*a*b*c^8*d^6*f^2*z^2 - 20*A*B*a*b^4*c^5*d*e^7*z^2 + 4*A*B*a*b*c^8*d^4*e^4*z^2 + 2144*B^2*a^4*b*c^5*d^3*e*f^4*z^2 - 1120*B^2*a^3*b*c^6*d^4*e*f^3*z^2 - 688*B^2*a^5*b*c^4*d^2*e*f^5*z^2 - 256*B^2*a^3*b*c^6*d^2*e^5*f*z^2 + 152*B^2*a*b^3*c^6*d^5*e*f^2*z^2 + 120*B^2*a^5*b^3*c^2*d*e*f^6*z^2 - 116*B^2*a^5*b*c^4*d*e^3*f^4*z^2 + 110*B^2*a*b^7*c^2*d^3*e*f^4*z^2 - 80*B^2*a^2*b*c^7*d^5*e*f^2*z^2 - 72*B^2*a*b^5*c^4*d^4*e*f^3*z^2 - 48*B^2*a^4*b*c^5*d*e^5*f^2*z^2 - 46*B^2*a*b^3*c^6*d^4*e^3*f*z^2 - 44*B^2*a*b^4*c^5*d^3*e^4*f*z^2 - 34*B^2*a*b^5*c^4*d^2*e^5*f*z^2 + 20*B^2*a^2*b*c^7*d^4*e^3*f*z^2 - 10*B^2*a^3*b^6*c*d*e^2*f^5*z^2 - 10*B^2*a^2*b^7*c*d^2*e*f^5*z^2 - 10*B^2*a*b^2*c^7*d^5*e^2*f*z^2 - 7*B^2*a^2*b^4*c^4*d*e^6*f*z^2 - 6*B^2*a^3*b^2*c^5*d*e^6*f*z^2 + 4*B^2*a*b^8*c*d^2*e^2*f^4*z^2 - 2*B^2*a^2*b^7*c*d*e^3*f^4*z^2 + 3196*A^2*a^4*b*c^5*d*e^3*f^4*z^2 - 3184*A^2*a^4*b*c^5*d^2*e*f^5*z^2 + 1568*A^2*a^3*b*c^6*d^3*e*f^4*z^2 + 1504*A^2*a^3*b*c^6*d*e^5*f^2*z^2 - 656*A^2*a^4*b^3*c^3*d*e*f^6*z^2 - 400*A^2*a*b^6*c^3*d*e^4*f^3*z^2 + 314*A^2*a*b^5*c^4*d*e^5*f^2*z^2 - 264*A^2*a^3*b^5*c^2*d*e*f^6*z^2 + 240*A^2*a^2*b^2*c^6*d*e^6*f*z^2 - 224*A^2*a^2*b*c^7*d^4*e*f^3*z^2 + 216*A^2*a*b^5*c^4*d^3*e*f^4*z^2 - 192*A^2*a^2*b*c^7*d^2*e^5*f*z^2 + 178*A^2*a*b^7*c^2*d*e^3*f^4*z^2 - 154*A^2*a*b^7*c^2*d^2*e*f^5*z^2 + 128*A^2*a*b^3*c^6*d^4*e*f^3*z^2 + 106*A^2*a*b^3*c^6*d^2*e^5*f*z^2 - 12*A^2*a*b^2*c^7*d^3*e^4*f*z^2 - 58*A*B*b^8*c^2*d^2*e^3*f^3*z^2 + 40*A*B*b^7*c^3*d^2*e^4*f^2*z^2 - 28*A*B*b^7*c^3*d^3*e^2*f^3*z^2 - 24*A*B*b^5*c^5*d^4*e^2*f^2*z^2 - 20*A*B*b^6*c^4*d^3*e^3*f^2*z^2 + 2768*A*B*a^4*c^6*d^2*e^3*f^3*z^2 - 1712*A*B*a^3*c^7*d^3*e^3*f^2*z^2 - 156*A*B*a^4*b^2*c^4*e^5*f^3*z^2 + 146*A*B*a^4*b^3*c^3*e^4*f^4*z^2 - 106*A*B*a^5*b^2*c^3*e^3*f^5*z^2 + 90*A*B*a^5*b^3*c^2*e^2*f^6*z^2 + 38*A*B*a^3*b^3*c^4*e^6*f^2*z^2 - 36*A*B*a^3*b^5*c^2*e^4*f^4*z^2 + 16*A*B*a^3*b^4*c^3*e^5*f^3*z^2 - 9*A*B*a^4*b^4*c^2*e^3*f^5*z^2 - 8*A*B*a^2*b^5*c^3*e^6*f^2*z^2 + 2*A*B*a^2*b^6*c^2*e^5*f^3*z^2 + 920*A*B*a^4*b^3*c^3*d^2*f^6*z^2 - 480*A*B*a^2*b^5*c^3*d^3*f^5*z^2 - 336*A*B*a^2*b^3*c^5*d^4*f^4*z^2 - 272*A*B*a^3*b^3*c^4*d^3*f^5*z^2 + 240*A*B*a^3*b^5*c^2*d^2*f^6*z^2 - 32*A*B*a*c^9*d^6*e*f*z^2 - 792*B^2*a^2*b^3*c^5*d^3*e^3*f^2*z^2 + 714*B^2*a^2*b^4*c^4*d^3*e^2*f^3*z^2 - 572*B^2*a^3*b^2*c^5*d^3*e^2*f^3*z^2 - 475*B^2*a^2*b^2*c^6*d^4*e^2*f^2*z^2 + 265*B^2*a^4*b^2*c^4*d^2*e^2*f^4*z^2 + 260*B^2*a^3*b^3*c^4*d^2*e^3*f^3*z^2 - 212*B^2*a^3*b^4*c^3*d^2*e^2*f^4*z^2 + 180*B^2*a^3*b^2*c^5*d^2*e^4*f^2*z^2 - 158*B^2*a^2*b^4*c^4*d^2*e^4*f^2*z^2 + 47*B^2*a^2*b^6*c^2*d^2*e^2*f^4*z^2 + 16*B^2*a^2*b^5*c^3*d^2*e^3*f^3*z^2 + 2752*A^2*a^3*b^2*c^5*d^2*e^2*f^4*z^2 - 2148*A^2*a^2*b^4*c^4*d^2*e^2*f^4*z^2 + 2064*A^2*a^2*b^3*c^5*d^2*e^3*f^3

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*z^2 - 424*A^2*a^2*b^2*c^6*d^3*e^2*f^3*z^2 - 198*A^2*a^2*b^2*c^6*d^2*e^4*f^
2*z^2 - 272*B^2*a^6*b*c^3*d*e*f^6*z^2 - 24*B^2*a^4*b^5*c*d*e*f^6*z^2 + 1808
*A^2*a^5*b*c^4*d*e*f^6*z^2 - 244*A^2*a*b*c^8*d^4*e^3*f*z^2 + 208*A^2*a*b*c^
8*d^5*e*f^2*z^2 + 134*A^2*a^2*b^7*c*d*e*f^6*z^2 - 76*A^2*a*b^4*c^5*d*e^6*f*
z^2 + 4*A^2*a*b^8*c*d*e^2*f^5*z^2 + 148*A*B*b^4*c^6*d^5*e*f^2*z^2 + 65*A*B*
b^6*c^4*d^4*e*f^3*z^2 + 46*A*B*b^8*c^2*d^3*e*f^4*z^2 - 38*A*B*b^3*c^7*d^5*e
^2*f*z^2 + 34*A*B*b^9*c*d^2*e^2*f^4*z^2 - 29*A*B*b^4*c^6*d^4*e^3*f*z^2 + 20
*A*B*b^5*c^5*d^3*e^4*f*z^2 + 12*A*B*b^8*c^2*d*e^5*f^2*z^2 - 7*A*B*b^6*c^4*d
^2*e^5*f*z^2 - 2880*A*B*a^4*c^6*d^3*e*f^4*z^2 + 2784*A*B*a^5*c^5*d^2*e*f^5*
z^2 - 1112*A*B*a^5*c^5*d*e^3*f^4*z^2 + 896*A*B*a^3*c^7*d^4*e*f^3*z^2 + 848*
A*B*a^3*c^7*d^2*e^5*f*z^2 - 560*A*B*a^4*c^6*d*e^5*f^2*z^2 + 96*A*B*a^2*c^8*
d^5*e*f^2*z^2 - 88*A*B*a^2*c^8*d^4*e^3*f*z^2 - 100*A*B*a^6*b*c^3*e^2*f^6*z^
2 - 76*A*B*a^5*b*c^4*e^4*f^4*z^2 + 48*A*B*a^6*b^2*c^2*e*f^7*z^2 - 42*A*B*a^
3*b^2*c^5*e^7*f*z^2 + 36*A*B*a^4*b*c^5*e^6*f^2*z^2 - 24*A*B*a^4*b^5*c*e^2*f
^6*z^2 + 10*A*B*a^3*b^6*c*e^3*f^5*z^2 + 7*A*B*a^2*b^4*c^4*e^7*f*z^2 + 2*A*B
*a^2*b^7*c*e^4*f^4*z^2 - 2496*A*B*a^5*b*c^4*d^2*f^6*z^2 + 1872*A*B*a^4*b*c^
5*d^3*f^5*z^2 - 744*A*B*a^5*b^3*c^2*d*f^7*z^2 - 720*A*B*a^2*b*c^7*d^5*f^3*z
^2 + 504*A*B*a*b^3*c^6*d^5*f^3*z^2 + 256*A*B*a^3*b*c^6*d^4*f^4*z^2 + 168*A*
B*a*b^7*c^2*d^3*f^5*z^2 - 144*A*B*a^2*b^7*c*d^2*f^6*z^2 + 144*A*B*a*b^5*c^4
*d^4*f^4*z^2 + 66*A*B*a^2*b^2*c^6*d*e^7*z^2 - 36*A*B*a*b^2*c^7*d^3*e^5*z^2
 + 20*A*B*a*b^3*c^6*d^2*e^6*z^2 + 12*A*B*a^2*b*c^7*d^2*e^6*z^2 + 1208*B^2*a^
3*b*c^6*d^3*e^3*f^2*z^2 - 848*B^2*a^3*b^3*c^4*d^3*e*f^4*z^2 + 672*B^2*a^2*b
^3*c^5*d^4*e*f^3*z^2 - 632*B^2*a^4*b*c^5*d^2*e^3*f^3*z^2 + 432*B^2*a^4*b^3*
c^3*d^2*e*f^5*z^2 + 276*B^2*a^2*b^2*c^6*d^3*e^4*f*z^2 - 196*B^2*a*b^6*c^3*d
^3*e^2*f^3*z^2 - 168*B^2*a^2*b^5*c^3*d^3*e*f^4*z^2 + 154*B^2*a^2*b^3*c^5*d^
2*e^5*f*z^2 + 148*B^2*a*b^5*c^4*d^3*e^3*f^2*z^2 + 96*B^2*a*b^4*c^5*d^4*e^2*
f^2*z^2 - 72*B^2*a^3*b^5*c^2*d^2*e*f^5*z^2 + 70*B^2*a^5*b^2*c^3*d*e^2*f^5*z
^2 - 60*B^2*a^4*b^3*c^3*d*e^3*f^4*z^2 + 52*B^2*a*b^6*c^3*d^2*e^4*f^2*z^2 +
36*B^2*a^4*b^2*c^4*d*e^4*f^3*z^2 - 32*B^2*a*b^7*c^2*d^2*e^3*f^3*z^2 + 24*B^
2*a^3*b^5*c^2*d*e^3*f^4*z^2 + 15*B^2*a^4*b^4*c^2*d*e^2*f^5*z^2 - 8*B^2*a^3*
b^4*c^3*d*e^4*f^3*z^2 + 8*B^2*a^2*b^5*c^3*d*e^5*f^2*z^2 - 2*B^2*a^3*b^3*c^4
*d*e^5*f^2*z^2 - 2*B^2*a^2*b^6*c^2*d*e^4*f^3*z^2 - 3176*A^2*a^3*b*c^6*d^2*e
^3*f^3*z^2 - 2252*A^2*a^4*b^2*c^4*d*e^2*f^5*z^2 + 1952*A^2*a^3*b^4*c^3*d*e^
2*f^5*z^2 - 1496*A^2*a^3*b^3*c^4*d*e^3*f^4*z^2 + 1378*A^2*a^2*b^4*c^4*d*e^4
*f^3*z^2 + 1184*A^2*a^3*b^3*c^4*d^2*e*f^5*z^2 - 1166*A^2*a^2*b^3*c^5*d*e^5*
f^2*z^2 - 1164*A^2*a^3*b^2*c^5*d*e^4*f^3*z^2 - 1152*A^2*a^2*b^3*c^5*d^3*e*f
^4*z^2 + 578*A^2*a*b^6*c^3*d^2*e^2*f^4*z^2 - 548*A^2*a*b^5*c^4*d^2*e^3*f^3*
z^2 + 440*A^2*a*b^2*c^7*d^4*e^2*f^2*z^2 - 412*A^2*a^2*b^6*c^2*d*e^2*f^5*z^2
 - 360*A^2*a*b^3*c^6*d^3*e^3*f^2*z^2 + 312*A^2*a*b^4*c^5*d^3*e^2*f^3*z^2 +
248*A^2*a^2*b*c^7*d^3*e^3*f^2*z^2 - 224*A^2*a^2*b^5*c^3*d*e^3*f^4*z^2 + 216
*A^2*a^2*b^5*c^3*d^2*e*f^5*z^2 + 52*A^2*a*b^4*c^5*d^2*e^4*f^2*z^2 - 16*B^2*
b^3*c^7*d^6*e*f*z^2 - 14*B^2*b^9*c*d^3*e*f^4*z^2 + 32*B^2*a^4*c^6*d*e^6*f*z
^2 - 20*A^2*b^9*c*d*e^3*f^4*z^2 + 18*A^2*b^9*c*d^2*e*f^5*z^2 + 8*A^2*b^6*c^
4*d*e^6*f*z^2 - 360*A^2*a^3*c^7*d*e^6*f*z^2 + 136*A^2*a*c^9*d^5*e^2*f*z^2 +
 2*B^2*a^3*b^7*d*e*f^6*z^2 + 2*B^2*a*b^9*d^2*e*f^5*z^2 + 12*B^2*a^4*b*c^5*e
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^7*f*z^2 - 204*A^2*a^3*b*c^6*e^7*f*z^2 - 128*A^2*a^6*b*c^3*e*f^7*z^2 - 48*A
^2*a*b^5*c^4*e^7*f*z^2 - 36*B^2*a^5*b^4*c*d*f^7*z^2 - 24*A^2*a^4*b^5*c*e*f^
7*z^2 - 16*B^2*a*b^8*c*d^3*f^5*z^2 - 164*A^2*a^3*b^6*c*d*f^7*z^2 - 16*A^2*a
*b^8*c*d^2*f^6*z^2 + 4*B^2*a^3*b*c^6*d*e^7*z^2 - 4*B^2*a*b*c^8*d^5*e^3*z^2
+ 48*A^2*a*b*c^8*d^3*e^5*z^2 + 36*A^2*a^2*b*c^7*d*e^7*z^2 - 6*A^2*a*b^3*c^6
*d*e^7*z^2 + 136*A*B*a^6*c^4*e^3*f^5*z^2 - 96*A*B*b^5*c^5*d^5*f^3*z^2 + 80*
A*B*a^5*c^5*e^5*f^3*z^2 - 72*A*B*b^3*c^7*d^6*f^2*z^2 - 24*A*B*b^7*c^3*d^4*f
^4*z^2 + 14*A*B*b^3*c^7*d^4*e^4*z^2 - 14*A*B*b^2*c^8*d^5*e^3*z^2 - 2*A*B*b^
5*c^5*d^2*e^6*z^2 - 2*A*B*b^4*c^6*d^3*e^5*z^2 + 2*A*B*a^3*b^7*e^2*f^6*z^2 -
 A*B*a^2*b^8*e^3*f^5*z^2 + 16*A*B*a^2*c^8*d^3*e^5*z^2 - 2*A*B*a^2*b^3*c^5*e
^8*z^2 + 22*B^2*b^8*c^2*d^3*e^2*f^3*z^2 - 12*B^2*b^7*c^3*d^3*e^3*f^2*z^2 +
12*B^2*b^6*c^4*d^4*e^2*f^2*z^2 - 6*B^2*b^8*c^2*d^2*e^4*f^2*z^2 - 864*B^2*a^
4*c^6*d^3*e^2*f^3*z^2 + 496*B^2*a^3*c^7*d^4*e^2*f^2*z^2 + 224*B^2*a^5*c^5*d
^2*e^2*f^4*z^2 + 136*B^2*a^4*c^6*d^2*e^4*f^2*z^2 - 53*A^2*b^8*c^2*d^2*e^2*f
^4*z^2 + 52*A^2*b^7*c^3*d^2*e^3*f^3*z^2 + 52*A^2*b^5*c^5*d^3*e^3*f^2*z^2 -
36*A^2*b^6*c^4*d^3*e^2*f^3*z^2 - 12*A^2*b^4*c^6*d^4*e^2*f^2*z^2 - 9*A^2*b^6
*c^4*d^2*e^4*f^2*z^2 + 836*A^2*a^4*c^6*d^2*e^2*f^4*z^2 - 668*A^2*a^2*c^8*d^
4*e^2*f^2*z^2 + 656*A^2*a^3*c^7*d^2*e^4*f^2*z^2 + 368*A^2*a^3*c^7*d^3*e^2*f
^3*z^2 - 45*B^2*a^6*b^2*c^2*e^2*f^6*z^2 - 18*B^2*a^5*b^2*c^3*e^4*f^4*z^2 -
9*B^2*a^4*b^2*c^4*e^6*f^2*z^2 - 6*B^2*a^5*b^3*c^2*e^3*f^5*z^2 + 3*B^2*a^4*b
^4*c^2*e^4*f^4*z^2 - 2*B^2*a^4*b^3*c^3*e^5*f^3*z^2 - 580*B^2*a^4*b^2*c^4*d^
3*f^5*z^2 + 536*B^2*a^3*b^4*c^3*d^3*f^5*z^2 + 471*A^2*a^4*b^2*c^4*e^4*f^4*z
^2 - 436*A^2*a^3*b^4*c^3*e^4*f^4*z^2 - 348*B^2*a^4*b^4*c^2*d^2*f^6*z^2 + 31
6*B^2*a^2*b^2*c^6*d^5*f^3*z^2 + 310*A^2*a^3*b^3*c^4*e^5*f^3*z^2 + 232*A^2*a
^5*b^2*c^3*e^2*f^6*z^2 - 229*A^2*a^2*b^4*c^4*e^6*f^2*z^2 - 216*A^2*a^4*b^4*
c^2*e^2*f^6*z^2 + 204*A^2*a^4*b^3*c^3*e^3*f^5*z^2 + 200*B^2*a^5*b^2*c^3*d^2
*f^6*z^2 + 150*A^2*a^3*b^2*c^5*e^6*f^2*z^2 - 120*B^2*a^2*b^4*c^4*d^4*f^4*z^
2 + 91*A^2*a^2*b^6*c^2*e^4*f^4*z^2 + 72*A^2*a^3*b^5*c^2*e^3*f^5*z^2 - 66*B^
2*a^2*b^6*c^2*d^3*f^5*z^2 + 44*A^2*a^2*b^5*c^3*e^5*f^3*z^2 - 16*B^2*a^3*b^2
*c^5*d^4*f^4*z^2 + 1952*A^2*a^4*b^2*c^4*d^2*f^6*z^2 - 1792*A^2*a^3*b^2*c^5*
d^3*f^5*z^2 - 1272*A^2*a^3*b^4*c^3*d^2*f^6*z^2 + 976*A^2*a^2*b^2*c^6*d^4*f^
4*z^2 + 960*A^2*a^2*b^4*c^4*d^3*f^5*z^2 + 282*A^2*a^2*b^6*c^2*d^2*f^6*z^2 -
 45*B^2*a^2*b^2*c^6*d^2*e^6*z^2 - 48*A^2*b*c^9*d^6*e*f*z^2 - 14*A^2*a*b^9*d
*e*f^6*z^2 - 7*A*B*b^10*d^2*e*f^5*z^2 + 2*A*B*b^10*d*e^3*f^4*z^2 - 64*A*B*a
^7*c^3*e*f^7*z^2 - 16*A*B*b^9*c*d^3*f^5*z^2 + 8*A*B*a^4*c^6*e^7*f*z^2 + 4*A
*B*b*c^9*d^6*e^2*z^2 + 2*A*B*b^6*c^4*d*e^7*z^2 - 120*A*B*a^3*c^7*d*e^7*z^2
- 16*A*B*a^3*b^7*d*f^7*z^2 + 16*A*B*a*b^9*d^2*f^6*z^2 + 8*A*B*a*c^9*d^5*e^3
*z^2 + 12*A*B*a^3*b*c^6*e^8*z^2 - 48*B^2*b^5*c^5*d^5*e*f^2*z^2 + 15*B^2*b^4
*c^6*d^5*e^2*f*z^2 - 14*B^2*b^7*c^3*d^4*e*f^3*z^2 + 4*B^2*b^9*c*d^2*e^3*f^3
*z^2 + 4*B^2*b^7*c^3*d^2*e^5*f*z^2 + 4*B^2*b^5*c^5*d^4*e^3*f*z^2 - B^2*b^6*
c^4*d^3*e^4*f*z^2 - 336*B^2*a^3*c^7*d^3*e^4*f*z^2 + 112*B^2*a^5*c^5*d*e^4*f
^3*z^2 - 112*A^2*b^3*c^7*d^5*e*f^2*z^2 + 80*B^2*a^6*c^4*d*e^2*f^5*z^2 - 48*
A^2*b^5*c^5*d^4*e*f^3*z^2 + 36*A^2*b^8*c^2*d*e^4*f^3*z^2 + 36*A^2*b^3*c^7*d
^4*e^3*f*z^2 - 28*A^2*b^7*c^3*d*e^5*f^2*z^2 + 20*A^2*b^2*c^8*d^5*e^2*f*z^2
+ 16*B^2*a^2*c^8*d^5*e^2*f*z^2 - 14*A^2*b^7*c^3*d^3*e*f^4*z^2 - 14*A^2*b^4*
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c^6*d^3*e^4*f*z^2 - 10*A^2*b^5*c^5*d^2*e^5*f*z^2 - 1008*A^2*a^4*c^6*d*e^4*f
^3*z^2 - 760*A^2*a^5*c^5*d*e^2*f^5*z^2 + 272*A^2*a^2*c^8*d^3*e^4*f*z^2 + 48
*B^2*a^5*b*c^4*e^5*f^3*z^2 + 36*B^2*a^6*b*c^3*e^3*f^5*z^2 + 12*B^2*a^5*b^4*
c*e^2*f^6*z^2 - 624*A^2*a^4*b*c^5*e^5*f^3*z^2 - 548*A^2*a^5*b*c^4*e^3*f^5*z
^2 + 182*A^2*a^2*b^3*c^5*e^7*f*z^2 - 180*B^2*a*b^4*c^5*d^5*f^3*z^2 + 132*B^
2*a^6*b^2*c^2*d*f^7*z^2 + 108*B^2*a^3*b^6*c*d^2*f^6*z^2 + 96*A^2*a^5*b^3*c^
2*e*f^7*z^2 + 68*A^2*a*b^6*c^3*e^6*f^2*z^2 + 58*A^2*a^3*b^6*c*e^2*f^6*z^2 -
56*B^2*a*b^2*c^7*d^6*f^2*z^2 - 38*A^2*a^2*b^7*c*e^3*f^5*z^2 - 36*A^2*a*b^7
*c^2*e^5*f^3*z^2 + 20*B^2*a*b^6*c^3*d^4*f^4*z^2 - 736*A^2*a^5*b^2*c^3*d*f^7
*z^2 + 624*A^2*a^4*b^4*c^2*d*f^7*z^2 - 416*A^2*a*b^2*c^7*d^5*f^3*z^2 - 276*
A^2*a*b^4*c^5*d^4*f^4*z^2 - 196*A^2*a*b^6*c^3*d^3*f^5*z^2 + 8*B^2*a*b^4*c^5
*d^2*e^6*z^2 + 6*B^2*a*b^2*c^7*d^4*e^4*z^2 + 2*B^2*a^2*b^3*c^5*d*e^7*z^2 +
2*B^2*a*b^3*c^6*d^3*e^5*z^2 - 18*A^2*a*b^2*c^7*d^2*e^6*z^2 - 16*A*B*b*c^9*d
^7*f*z^2 - B^2*b^10*d^2*e^2*f^4*z^2 + 48*B^2*a^7*c^3*e^2*f^6*z^2 - 36*B^2*a
^6*c^4*e^4*f^4*z^2 + 31*B^2*b^6*c^4*d^5*f^3*z^2 - 24*B^2*a^5*c^5*e^6*f^2*z^
2 + 20*B^2*b^4*c^6*d^6*f^2*z^2 - 6*A^2*b^8*c^2*e^6*f^2*z^2 + 2*B^2*b^8*c^2*
d^4*f^4*z^2 - 768*B^2*a^5*c^5*d^3*f^5*z^2 + 512*B^2*a^6*c^4*d^2*f^6*z^2 + 5
12*B^2*a^4*c^6*d^4*f^4*z^2 + 232*A^2*a^5*c^5*e^4*f^4*z^2 + 188*A^2*a^4*c^6*
e^6*f^2*z^2 - 128*B^2*a^3*c^7*d^5*f^3*z^2 + 92*A^2*a^6*c^4*e^2*f^6*z^2 + 80
*A^2*b^4*c^6*d^5*f^3*z^2 + 64*A^2*b^2*c^8*d^6*f^2*z^2 + 31*A^2*b^6*c^4*d^4*
f^4*z^2 + 14*A^2*b^8*c^2*d^3*f^5*z^2 - 5*B^2*b^4*c^6*d^4*e^4*z^2 + 4*B^2*b^
3*c^7*d^5*e^3*z^2 + 2*B^2*b^5*c^5*d^3*e^5*z^2 - B^2*b^6*c^4*d^2*e^6*z^2 - B
^2*b^2*c^8*d^6*e^2*z^2 - B^2*a^4*b^6*e^2*f^6*z^2 - 1152*A^2*a^3*c^7*d^4*f^4
*z^2 + 1008*A^2*a^4*c^6*d^3*f^5*z^2 + 624*A^2*a^2*c^8*d^5*f^3*z^2 - 288*A^2
*a^5*c^5*d^2*f^6*z^2 + 56*B^2*a^3*c^7*d^2*e^6*z^2 - 10*B^2*a^2*b^8*d^2*f^6*
z^2 - 9*A^2*b^2*c^8*d^4*e^4*z^2 - 5*A^2*a^2*b^8*e^2*f^6*z^2 - 4*B^2*a^2*c^8
*d^4*e^4*z^2 + 3*A^2*b^4*c^6*d^2*e^6*z^2 - 2*A^2*b^3*c^7*d^3*e^5*z^2 - 36*A
^2*a^2*c^8*d^2*e^6*z^2 - 48*A^2*a^6*b^2*c^2*f^8*z^2 - 45*A^2*a^2*b^2*c^6*e^
8*z^2 + 4*A^2*b^10*d*e^2*f^5*z^2 + 4*B^2*b^2*c^8*d^7*f*z^2 + 4*A^2*b^9*c*e^
5*f^3*z^2 + 4*A^2*b^7*c^3*e^7*f*z^2 - 128*B^2*a^7*c^3*d*f^7*z^2 - 160*A^2*a
*c^9*d^6*f^2*z^2 - 112*A^2*a^6*c^4*d*f^7*z^2 + 12*A^2*b*c^9*d^5*e^3*z^2 + 4
*A^2*a*b^9*e^3*f^5*z^2 + 3*B^2*a^4*b^6*d*f^7*z^2 + 2*A^2*a^3*b^7*e*f^7*z^2
- 24*A^2*a*c^9*d^4*e^4*z^2 + 14*A^2*a^2*b^8*d*f^7*z^2 + 12*A^2*a^5*b^4*c*f^
8*z^2 + 12*A^2*a*b^4*c^5*e^8*z^2 + A*B*a^4*b^6*e*f^7*z^2 + B^2*a^2*b^8*d*e^
2*f^5*z^2 + 16*A^2*c^10*d^7*f*z^2 + 3*B^2*b^10*d^3*f^5*z^2 - A^2*b^10*e^4*f
^4*z^2 - 4*A^2*c^10*d^6*e^2*z^2 - A^2*b^10*d^2*f^6*z^2 + 64*A^2*a^7*c^3*f^8
*z^2 - 4*B^2*a^4*c^6*e^8*z^2 - A^2*b^6*c^4*e^8*z^2 + 48*A^2*a^3*c^7*e^8*z^2
- A^2*a^4*b^6*f^8*z^2 + 720*A^2*B*a*b^2*c^5*d^2*e^2*f^3*z - 600*A^2*B*a^2*
b^2*c^4*d*e^2*f^4*z + 576*A*B^2*a^2*b^2*c^4*d^2*e*f^4*z + 348*A*B^2*a*b^2*c
^5*d^2*e^3*f^2*z - 336*A*B^2*a^2*b*c^5*d^2*e^2*f^3*z - 260*A*B^2*a*b^3*c^4*
d^2*e^2*f^3*z - 240*A*B^2*a^2*b^2*c^4*d*e^3*f^3*z + 196*A*B^2*a^2*b^3*c^3*d
*e^2*f^4*z + 172*A^2*B*a*b*c^6*d*e^5*f*z + 20*A*B^2*a*b^6*c*d*e*f^5*z - 912
*A^2*B*a^2*b*c^5*d^2*e*f^4*z - 644*A^2*B*a*b*c^6*d^2*e^3*f^2*z - 432*A*B^2*
a*b^2*c^5*d^3*e*f^3*z + 372*A^2*B*a^2*b*c^5*d*e^3*f^3*z - 330*A^2*B*a*b^2*c
^5*d*e^4*f^2*z + 312*A*B^2*a*b*c^6*d^3*e^2*f^2*z - 208*A*B^2*a^3*b^2*c^3*d*

e*f^5*z + 192*A^2*B*a^2*b^3*c^3*d*e*f^5*z + 172*A^2*B*a*b^3*c^4*d*e^3*f^3*z + 108*A*B^2*a^2*b*c^5*d*e^4*f^2*z + 104*A*B^2*a^3*b*c^4*d*e^2*f^4*z - 80*A^2*B*a*b^3*c^4*d^2*e*f^4*z + 68*A^2*B*a*b^4*c^3*d*e^2*f^4*z - 60*A*B^2*a*b^5*c^2*d*e^2*f^4*z + 58*A*B^2*a*b^3*c^4*d*e^4*f^2*z - 36*A*B^2*a*b^4*c^3*d^2*e*f^4*z - 24*A*B^2*a^2*b^4*c^2*d*e*f^5*z + 24*A*B^2*a*b^4*c^3*d*e^3*f^3*z + 592*A^2*B*a*b*c^6*d^3*e*f^3*z + 240*A^2*B*a^3*b*c^4*d*e*f^5*z - 132*A*B^2*a*b*c^6*d^2*e^4*f*z - 60*A*B^2*a*b^2*c^5*d*e^5*f*z - 48*A^2*B*a*b^5*c^2*d*e*f^5*z + 20*B^3*a*b*c^6*d^3*e^3*f*z + 16*B^3*a^4*b*c^3*d*e*f^5*z - 16*B^3*a*b*c^6*d^4*e*f^2*z + 12*B^3*a^2*b*c^5*d*e^5*f*z + 320*A^3*a*b*c^6*d*e^4*f^2*z + 40*A^3*a*b^4*c^3*d*e*f^5*z - 48*A^2*B*b*c^7*d^4*e*f^2*z - 44*A^2*B*b^3*c^5*d*e^5*f*z - 20*A*B^2*b*c^7*d^4*e^2*f*z + 14*A*B^2*b^4*c^4*d*e^5*f*z + 12*A^2*B*b*c^7*d^3*e^3*f*z + 4*A*B^2*b^7*c*d*e^2*f^4*z + 160*A*B^2*a^4*c^4*d*e*f^5*z + 152*A^2*B*a*c^7*d^2*e^4*f*z - 40*A*B^2*a*c^7*d^3*e^3*f*z + 32*A*B^2*a*c^7*d^4*e*f^2*z - 16*A*B^2*a^2*c^6*d*e^5*f*z + 128*A^2*B*a^4*b*c^3*e*f^6*z + 42*A^2*B*a*b^2*c^5*e^6*f*z + 24*A^2*B*a^2*b^5*c*e*f^6*z - 12*A*B^2*a^3*b^4*c*e*f^6*z - 12*A*B^2*a^2*b*c^5*e^6*f*z - 10*A^2*B*a*b^6*c*e^2*f^5*z - 160*A*B^2*a*b*c^6*d^4*f^3*z + 112*A*B^2*a^4*b*c^3*d*f^6*z - 24*A*B^2*a^2*b^5*c*d*f^6*z - 84*B^3*a*b^2*c^5*d^3*e^2*f^2*z - 80*B^3*a^2*b^3*c^3*d^2*e*f^4*z - 60*B^3*a^2*b*c^5*d^2*e^3*f^2*z - 20*B^3*a^3*b^2*c^3*d*e^2*f^4*z - 20*B^3*a*b^3*c^4*d^2*e^3*f^2*z - 9*B^3*a^2*b^2*c^4*d*e^4*f^2*z - 8*B^3*a*b^4*c^3*d^2*e^2*f^3*z + 6*B^3*a^2*b^4*c^2*d*e^2*f^4*z - 4*B^3*a^2*b^3*c^3*d*e^3*f^3*z - 216*A^2*B*b^4*c^4*d^2*e^2*f^3*z + 196*A^2*B*b^3*c^5*d^2*e^3*f^2*z - 108*A*B^2*b^3*c^5*d^3*e^2*f^2*z - 94*A*B^2*b^4*c^4*d^2*e^3*f^2*z + 88*A^2*B*b^2*c^6*d^3*e^2*f^2*z + 80*A*B^2*b^5*c^3*d^2*e^2*f^3*z + 360*A^2*B*a^2*c^6*d^2*e^2*f^3*z + 8*A*B^2*a^2*c^6*d^2*e^3*f^2*z + 153*A^2*B*a^2*b^2*c^4*e^4*f^3*z - 144*A^2*B*a^2*b^3*c^3*e^3*f^4*z + 80*A^2*B*a^3*b^2*c^3*e^2*f^5*z + 36*A*B^2*a^3*b^2*c^3*e^3*f^4*z + 12*A^2*B*a^2*b^4*c^2*e^2*f^5*z + 12*A*B^2*a^3*b^3*c^2*e^2*f^5*z + 9*A*B^2*a^2*b^2*c^4*e^5*f^2*z - 6*A*B^2*a^2*b^4*c^2*e^3*f^4*z + 4*A*B^2*a^2*b^3*c^3*e^4*f^3*z + 480*A^2*B*a^2*b^2*c^4*d^2*f^5*z - 176*A*B^2*a^2*b^3*c^3*d^2*f^5*z - 10*A^2*B*a*b^6*c*d*f^6*z + 16*A*B^2*a*b*c^6*d*e^6*z + 80*B^3*a*b^3*c^4*d^3*e*f^3*z - 48*B^3*a^3*b*c^4*d^2*e*f^4*z + 48*B^3*a^2*b*c^5*d^3*e*f^3*z + 44*B^3*a^3*b*c^4*d*e^3*f^3*z + 24*B^3*a*b^5*c^2*d^2*e*f^4*z + 18*B^3*a*b^2*c^5*d^2*e^4*f*z + 696*A^3*a^2*b*c^5*d*e^2*f^4*z - 504*A^3*a*b*c^6*d^2*e^2*f^3*z - 192*A^3*a*b^2*c^5*d*e^3*f^3*z - 144*A^3*a^2*b^2*c^4*d*e*f^5*z + 96*A^3*a*b^2*c^5*d^2*e*f^4*z - 72*A^3*a*b^3*c^4*d*e^2*f^4*z - 208*A^2*B*b^3*c^5*d^3*e*f^3*z + 152*A*B^2*b^4*c^4*d^3*e*f^3*z + 80*A^2*B*b^5*c^3*d^2*e*f^4*z + 75*A^2*B*b^4*c^4*d*e^4*f^2*z - 59*A^2*B*b^2*c^6*d^2*e^4*f*z - 52*A^2*B*b^5*c^3*d*e^3*f^3*z + 42*A*B^2*b^3*c^5*d^2*e^4*f*z - 21*A*B^2*b^6*c^2*d^2*e*f^4*z - 16*A*B^2*b^5*c^3*d*e^4*f^2*z + 16*A*B^2*b^2*c^6*d^4*e*f^2*z + 16*A*B^2*b^2*c^6*d^3*e^3*f*z + 11*A^2*B*b^6*c^2*d*e^2*f^4*z + 4*A*B^2*b^6*c^2*d*e^3*f^3*z - 256*A^2*B*a*c^7*d^3*e^2*f^2*z - 96*A*B^2*a^3*c^5*d^2*e*f^4*z - 36*A^2*B*a^2*c^6*d*e^4*f^2*z - 32*A^2*B*a^3*c^5*d*e^2*f^4*z - 32*A*B^2*a^2*c^6*d^3*e*f^3*z + 8*A*B^2*a^3*c^5*d*e^3*f^3*z - 96*A^2*B*a^3*b^3*c^2*e*f^6*z + 68*A^2*B*a^3*b*c^4*e^3*f^4*z - 60*A*B^2*a^4*b*c^3*e^2*f^5*z - 60*A*B^2*a^3*b*c^4*e^4*f^3*z + 48*A*B^2*a^4*

b^2*c^2*e*f^6*z - 38*A^2*B*a*b^3*c^4*e^5*f^2*z - 36*A^2*B*a^2*b*c^5*e^5*f^2
*z + 36*A^2*B*a*b^5*c^2*e^3*f^4*z - 16*A^2*B*a*b^4*c^3*e^4*f^3*z + 384*A*B^
2*a^2*b*c^5*d^3*f^4*z - 352*A*B^2*a^3*b*c^4*d^2*f^5*z - 288*A^2*B*a*b^2*c^5
*d^3*f^4*z - 160*A^2*B*a^3*b^2*c^3*d*f^6*z - 148*A^2*B*a*b^4*c^3*d^2*f^5*z
+ 112*A*B^2*a*b^3*c^4*d^3*f^4*z + 72*A^2*B*a^2*b^4*c^2*d*f^6*z + 72*A*B^2*a
*b^5*c^2*d^2*f^5*z + 48*A*B^2*a^3*b^3*c^2*d*f^6*z + 102*B^3*a^2*b^2*c^4*d^2
*e^2*f^3*z - 32*B^3*b^5*c^3*d^3*e*f^3*z - 8*B^3*b^3*c^5*d^3*e^3*f*z - 7*B^3
*b^4*c^4*d^2*e^4*f*z + 5*B^3*b^2*c^6*d^4*e^2*f*z + 80*A^3*b^2*c^6*d^3*e*f^3
*z - 74*A^3*b^3*c^5*d*e^4*f^2*z - 64*A^3*b^4*c^4*d^2*e*f^4*z + 60*A^3*b^4*c
^4*d*e^3*f^3*z - 48*B^3*a^4*c^4*d*e^2*f^4*z - 24*B^3*a^3*c^5*d*e^4*f^2*z +
20*B^3*a^2*c^6*d^2*e^4*f*z - 16*A^3*b^5*c^3*d*e^2*f^4*z + 8*A^3*b*c^7*d^3*e
^2*f^2*z + 480*A^3*a^2*c^6*d^2*e*f^4*z - 392*A^3*a^2*c^6*d*e^3*f^3*z + 280*
A^3*a*c^7*d^2*e^3*f^2*z - 4*B^3*a^4*b*c^3*e^3*f^4*z - 200*A^3*a^3*b*c^4*e^2
*f^5*z - 144*A^3*a^2*b*c^5*e^4*f^3*z + 48*B^3*a*b^2*c^5*d^4*f^3*z + 42*A^3*
a*b^2*c^5*e^5*f^2*z - 36*B^3*a^4*b^2*c^2*d*f^6*z - 32*A^3*a^3*b^2*c^3*e*f^6
*z - 24*A^3*a^2*b^4*c^2*e*f^6*z - 24*A^3*a*b^5*c^2*e^2*f^5*z + 10*A^3*a*b^3
*c^4*e^4*f^3*z - 4*B^3*a*b^4*c^3*d^3*f^4*z - 4*A^3*a*b^4*c^3*e^3*f^4*z - 48
0*A^3*a^2*b*c^5*d^2*f^5*z - 160*A^3*a^2*b^3*c^3*d*f^6*z + 128*A^3*a*b^3*c^4
*d^2*f^5*z + 8*A^2*B*b^5*c^3*e^5*f^2*z - 2*A^2*B*b^6*c^2*e^4*f^3*z + 112*A^
2*B*b^4*c^4*d^3*f^4*z - 92*A^2*B*a^4*c^4*e^2*f^5*z - 64*A^2*B*a^3*c^5*e^4*f
^3*z - 64*A*B^2*b^5*c^3*d^3*f^4*z + 24*A*B^2*a^4*c^4*e^3*f^4*z + 24*A*B^2*a
^3*c^5*e^5*f^2*z + 16*A^2*B*b^2*c^6*d^4*f^3*z + 16*A*B^2*b^3*c^5*d^4*f^3*z
- A^2*B*b^6*c^2*d^2*f^5*z + 448*A^2*B*a^3*c^5*d^2*f^5*z - 352*A^2*B*a^2*c^6
*d^3*f^4*z - 5*A*B^2*b^2*c^6*d^2*e^5*z - 48*A^2*B*a^4*b^2*c^2*f^7*z - 2*B^3
*b^7*c*d^2*e*f^4*z + 34*A^3*b^2*c^6*d*e^5*f*z + 16*A^3*b*c^7*d^2*e^4*f*z +
2*A^3*b^6*c^2*d*e*f^5*z - 416*A^3*a^3*c^5*d*e*f^5*z - 224*A^3*a*c^7*d^3*e*f
^3*z + 12*B^3*a^3*b^4*c*d*f^6*z - 10*B^3*a*b^6*c*d^2*f^5*z + 416*A^3*a^3*b*
c^4*d*f^6*z + 224*A^3*a*b*c^6*d^3*f^4*z + 24*A^3*a*b^5*c^2*d*f^6*z - 4*B^3*
a*b*c^6*d^2*e^5*z + 20*A^2*B*c^8*d^4*e^2*f*z - 7*A^2*B*b^4*c^4*e^6*f*z - 2*
A^2*B*b^7*c*e^3*f^4*z - 64*A*B^2*a^5*c^3*e*f^6*z + 16*A*B^2*b*c^7*d^5*f^2*z
- 8*A^2*B*a^2*c^6*e^6*f*z - 2*A*B^2*b^7*c*d^2*f^5*z - 272*A^2*B*a^4*c^4*d*
f^6*z + 128*A^2*B*a*c^7*d^4*f^3*z + 9*A^2*B*b^2*c^6*d*e^6*z - 4*A*B^2*b^3*c
^5*d*e^6*z + 4*A*B^2*b*c^7*d^3*e^4*z + 8*A*B^2*a*c^7*d^2*e^5*z + 12*A^2*B*a
^3*b^4*c*f^7*z + 30*B^3*b^4*c^4*d^3*e^2*f^2*z + 8*B^3*b^5*c^3*d^2*e^3*f^2*z
- 2*B^3*b^6*c^2*d^2*e^2*f^3*z + 152*A^3*b^3*c^5*d^2*e^2*f^3*z - 108*A^3*b^
2*c^6*d^2*e^3*f^2*z + 48*B^3*a^3*c^5*d^2*e^2*f^3*z - 16*B^3*a^2*c^6*d^3*e^2
*f^2*z - 3*B^3*a^4*b^2*c^2*e^2*f^5*z - 120*B^3*a^2*b^2*c^4*d^3*f^4*z + 112*
B^3*a^3*b^2*c^3*d^2*f^5*z + 112*A^3*a^2*b^3*c^3*e^2*f^5*z + 12*A^3*a^2*b^2*
c^4*e^3*f^4*z - 120*A^3*a*c^7*d*e^5*f*z - 52*A^3*a*b*c^6*e^6*f*z + 10*A^3*a
*b^6*c*e*f^6*z - 2*A*B^2*b^8*d*e*f^5*z - 2*A^2*B*a*b^7*e*f^6*z - 24*A^2*B*a
*c^7*d*e^6*z + 2*A*B^2*a*b^7*d*f^6*z - 12*A^2*B*a*b*c^6*e^7*z - 2*A^3*b^7*c
*d*f^6*z - 4*A^3*b*c^7*d*e^6*z + 16*B^3*a^5*c^3*e^2*f^5*z + 11*B^3*b^6*c^2*
d^3*f^4*z - 11*A^3*b^4*c^4*e^5*f^2*z - 8*B^3*b^4*c^4*d^4*f^3*z - 4*B^3*b^2*
c^6*d^5*f^2*z + 4*B^3*a^4*c^4*e^4*f^3*z + 4*A^3*b^5*c^3*e^4*f^3*z - A^3*b^6
*c^2*e^3*f^4*z + 136*A^3*a^3*c^5*e^3*f^4*z + 68*A^3*a^2*c^6*e^5*f^2*z - 64*

A^3*b^3*c^5*d^3*f^4*z + 2*B^3*b^3*c^5*d^2*e^5*z - B^3*b^2*c^6*d^3*e^4*z + 96*A^3*a^3*b^3*c^2*f^7*z + A*B^2*a^2*b^6*e*f^6*z + 32*A^3*c^8*d^4*e*f^2*z - 24*A^3*c^8*d^3*e^3*f*z + 10*A^3*b^3*c^5*e^6*f*z + 2*A^3*b^7*c*e^2*f^5*z + 128*A^3*a^4*c^4*e*f^6*z - 32*A^3*b*c^7*d^4*f^3*z - 4*B^3*a^2*c^6*d*e^6*z - B^3*a^2*b^6*d*f^6*z - 128*A^3*a^4*b*c^3*f^7*z - 24*A^3*a^2*b^5*c*f^7*z - 16*A^2*B*c^8*d^5*f^2*z - 4*A^2*B*c^8*d^3*e^4*z + 64*A^2*B*a^5*c^3*f^7*z + 2*A^2*B*b^3*c^5*e^7*z + 4*A*B^2*a^2*c^6*e^7*z - A^2*B*a^2*b^6*f^7*z + 4*A^3*c^8*d^2*e^5*z - 3*A^3*b^2*c^6*e^7*z + A^2*B*b^8*d*f^6*z - A^3*b^8*e*f^6*z + 16*A^3*a*c^7*e^7*z + 2*A^3*a*b^7*f^7*z + A^2*B*b^8*e^2*f^5*z + B^3*b^8*d^2*f^5*z - 48*A^2*B^2*a*b*c^4*d*e*f^4 + 28*A*B^3*a*b^2*c^3*d*e*f^4 - 16*A*B^3*a*b*c^4*d*e^2*f^3 + 16*A^3*B*a*c^5*d*e*f^4 + 32*A^3*B*a*b*c^4*d*f^5 + 12*A^2*B^2*b^3*c^3*d*e*f^4 + 5*A*B^3*b^2*c^4*d^2*e*f^3 + 4*A*B^3*b^3*c^3*d*e^2*f^3 + 24*A^2*B^2*a*c^5*d*e^2*f^3 + 24*A^2*B^2*a^2*b*c^3*e*f^5 + 12*A^2*B^2*a*b*c^4*e^3*f^3 - 6*A^2*B^2*a*b^3*c^2*e*f^5 + 4*A*B^3*a^2*b*c^3*e^2*f^4 + 3*A*B^3*a^2*b^2*c^2*e*f^5 - 18*A^2*B^2*a*b^2*c^3*d*f^5 - 4*B^4*a^2*b*c^3*d*e*f^4 + 4*B^4*a*b*c^4*d^2*e*f^3 - 6*A*B^3*b^4*c^2*d*e*f^4 + 4*A^3*B*b*c^5*d*e^2*f^3 - 2*A^3*B*b^2*c^4*d*e*f^4 - 8*A*B^3*a^2*c^4*d*e*f^4 - 8*A*B^3*a*c^5*d^2*e*f^3 + 26*A^3*B*a*b^2*c^3*e*f^5 + 8*A^3*B*a*b*c^4*e^2*f^4 + 32*A*B^3*a*b*c^4*d^2*f^4 - 28*A*B^3*a^2*b*c^3*d*f^5 + 6*A*B^3*a*b^3*c^2*d*f^5 - 9*A^2*B^2*b^2*c^4*d*e^2*f^3 - 18*A^2*B^2*a*b^2*c^3*e^2*f^4 - 4*A^3*B*c^6*d^2*e*f^3 - 3*A^3*B*b^4*c^2*e*f^5 - 44*A^3*B*a^2*c^4*e*f^5 - 16*A^3*B*a*c^5*e^3*f^3 - 16*A*B^3*a^3*c^3*e*f^5 - 10*A^3*B*b^3*c^3*d*f^5 - 4*A^3*B*b*c^5*d^2*f^4 - 4*A*B^3*b*c^5*d^3*f^3 - 28*A^3*B*a^2*b*c^3*f^6 + 6*A^3*B*a*b^3*c^2*f^6 - 4*A^4*b*c^5*d*e*f^4 - 20*A^4*a*b*c^4*e*f^5 + 3*A^2*B^2*b^4*c^2*e^2*f^4 - 2*A^2*B^2*b^3*c^3*e^3*f^3 + 12*A^2*B^2*a^2*c^4*e^2*f^4 + 9*A^2*B^2*b^2*c^4*d^2*f^4 - 3*A^2*B^2*a^2*b^2*c^2*f^6 - 2*B^4*b^3*c^3*d^2*e*f^3 + 4*B^4*a^2*c^4*d*e^2*f^3 - 10*B^4*a*b^2*c^3*d^2*f^4 - 3*B^4*a^2*b^2*c^2*d*f^5 + 3*A^3*B*b^2*c^4*e^3*f^3 - 2*A^3*B*b^3*c^3*e^2*f^4 - 10*A*B^3*b^3*c^3*d^2*f^4 - 4*A*B^3*a^2*c^4*e^3*f^3 + 3*A^2*B^2*b^4*c^2*d*f^5 + 36*A^2*B^2*a^2*c^4*d*f^5 - 24*A^2*B^2*a*c^5*d^2*f^4 + 4*A^2*B^2*c^6*d^3*f^3 + 16*A^2*B^2*a^3*c^3*f^6 + 4*A^4*b^3*c^3*e*f^5 + 16*B^4*a^3*c^3*d*f^5 + 16*A^4*a*c^5*e^2*f^4 + 8*A^4*b^2*c^4*d*f^5 - 8*A^4*a*b^2*c^3*f^6 - 24*A^4*a*c^5*d*f^5 + 3*B^4*b^4*c^2*d^2*f^4 - 3*A^4*b^2*c^4*e^2*f^4 + 4*A^4*c^6*d^2*f^4 + 36*A^4*a^2*c^4*f^6 + B^4*b^2*c^4*d^3*f^3, z, k), k, 1, 4) - ((A*b^3*f + 2*A*a*c^2*e + A*b*c^2*d - 2*B*a*c^2*d - A*b^2*c*e - B*a*b^2*f + 2*B*a^2*c*f - 3*A*a*b*c*f + B*a*b*c*e)/(a^2*b^2*f^2 - 4*a^3*c*f^2 - 4*a*c^3*d^2 - 4*a^2*c^2*e^2 + b^2*c^2*d^2 + b^4*d*f - a*b^3*e*f - b^3*c*d*e + a*b^2*c*e^2 + 8*a^2*c^2*d*f + 4*a*b*c^2*d*e - 6*a*b^2*c*d*f + 4*a^2*b*c*e*f) - (x*(2*A*a*c^2*f - 2*A*c^3*d + A*b*c^2*e - 2*B*a*c^2*e + B*b*c^2*d - A*b^2*c*f + B*a*b*c*f))/(a^2*b^2*f^2 - 4*a^3*c*f^2 - 4*a*c^3*d^2 - 4*a^2*c^2*e^2 + b^2*c^2*d^2 + b^4*d*f - a*b^3*e*f - b^3*c*d*e + a*b^2*c*e^2 + 8*a^2*c^2*d*f + 4*a*b*c^2*d*e - 6*a*b^2*c*d*f + 4*a^2*b*c*e*f))/(a + b*x + c*x^2)

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)**2/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

## 3.17 $\int \frac{g+hx}{(a+bx+cx^2)(ad+bdx+cdx^2)^2}\,dx$

**Optimal**. Leaf size=140

$$\frac{3(b+2cx)(2cg-bh)}{2d^2(b^2-4ac)^2(a+bx+cx^2)}-\frac{-2ah+x(2cg-bh)+bg}{2d^2(b^2-4ac)(a+bx+cx^2)^2}-\frac{6c(2cg-bh)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{d^2(b^2-4ac)^{5/2}}$$

**Rubi [A]** time = 0.13, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {998, 638, 614, 618, 206}

$$\frac{3(b+2cx)(2cg-bh)}{2d^2(b^2-4ac)^2(a+bx+cx^2)}-\frac{-2ah+x(2cg-bh)+bg}{2d^2(b^2-4ac)(a+bx+cx^2)^2}-\frac{6c(2cg-bh)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{d^2(b^2-4ac)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)/((a + b*x + c*x^2)*(a*d + b*d*x + c*d*x^2)^2),x]
```

```
[Out] -(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*(b^2 - 4*a*c)*d^2*(a + b*x + c*x^2)^2)
+ (3*(2*c*g - b*h)*(b + 2*c*x))/(2*(b^2 - 4*a*c)^2*d^2*(a + b*x + c*x^2)) -
 (6*c*(2*c*g - b*h)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^
(5/2)*d^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 998

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(c/f)^p, Int[(g + h*
x)^m*(d + e*x + f*x^2)^(p + q), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p,
 q}, x] && EqQ[c*d - a*f, 0] && EqQ[b*d - a*e, 0] && (IntegerQ[p] || GtQ[c/
f, 0]) && ( !IntegerQ[q] || LeafCount[d + e*x + f*x^2] <= LeafCount[a + b*x
 + c*x^2])
```

Rubi steps

$$\int \frac{g+hx}{\left(a+bx+cx^2\right)\left(ad+bdx+cdx^2\right)^2}\,dx = \frac{\int \frac{g+hx}{\left(a+bx+cx^2\right)^3}\,dx}{d^2}$$

$$= -\frac{bg-2ah+(2cg-bh)x}{2\left(b^2-4ac\right)d^2\left(a+bx+cx^2\right)^2} - \frac{(3(2cg-bh))\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)d^2}$$

$$= -\frac{bg-2ah+(2cg-bh)x}{2\left(b^2-4ac\right)d^2\left(a+bx+cx^2\right)^2} + \frac{3(2cg-bh)(b+2cx)}{2\left(b^2-4ac\right)^2d^2\left(a+bx+cx^2\right)} +$$

$$= -\frac{bg-2ah+(2cg-bh)x}{2\left(b^2-4ac\right)d^2\left(a+bx+cx^2\right)^2} + \frac{3(2cg-bh)(b+2cx)}{2\left(b^2-4ac\right)^2d^2\left(a+bx+cx^2\right)} -$$

$$= -\frac{bg-2ah+(2cg-bh)x}{2\left(b^2-4ac\right)d^2\left(a+bx+cx^2\right)^2} + \frac{3(2cg-bh)(b+2cx)}{2\left(b^2-4ac\right)^2d^2\left(a+bx+cx^2\right)} -$$

**Mathematica [A]** time = 0.14, size = 131, normalized size = 0.94

$$\frac{\frac{(b^2-4ac)(2ah-bg+bhx-2cgx)}{(a+x(b+cx))^2}-\frac{12c(bh-2cg)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{3(b+2cx)(2cg-bh)}{a+x(b+cx)}}{2d^2(b^2-4ac)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)/((a + b*x + c*x^2)*(a*d + b*d*x + c*d*x^2)^2),x]
```

```
[Out] (((b^2 - 4*a*c)*(-(b*g) + 2*a*h - 2*c*g*x + b*h*x))/(a + x*(b + c*x))^2 + (
3*(2*c*g - b*h)*(b + 2*c*x))/(a + x*(b + c*x)) - (12*c*(-2*c*g + b*h)*ArcTa
n[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(2*(b^2 - 4*a*c)^2*d
^2)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{g+hx}{(a+bx+cx^2)(ad+bdx+cdx^2)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(g + h*x)/((a + b*x + c*x^2)*(a*d + b*d*x + c*d*x^2)^2),
x]
```

```
[Out] IntegrateAlgebraic[(g + h*x)/((a + b*x + c*x^2)*(a*d + b*d*x + c*d*x^2)^2),
 x]
```

**fricas [B]** time = 0.64, size = 1150, normalized size = 8.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(c*x^2+b*x+a)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="fricas"
)
```

```
[Out] [1/2*(6*(2*(b^2*c^3 - 4*a*c^4)*g - (b^3*c^2 - 4*a*b*c^3)*h)*x^3 + 9*(2*(b^3
*c^2 - 4*a*b*c^3)*g - (b^4*c - 4*a*b^2*c^2)*h)*x^2 - 6*(2*a^2*c^2*g - a^2*b
*c*h + (2*c^4*g - b*c^3*h)*x^4 + 2*(2*b*c^3*g - b^2*c^2*h)*x^3 + (2*(b^2*c^
2 + 2*a*c^3)*g - (b^3*c + 2*a*b*c^2)*h)*x^2 + 2*(2*a*b*c^2*g - a*b^2*c*h)*x
)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a
*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (b^5 - 14*a*b^3*c + 40*a^2*b*c^2)*g -
 (a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2)*h + 2*(2*(b^4*c + a*b^2*c^2 - 20*a^2*c^
3)*g - (b^5 + a*b^3*c - 20*a^2*b*c^2)*h)*x)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5)*d^2*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3
```

```
 - 64*a^3*b*c^4)*d^2*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*
c^3 - 128*a^4*c^4)*d^2*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*
a^4*b*c^3)*d^2*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*d
^2), 1/2*(6*(2*(b^2*c^3 - 4*a*c^4)*g - (b^3*c^2 - 4*a*b*c^3)*h)*x^3 + 9*(2*
(b^3*c^2 - 4*a*b*c^3)*g - (b^4*c - 4*a*b^2*c^2)*h)*x^2 - 12*(2*a^2*c^2*g -
a^2*b*c*h + (2*c^4*g - b*c^3*h)*x^4 + 2*(2*b*c^3*g - b^2*c^2*h)*x^3 + (2*(b
^2*c^2 + 2*a*c^3)*g - (b^3*c + 2*a*b*c^2)*h)*x^2 + 2*(2*a*b*c^2*g - a*b^2*c
*h)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a
*c)) - (b^5 - 14*a*b^3*c + 40*a^2*b*c^2)*g - (a*b^4 + 4*a^2*b^2*c - 32*a^3*
c^2)*h + 2*(2*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*g - (b^5 + a*b^3*c - 20*a^2*
b*c^2)*h)*x)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2*x^
4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2*x^3 + (b^8
 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2*x^2 + 2*
(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2*x + (a^2*b^6 - 1
2*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*d^2)]
```

**giac [A]** time = 0.17, size = 219, normalized size = 1.56

$$\frac{6\left(2c^2g - bch\right)\arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\left(b^4d^2 - 8ab^2cd^2 + 16a^2c^2d^2\right)\sqrt{-b^2+4ac}} + \frac{12c^3gx^3 - 6bc^2hx^3 + 18bc^2gx^2 - 9b^2chx^2 + 4b^2cgx + 20ac^2gx - 2b^3hx - 10abchx - b^3g + 10abcg - ab^2h - 8a^2ch}{2\left(b^4d^2 - 8ab^2cd^2 + 16a^2c^2d^2\right)\left(cx^2 + bx + a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(c*x^2+b*x+a)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="giac")
```

```
[Out] 6*(2*c^2*g - b*c*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4*d^2 - 8*a*
b^2*c*d^2 + 16*a^2*c^2*d^2)*sqrt(-b^2 + 4*a*c)) + 1/2*(12*c^3*g*x^3 - 6*b*c
^2*h*x^3 + 18*b*c^2*g*x^2 - 9*b^2*c*h*x^2 + 4*b^2*c*g*x + 20*a*c^2*g*x - 2*
b^3*h*x - 10*a*b*c*h*x - b^3*g + 10*a*b*c*g - a*b^2*h - 8*a^2*c*h)/((b^4*d^
2 - 8*a*b^2*c*d^2 + 16*a^2*c^2*d^2)*(c*x^2 + b*x + a)^2)
```

**maple [B]** time = 0.01, size = 340, normalized size = 2.43

$$-\frac{3bchx}{\left(4ac-b^2\right)^2\left(cx^2+bx+a\right)d^2} - \frac{6bch\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{5}{2}}d^2} + \frac{6c^2gx}{\left(4ac-b^2\right)^2\left(cx^2+bx+a\right)d^2} + \frac{12c^2g\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{5}{2}}d^2} - \frac{3b^2h}{2\left(4ac-b^2\right)^2\left(cx^2+bx+a\right)d^2} + \frac{3bcg}{\left(4ac-b^2\right)^2\left(cx^2+bx+a\right)d^2} - \frac{bhx}{2\left(4ac-b^2\right)\left(cx^2+bx+a\right)^2d^2} + \frac{cgx}{\left(4ac-b^2\right)\left(cx^2+bx+a\right)^2d^2} - \frac{ah}{\left(4ac-b^2\right)\left(cx^2+bx+a\right)^2d^2} + \frac{bg}{2\left(4ac-b^2\right)\left(cx^2+bx+a\right)^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)/(c*x^2+b*x+a)/(c*d*x^2+b*d*x+a*d)^2,x)
```

```
[Out] -1/2/d^2/(4*a*c-b^2)/(c*x^2+b*x+a)^2*x*b*h+1/d^2/(4*a*c-b^2)/(c*x^2+b*x+a)^
2*x*c*g-1/d^2/(4*a*c-b^2)/(c*x^2+b*x+a)^2*a*h+1/2/d^2/(4*a*c-b^2)/(c*x^2+b*
x+a)^2*b*g-3/d^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)*c*x*b*h+6/d^2/(4*a*c-b^2)^2/(c
*x^2+b*x+a)*c^2*x*g-3/2/d^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)*b^2*h+3/d^2/(4*a*c-
b^2)^2/(c*x^2+b*x+a)*b*c*g-6/d^2/(4*a*c-b^2)^(5/2)*c*arctan((2*c*x+b)/(4*a*
c-b^2)^(1/2))*b*h+12/d^2/(4*a*c-b^2)^(5/2)*c^2*arctan((2*c*x+b)/(4*a*c-b^2)
^(1/2))*g
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(c*x^2+b*x+a)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

**mupad [B]** time = 0.42, size = 395, normalized size = 2.82

$$\frac{6\,c\,\mathrm{atan}\left(\frac{d^2\left(\frac{6c^2x(bh-2cg)}{d^2(4ac-b^2)^{5/2}}+\frac{3c(bh-2cg)(16a^2bc^2d^2-8ab^3cd^2+b^5d^2)}{d^4(4ac-b^2)^{5/2}(16a^2c^2-8ab^2c+b^4)}\right)(16a^2c^2-8ab^2c+b^4)}{6c^2g-3bch}\right)(bh-2cg)}{d^2\left(4ac-b^2\right)^{5/2}} - \frac{\frac{8cha^2+hab^2-10cgab+gb^3}{2(16a^2c^2-8ab^2c+b^4)}+\frac{x(b^2+5ac)(bh-2cg)}{16a^2c^2-8ab^2c+b^4}+\frac{3c^2x^3(bh-2cg)}{16a^2c^2-8ab^2c+b^4}+\frac{9bcx^2(bh-2cg)}{2(16a^2c^2-8ab^2c+b^4)}}{x^2\left(b^2d^2+2acd^2\right)+a^2d^2+c^2d^2x^4+2abd^2x+2bcd^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)/((a*d + b*d*x + c*d*x^2)^2*(a + b*x + c*x^2)),x)
```

```
[Out] (6*c*atan((d^2*((6*c^2*x*(b*h - 2*c*g))/(d^2*(4*a*c - b^2)^(5/2)) + (3*c*(b
*h - 2*c*g)*(b^5*d^2 + 16*a^2*b*c^2*d^2 - 8*a*b^3*c*d^2))/(d^4*(4*a*c - b^2
)^(5/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(6
*c^2*g - 3*b*c*h))*(b*h - 2*c*g))/(d^2*(4*a*c - b^2)^(5/2)) - ((b^3*g + a*b
^2*h + 8*a^2*c*h - 10*a*b*c*g)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x*(5*a
*c + b^2)*(b*h - 2*c*g))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (3*c^2*x^3*(b*h -
 2*c*g))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (9*b*c*x^2*(b*h - 2*c*g))/(2*(b^4
 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^2*(b^2*d^2 + 2*a*c*d^2) + a^2*d^2 + c^2*d^2
*x^4 + 2*a*b*d^2*x + 2*b*c*d^2*x^3)
```

**sympy [B]** time = 2.44, size = 709, normalized size = 5.06

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(c*x**2+b*x+a)/(c*d*x**2+b*d*x+a*d)**2,x)
```

```
[Out] 3*c*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g)*log(x + (-192*a**3*c**4*sqrt(-
1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 144*a**2*b**2*c**3*sqrt(-1/(4*a*c - b*
*2)**5)*(b*h - 2*c*g) - 36*a*b**4*c**2*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*
c*g) + 3*b**6*c*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 3*b**2*c*h - 6*b
```

```
*c**2*g)/(6*b*c**2*h - 12*c**3*g))/d**2 - 3*c*sqrt(-1/(4*a*c - b**2)**5)*(b
*h - 2*c*g)*log(x + (192*a**3*c**4*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g)
 - 144*a**2*b**2*c**3*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 36*a*b**4*
c**2*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) - 3*b**6*c*sqrt(-1/(4*a*c - b
**2)**5)*(b*h - 2*c*g) + 3*b**2*c*h - 6*b*c**2*g)/(6*b*c**2*h - 12*c**3*g))
/d**2 + (-8*a**2*c*h - a*b**2*h + 10*a*b*c*g - b**3*g + x**3*(-6*b*c**2*h +
 12*c**3*g) + x**2*(-9*b**2*c*h + 18*b*c**2*g) + x*(-10*a*b*c*h + 20*a*c**2
*g - 2*b**3*h + 4*b**2*c*g))/(32*a**4*c**2*d**2 - 16*a**3*b**2*c*d**2 + 2*a
**2*b**4*d**2 + x**4*(32*a**2*c**4*d**2 - 16*a*b**2*c**3*d**2 + 2*b**4*c**2
*d**2) + x**3*(64*a**2*b*c**3*d**2 - 32*a*b**3*c**2*d**2 + 4*b**5*c*d**2) +
 x**2*(64*a**3*c**3*d**2 - 12*a*b**4*c*d**2 + 2*b**6*d**2) + x*(64*a**3*b*c
**2*d**2 - 32*a**2*b**3*c*d**2 + 4*a*b**5*d**2))
```

## 3.18 $\int \frac{g+hx}{(a+bx+cx^2)^2(ad+bdx+cdx^2)}\,dx$

**Optimal**. Leaf size=140

$$\frac{3(b+2cx)(2cg-bh)}{2d\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{-2ah+x(2cg-bh)+bg}{2d\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{6c(2cg-bh)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{d\left(b^2-4ac\right)^{5/2}}$$

**Rubi [A]** time = 0.10, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {998, 638, 614, 618, 206}

$$\frac{3(b+2cx)(2cg-bh)}{2d\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{-2ah+x(2cg-bh)+bg}{2d\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{6c(2cg-bh)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{d\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)/((a + b*x + c*x^2)^2*(a*d + b*d*x + c*d*x^2)),x]
```

```
[Out] -(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*(b^2 - 4*a*c)*d*(a + b*x + c*x^2)^2) +
(3*(2*c*g - b*h)*(b + 2*c*x))/(2*(b^2 - 4*a*c)^2*d*(a + b*x + c*x^2)) - (6*
c*(2*c*g - b*h)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(5/2
)*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 998

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(c/f)^p, Int[(g + h*
x)^m*(d + e*x + f*x^2)^(p + q), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p,
 q}, x] && EqQ[c*d - a*f, 0] && EqQ[b*d - a*e, 0] && (IntegerQ[p] || GtQ[c/
f, 0]) && ( !IntegerQ[q] || LeafCount[d + e*x + f*x^2] <= LeafCount[a + b*x
 + c*x^2])
```

Rubi steps

$$\int \frac{g+hx}{\left(a+bx+cx^2\right)^2\left(ad+bdx+cdx^2\right)}\,dx=\frac{\int \frac{g+hx}{\left(a+bx+cx^2\right)^3}\,dx}{d}$$

$$=-\frac{bg-2ah+(2cg-bh)x}{2\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^2}-\frac{(3(2cg-bh))\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)d}$$

$$=-\frac{bg-2ah+(2cg-bh)x}{2\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^2}+\frac{3(2cg-bh)(b+2cx)}{2\left(b^2-4ac\right)^2d\left(a+bx+cx^2\right)}+$$

$$=-\frac{bg-2ah+(2cg-bh)x}{2\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^2}+\frac{3(2cg-bh)(b+2cx)}{2\left(b^2-4ac\right)^2d\left(a+bx+cx^2\right)}-$$

$$=-\frac{bg-2ah+(2cg-bh)x}{2\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^2}+\frac{3(2cg-bh)(b+2cx)}{2\left(b^2-4ac\right)^2d\left(a+bx+cx^2\right)}-$$

**Mathematica [A]** time = 0.03, size = 131, normalized size = 0.94

$$\frac{\frac{(b^2-4ac)(2ah-bg+bhx-2cgx)}{(a+x(b+cx))^2}-\frac{12c(bh-2cg)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{3(b+2cx)(2cg-bh)}{a+x(b+cx)}}{2d\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)/((a + b*x + c*x^2)^2*(a*d + b*d*x + c*d*x^2)),x]
```

```
[Out] (((b^2 - 4*a*c)*(-(b*g) + 2*a*h - 2*c*g*x + b*h*x))/(a + x*(b + c*x))^2 + (
3*(2*c*g - b*h)*(b + 2*c*x))/(a + x*(b + c*x)) - (12*c*(-2*c*g + b*h)*ArcTa
n[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(2*(b^2 - 4*a*c)^2*d
)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{g+hx}{\left(a+bx+cx^2\right)^2\left(ad+bdx+cdx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(g + h*x)/((a + b*x + c*x^2)^2*(a*d + b*d*x + c*d*x^2)),
x]
```

```
[Out] IntegrateAlgebraic[(g + h*x)/((a + b*x + c*x^2)^2*(a*d + b*d*x + c*d*x^2)),
 x]
```

**fricas [B]** time = 0.61, size = 1130, normalized size = 8.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(c*x^2+b*x+a)^2/(c*d*x^2+b*d*x+a*d),x, algorithm="fricas"
)
```

```
[Out] [1/2*(6*(2*(b^2*c^3 - 4*a*c^4)*g - (b^3*c^2 - 4*a*b*c^3)*h)*x^3 + 9*(2*(b^3
*c^2 - 4*a*b*c^3)*g - (b^4*c - 4*a*b^2*c^2)*h)*x^2 - 6*(2*a^2*c^2*g - a^2*b
*c*h + (2*c^4*g - b*c^3*h)*x^4 + 2*(2*b*c^3*g - b^2*c^2*h)*x^3 + (2*(b^2*c^
2 + 2*a*c^3)*g - (b^3*c + 2*a*b*c^2)*h)*x^2 + 2*(2*a*b*c^2*g - a*b^2*c*h)*x
)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a
*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (b^5 - 14*a*b^3*c + 40*a^2*b*c^2)*g -
 (a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2)*h + 2*(2*(b^4*c + a*b^2*c^2 - 20*a^2*c^
3)*g - (b^5 + a*b^3*c - 20*a^2*b*c^2)*h)*x)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5)*d*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 -
```

```
 64*a^3*b*c^4)*d*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3
- 128*a^4*c^4)*d*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*
c^3)*d*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*d), 1/2*(
6*(2*(b^2*c^3 - 4*a*c^4)*g - (b^3*c^2 - 4*a*b*c^3)*h)*x^3 + 9*(2*(b^3*c^2 -
 4*a*b*c^3)*g - (b^4*c - 4*a*b^2*c^2)*h)*x^2 - 12*(2*a^2*c^2*g - a^2*b*c*h
+ (2*c^4*g - b*c^3*h)*x^4 + 2*(2*b*c^3*g - b^2*c^2*h)*x^3 + (2*(b^2*c^2 + 2
*a*c^3)*g - (b^3*c + 2*a*b*c^2)*h)*x^2 + 2*(2*a*b*c^2*g - a*b^2*c*h)*x)*sqr
t(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (b^
5 - 14*a*b^3*c + 40*a^2*b*c^2)*g - (a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2)*h + 2
*(2*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*g - (b^5 + a*b^3*c - 20*a^2*b*c^2)*h)*
x)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*x^4 + 2*(b^7*c
 - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d*x^3 + (b^8 - 10*a*b^6*c
+ 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d*x^2 + 2*(a*b^7 - 12*a^2*
b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a
^4*b^2*c^2 - 64*a^5*c^3)*d)]
```

**giac [A]** time = 0.17, size = 207, normalized size = 1.48

$$\frac{6\left(2c^2g-bch\right)\arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\left(b^4d-8ab^2cd+16a^2c^2d\right)\sqrt{-b^2+4ac}}+\frac{12c^3gx^3-6bc^2hx^3+18bc^2gx^2-9b^2chx^2+4b^2cgx+20ac^2gx-2b^3hx-10abchx-b^3g+10abcg-ab^2h-8a^2ch}{2\left(b^4d-8ab^2cd+16a^2c^2d\right)\left(cx^2+bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(c*x^2+b*x+a)^2/(c*d*x^2+b*d*x+a*d),x, algorithm="giac")
```

```
[Out] 6*(2*c^2*g - b*c*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4*d - 8*a*b^
2*c*d + 16*a^2*c^2*d)*sqrt(-b^2 + 4*a*c)) + 1/2*(12*c^3*g*x^3 - 6*b*c^2*h*x
^3 + 18*b*c^2*g*x^2 - 9*b^2*c*h*x^2 + 4*b^2*c*g*x + 20*a*c^2*g*x - 2*b^3*h*
x - 10*a*b*c*h*x - b^3*g + 10*a*b*c*g - a*b^2*h - 8*a^2*c*h)/((b^4*d - 8*a*
b^2*c*d + 16*a^2*c^2*d)*(c*x^2 + b*x + a)^2)
```

**maple [B]** time = 0.01, size = 340, normalized size = 2.43

$$-\frac{3bchx}{\left(4ac-b^2\right)^2\left(cx^2+bx+a\right)d}-\frac{6bch\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{5}{2}}d}+\frac{6c^2gx}{\left(4ac-b^2\right)^2\left(cx^2+bx+a\right)d}+\frac{12c^2g\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{5}{2}}d}-\frac{3b^2h}{2\left(4ac-b^2\right)^2\left(cx^2+bx+a\right)d}+\frac{3bcg}{\left(4ac-b^2\right)^2\left(cx^2+bx+a\right)d}-\frac{bhx}{2\left(4ac-b^2\right)\left(cx^2+bx+a\right)^2d}+\frac{cgx}{\left(4ac-b^2\right)\left(cx^2+bx+a\right)^2d}-\frac{ah}{\left(4ac-b^2\right)\left(cx^2+bx+a\right)^2d}+\frac{bg}{2\left(4ac-b^2\right)\left(cx^2+bx+a\right)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)/(c*x^2+b*x+a)^2/(c*d*x^2+b*d*x+a*d),x)
```

```
[Out] -1/2/d/(4*a*c-b^2)/(c*x^2+b*x+a)^2*x*b*h+1/d/(4*a*c-b^2)/(c*x^2+b*x+a)^2*x*
c*g-1/d/(4*a*c-b^2)/(c*x^2+b*x+a)^2*a*h+1/2/d/(4*a*c-b^2)/(c*x^2+b*x+a)^2*b
*g-3/d/(4*a*c-b^2)^2/(c*x^2+b*x+a)*c*x*b*h+6/d/(4*a*c-b^2)^2/(c*x^2+b*x+a)*
c^2*x*g-3/2/d/(4*a*c-b^2)^2/(c*x^2+b*x+a)*b^2*h+3/d/(4*a*c-b^2)^2/(c*x^2+b*
x+a)*b*c*g-6/d/(4*a*c-b^2)^(5/2)*c*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*h+
12/d/(4*a*c-b^2)^(5/2)*c^2*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*g
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(c*x^2+b*x+a)^2/(c*d*x^2+b*d*x+a*d),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

**mupad [B]** time = 4.02, size = 375, normalized size = 2.68

$$\frac{6c\,\mathrm{atan}\left(\frac{d\left(\frac{6c^2x\,(bh-2cg)}{d\,(4ac-b^2)^{5/2}}+\frac{3c\,(bh-2cg)\,(16da^2bc^2-8dab^3c+db^5)}{d^2\,(4ac-b^2)^{5/2}\,(16a^2c^2-8ab^2c+b^4)}\right)(16a^2c^2-8ab^2c+b^4)}{6c^2g-3bch}\right)(bh-2cg)}{d\,(4ac-b^2)^{5/2}}-\frac{\frac{8cha^2+hab^2-10cgab+gb^3}{2\,(16a^2c^2-8ab^2c+b^4)}+\frac{x\,(b^2+5ac)\,(bh-2cg)}{16a^2c^2-8ab^2c+b^4}+\frac{3c^2x^3\,(bh-2cg)}{16a^2c^2-8ab^2c+b^4}+\frac{9bcx^2\,(bh-2cg)}{2\,(16a^2c^2-8ab^2c+b^4)}}{a^2d+x^2\,(db^2+2acd)+c^2dx^4+2bcdx^3+2abdx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)/((a*d + b*d*x + c*d*x^2)*(a + b*x + c*x^2)^2),x)
```

```
[Out] (6*c*atan((d*((6*c^2*x*(b*h - 2*c*g))/(d*(4*a*c - b^2)^(5/2)) + (3*c*(b*h -
 2*c*g)*(b^5*d - 8*a*b^3*c*d + 16*a^2*b*c^2*d))/(d^2*(4*a*c - b^2)^(5/2)*(b
^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(6*c^2*g - 3
*b*c*h))*(b*h - 2*c*g))/(d*(4*a*c - b^2)^(5/2)) - ((b^3*g + a*b^2*h + 8*a^2
*c*h - 10*a*b*c*g)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x*(5*a*c + b^2)*(b
*h - 2*c*g))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (3*c^2*x^3*(b*h - 2*c*g))/(b^
4 + 16*a^2*c^2 - 8*a*b^2*c) + (9*b*c*x^2*(b*h - 2*c*g))/(2*(b^4 + 16*a^2*c^
2 - 8*a*b^2*c)))/(a^2*d + x^2*(b^2*d + 2*a*c*d) + c^2*d*x^4 + 2*b*c*d*x^3 +
 2*a*b*d*x)
```

**sympy [B]** time = 2.26, size = 680, normalized size = 4.86

$$\frac{3c\sqrt{-\frac{1}{(4ac-b^2)^5}}\,(bh-2cg)\log\left(x+\frac{-192a^3c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}\,(bh-2cg)+144a^2b^2c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}\,(bh-2cg)-36ab^4c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\,(bh-2cg)+3b^6c\sqrt{-\frac{1}{(4ac-b^2)^5}}\,(bh-2cg)+3b^2ch-6bc^2g}{6bc^2h-12c^3g}\right)}{d}-\frac{3c\sqrt{-\frac{1}{(4ac-b^2)^5}}\,(bh-2cg)\log\left(x+\frac{192a^3c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}\,(bh-2cg)-144a^2b^2c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}\,(bh-2cg)+36ab^4c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\,(bh-2cg)-3b^6c\sqrt{-\frac{1}{(4ac-b^2)^5}}\,(bh-2cg)+3b^2ch-6bc^2g}{6bc^2h-12c^3g}\right)}{d}+\frac{-8a^2ch-ab^2h+10abcg-b^3g+x^3\left(-6bc^2h+12c^3g\right)+x^2\left(-9b^2ch+18bc^2g\right)+x\left(-10abch+20ac^2g-2b^3h+4b^2cg\right)}{32a^4c^2d-16a^3b^2cd+2a^2b^4d+x^4\left(32a^2c^4d-16ab^2c^3d+2b^4c^2d\right)+x^3\left(64a^2bc^3d-32ab^3c^2d+4b^5cd\right)+x^2\left(64a^3c^3d-12ab^4cd+2b^6d\right)+x\left(64a^3bc^2d-32a^2b^3cd+4ab^5d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(c*x**2+b*x+a)**2/(c*d*x**2+b*d*x+a*d),x)
```

```
[Out] 3*c*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g)*log(x + (-192*a**3*c**4*sqrt(-
1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 144*a**2*b**2*c**3*sqrt(-1/(4*a*c - b*
*2)**5)*(b*h - 2*c*g) - 36*a*b**4*c**2*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*
c*g) + 3*b**6*c*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 3*b**2*c*h - 6*b
```

```
*c**2*g)/(6*b*c**2*h - 12*c**3*g))/d - 3*c*sqrt(-1/(4*a*c - b**2)**5)*(b*h
- 2*c*g)*log(x + (192*a**3*c**4*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) -
144*a**2*b**2*c**3*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 36*a*b**4*c**
2*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) - 3*b**6*c*sqrt(-1/(4*a*c - b**2
)**5)*(b*h - 2*c*g) + 3*b**2*c*h - 6*b*c**2*g)/(6*b*c**2*h - 12*c**3*g))/d
+ (-8*a**2*c*h - a*b**2*h + 10*a*b*c*g - b**3*g + x**3*(-6*b*c**2*h + 12*c*
*3*g) + x**2*(-9*b**2*c*h + 18*b*c**2*g) + x*(-10*a*b*c*h + 20*a*c**2*g - 2
*b**3*h + 4*b**2*c*g))/(32*a**4*c**2*d - 16*a**3*b**2*c*d + 2*a**2*b**4*d +
 x**4*(32*a**2*c**4*d - 16*a*b**2*c**3*d + 2*b**4*c**2*d) + x**3*(64*a**2*b
*c**3*d - 32*a*b**3*c**2*d + 4*b**5*c*d) + x**2*(64*a**3*c**3*d - 12*a*b**4
*c*d + 2*b**6*d) + x*(64*a**3*b*c**2*d - 32*a**2*b**3*c*d + 4*a*b**5*d))
```

## 3.19 $\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$

**Optimal**. Leaf size=617

$$\frac{\left(2f(Af(cd-af)-Bd(ce-bf))-\left(e-\sqrt{e^2-4df}\right)\left(B\left(f(be-af)-c\left(e^2-df\right)\right)+Af(ce-bf)\right)\right)\tanh^{-1}\left(\frac{4a\cdots}{2\sqrt{2}\sqrt{a\cdots}}\cdots\right.}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

**Rubi [A]** time = 9.00, antiderivative size = 615, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1019, 1076, 621, 206, 1032, 724}

$$\frac{\left(2f(Af(cd-af)-Bd(ce-bf))-\left(e-\sqrt{e^2-4df}\right)\left(Bf(be-af)+Af(ce-bf)-Bc\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{\left(2f(Af(cd-af)-Bd(ce-bf))-\left(\sqrt{e^2-4df}+e\right)\left(Bf(be-af)+Af(ce-bf)-Bc\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-2Acf-bBf+2Bce)}{2\sqrt{c}f^2}+\frac{B\sqrt{a+bx+cx^2}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] (B*Sqrt[a + b*x + c*x^2])/f - ((2*B*c*e - b*B*f - 2*A*c*f)*ArcTanh[(b + 2*c
*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*f^2) + ((2*f*(A*f*(c*d -
 a*f) - B*d*(c*e - b*f)) - (e - Sqrt[e^2 - 4*d*f])*(B*f*(b*e - a*f) + A*f*(
c*e - b*f) - B*c*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) +
 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b
*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(S
qrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e
- b*f)*Sqrt[e^2 - 4*d*f]]) - ((2*f*(A*f*(c*d - a*f) - B*d*(c*e - b*f)) - (e
 + Sqrt[e^2 - 4*d*f])*(B*f*(b*e - a*f) + A*f*(c*e - b*f) - B*c*(e^2 - d*f))
)*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4
*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*
Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*
Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1019

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d
+ e*x + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), I
nt[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[h*p*(b*d - a*e) + a*(
h*e - 2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(h*e - 2*g*f)*(p + q + 1)
)*x + (h*p*(c*e - b*f) + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d
*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \frac{B\sqrt{a+bx+cx^2}}{f} - \frac{\int \frac{\frac{1}{2}(bBd-2aAf)-\frac{1}{2}(2Abf-B(2cd+be-2af))x+\frac{1}{2}(2Bce-bBf-2Acf)x^2}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx}{f}$$

$$= \frac{B\sqrt{a+bx+cx^2}}{f} - \frac{\int \frac{\frac{1}{2}f(bBd-2aAf)-\frac{1}{2}d(2Bce-bBf-2Acf)+\left(-\frac{1}{2}e(2Bce-bBf-2Acf)+\frac{1}{2}f(-2Abf\cdots\right.}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}}{f^2}$$

$$= \frac{B\sqrt{a+bx+cx^2}}{f} - \frac{(2Bce-bBf-2Acf)\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f^2} + \frac{(2f\cdots}{\cdots}$$

$$= \frac{B\sqrt{a+bx+cx^2}}{f} - \frac{(2Bce-bBf-2Acf)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\,f^2} - \frac{\left(2\left(2f(Af(c\cdots\right.\right.}{\cdots}$$

$$= \frac{B\sqrt{a+bx+cx^2}}{f} - \frac{(2Bce-bBf-2Acf)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\,f^2} + \frac{\left(2f(Af(cd\cdots\right.}{\cdots}$$

**Mathematica [A]** time = 2.09, size = 517, normalized size = 0.84

$$\frac{-\sqrt{2}\left(B\left(\sqrt{e^2-4df}+e\right)-2Af\right)\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af-b\left(\sqrt{e^2-4df}+e-2fx\right)-2cx\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)-\sqrt{2}\left(2Af+B\left(\sqrt{e^2-4df}-e\right)\right)\sqrt{f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af+b\left(\sqrt{e^2-4df}-e+2fx\right)+2cx\left(\sqrt{e^2-4df}-e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)+4Bf\sqrt{e^2-4df}\sqrt{a+x(b+cx)}}{4f^2\sqrt{e^2-4df}}+\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)(2Acf+bBf-2Bce)}{2\sqrt{c}\,f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] ((-2*B*c*e + b*B*f + 2*A*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b
+ c*x)])])/(2*Sqrt[c]*f^2) + (4*B*f*Sqrt[e^2 - 4*d*f]*Sqrt[a + x*(b + c*x)]
 - Sqrt[2]*(-2*A*f + B*(e + Sqrt[e^2 - 4*d*f]))*Sqrt[c*(e^2 - 2*d*f + e*Sqr
t[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*ArcTanh[(4*a*f - 2
*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2
]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 -
 4*d*f]))]*Sqrt[a + x*(b + c*x)])] - Sqrt[2]*(2*A*f + B*(-e + Sqrt[e^2 - 4*
d*f]))*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt
[e^2 - 4*d*f]))]*ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e +
Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4
*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/(
4*f^2*Sqrt[e^2 - 4*d*f])
```

**IntegrateAlgebraic [C]** time = 1.21, size = 893, normalized size = 1.45

$$\frac{\sqrt{cx^2+bx+a}\,B}{f}+\frac{(2Bce-bBf-2Acf)\log\left(b+2cx-2\sqrt{c}\sqrt{cx^2+bx+a}\right)}{2\sqrt{c}\,f^2}+\frac{\mathrm{RootSum}\left[f\#1^4-2\sqrt{c}e\#1^3+4cd\#1^2+be\#1^2-2af\#1^2-4b\sqrt{c}d\#1+2a\sqrt{c}e\#1+b^2d-abe+a^2f\&,\ \cdots\right]}{f^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((A + B*x)*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] (B*Sqrt[a + b*x + c*x^2])/f + ((2*B*c*e - b*B*f - 2*A*c*f)*Log[b + 2*c*x -
2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(2*Sqrt[c]*f^2) + RootSum[b^2*d - a*b*e +
 a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*
f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (-(b*B*c*d*e*Log[-(Sqrt[c]*x) + Sqrt
[a + b*x + c*x^2] - #1]) + a*B*c*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^
2] - #1] + b^2*B*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + A*b*c
*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*B*c*d*f*Log[-(Sqrt[
c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*b*B*e*f*Log[-(Sqrt[c]*x) + Sqrt[a +
 b*x + c*x^2] - #1] - a*A*c*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] -
#1] + a^2*B*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*B*c^(3/2
)*d*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*b*B*Sqrt[c]*d*f
*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*A*c^(3/2)*d*f*Log[-(
Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*a*A*Sqrt[c]*f^2*Log[-(Sqrt[
c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - B*c*e^2*Log[-(Sqrt[c]*x) + Sqrt[a
+ b*x + c*x^2] - #1]*#1^2 + B*c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2
] - #1]*#1^2 + b*B*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2
+ A*c*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 - A*b*f^2*Log
[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 - a*B*f^2*Log[-(Sqrt[c]*x)
 + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(2*b*Sqrt[c]*d - a*Sqrt[c]*e - 4*c*d*#
1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*#1^2 - 2*f*#1^3) & ]/f^2
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

**maple [B]** time = 0.05, size = 16209, normalized size = 26.27

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,x)\,\sqrt{c\,x^2+b\,x+a}}{f\,x^2+e\,x+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/(d + e*x + f*x^2),x)
```

```
[Out] int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/(d + e*x + f*x^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral((A + B*x)*sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)
```

## 3.20 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx$

**Optimal**. Leaf size=1092

$$\frac{B\left(cx^2+bx+a\right)^{3/2}}{3f}-\frac{\left(2Acf(4ce-5bf)-B\left(8\left(e^2-df\right)c^2-2f(5be-4af)c+b^2f^2\right)+2cf(2Bce-bBf-2Acf\right.}{8cf^3}$$

**Rubi [A]** time = 18.87, antiderivative size = 1092, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1019, 1066, 1076, 621, 206, 1032, 724}

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
```

```
[Out] -((2*A*c*f*(4*c*e - 5*b*f) - B*(b^2*f^2 - 2*c*f*(5*b*e - 4*a*f) + 8*c^2*(e^
2 - d*f)) + 2*c*f*(2*B*c*e - b*B*f - 2*A*c*f)*x)*Sqrt[a + b*x + c*x^2])/(8*
c*f^3) + (B*(a + b*x + c*x^2)^(3/2))/(3*f) + ((2*A*c*f*(3*b^2*f^2 - 12*c*f*
(b*e - a*f) + 8*c^2*(e^2 - d*f)) - B*(b^3*f^3 + 6*b*c*f^2*(b*e - 2*a*f) - 2
4*c^2*f*(b*e^2 - b*d*f - a*e*f) + 16*c^3*(e^3 - 2*d*e*f)))*ArcTanh[(b + 2*c
*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*f^4) - ((2*c*f*(B*d*(c*
e - b*f)*(c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2) + A*f*(2*c*d*f*(b*e - a*f) - f
^2*(b^2*d - a^2*f) - c^2*d*(e^2 - d*f))) - c*(e - Sqrt[e^2 - 4*d*f])*(A*f*(
c*e - b*f)*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)) + B*(c^2*(e^4 - 3*d*e^2*f +
d^2*f^2) - f^2*(2*a*b*e*f - a^2*f^2 - b^2*(e^2 - d*f)) + 2*c*f*(a*f*(e^2 -
d*f) - b*(e^3 - 2*d*e*f)))))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2
*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e
*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqr
t[2]*c*f^4*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e
- b*f)*Sqrt[e^2 - 4*d*f]]) + ((2*f*(B*d*(c*e - b*f)*(c*e^2 - 2*c*d*f - b*e*
f + 2*a*f^2) + A*f*(2*c*d*f*(b*e - a*f) - f^2*(b^2*d - a^2*f) - c^2*d*(e^2
- d*f))) - (e + Sqrt[e^2 - 4*d*f])*(A*f*(c*e - b*f)*(f*(b*e - 2*a*f) - c*(e
^2 - 2*d*f)) + B*(c^2*(e^4 - 3*d*e^2*f + d^2*f^2) - f^2*(2*a*b*e*f - a^2*f^
2 - b^2*(e^2 - d*f)) + 2*c*f*(a*f*(e^2 - d*f) - b*(e^3 - 2*d*e*f)))))*ArcTa
nh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))
*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^
2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f^4*Sqrt[e^2 - 4*d*f]*Sqrt[c*
e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1019

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d
+ e*x + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), I
nt[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[h*p*(b*d - a*e) + a*(
h*e - 2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(h*e - 2*g*f)*(p + q + 1)
)*x + (h*p*(c*e - b*f) + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d
*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1066

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*
p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a +
b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
```

```
 c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx = \frac{B\left(a+bx+cx^2\right)^{3/2}}{3f} - \frac{\int \frac{\sqrt{a+bx+cx^2}\left(\frac{3}{2}(bBd-2aAf)-\frac{3}{2}(2Abf-B(2cd+be-2af))x+\frac{3}{2}(2Bce-bBf}{d+ex+fx^2}}{3f}$$

$$= -\frac{\left(2Acf(4ce-5bf)-B\left(b^2f^2-2cf(5be-4af)+8c^2\left(e^2-df\right)\right)+2cf(2Bce}{8cf^3}$$

$$= -\frac{\left(2Acf(4ce-5bf)-B\left(b^2f^2-2cf(5be-4af)+8c^2\left(e^2-df\right)\right)+2cf(2Bce}{8cf^3}$$

$$= -\frac{\left(2Acf(4ce-5bf)-B\left(b^2f^2-2cf(5be-4af)+8c^2\left(e^2-df\right)\right)+2cf(2Bce}{8cf^3}$$

$$= -\frac{\left(2Acf(4ce-5bf)-B\left(b^2f^2-2cf(5be-4af)+8c^2\left(e^2-df\right)\right)+2cf(2Bce}{8cf^3}$$

$$= -\frac{\left(2Acf(4ce-5bf)-B\left(b^2f^2-2cf(5be-4af)+8c^2\left(e^2-df\right)\right)+2cf(2Bce}{8cf^3}$$

**Mathematica [A]** time = 6.56, size = 1627, normalized size = 1.49

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
```

```
[Out] ((B - (B*e - 2*A*f)/Sqrt[e^2 - 4*d*f])*(a + x*(b + c*x))^(3/2))/(6*f) + ((B
 + (B*e - 2*A*f)/Sqrt[e^2 - 4*d*f])*(a + x*(b + c*x))^(3/2))/(6*f) - ((B +
(-(B*e) + 2*A*f)/Sqrt[e^2 - 4*d*f])*(a + x*(b + c*x))^(3/2)*(((4*c*f*(-4*a*
f + b*(e - Sqrt[e^2 - 4*d*f])) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*(-(b*f
) + 2*c*(e - Sqrt[e^2 - 4*d*f])) - 4*c*f*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*
x)*Sqrt[a + b*x + c*x^2])/(8*c*f^2) - ((-2*(b*f - c*(e - Sqrt[e^2 - 4*d*f])
)*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) - 4*c*f*(3*a*f - b*(
e - Sqrt[e^2 - 4*d*f])))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^
2])])/(Sqrt[c]*f) - (2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e
*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*(4*(e - Sqrt[e^2 - 4*d*f])*(b*f
 - c*(e - Sqrt[e^2 - 4*d*f]))*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f - e*Sqrt[e^2 -
```

```
4*d*f]) - 4*c*f*(3*a*f - b*(e - Sqrt[e^2 - 4*d*f]))) + 4*f*(2*c*f*(4*a*f -
b*(e - Sqrt[e^2 - 4*d*f]))^2 - (e - Sqrt[e^2 - 4*d*f])*(b*f - c*(e - Sqrt[e
^2 - 4*d*f]))*(b^2*f + 4*a*c*f - 2*b*c*(e - Sqrt[e^2 - 4*d*f]))))*ArcTanh[(
4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) - (-2*b*f + 2*c*(e - Sqrt[e^2 - 4*d*f]))*
x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f
] + b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(f*(16*a*f^2 - 8*b*f*(e
 - Sqrt[e^2 - 4*d*f]) + 4*c*(e - Sqrt[e^2 - 4*d*f])^2)))/(16*c*f^2)))/(4*f*
(a + b*x + c*x^2)^(3/2)) - ((B - (-(B*e) + 2*A*f)/Sqrt[e^2 - 4*d*f])*(a + x
*(b + c*x))^(3/2)*(((4*c*f*(-4*a*f + b*(e + Sqrt[e^2 - 4*d*f])) + 2*(b*f -
c*(e + Sqrt[e^2 - 4*d*f]))*(-(b*f) + 2*c*(e + Sqrt[e^2 - 4*d*f])) - 4*c*f*(
b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^2) - ((-2
*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f + e*Sqrt[e
^2 - 4*d*f]) - 4*c*f*(3*a*f - b*(e + Sqrt[e^2 - 4*d*f])))*ArcTanh[(b + 2*c*
x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*f) - (2*Sqrt[2]*Sqrt[c*e^2
- 2*c*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]
]*(4*(e + Sqrt[e^2 - 4*d*f])*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*(b^2*f^2 - 4
*c^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) - 4*c*f*(3*a*f - b*(e + Sqrt[e^2 -
 4*d*f]))) + 4*f*(2*c*f*(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]))^2 - (e + Sqrt[e
^2 - 4*d*f])*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*(b^2*f + 4*a*c*f - 2*b*c*(e
+ Sqrt[e^2 - 4*d*f]))))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) - (-2*b*
f + 2*c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f
 + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x +
c*x^2])])/(f*(16*a*f^2 - 8*b*f*(e + Sqrt[e^2 - 4*d*f]) + 4*c*(e + Sqrt[e^2
- 4*d*f])^2)))/(16*c*f^2)))/(4*f*(a + b*x + c*x^2)^(3/2))
```

**IntegrateAlgebraic [C]** time = 7.50, size = 2787, normalized size = 2.55

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((A + B*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + b*x + c*x^2]*(24*B*c^2*e^2 - 24*B*c^2*d*f - 30*b*B*c*e*f - 24*A*c
^2*e*f + 3*b^2*B*f^2 + 30*A*b*c*f^2 + 32*a*B*c*f^2 - 12*B*c^2*e*f*x + 14*b*
B*c*f^2*x + 12*A*c^2*f^2*x + 8*B*c^2*f^2*x^2))/(24*c*f^3) + ((16*B*c^3*e^3
- 32*B*c^3*d*e*f - 24*b*B*c^2*e^2*f - 16*A*c^3*e^2*f + 24*b*B*c^2*d*f^2 + 1
6*A*c^3*d*f^2 + 6*b^2*B*c*e*f^2 + 24*A*b*c^2*e*f^2 + 24*a*B*c^2*e*f^2 + b^3
*B*f^3 - 6*A*b^2*c*f^3 - 12*a*b*B*c*f^3 - 24*a*A*c^2*f^3)*Log[b*c + 2*c^2*x
 - 2*c^(3/2)*Sqrt[a + b*x + c*x^2]])/(16*c^(3/2)*f^4) - RootSum[b^2*d - a*b
*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 -
2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (b*B*c^2*d*e^3*Log[-(Sqrt[c]*x)
+ Sqrt[a + b*x + c*x^2] - #1] - a*B*c^2*e^4*Log[-(Sqrt[c]*x) + Sqrt[a + b*x
 + c*x^2] - #1] - 2*b*B*c^2*d^2*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2
] - #1] - 2*b^2*B*c*d*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]
- A*b*c^2*d*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 3*a*B*c^
```

2*d*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*a*b*B*c*e^3*f*
Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a*A*c^2*e^3*f*Log[-(Sqrt[c
]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*b^2*B*c*d^2*f^2*Log[-(Sqrt[c]*x) + S
qrt[a + b*x + c*x^2] - #1] + A*b*c^2*d^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*
x + c*x^2] - #1] - a*B*c^2*d^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2]
 - #1] + b^3*B*d*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*A
*b^2*c*d*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*a*b*B*c*d
*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*a*A*c^2*d*e*f^2*L
og[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*b^2*B*e^2*f^2*Log[-(Sqrt[
c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*a*A*b*c*e^2*f^2*Log[-(Sqrt[c]*x) +
Sqrt[a + b*x + c*x^2] - #1] - 2*a^2*B*c*e^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a +
 b*x + c*x^2] - #1] - A*b^3*d*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2]
- #1] - a*b^2*B*d*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*a^
2*B*c*d*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a*A*b^2*e*f^3*
Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*a^2*b*B*e*f^3*Log[-(Sqrt
[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*a^2*A*c*e*f^3*Log[-(Sqrt[c]*x) + S
qrt[a + b*x + c*x^2] - #1] - a^2*A*b*f^4*Log[-(Sqrt[c]*x) + Sqrt[a + b*x +
c*x^2] - #1] - a^3*B*f^4*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2
*B*c^(5/2)*d*e^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 4*B*c^
(5/2)*d^2*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 4*b*B*c^(
3/2)*d*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*A*c^(5/2
)*d*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 4*b*B*c^(3/2)
*d^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*A*c^(5/2)*d^
2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*b^2*B*Sqrt[c]*d
*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 4*A*b*c^(3/2)*d*
e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 4*a*B*c^(3/2)*d*e
*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*A*b^2*Sqrt[c]*d*
f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 4*a*b*B*Sqrt[c]*d*f
^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 4*a*A*c^(3/2)*d*f^3*
Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*a^2*A*Sqrt[c]*f^4*Log
[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + B*c^2*e^4*Log[-(Sqrt[c]*x)
 + Sqrt[a + b*x + c*x^2] - #1]*#1^2 - 3*B*c^2*d*e^2*f*Log[-(Sqrt[c]*x) + Sq
rt[a + b*x + c*x^2] - #1]*#1^2 - 2*b*B*c*e^3*f*Log[-(Sqrt[c]*x) + Sqrt[a +
b*x + c*x^2] - #1]*#1^2 - A*c^2*e^3*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x
^2] - #1]*#1^2 + B*c^2*d^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #
1]*#1^2 + 4*b*B*c*d*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1
^2 + 2*A*c^2*d*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 +
b^2*B*e^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + 2*A*b*c
*e^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + 2*a*B*c*e^2*
f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 - b^2*B*d*f^3*Log[-
(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 - 2*A*b*c*d*f^3*Log[-(Sqrt[c
]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 - 2*a*B*c*d*f^3*Log[-(Sqrt[c]*x) +
Sqrt[a + b*x + c*x^2] - #1]*#1^2 - A*b^2*e*f^3*Log[-(Sqrt[c]*x) + Sqrt[a +
b*x + c*x^2] - #1]*#1^2 - 2*a*b*B*e*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c
*x^2] - #1]*#1^2 - 2*a*A*c*e*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] -

```
 #1]*#1^2 + 2*a*A*b*f^4*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2
 + a^2*B*f^4*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(2*b*Sqrt
[c]*d - a*Sqrt[c]*e - 4*c*d*#1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*#1^2 - 2*f
*#1^3) & ]/f^4
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

**maple [B]** time = 0.03, size = 59465, normalized size = 54.46

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+Bx)\left(cx^2+bx+a\right)^{3/2}}{fx^2+ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x + f*x^2),x)
[Out] int(((A + B*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x + f*x^2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
[Out] Timed out
```

## 3.21 $\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+ex+fx^2}} \, dx$

**Optimal**. Leaf size=416

$$\frac{\left(-B\sqrt{b^2-4ac}-2Ac+bB\right)\tanh^{-1}\left(\frac{2x\left(ce-f\left(b-\sqrt{b^2-4ac}\right)\right)-e\left(b-\sqrt{b^2-4ac}\right)+4cd}{2\sqrt{2}\sqrt{d+ex+fx^2}\sqrt{\sqrt{b^2-4ac}(ce-bf)-2acf+b^2f-bce+2c^2d}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}(ce-bf)-2acf+b^2f-bce+2c^2d}}+\frac{\left(2Ac-B\left(\sqrt{b^2-4ac}+b\right.\right.}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\cdots}}$$

**Rubi** [A] time = 2.70, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1032, 724, 206}

$$\frac{\left(-B\sqrt{b^2-4ac}-2Ac+bB\right)\tanh^{-1}\left(\frac{2x\left(ce-f\left(b-\sqrt{b^2-4ac}\right)\right)-e\left(b-\sqrt{b^2-4ac}\right)+4cd}{2\sqrt{2}\sqrt{d+ex+fx^2}\sqrt{\sqrt{b^2-4ac}(ce-bf)-2acf+b^2f-bce+2c^2d}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}(ce-bf)-2acf+b^2f-bce+2c^2d}}+\frac{\left(2Ac-B\left(\sqrt{b^2-4ac}+b\right)\right)\tanh^{-1}\left(\frac{2x\left(ce-f\left(\sqrt{b^2-4ac}+b\right)\right)-e\left(\sqrt{b^2-4ac}+b\right)+4cd}{2\sqrt{2}\sqrt{d+ex+fx^2}\sqrt{-\sqrt{b^2-4ac}(ce-bf)-2acf+b^2f-bce+2c^2d}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}(ce-bf)-2acf+b^2f-bce+2c^2d}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]),x]
```

```
[Out] ((b*B - 2*A*c - B*Sqrt[b^2 - 4*a*c])*ArcTanh[(4*c*d - (b - Sqrt[b^2 - 4*a*c
])*e + 2*(c*e - (b - Sqrt[b^2 - 4*a*c])*f)*x)/(2*Sqrt[2]*Sqrt[2*c^2*d - b*c
*e + b^2*f - 2*a*c*f + Sqrt[b^2 - 4*a*c]*(c*e - b*f)]*Sqrt[d + e*x + f*x^2]
)])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2*d - b*c*e + b^2*f - 2*a*c*f + Sqr
t[b^2 - 4*a*c]*(c*e - b*f)]) + ((2*A*c - B*(b + Sqrt[b^2 - 4*a*c]))*ArcTanh
[(4*c*d - (b + Sqrt[b^2 - 4*a*c])*e + 2*(c*e - (b + Sqrt[b^2 - 4*a*c])*f)*x
)/(2*Sqrt[2]*Sqrt[2*c^2*d - b*c*e + b^2*f - 2*a*c*f - Sqrt[b^2 - 4*a*c]*(c*
e - b*f)]*Sqrt[d + e*x + f*x^2])])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2*d
- b*c*e + b^2*f - 2*a*c*f - Sqrt[b^2 - 4*a*c]*(c*e - b*f)])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx}{\left(a+bx+cx^2\right)\sqrt{d+ex+fx^2}}\,dx = \frac{\left(2Ac-B\left(b-\sqrt{b^2-4ac}\right)\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex+fx^2}}\,dx}{\sqrt{b^2-4ac}} - \frac{\left(2Ac-B\left(b\right.\right.}{}$$

$$= \frac{\left(2\left(bB-2Ac-B\sqrt{b^2-4ac}\right)\right)\text{Subst}\left(\int \frac{1}{16c^2d-8c\left(b-\sqrt{b^2-4ac}\right)e+4\left(b-\sqrt{b^2-4ac}\right)^2 f}\right.}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(bB-2Ac-B\sqrt{b^2-4ac}\right)\tanh^{-1}\left(\frac{4cd-\left(b-\sqrt{b^2-4ac}\right)e+2\left(ce-\left(b-\sqrt{b^2-4ac}\right)f\right)}{2\sqrt{2}\sqrt{2c^2d-bce+b^2f-2acf+\sqrt{b^2-4ac}\,(ce-bf)}\sqrt{d}}\right.}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{2c^2d-bce+b^2f-2acf+\sqrt{b^2-4ac}\,(ce-bf)}}$$

**Mathematica [A]** time = 4.18, size = 393, normalized size = 0.94

$$\frac{-\frac{\left(B\sqrt{b^2-4ac}+2Ac-bB\right)\tanh^{-1}\left(\frac{\left(\sqrt{b^2-4ac}-b\right)(e+2fx)+2c(2d+ex)}{2\sqrt{2}\sqrt{d+x(e+fx)}\sqrt{c\left(e\sqrt{b^2-4ac}-2af-be\right)+bf\left(b-\sqrt{b^2-4ac}\right)+2c^2d}}\right)}{\sqrt{c\left(e\sqrt{b^2-4ac}-2af-be\right)+bf\left(b-\sqrt{b^2-4ac}\right)+2c^2d}}-\frac{\left(B\sqrt{b^2-4ac}-2Ac+bB\right)\tanh^{-1}\left(\frac{2c(2d+ex)-\left(\sqrt{b^2-4ac}+b\right)(e+2fx)}{2\sqrt{d+x(e+fx)}\sqrt{-2c\left(e\sqrt{b^2-4ac}+2af+be\right)+2bf\left(\sqrt{b^2-4ac}+b\right)+4c^2d}}\right)}{\sqrt{-c\left(e\sqrt{b^2-4ac}+2af+be\right)+bf\left(\sqrt{b^2-4ac}+b\right)+2c^2d}}}{\sqrt{2}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]),x]
```

```
[Out] (-(((-(b*B) + 2*A*c + B*Sqrt[b^2 - 4*a*c])*ArcTanh[(2*c*(2*d + e*x) + (-b +
 Sqrt[b^2 - 4*a*c])*(e + 2*f*x))/(2*Sqrt[2]*Sqrt[2*c^2*d + b*(b - Sqrt[b^2
- 4*a*c])*f + c*(-(b*e) + Sqrt[b^2 - 4*a*c]*e - 2*a*f)]*Sqrt[d + x*(e + f*x
)])])/Sqrt[2*c^2*d + b*(b - Sqrt[b^2 - 4*a*c])*f + c*(-(b*e) + Sqrt[b^2 - 4
*a*c]*e - 2*a*f)]) - ((b*B - 2*A*c + B*Sqrt[b^2 - 4*a*c])*ArcTanh[(2*c*(2*d
 + e*x) - (b + Sqrt[b^2 - 4*a*c])*(e + 2*f*x))/(2*Sqrt[4*c^2*d + 2*b*(b + S
qrt[b^2 - 4*a*c])*f - 2*c*(b*e + Sqrt[b^2 - 4*a*c]*e + 2*a*f)]*Sqrt[d + x*(
e + f*x)])])/Sqrt[2*c^2*d + b*(b + Sqrt[b^2 - 4*a*c])*f - c*(b*e + Sqrt[b^2
 - 4*a*c]*e + 2*a*f)])/(Sqrt[2]*Sqrt[b^2 - 4*a*c])
```

**IntegrateAlgebraic [C]** time = 0.53, size = 278, normalized size = 0.67

$$-\text{RootSum}\left[\#1^4c-2\#1^3b\sqrt{f}+4\#1^2af+\#1^2be-2\#1^2cd-4\#1ae\sqrt{f}+2\#1bd\sqrt{f}+ae^2-bde+cd^2\&,\frac{\#1^2(-B)\log\left(-\#1+\sqrt{d+ex+fx^2}-\sqrt{f}x\right)+2\#1A\sqrt{f}\log\left(-\#1+\sqrt{d+ex+fx^2}-\sqrt{f}x\right)-Ae\log\left(-\#1+\sqrt{d+ex+fx^2}-\sqrt{f}x\right)+Bd\log\left(-\#1+\sqrt{d+ex+fx^2}-\sqrt{f}x\right)}{2\#1^3c-3\#1^2b\sqrt{f}+4\#1af+\#1be-2\#1cd-2ae\sqrt{f}+bd\sqrt{f}}\&\right]$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]),x]
```

```
[Out] -RootSum[c*d^2 - b*d*e + a*e^2 + 2*b*d*Sqrt[f]*#1 - 4*a*e*Sqrt[f]*#1 - 2*c*
d*#1^2 + b*e*#1^2 + 4*a*f*#1^2 - 2*b*Sqrt[f]*#1^3 + c*#1^4 & , (B*d*Log[-(S
qrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1] - A*e*Log[-(Sqrt[f]*x) + Sqrt[d + e
*x + f*x^2] - #1] + 2*A*Sqrt[f]*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] -
#1]*#1 - B*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1]*#1^2)/(b*d*Sqrt[f
] - 2*a*e*Sqrt[f] - 2*c*d*#1 + b*e*#1 + 4*a*f*#1 - 3*b*Sqrt[f]*#1^2 + 2*c*#
1^3) & ]
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%{poly1[%%%{-4,[3,2,0]%%%}+%%%{16,[1,3,1]%%%},%%%{4,[4,2,0]%%%}+%%%
{-24,[2,3,1]%%%}+%%%{32,[0,4,2]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%
%%}]%%},[2,1,0,0,0]%%%}+%%%{%%{[%%%{-4,[5,0,0]%%%}+%%%{24,[3,1,1]%%%}+%%%{-
32,[1,2,2]%%%},%%%{4,[6,0,0]%%%}+%%%{-32,[4,1,1]%%%}+%%%{72,[2,2,2]%%%}+%%%
{-32,[0,3,3]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%}]%%},[2,0,0,1,0]
%%%}+%%%{%%{[%%%{4,[4,1,0]%%%}+%%%{-20,[2,2,1]%%%}+%%%{16,[0,3,2]%%%},%%%{-
4,[5,1,0]%%%}+%%%{28,[3,2,1]%%%}+%%%{-48,[1,3,2]%%%}]:[1,0,%%%{-1,[2,0,0]%%
%}+%%%{4,[0,1,1]%%%}]%%},[2,0,0,0,1]%%%}+%%%{%%{poly1[%%%{8,[2,3,0]%%%}+%%%
{-32,[0,4,1]%%%},%%%{-8,[3,3,0]%%%}+%%%{32,[1,4,1]%%%}]:[1,0,%%%{-1,[2,0,0]
%%%}+%%%{4,[0,1,1]%%%}]%%},[1,1,1,0,0]%%%}+%%%{%%{[%%%{8,[4,1,0]%%%}+%%%{-4
0,[2,2,1]%%%}+%%%{32,[0,3,2]%%%},%%%{-8,[5,1,0]%%%}+%%%{56,[3,2,1]%%%}+%%%{
```

-96,[1,3,2]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%}]%%},[1,0,1,1,0]%
%%}+%%%{%%{[%%%{-8,[3,2,0]%%%}+%%%{32,[1,3,1]%%%},%%%{8,[4,2,0]%%%}+%%%{-48
,[2,3,1]%%%}+%%%{64,[0,4,2]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%}]
%%},[1,0,1,0,1]%%%}+%%%{%%%{8,[2,4,0]%%%}+%%%{-32,[0,5,1]%%%},[0,1,2,0,0]%%
%}+%%%{%%{poly1[%%%{-4,[3,2,0]%%%}+%%%{16,[1,3,1]%%%},%%%{4,[4,2,0]%%%}+%%%
{-24,[2,3,1]%%%}+%%%{32,[0,4,2]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%
%%}]%%},[0,0,2,1,0]%%%}+%%%{%%{poly1[%%%{4,[2,3,0]%%%}+%%%{-16,[0,4,1]%%%},
%%%{-4,[3,3,0]%%%}+%%%{16,[1,4,1]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1
]%%%}]%%},[0,0,2,0,1]%%%} / %%%{%%{[%%%{4,[3,2,0]%%%}+%%%{-16,[1,3,1]%%%},%
%%{-4,[4,2,0]%%%}+%%%{24,[2,3,1]%%%}+%%%{-32,[0,4,2]%%%}]:[1,0,%%%{-1,[2,0,
0]%%%}+%%%{4,[0,1,1]%%%}]%%},[0,1,0,0,0]%%%}+%%%{%%{[%%%{4,[5,0,0]%%%}+%%%{
-24,[3,1,1]%%%}+%%%{32,[1,2,2]%%%},%%%{-4,[6,0,0]%%%}+%%%{32,[4,1,1]%%%}+%%
%{-72,[2,2,2]%%%}+%%%{32,[0,3,3]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]
%%%}]%%},[0,0,0,1,0]%%%}+%%%{%%{[%%%{-4,[4,1,0]%%%}+%%%{20,[2,2,1]%%%}+%%%{
-16,[0,3,2]%%%},%%%{4,[5,1,0]%%%}+%%%{-28,[3,2,1]%%%}+%%%{48,[1,3,2]%%%}]:[
1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%}]%%},[0,0,0,0,1]%%%} Error: Bad Arg
ument ValueEvaluation time: 0.92Unable to divide, perhaps due to rounding e
rror%%%{%%{[%%%{1,[3,0,0]%%%}+%%%{-4,[1,1,1]%%%},%%%{1,[4,0,0]%%%}+%%%{-6,[
2,1,1]%%%}+%%%{8,[0,2,2]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%}]%%}
/%%%{4,[0,0,2]%%%},[2,1,0,0,0]%%%}+%%%{%%%{1,[2,0,0]%%%}+%%%{-4,[0,1,1]%%%}
/2,[2,0,0,1,0]%%%}+%%%{%%{[%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%},%%%{-1,[3,0
,0]%%%}+%%%{4,[1,1,1]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%}]%%}/%%
%{4,[0,0,1]%%%},[2,0,0,0,1]%%%}+%%%{%%{[%%%{-1,[4,0,0]%%%}+%%%{5,[2,1,1]%%%
}+%%%{-4,[0,2,2]%%%},%%%{-1,[5,0,0]%%%}+%%%{7,[3,1,1]%%%}+%%%{-12,[1,2,2]%%
%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%}]%%}/%%%{2,[0,0,3]%%%},[1,1,1,
0,0]%%%}+%%%{%%{[%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%},%%%{-1,[3,0,0]%%%}+%%
%{4,[1,1,1]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%}]%%}/%%%{2,[0,0,1
]%%%},[1,0,1,1,0]%%%}+%%%{%%{[%%%{1,[3,0,0]%%%}+%%%{-4,[1,1,1]%%%},%%%{1,[4
,0,0]%%%}+%%%{-6,[2,1,1]%%%}+%%%{8,[0,2,2]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%
{4,[0,1,1]%%%}]%%}/%%%{2,[0,0,2]%%%},[1,0,1,0,1]%%%}+%%%{%%{[%%%{1,[5,0,0]%
%%}+%%%{-6,[3,1,1]%%%}+%%%{8,[1,2,2]%%%},%%%{1,[6,0,0]%%%}+%%%{-8,[4,1,1]%%
%}+%%%{18,[2,2,2]%%%}+%%%{-8,[0,3,3]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,
1,1]%%%}]%%}/%%%{4,[0,0,4]%%%},[0,1,2,0,0]%%%}+%%%{%%{[%%%{1,[3,0,0]%%%}+%%
%{-4,[1,1,1]%%%},%%%{1,[4,0,0]%%%}+%%%{-6,[2,1,1]%%%}+%%%{8,[0,2,2]%%%}]:[1
,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%}]%%}/%%%{4,[0,0,2]%%%},[0,0,2,1,0]%%
%}+%%%{%%{[%%%{-1,[4,0,0]%%%}+%%%{5,[2,1,1]%%%}+%%%{-4,[0,2,2]%%%},%%%{-1,[
5,0,0]%%%}+%%%{7,[3,1,1]%%%}+%%%{-12,[1,2,2]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%
%%{4,[0,1,1]%%%}]%%}/%%%{4,[0,0,3]%%%},[0,0,2,0,1]%%%} / %%%{%%{[%%%{-1,[3,
0,0]%%%}+%%%{4,[1,1,1]%%%},%%%{-1,[4,0,0]%%%}+%%%{6,[2,1,1]%%%}+%%%{-8,[0,2
,2]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%}]%%}/%%%{4,[0,0,4]%%%},[0
,1,0,0,0]%%%}+%%%{%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%}/%%%{2,[0,0,2]%%%},[0
,0,0,1,0]%%%}+%%%{%%{[%%%{1,[2,0,0]%%%}+%%%{-4,[0,1,1]%%%},%%%{1,[3,0,0]%%%
}+%%%{-4,[1,1,1]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%}]%%}/%%%{4,[
0,0,3]%%%},[0,0,0,0,1]%%%} Error: Bad Argument Value

**maple [B]** time = 0.04, size = 2269, normalized size = 5.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d)^(1/2),x)
```

```
[Out] 2/(-4*a*c+b^2)^(1/2)/(-2*(-(-4*a*c+b^2)^(1/2)*b*f+(-4*a*c+b^2)^(1/2)*c*e+2*
a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2)*ln((-(-(-4*a*c+b^2)^(1/2)*b*f+(-4*a*c
+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2-(f*(-4*a*c+b^2)^(1/2)+b*f-
c*e)/c*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)+1/2*(-2*(-(-4*a*c+b^2)^(1/2)*b*f+(-
4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2)*(4*(x+1/2*(b+(
-4*a*c+b^2)^(1/2))/c)^2*f-4*(f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x+1/2*(b+(-4*
a*c+b^2)^(1/2))/c)-2*(-(-4*a*c+b^2)^(1/2)*b*f+(-4*a*c+b^2)^(1/2)*c*e+2*a*c*
f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2))/(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c))*A-1/c/
(-2*(-(-4*a*c+b^2)^(1/2)*b*f+(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c
^2*d)/c^2)^(1/2)*ln((-(-(-4*a*c+b^2)^(1/2)*b*f+(-4*a*c+b^2)^(1/2)*c*e+2*a*c
*f-b^2*f+b*c*e-2*c^2*d)/c^2-(f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x+1/2*(b+(-4*
a*c+b^2)^(1/2))/c)+1/2*(-2*(-(-4*a*c+b^2)^(1/2)*b*f+(-4*a*c+b^2)^(1/2)*c*e+
2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2)*(4*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)
^2*f-4*(f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)-2*
(-(-4*a*c+b^2)^(1/2)*b*f+(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d
)/c^2)^(1/2))/(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c))*B-1/(-4*a*c+b^2)^(1/2)/c/(-
2*(-(-4*a*c+b^2)^(1/2)*b*f+(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2
*d)/c^2)^(1/2)*ln((-(-(-4*a*c+b^2)^(1/2)*b*f+(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f
-b^2*f+b*c*e-2*c^2*d)/c^2-(f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x+1/2*(b+(-4*a*
c+b^2)^(1/2))/c)+1/2*(-2*(-(-4*a*c+b^2)^(1/2)*b*f+(-4*a*c+b^2)^(1/2)*c*e+2*
a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2)*(4*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)^2
*f-4*(f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)-2*(-
(-4*a*c+b^2)^(1/2)*b*f+(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/
c^2)^(1/2))/(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c))*b*B-2/(-4*a*c+b^2)^(1/2)/(-2*
((-4*a*c+b^2)^(1/2)*b*f-(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)
/c^2)^(1/2)*ln((-((-4*a*c+b^2)^(1/2)*b*f-(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2
*f+b*c*e-2*c^2*d)/c^2-(-f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x-1/2*(-b+(-4*a*c+
b^2)^(1/2))/c)+1/2*(-2*((-4*a*c+b^2)^(1/2)*b*f-(-4*a*c+b^2)^(1/2)*c*e+2*a*c
*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2)*(4*(x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)^2*f
-4*(-f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)-2*((
-4*a*c+b^2)^(1/2)*b*f-(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c
^2)^(1/2))/(x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c))*A-1/c/(-2*((-4*a*c+b^2)^(1/2)
*b*f-(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2)*ln((-((
-4*a*c+b^2)^(1/2)*b*f-(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c
^2-(-f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)+1/2*
(-2*((-4*a*c+b^2)^(1/2)*b*f-(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^
2*d)/c^2)^(1/2)*(4*(x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)^2*f-4*(-f*(-4*a*c+b^2)
^(1/2)+b*f-c*e)/c*(x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)-2*((-4*a*c+b^2)^(1/2)*b
```

```
*f-(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2))/(x-1/2*(
-b+(-4*a*c+b^2)^(1/2))/c))*B+1/(-4*a*c+b^2)^(1/2)/c/(-2*((-4*a*c+b^2)^(1/2)
*b*f-(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2)*ln((-((
-4*a*c+b^2)^(1/2)*b*f-(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c
^2-(-f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)+1/2*
(-2*((-4*a*c+b^2)^(1/2)*b*f-(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^
2*d)/c^2)^(1/2)*(4*(x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)^2*f-4*(-f*(-4*a*c+b^2)
^(1/2)+b*f-c*e)/c*(x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)-2*((-4*a*c+b^2)^(1/2)*b
*f-(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2))/(x-1/2*(
-b+(-4*a*c+b^2)^(1/2))/c))*b*B
```

**maxima** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

**mupad** [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,x}{\left(c\,x^2+b\,x+a\right)\,\sqrt{f\,x^2+e\,x+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/((a + b*x + c*x^2)*(d + e*x + f*x^2)^(1/2)),x)
```

```
[Out] int((A + B*x)/((a + b*x + c*x^2)*(d + e*x + f*x^2)^(1/2)), x)
```

**sympy** [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx}{\left(a+bx+cx^2\right)\sqrt{d+ex+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)/(f*x**2+e*x+d)**(1/2),x)
```

```
[Out] Integral((A + B*x)/((a + b*x + c*x**2)*sqrt(d + e*x + f*x**2)), x)
```

## 3.22 $\int \frac{A+Bx}{(a+cx^2)\sqrt{d+ex+fx^2}}\,dx$

**Optimal**. Leaf size=780

$$\frac{\sqrt{A\left(-\sqrt{a^2f^2+ac\left(e^2-2df\right)+c^2d^2}-af+cd\right)+aBe}\sqrt{B\left(\sqrt{a^2f^2+ac\left(e^2-2df\right)+c^2d^2}-af+cd\right)-Ace}\ \mathrm{t}}{\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{e}\sqrt{a^2f^2+ac\left(e^2\right.}}$$

**Rubi** [A] time = 5.16, antiderivative size = 780, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1036, 1030, 208}

$$\frac{\sqrt{A\left(-\sqrt{a^2f^2+ac\left(e^2-2df\right)+c^2d^2}-af+cd\right)+aBe}\sqrt{B\left(\sqrt{a^2f^2+ac\left(e^2-2df\right)+c^2d^2}-af+cd\right)-Ace}\tanh^{-1}\left(\frac{\sqrt{e}\left(a\left(Ace-B\left(\sqrt{a^2f^2+ac(e^2-2df)+c^2d^2}-af+cd\right)\right)-cx\left(A\left(-\sqrt{a^2f^2+ac(e^2-2df)+c^2d^2}-af+cd\right)+aBe\right)\right)}{\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{d+ex+fx^2}\sqrt{A\left(-\sqrt{a^2f^2+ac(e^2-2df)+c^2d^2}-af+cd\right)+aBe}\sqrt{B\left(\sqrt{a^2f^2+ac(e^2-2df)+c^2d^2}-af+cd\right)-Ace}}\right)}{\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{e}\sqrt{a^2f^2+ac\left(e^2-2df\right)+c^2d^2}}-\frac{\sqrt{B\left(-\sqrt{a^2f^2+ac\left(e^2-2df\right)+c^2d^2}-af+cd\right)-Ace}\sqrt{A\left(\sqrt{a^2f^2+ac\left(e^2-2df\right)+c^2d^2}-af+cd\right)+aBe}\tanh^{-1}\left(\frac{\sqrt{e}\left(a\left(Ace-B\left(-\sqrt{a^2f^2+ac(e^2-2df)+c^2d^2}-af+cd\right)\right)-cx\left(A\left(\sqrt{a^2f^2+ac(e^2-2df)+c^2d^2}-af+cd\right)+aBe\right)\right)}{\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{d+ex+fx^2}\sqrt{B\left(-\sqrt{a^2f^2+ac(e^2-2df)+c^2d^2}-af+cd\right)-Ace}\sqrt{A\left(\sqrt{a^2f^2+ac(e^2-2df)+c^2d^2}-af+cd\right)+aBe}}\right)}{\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{e}\sqrt{a^2f^2+ac\left(e^2-2df\right)+c^2d^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]),x]
```

```
[Out] (Sqrt[a*B*e + A*(c*d - a*f - Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])]*
Sqrt[-(A*c*e) + B*(c*d - a*f + Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])
]*ArcTanh[(Sqrt[e]*(a*(A*c*e - B*(c*d - a*f + Sqrt[c^2*d^2 + a^2*f^2 + a*c*
(e^2 - 2*d*f)])) - c*(a*B*e + A*(c*d - a*f - Sqrt[c^2*d^2 + a^2*f^2 + a*c*(
e^2 - 2*d*f)]))*x))/(Sqrt[2]*Sqrt[a]*Sqrt[c]*Sqrt[a*B*e + A*(c*d - a*f - Sq
rt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])]*Sqrt[-(A*c*e) + B*(c*d - a*f +
Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])]*Sqrt[d + e*x + f*x^2])])/(Sqr
t[2]*Sqrt[a]*Sqrt[c]*Sqrt[e]*Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)]) -
 (Sqrt[-(A*c*e) + B*(c*d - a*f - Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)
])]*Sqrt[a*B*e + A*(c*d - a*f + Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)]
)]*ArcTanh[(Sqrt[e]*(a*(A*c*e - B*(c*d - a*f - Sqrt[c^2*d^2 + a^2*f^2 + a*c
*(e^2 - 2*d*f)])) - c*(a*B*e + A*(c*d - a*f + Sqrt[c^2*d^2 + a^2*f^2 + a*c*
(e^2 - 2*d*f)]))*x))/(Sqrt[2]*Sqrt[a]*Sqrt[c]*Sqrt[-(A*c*e) + B*(c*d - a*f
- Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])]*Sqrt[a*B*e + A*(c*d - a*f +
 Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])]*Sqrt[d + e*x + f*x^2])])/(Sq
rt[2]*Sqrt[a]*Sqrt[c]*Sqrt[e]*Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1030

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1036

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[-(a*h*e) - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[-(a*h*e) - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[-(a*c)]
```

Rubi steps

$$\int \frac{A+Bx}{\left(a+cx^2\right)\sqrt{d+ex+fx^2}}\,dx = -\frac{\int \frac{-aBe-A\left(cd-af+\sqrt{c^2d^2+a^2f^2+ac\left(e^2-2df\right)}\right)+\left(-Ace+B\left(cd-af-\sqrt{c^2d^2+a^2f^2+ac\left(e^2-2df\right)}\right)\right)x}{\left(a+cx^2\right)\sqrt{d+ex+fx^2}}\,dx}{2\sqrt{c^2d^2+a^2f^2+ac\left(e^2-2df\right)}}$$

$$= \frac{\left(a\left(Ace-B\left(cd-af-\sqrt{c^2d^2+a^2f^2+ac\left(e^2-2df\right)}\right)\right)\left(aBe+A\left(cd-af+\sqrt{\ldots}\right.\right.\right.}{\ldots}$$

$$= \frac{\sqrt{aBe+A\left(cd-af-\sqrt{c^2d^2+a^2f^2+ac\left(e^2-2df\right)}\right)}\sqrt{-Ace+B\left(cd-af+\ldots\right.}}{\ldots}$$

**Mathematica [A]** time = 0.41, size = 254, normalized size = 0.33

$$\frac{\frac{\left(A\sqrt{c}-\sqrt{-a}B\right)\tanh^{-1}\left(\frac{\sqrt{c}(2d+ex)-\sqrt{-a}(e+2fx)}{2\sqrt{d+x(e+fx)}\sqrt{-\sqrt{-a}\sqrt{c}e-af+cd}}\right)}{\sqrt{-\sqrt{-a}\sqrt{c}e-af+cd}}-\frac{\left(\sqrt{-a}B+A\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{-a}(e+2fx)+\sqrt{c}(2d+ex)}{2\sqrt{d+x(e+fx)}\sqrt{\sqrt{-a}\sqrt{c}e-af+cd}}\right)}{\sqrt{\sqrt{-a}\sqrt{c}e-af+cd}}}{2\sqrt{-a}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]),x]
```

```
[Out] (((-(Sqrt[-a]*B) + A*Sqrt[c])*ArcTanh[(Sqrt[c]*(2*d + e*x) - Sqrt[-a]*(e +
2*f*x))/(2*Sqrt[c*d - Sqrt[-a]*Sqrt[c]*e - a*f]*Sqrt[d + x*(e + f*x)])]])/Sq
rt[c*d - Sqrt[-a]*Sqrt[c]*e - a*f] - ((Sqrt[-a]*B + A*Sqrt[c])*ArcTanh[(Sqr
t[c]*(2*d + e*x) + Sqrt[-a]*(e + 2*f*x))/(2*Sqrt[c*d + Sqrt[-a]*Sqrt[c]*e -
 a*f]*Sqrt[d + x*(e + f*x)])]])/Sqrt[c*d + Sqrt[-a]*Sqrt[c]*e - a*f])/(2*Sqr
t[-a]*Sqrt[c])
```

**IntegrateAlgebraic [C]** time = 0.44, size = 218, normalized size = 0.28

$$\frac{1}{2}\text{RootSum}\left[\#1^4c+4\#1^2af-2\#1^2cd-4\#1ae\sqrt{f}+ae^2+cd^2\&,\frac{\#1^2(-B)\log\left(-\#1+\sqrt{d+ex+fx^2}-\sqrt{f}x\right)+2\#1A\sqrt{f}\log\left(-\#1+\sqrt{d+ex+fx^2}-\sqrt{f}x\right)-Ae\log\left(-\#1+\sqrt{d+ex+fx^2}-\sqrt{f}x\right)+Bd\log\left(-\#1+\sqrt{d+ex+fx^2}-\sqrt{f}x\right)}{\#1^3(-c)-2\#1af+\#1cd+ae\sqrt{f}}\&\right]$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]),x]
```

```
[Out] RootSum[c*d^2 + a*e^2 - 4*a*e*Sqrt[f]*#1 - 2*c*d*#1^2 + 4*a*f*#1^2 + c*#1^4
 & , (B*d*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1] - A*e*Log[-(Sqrt[f
]*x) + Sqrt[d + e*x + f*x^2] - #1] + 2*A*Sqrt[f]*Log[-(Sqrt[f]*x) + Sqrt[d
+ e*x + f*x^2] - #1]*#1 - B*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1]*
#1^2)/(a*e*Sqrt[f] + c*d*#1 - 2*a*f*#1 - c*#1^3) & ]/2
```

**fricas [B]** time = 47.87, size = 6861, normalized size = 8.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-(2*A*B*a*c*e - (B^2*a*c - A^2*c^2)*d + (B^2*a^2 - A^2*a*c)*f + (
a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2)*sqrt(-(4*A^2*B^2*c^2*d
^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*A^2*B
^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e)*f)/
(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c*f^4
+ 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2)*f))
)/(a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2))*log(-(2*(A*B^3*a*c
 + A^3*B*c^2)*d*e + (B^4*a^2 - A^4*c^2)*e^2 - 2*(A*B^3*a^2 + A^3*B*a*c)*e*f
 - 2*(2*(A*B^3*a^2 + A^3*B*a*c)*f^2 - (2*(A*B^3*a*c + A^3*B*c^2)*d + (B^4*a
^2 - A^4*c^2)*e)*f)*x + 2*(2*A^2*B*c^3*d^2 + 2*A^2*B*a^2*c*f^2 + (3*A*B^2*a
*c^2 - A^3*c^3)*d*e + (B^3*a^2*c - A^2*B*a*c^2)*e^2 - (4*A^2*B*a*c^2*d + (3
*A*B^2*a^2*c - A^3*a*c^2)*e)*f - (B*a*c^4*d^3 - A*a*c^4*d^2*e + B*a^2*c^3*d
*e^2 - A*a^2*c^3*e^3 - B*a^4*c*f^3 + (3*B*a^3*c^2*d - A*a^3*c^2*e)*f^2 - (3
*B*a^2*c^3*d^2 - 2*A*a^2*c^3*d*e + B*a^3*c^2*e^2)*f)*sqrt(-(4*A^2*B^2*c^2*d
^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*A^2*B
^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e)*f)/
(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c*f^4
```

```
+ 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2)*f))
)*sqrt(f*x^2 + e*x + d)*sqrt(-(2*A*B*a*c*e - (B^2*a*c - A^2*c^2)*d + (B^2*a
^2 - A^2*a*c)*f + (a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2)*sqr
t(-(4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e +
 (B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2
- A^3*B*a*c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2
*d*f^3 + a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 +
 a^3*c^3*d*e^2)*f)))/(a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2))
 - (2*(B^2*a*c^3 + A^2*c^4)*d^3 + 2*(B^2*a^2*c^2 + A^2*a*c^3)*d*e^2 - 4*(B^
2*a^2*c^2 + A^2*a*c^3)*d^2*f + 2*(B^2*a^3*c + A^2*a^2*c^2)*d*f^2 + ((B^2*a*
c^3 + A^2*c^4)*d^2*e + (B^2*a^2*c^2 + A^2*a*c^3)*e^3 - 2*(B^2*a^2*c^2 + A^2
*a*c^3)*d*e*f + (B^2*a^3*c + A^2*a^2*c^2)*e*f^2)*x)*sqrt(-(4*A^2*B^2*c^2*d^
2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*A^2*B^
2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e)*f)/(
a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c*f^4 +
 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2)*f)))
/x) + 1/4*sqrt(-(2*A*B*a*c*e - (B^2*a*c - A^2*c^2)*d + (B^2*a^2 - A^2*a*c)*
f + (a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2)*sqrt(-(4*A^2*B^2*
c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*
A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e
)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c
*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2
)*f)))/(a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2))*log(-(2*(A*B^
3*a*c + A^3*B*c^2)*d*e + (B^4*a^2 - A^4*c^2)*e^2 - 2*(A*B^3*a^2 + A^3*B*a*c
)*e*f - 2*(2*(A*B^3*a^2 + A^3*B*a*c)*f^2 - (2*(A*B^3*a*c + A^3*B*c^2)*d + (
B^4*a^2 - A^4*c^2)*e)*f)*x - 2*(2*A^2*B*c^3*d^2 + 2*A^2*B*a^2*c*f^2 + (3*A*
B^2*a*c^2 - A^3*c^3)*d*e + (B^3*a^2*c - A^2*B*a*c^2)*e^2 - (4*A^2*B*a*c^2*d
 + (3*A*B^2*a^2*c - A^3*a*c^2)*e)*f - (B*a*c^4*d^3 - A*a*c^4*d^2*e + B*a^2*
c^3*d*e^2 - A*a^2*c^3*e^3 - B*a^4*c*f^3 + (3*B*a^3*c^2*d - A*a^3*c^2*e)*f^2
 - (3*B*a^2*c^3*d^2 - 2*A*a^2*c^3*d*e + B*a^3*c^2*e^2)*f)*sqrt(-(4*A^2*B^2*
c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*
A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e
)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c
*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2
)*f)))*sqrt(f*x^2 + e*x + d)*sqrt(-(2*A*B*a*c*e - (B^2*a*c - A^2*c^2)*d + (
B^2*a^2 - A^2*a*c)*f + (a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2
)*sqrt(-(4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*
d*e + (B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3
*a^2 - A^3*B*a*c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^
4*c^2*d*f^3 + a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*
d^3 + a^3*c^3*d*e^2)*f)))/(a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*
f^2)) - (2*(B^2*a*c^3 + A^2*c^4)*d^3 + 2*(B^2*a^2*c^2 + A^2*a*c^3)*d*e^2 -
4*(B^2*a^2*c^2 + A^2*a*c^3)*d^2*f + 2*(B^2*a^3*c + A^2*a^2*c^2)*d*f^2 + ((B
^2*a*c^3 + A^2*c^4)*d^2*e + (B^2*a^2*c^2 + A^2*a*c^3)*e^3 - 2*(B^2*a^2*c^2
+ A^2*a*c^3)*d*e*f + (B^2*a^3*c + A^2*a^2*c^2)*e*f^2)*x)*sqrt(-(4*A^2*B^2*c
```

```
^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*A
^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e)
*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c*
f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2)
*f)))/x) - 1/4*sqrt(-(2*A*B*a*c*e - (B^2*a*c - A^2*c^2)*d + (B^2*a^2 - A^2*
a*c)*f - (a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2)*sqrt(-(4*A^2
*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2
 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a
*c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 +
a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*
d*e^2)*f)))/(a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2))*log(-(2*
(A*B^3*a*c + A^3*B*c^2)*d*e + (B^4*a^2 - A^4*c^2)*e^2 - 2*(A*B^3*a^2 + A^3*
B*a*c)*e*f - 2*(2*(A*B^3*a^2 + A^3*B*a*c)*f^2 - (2*(A*B^3*a*c + A^3*B*c^2)*
d + (B^4*a^2 - A^4*c^2)*e)*f)*x + 2*(2*A^2*B*c^3*d^2 + 2*A^2*B*a^2*c*f^2 +
(3*A*B^2*a*c^2 - A^3*c^3)*d*e + (B^3*a^2*c - A^2*B*a*c^2)*e^2 - (4*A^2*B*a*
c^2*d + (3*A*B^2*a^2*c - A^3*a*c^2)*e)*f + (B*a*c^4*d^3 - A*a*c^4*d^2*e + B
*a^2*c^3*d*e^2 - A*a^2*c^3*e^3 - B*a^4*c*f^3 + (3*B*a^3*c^2*d - A*a^3*c^2*e
)*f^2 - (3*B*a^2*c^3*d^2 - 2*A*a^2*c^3*d*e + B*a^3*c^2*e^2)*f)*sqrt(-(4*A^2
*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2
 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a
*c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 +
a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*
d*e^2)*f)))*sqrt(f*x^2 + e*x + d)*sqrt(-(2*A*B*a*c*e - (B^2*a*c - A^2*c^2)*
d + (B^2*a^2 - A^2*a*c)*f - (a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*
c*f^2)*sqrt(-(4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*
c^2)*d*e + (B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (
A*B^3*a^2 - A^3*B*a*c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 -
 4*a^4*c^2*d*f^3 + a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2
*c^4*d^3 + a^3*c^3*d*e^2)*f)))/(a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a
^3*c*f^2)) + (2*(B^2*a*c^3 + A^2*c^4)*d^3 + 2*(B^2*a^2*c^2 + A^2*a*c^3)*d*e
^2 - 4*(B^2*a^2*c^2 + A^2*a*c^3)*d^2*f + 2*(B^2*a^3*c + A^2*a^2*c^2)*d*f^2
+ ((B^2*a*c^3 + A^2*c^4)*d^2*e + (B^2*a^2*c^2 + A^2*a*c^3)*e^3 - 2*(B^2*a^2
*c^2 + A^2*a*c^3)*d*e*f + (B^2*a^3*c + A^2*a^2*c^2)*e*f^2)*x)*sqrt(-(4*A^2*
B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2
- 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*
c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a
^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d
*e^2)*f)))/x) + 1/4*sqrt(-(2*A*B*a*c*e - (B^2*a*c - A^2*c^2)*d + (B^2*a^2 -
 A^2*a*c)*f - (a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2)*sqrt(-(
4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^
4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^
3*B*a*c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f
^3 + a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3
*c^3*d*e^2)*f)))/(a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2))*log
(-(2*(A*B^3*a*c + A^3*B*c^2)*d*e + (B^4*a^2 - A^4*c^2)*e^2 - 2*(A*B^3*a^2 +
```

```
 A^3*B*a*c)*e*f - 2*(2*(A*B^3*a^2 + A^3*B*a*c)*f^2 - (2*(A*B^3*a*c + A^3*B*
c^2)*d + (B^4*a^2 - A^4*c^2)*e)*f)*x - 2*(2*A^2*B*c^3*d^2 + 2*A^2*B*a^2*c*f
^2 + (3*A*B^2*a*c^2 - A^3*c^3)*d*e + (B^3*a^2*c - A^2*B*a*c^2)*e^2 - (4*A^2
*B*a*c^2*d + (3*A*B^2*a^2*c - A^3*a*c^2)*e)*f + (B*a*c^4*d^3 - A*a*c^4*d^2*
e + B*a^2*c^3*d*e^2 - A*a^2*c^3*e^3 - B*a^4*c*f^3 + (3*B*a^3*c^2*d - A*a^3*
c^2*e)*f^2 - (3*B*a^2*c^3*d^2 - 2*A*a^2*c^3*d*e + B*a^3*c^2*e^2)*f)*sqrt(-(
4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^
4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^
3*B*a*c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f
^3 + a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3
*c^3*d*e^2)*f)))*sqrt(f*x^2 + e*x + d)*sqrt(-(2*A*B*a*c*e - (B^2*a*c - A^2*
c^2)*d + (B^2*a^2 - A^2*a*c)*f - (a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f +
 a^3*c*f^2)*sqrt(-(4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A
^3*B*c^2)*d*e + (B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*
d + (A*B^3*a^2 - A^3*B*a*c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*
e^4 - 4*a^4*c^2*d*f^3 + a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4
*(a^2*c^4*d^3 + a^3*c^3*d*e^2)*f)))/(a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*
f + a^3*c*f^2)) + (2*(B^2*a*c^3 + A^2*c^4)*d^3 + 2*(B^2*a^2*c^2 + A^2*a*c^3
)*d*e^2 - 4*(B^2*a^2*c^2 + A^2*a*c^3)*d^2*f + 2*(B^2*a^3*c + A^2*a^2*c^2)*d
*f^2 + ((B^2*a*c^3 + A^2*c^4)*d^2*e + (B^2*a^2*c^2 + A^2*a*c^3)*e^3 - 2*(B^
2*a^2*c^2 + A^2*a*c^3)*d*e*f + (B^2*a^3*c + A^2*a^2*c^2)*e*f^2)*x)*sqrt(-(4
*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4
*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3
*B*a*c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^
3 + a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*
c^3*d*e^2)*f)))/x)
```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument ValueConj Error: Bad Argument Typ
e
```

**maple** [A] time = 0.04, size = 784, normalized size = 1.01

$$-\frac{A \ln\left(\frac{-\frac{2\left(af-cd-\sqrt{-ac}\,e\right)}{c}+\frac{\left(ce+2\sqrt{-ac}\,f\right)\left(x-\frac{\sqrt{-ac}}{c}\right)}{c}+2\sqrt{-\frac{af-cd-\sqrt{-ac}\,e}{c}}\,\sqrt{\left(x-\frac{\sqrt{-ac}}{c}\right)^{2} f+\frac{\left(ce+2\sqrt{-ac}\,f\right)\left(x-\frac{\sqrt{-ac}}{c}\right)}{c}-\frac{af-cd-\sqrt{-ac}\,e}{c}}}{x-\frac{\sqrt{-ac}}{c}}\right)}{2\sqrt{-ac}\,\sqrt{-\frac{af-cd-\sqrt{-ac}\,e}{c}}}+\frac{A \ln\left(\frac{-\frac{2\left(af-cd+\sqrt{-ac}\,e\right)}{c}+\frac{\left(ce-2\sqrt{-ac}\,f\right)\left(x+\frac{\sqrt{-ac}}{c}\right)}{c}+2\sqrt{-\frac{af-cd+\sqrt{-ac}\,e}{c}}\,\sqrt{\left(x+\frac{\sqrt{-ac}}{c}\right)^{2} f+\frac{\left(ce-2\sqrt{-ac}\,f\right)\left(x+\frac{\sqrt{-ac}}{c}\right)}{c}-\frac{af-cd+\sqrt{-ac}\,e}{c}}}{x+\frac{\sqrt{-ac}}{c}}\right)}{2\sqrt{-ac}\,\sqrt{-\frac{af-cd+\sqrt{-ac}\,e}{c}}}-\frac{B \ln\left(\frac{-\frac{2\left(af-cd-\sqrt{-ac}\,e\right)}{c}+\frac{\left(ce+2\sqrt{-ac}\,f\right)\left(x-\frac{\sqrt{-ac}}{c}\right)}{c}+2\sqrt{-\frac{af-cd-\sqrt{-ac}\,e}{c}}\,\sqrt{\left(x-\frac{\sqrt{-ac}}{c}\right)^{2} f+\frac{\left(ce+2\sqrt{-ac}\,f\right)\left(x-\frac{\sqrt{-ac}}{c}\right)}{c}-\frac{af-cd-\sqrt{-ac}\,e}{c}}}{x-\frac{\sqrt{-ac}}{c}}\right)}{2\sqrt{-\frac{af-cd-\sqrt{-ac}\,e}{c}}\,c}-\frac{B \ln\left(\frac{-\frac{2\left(af-cd+\sqrt{-ac}\,e\right)}{c}+\frac{\left(ce-2\sqrt{-ac}\,f\right)\left(x+\frac{\sqrt{-ac}}{c}\right)}{c}+2\sqrt{-\frac{af-cd+\sqrt{-ac}\,e}{c}}\,\sqrt{\left(x+\frac{\sqrt{-ac}}{c}\right)^{2} f+\frac{\left(ce-2\sqrt{-ac}\,f\right)\left(x+\frac{\sqrt{-ac}}{c}\right)}{c}-\frac{af-cd+\sqrt{-ac}\,e}{c}}}{x+\frac{\sqrt{-ac}}{c}}\right)}{2\sqrt{-\frac{af-cd+\sqrt{-ac}\,e}{c}}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(c*x^2+a)/(f*x^2+e*x+d)^(1/2),x)
```

```
[Out] -1/2/(-a*c)^(1/2)/(-(-e*(-a*c)^(1/2)+a*f-c*d)/c)^(1/2)*ln((-2*(-e*(-a*c)^(1
/2)+a*f-c*d)/c+(2*f*(-a*c)^(1/2)+c*e)/c*(x-(-a*c)^(1/2)/c)+2*(-(-e*(-a*c)^(
1/2)+a*f-c*d)/c)^(1/2)*((x-(-a*c)^(1/2)/c)^2*f+(2*f*(-a*c)^(1/2)+c*e)/c*(x-
(-a*c)^(1/2)/c)-(-e*(-a*c)^(1/2)+a*f-c*d)/c)^(1/2))/(x-(-a*c)^(1/2)/c))*A-1
/2/c/(-(-e*(-a*c)^(1/2)+a*f-c*d)/c)^(1/2)*ln((-2*(-e*(-a*c)^(1/2)+a*f-c*d)/
c+(2*f*(-a*c)^(1/2)+c*e)/c*(x-(-a*c)^(1/2)/c)+2*(-(-e*(-a*c)^(1/2)+a*f-c*d)
/c)^(1/2)*((x-(-a*c)^(1/2)/c)^2*f+(2*f*(-a*c)^(1/2)+c*e)/c*(x-(-a*c)^(1/2)/
c)-(-e*(-a*c)^(1/2)+a*f-c*d)/c)^(1/2))/(x-(-a*c)^(1/2)/c))*B+1/2/(-a*c)^(1/
2)/(-(e*(-a*c)^(1/2)+a*f-c*d)/c)^(1/2)*ln((-2*(e*(-a*c)^(1/2)+a*f-c*d)/c+1/
c*(-2*f*(-a*c)^(1/2)+c*e)*(x+(-a*c)^(1/2)/c)+2*(-(e*(-a*c)^(1/2)+a*f-c*d)/c
)^(1/2)*((x+(-a*c)^(1/2)/c)^2*f+1/c*(-2*f*(-a*c)^(1/2)+c*e)*(x+(-a*c)^(1/2)
/c)-(e*(-a*c)^(1/2)+a*f-c*d)/c)^(1/2))/(x+(-a*c)^(1/2)/c))*A-1/2/c/(-(e*(-a
*c)^(1/2)+a*f-c*d)/c)^(1/2)*ln((-2*(e*(-a*c)^(1/2)+a*f-c*d)/c+1/c*(-2*f*(-a
*c)^(1/2)+c*e)*(x+(-a*c)^(1/2)/c)+2*(-(e*(-a*c)^(1/2)+a*f-c*d)/c)^(1/2)*((x
+(-a*c)^(1/2)/c)^2*f+1/c*(-2*f*(-a*c)^(1/2)+c*e)*(x+(-a*c)^(1/2)/c)-(e*(-a*
c)^(1/2)+a*f-c*d)/c)^(1/2))/(x+(-a*c)^(1/2)/c))*B
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx + A}{\left(cx^2 + a\right)\sqrt{fx^2 + ex + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x + A)/((c*x^2 + a)*sqrt(f*x^2 + e*x + d)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B\,x}{\left(c\,x^2 + a\right)\,\sqrt{f\,x^2 + e\,x + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/((a + c*x^2)*(d + e*x + f*x^2)^(1/2)),x)
```

```
[Out] int((A + B*x)/((a + c*x^2)*(d + e*x + f*x^2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx}{\left(a + cx^2\right)\sqrt{d + ex + fx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+a)/(f*x**2+e*x+d)**(1/2),x)

[Out] Integral((A + B*x)/((a + c*x**2)*sqrt(d + e*x + f*x**2)), x)
```

## 3.23 $\int \frac{A+Bx}{\left(a+bx+cx^2\right)\sqrt{d+fx^2}}\,dx$

**Optimal**. Leaf size=302

$$\frac{\left(-B\sqrt{b^2-4ac}-2Ac+bB\right)\tanh^{-1}\left(\frac{2cd-fx\left(b-\sqrt{b^2-4ac}\right)}{\sqrt{2}\sqrt{d+fx^2}\sqrt{bf\left(b-\sqrt{b^2-4ac}\right)-2acf+2c^2d}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{bf\left(b-\sqrt{b^2-4ac}\right)-2acf+2c^2d}}+\frac{\left(2Ac-B\left(\sqrt{b^2-4ac}+b\right)\right)\tanh^{-1}\left(\ldots\right.}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{bf\left(\sqrt{b^2-\ldots\right.}}}$$

**Rubi [A]** time = 0.84, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1034, 725, 206}

$$\frac{\left(-B\sqrt{b^2-4ac}-2Ac+bB\right)\tanh^{-1}\left(\frac{2cd-fx\left(b-\sqrt{b^2-4ac}\right)}{\sqrt{2}\sqrt{d+fx^2}\sqrt{bf\left(b-\sqrt{b^2-4ac}\right)-2acf+2c^2d}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{bf\left(b-\sqrt{b^2-4ac}\right)-2acf+2c^2d}}+\frac{\left(2Ac-B\left(\sqrt{b^2-4ac}+b\right)\right)\tanh^{-1}\left(\frac{2cd-fx\left(\sqrt{b^2-4ac}+b\right)}{\sqrt{2}\sqrt{d+fx^2}\sqrt{bf\left(\sqrt{b^2-4ac}+b\right)-2acf+2c^2d}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{bf\left(\sqrt{b^2-4ac}+b\right)-2acf+2c^2d}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]),x]
```

```
[Out] ((b*B - 2*A*c - B*Sqrt[b^2 - 4*a*c])*ArcTanh[(2*c*d - (b - Sqrt[b^2 - 4*a*c
])*f*x)/(Sqrt[2]*Sqrt[2*c^2*d - 2*a*c*f + b*(b - Sqrt[b^2 - 4*a*c])*f]*Sqrt
[d + f*x^2])])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2*d - 2*a*c*f + b*(b - S
qrt[b^2 - 4*a*c])*f]) + ((2*A*c - B*(b + Sqrt[b^2 - 4*a*c]))*ArcTanh[(2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*f*x)/(Sqrt[2]*Sqrt[2*c^2*d - 2*a*c*f + b*(b + Sq
rt[b^2 - 4*a*c])*f]*Sqrt[d + f*x^2])])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^
2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*f])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx}{\left(a+bx+cx^2\right)\sqrt{d+fx^2}}\,dx = \frac{\left(2Ac-B\left(b-\sqrt{b^2-4ac}\right)\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{d+fx^2}}\,dx}{\sqrt{b^2-4ac}} - \frac{\left(2Ac-B\left(b+\sqrt{b^2-\right.\right.}{}$$

$$= -\frac{\left(2Ac-B\left(b-\sqrt{b^2-4ac}\right)\right)\text{Subst}\left(\int \frac{1}{4c^2d+\left(b-\sqrt{b^2-4ac}\right)^2 f-x^2}\,dx, x, \frac{2cd-\left(b-\sqrt{b^2-4a\right.}{\sqrt{d+fx^2}}\right.}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(bB-2Ac-B\sqrt{b^2-4ac}\right)\tanh^{-1}\left(\frac{2cd-\left(b-\sqrt{b^2-4ac}\right)fx}{\sqrt{2}\sqrt{2c^2d-2acf+b\left(b-\sqrt{b^2-4ac}\right)f}\sqrt{d+fx^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{2c^2d-2acf+b\left(b-\sqrt{b^2-4ac}\right)f}} + \frac{\left(2Ac\right.}{}$$

**Mathematica [A]** time = 0.43, size = 283, normalized size = 0.94

$$\frac{\sqrt{2}\left(-\frac{\left(B\sqrt{b^2-4ac}+2Ac-bB\right)\tanh^{-1}\left(\frac{fx\left(\sqrt{b^2-4ac}-b\right)+2cd}{\sqrt{d+fx^2}\sqrt{2bf\left(b-\sqrt{b^2-4ac}\right)-4acf+4c^2d}}\right)}{2\sqrt{bf\left(b-\sqrt{b^2-4ac}\right)-2acf+2c^2d}}-\frac{\left(B\sqrt{b^2-4ac}-2Ac+bB\right)\tanh^{-1}\left(\frac{2cd-fx\left(\sqrt{b^2-4ac}+b\right)}{\sqrt{d+fx^2}\sqrt{2bf\left(\sqrt{b^2-4ac}+b\right)-4acf+4c^2d}}\right)}{2\sqrt{bf\left(\sqrt{b^2-4ac}+b\right)-2acf+2c^2d}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]),x]
```

```
[Out] (Sqrt[2]*(-1/2*((-(b*B) + 2*A*c + B*Sqrt[b^2 - 4*a*c])*ArcTanh[(2*c*d + (-b
 + Sqrt[b^2 - 4*a*c])*f*x)/(Sqrt[4*c^2*d - 4*a*c*f + 2*b*(b - Sqrt[b^2 - 4*
a*c])*f]*Sqrt[d + f*x^2])])/Sqrt[2*c^2*d - 2*a*c*f + b*(b - Sqrt[b^2 - 4*a*
c])*f] - ((b*B - 2*A*c + B*Sqrt[b^2 - 4*a*c])*ArcTanh[(2*c*d - (b + Sqrt[b^
2 - 4*a*c])*f*x)/(Sqrt[4*c^2*d - 4*a*c*f + 2*b*(b + Sqrt[b^2 - 4*a*c])*f]*S
qrt[d + f*x^2])])/(2*Sqrt[2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*f])
))/Sqrt[b^2 - 4*a*c]
```

**IntegrateAlgebraic [C]** time = 0.44, size = 195, normalized size = 0.65

$$-\text{RootSum}\left[\#1^4c-2\#1^3b\sqrt{f}+4\#1^2af-2\#1^2cd+2\#1bd\sqrt{f}+cd^2\&,\frac{\#1^2(-B)\log\left(-\#1+\sqrt{d+fx^2}-\sqrt{f}x\right)+2\#1A\sqrt{f}\log\left(-\#1+\sqrt{d+fx^2}-\sqrt{f}x\right)+Bd\log\left(-\#1+\sqrt{d+fx^2}-\sqrt{f}x\right)}{2\#1^3c-3\#1^2b\sqrt{f}+4\#1af-2\#1cd+bd\sqrt{f}}\&\right]$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]),x]
```

```
[Out] -RootSum[c*d^2 + 2*b*d*Sqrt[f]*#1 - 2*c*d*#1^2 + 4*a*f*#1^2 - 2*b*Sqrt[f]*#
1^3 + c*#1^4 & , (B*d*Log[-(Sqrt[f]*x) + Sqrt[d + f*x^2] - #1] + 2*A*Sqrt[f
]*Log[-(Sqrt[f]*x) + Sqrt[d + f*x^2] - #1]*#1 - B*Log[-(Sqrt[f]*x) + Sqrt[d
 + f*x^2] - #1]*#1^2)/(b*d*Sqrt[f] - 2*c*d*#1 + 4*a*f*#1 - 3*b*Sqrt[f]*#1^2
 + 2*c*#1^3) & ]
```

**fricas [B]** time = 60.52, size = 8977, normalized size = 29.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*sqrt(((B^2*b^2 + 2*A^2*c^2 - 2*(B^2*a + A*B*b)*c)*d + (2*B^2*a^
2 - 2*A*B*a*b + A^2*b^2 - 2*A^2*a*c)*f + ((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 -
6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2)*sqrt(((B^4*b^2 - 4*A*
B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*
a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^
4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a
*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c +
8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4)))/((b^2*c^2 - 4*a*c^3)*d^2 + (b
^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2))*log((2*(B^4*a*b
^2 - A*B^3*b^3 - 2*A^3*B*b*c^2 - (2*A*B^3*a*b - 3*A^2*B^2*b^2)*c)*d^2 + 2*(
2*A*B^3*a^2*b - 3*A^2*B^2*a*b^2 + A^3*B*b^3 + (2*A^3*B*a*b - A^4*b^2)*c)*d*
f + sqrt(2)*((B^3*b^4 - 8*A^2*B*a*c^3 + 2*(6*A*B^2*a*b + A^2*B*b^2)*c^2 - (
4*B^3*a*b^2 + 3*A*B^2*b^3)*c)*d^2 + (3*A*B^2*a*b^3 - A^2*B*b^4 + 4*(4*A^2*B
*a^2 - A^3*a*b)*c^2 - (12*A*B^2*a^2*b - A^3*b^3)*c)*d*f + (2*A^2*B*a^2*b^2
- A^3*a*b^3 - 4*(2*A^2*B*a^3 - A^3*a^2*b)*c)*f^2 - ((B*b^4*c^2 + 4*(2*B*a^2
 + A*a*b)*c^4 - (6*B*a*b^2 + A*b^3)*c^3)*d^3 + (B*b^6 - 4*(6*B*a^3 + A*a^2*
b)*c^3 + (22*B*a^2*b^2 + 5*A*a*b^3)*c^2 - (8*B*a*b^4 + A*b^5)*c)*d^2*f + (3
*B*a^2*b^4 - A*a*b^5 + 4*(6*B*a^4 - A*a^3*b)*c^2 - (18*B*a^3*b^2 - 5*A*a^2*
b^3)*c)*d*f^2 + (2*B*a^4*b^2 - A*a^3*b^3 - 4*(2*B*a^5 - A*a^4*b)*c)*f^3)*sq
rt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*
b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4
*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)
*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b
^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4)))*sqrt(f*x^2
 + d)*sqrt(((B^2*b^2 + 2*A^2*c^2 - 2*(B^2*a + A*B*b)*c)*d + (2*B^2*a^2 - 2*
```

A*B*a*b + A^2*b^2 - 2*A^2*a*c)*f + ((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4)))/((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2)) - 4*((B^4*a^2*b - A*B^3*a*b^2 - 2*A^3*B*a*c^2 - (2*A*B^3*a^2 - 3*A^2*B^2*a*b)*c)*d*f + (2*A*B^3*a^3 - 3*A^2*B^2*a^2*b + A^3*B*a*b^2 + (2*A^3*B*a^2 - A^4*a*b)*c)*f^2)*x + 2*((4*A^2*a*c^4 + (4*B^2*a^2 - 4*A*B*a*b - A^2*b^2)*c^3 - (B^2*a*b^2 - A*B*b^3)*c^2)*d^3 - (B^2*a*b^4 - A*B*b^5 + 8*A^2*a^2*c^3 + 2*(4*B^2*a^3 - 4*A*B*a^2*b - 3*A^2*a*b^2)*c^2 - (6*B^2*a^2*b^2 - 6*A*B*a*b^3 - A^2*b^4)*c)*d^2*f - (B^2*a^3*b^2 - A*B*a^2*b^3 - 4*A^2*a^3*c^2 - (4*B^2*a^4 - 4*A*B*a^3*b - A^2*a^2*b^2)*c)*d*f^2)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4)))/x) - 1/4*sqrt(2)*sqrt(((B^2*b^2 + 2*A^2*c^2 - 2*(B^2*a + A*B*b)*c)*d + (2*B^2*a^2 - 2*A*B*a*b + A^2*b^2 - 2*A^2*a*c)*f + ((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4)))/((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2))*log((2*(B^4*a*b^2 - A*B^3*b^3 - 2*A^3*B*b*c^2 - (2*A*B^3*a*b - 3*A^2*B^2*b^2)*c)*d^2 + 2*(2*A*B^3*a^2*b - 3*A^2*B^2*a*b^2 + A^3*B*b^3 + (2*A^3*B*a*b - A^4*b^2)*c)*d*f - sqrt(2)*((B^3*b^4 - 8*A^2*B*a*c^3 + 2*(6*A*B^2*a*b + A^2*B*b^2)*c^2 - (4*B^3*a*b^2 + 3*A*B^2*b^3)*c)*d^2 + (3*A*B^2*a*b^3 - A^2*B*b^4 + 4*(4*A^2*B*a^2 - A^3*a*b)*c^2 - (12*A*B^2*a^2*b - A^3*b^3)*c)*d*f + (2*A^2*B*a^2*b^2 - A^3*a*b^3 - 4*(2*A^2*B*a^3 - A^3*a^2*b)*c)*f^2 - ((B*b^4*c^2 + 4*(2*B*a^2 + A*a*b)*c^4 - (6*B*a*b^2 + A*b^3)*c^3)*d^3 + (B*b^6 - 4*(6*B*a^3 + A*a^2*b)*c^3 + (22*B*a^2*b^2 + 5*A*a*b^3)*c^2 - (8*B*a*b^4 + A*b^5)*c)*d^2*f + (3*B*a^2*b^4 - A*a*b^5 + 4*(6*B*a^4 - A*a^3*b)*c^2 - (18*B*a^3*b^2 - 5*A*a^2*b^3)*c)*d*f^2 + (2*B*a^4*b^2 - A*a^3*b^3 - 4*(2*B*a^5 - A*a^4*b)*c)*f^3)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4)))*sqrt(f*x^2 + d)*sqrt(((B^2*b^2 + 2*A^2*c^2 - 2*(B^2*a + A*B*b)*c)*d + (2*B^2*a^2 - 2*A*B*a*b + A^2*b^2 - 2*A^2*a*c)*f + ((b^2*c^2 - 4*a*c^3)

```
*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2)*sqrt(((
B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 -
 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)
*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*
f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 -
6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4)))/((b^2*c^2 - 4*a
*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2)) -
 4*((B^4*a^2*b - A*B^3*a*b^2 - 2*A^3*B*a*c^2 - (2*A*B^3*a^2 - 3*A^2*B^2*a*b
)*c)*d*f + (2*A*B^3*a^3 - 3*A^2*B^2*a^2*b + A^3*B*a*b^2 + (2*A^3*B*a^2 - A^
4*a*b)*c)*f^2)*x + 2*((4*A^2*a*c^4 + (4*B^2*a^2 - 4*A*B*a*b - A^2*b^2)*c^3
- (B^2*a*b^2 - A*B*b^3)*c^2)*d^3 - (B^2*a*b^4 - A*B*b^5 + 8*A^2*a^2*c^3 + 2
*(4*B^2*a^3 - 4*A*B*a^2*b - 3*A^2*a*b^2)*c^2 - (6*B^2*a^2*b^2 - 6*A*B*a*b^3
 - A^2*b^4)*c)*d^2*f - (B^2*a^3*b^2 - A*B*a^2*b^3 - 4*A^2*a^3*c^2 - (4*B^2*
a^4 - 4*A*B*a^3*b - A^2*a^2*b^2)*c)*d*f^2)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4
*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b
)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5
)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22
*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*
d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4)))/x) + 1/4*sqrt(2)*sqrt(((B^2*b^2 + 2*A^2*
c^2 - 2*(B^2*a + A*B*b)*c)*d + (2*B^2*a^2 - 2*A*B*a*b + A^2*b^2 - 2*A^2*a*c
)*f - ((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b
^2 - 4*a^3*c)*f^2)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2
*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^
2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a
*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^
3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5
*c)*f^4)))/((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (
a^2*b^2 - 4*a^3*c)*f^2))*log((2*(B^4*a*b^2 - A*B^3*b^3 - 2*A^3*B*b*c^2 - (2
*A*B^3*a*b - 3*A^2*B^2*b^2)*c)*d^2 + 2*(2*A*B^3*a^2*b - 3*A^2*B^2*a*b^2 + A
^3*B*b^3 + (2*A^3*B*a*b - A^4*b^2)*c)*d*f + sqrt(2)*((B^3*b^4 - 8*A^2*B*a*c
^3 + 2*(6*A*B^2*a*b + A^2*B*b^2)*c^2 - (4*B^3*a*b^2 + 3*A*B^2*b^3)*c)*d^2 +
 (3*A*B^2*a*b^3 - A^2*B*b^4 + 4*(4*A^2*B*a^2 - A^3*a*b)*c^2 - (12*A*B^2*a^2
*b - A^3*b^3)*c)*d*f + (2*A^2*B*a^2*b^2 - A^3*a*b^3 - 4*(2*A^2*B*a^3 - A^3*
a^2*b)*c)*f^2 + ((B*b^4*c^2 + 4*(2*B*a^2 + A*a*b)*c^4 - (6*B*a*b^2 + A*b^3)
*c^3)*d^3 + (B*b^6 - 4*(6*B*a^3 + A*a^2*b)*c^3 + (22*B*a^2*b^2 + 5*A*a*b^3)
*c^2 - (8*B*a*b^4 + A*b^5)*c)*d^2*f + (3*B*a^2*b^4 - A*a*b^5 + 4*(6*B*a^4 -
 A*a^3*b)*c^2 - (18*B*a^3*b^2 - 5*A*a^2*b^3)*c)*d*f^2 + (2*B*a^4*b^2 - A*a^
3*b^3 - 4*(2*B*a^5 - A*a^4*b)*c)*f^3)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*
B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*
d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4
 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2*
b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3
 + (a^4*b^2 - 4*a^5*c)*f^4)))*sqrt(f*x^2 + d)*sqrt(((B^2*b^2 + 2*A^2*c^2 -
2*(B^2*a + A*B*b)*c)*d + (2*B^2*a^2 - 2*A*B*a*b + A^2*b^2 - 2*A^2*a*c)*f -
((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4
```

```
*a^3*c)*f^2)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3
*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*
A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c
^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2
*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^
4)))/((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^
2 - 4*a^3*c)*f^2)) - 4*((B^4*a^2*b - A*B^3*a*b^2 - 2*A^3*B*a*c^2 - (2*A*B^3
*a^2 - 3*A^2*B^2*a*b)*c)*d*f + (2*A*B^3*a^3 - 3*A^2*B^2*a^2*b + A^3*B*a*b^2
 + (2*A^3*B*a^2 - A^4*a*b)*c)*f^2)*x - 2*((4*A^2*a*c^4 + (4*B^2*a^2 - 4*A*B
*a*b - A^2*b^2)*c^3 - (B^2*a*b^2 - A*B*b^3)*c^2)*d^3 - (B^2*a*b^4 - A*B*b^5
 + 8*A^2*a^2*c^3 + 2*(4*B^2*a^3 - 4*A*B*a^2*b - 3*A^2*a*b^2)*c^2 - (6*B^2*a
^2*b^2 - 6*A*B*a*b^3 - A^2*b^4)*c)*d^2*f - (B^2*a^3*b^2 - A*B*a^2*b^3 - 4*A
^2*a^3*c^2 - (4*B^2*a^4 - 4*A*B*a^3*b - A^2*a^2*b^2)*c)*d*f^2)*sqrt(((B^4*b
^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2
*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)
/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (
b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3
*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4)))/x) - 1/4*sqrt(2)*sqr
t(((B^2*b^2 + 2*A^2*c^2 - 2*(B^2*a + A*B*b)*c)*d + (2*B^2*a^2 - 2*A*B*a*b +
 A^2*b^2 - 2*A^2*a*c)*f - ((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a
^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2
*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)
*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^
4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2
*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^
3 + (a^4*b^2 - 4*a^5*c)*f^4)))/((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c
+ 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2))*log((2*(B^4*a*b^2 - A*B^3*b^3
- 2*A^3*B*b*c^2 - (2*A*B^3*a*b - 3*A^2*B^2*b^2)*c)*d^2 + 2*(2*A*B^3*a^2*b -
 3*A^2*B^2*a*b^2 + A^3*B*b^3 + (2*A^3*B*a*b - A^4*b^2)*c)*d*f - sqrt(2)*((B
^3*b^4 - 8*A^2*B*a*c^3 + 2*(6*A*B^2*a*b + A^2*B*b^2)*c^2 - (4*B^3*a*b^2 + 3
*A*B^2*b^3)*c)*d^2 + (3*A*B^2*a*b^3 - A^2*B*b^4 + 4*(4*A^2*B*a^2 - A^3*a*b)
*c^2 - (12*A*B^2*a^2*b - A^3*b^3)*c)*d*f + (2*A^2*B*a^2*b^2 - A^3*a*b^3 - 4
*(2*A^2*B*a^3 - A^3*a^2*b)*c)*f^2 + ((B*b^4*c^2 + 4*(2*B*a^2 + A*a*b)*c^4 -
 (6*B*a*b^2 + A*b^3)*c^3)*d^3 + (B*b^6 - 4*(6*B*a^3 + A*a^2*b)*c^3 + (22*B*
a^2*b^2 + 5*A*a*b^3)*c^2 - (8*B*a*b^4 + A*b^5)*c)*d^2*f + (3*B*a^2*b^4 - A*
a*b^5 + 4*(6*B*a^4 - A*a^3*b)*c^2 - (18*B*a^3*b^2 - 5*A*a^2*b^3)*c)*d*f^2 +
 (2*B*a^4*b^2 - A*a^3*b^3 - 4*(2*B*a^5 - A*a^4*b)*c)*f^3)*sqrt(((B^4*b^2 -
4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*
B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^
2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 -
 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*
c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4)))*sqrt(f*x^2 + d)*sqrt(((B^
2*b^2 + 2*A^2*c^2 - 2*(B^2*a + A*B*b)*c)*d + (2*B^2*a^2 - 2*A*B*a*b + A^2*b
^2 - 2*A^2*a*c)*f - ((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2
)*d*f + (a^2*b^2 - 4*a^3*c)*f^2)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c
```

```
^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f +
 (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*
(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c
^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a
^4*b^2 - 4*a^5*c)*f^4)))/((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^
2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2)) - 4*((B^4*a^2*b - A*B^3*a*b^2 - 2*A^
3*B*a*c^2 - (2*A*B^3*a^2 - 3*A^2*B^2*a*b)*c)*d*f + (2*A*B^3*a^3 - 3*A^2*B^2
*a^2*b + A^3*B*a*b^2 + (2*A^3*B*a^2 - A^4*a*b)*c)*f^2)*x - 2*((4*A^2*a*c^4
+ (4*B^2*a^2 - 4*A*B*a*b - A^2*b^2)*c^3 - (B^2*a*b^2 - A*B*b^3)*c^2)*d^3 -
(B^2*a*b^4 - A*B*b^5 + 8*A^2*a^2*c^3 + 2*(4*B^2*a^3 - 4*A*B*a^2*b - 3*A^2*a
*b^2)*c^2 - (6*B^2*a^2*b^2 - 6*A*B*a*b^3 - A^2*b^4)*c)*d^2*f - (B^2*a^3*b^2
 - A*B*a^2*b^3 - 4*A^2*a^3*c^2 - (4*B^2*a^4 - 4*A*B*a^3*b - A^2*a^2*b^2)*c)
*d*f^2)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b
- A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B
*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 +
8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2
+ 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4)))/
x)
```

**giac** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%{poly1[%%%{-4,[3,2,0]%%%}+%%%{16,[1,3,1]%%%},%%%{4,[4,2,0]%%%}+%%%
{-24,[2,3,1]%%%}+%%%{32,[0,4,2]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%
%%}]%%},[2,1,0,0]%%%}+%%%{%%{[%%%{-4,[5,0,0]%%%}+%%%{24,[3,1,1]%%%}+%%%{-32
,[1,2,2]%%%},%%%{4,[6,0,0]%%%}+%%%{-32,[4,1,1]%%%}+%%%{72,[2,2,2]%%%}+%%%{-
32,[0,3,3]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%}]%%},[2,0,0,1]%%%}
+%%%{%%{poly1[%%%{8,[2,3,0]%%%}+%%%{-32,[0,4,1]%%%},%%%{-8,[3,3,0]%%%}+%%%{
32,[1,4,1]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%}]%%},[1,1,1,0]%%%}
+%%%{%%{[%%%{8,[4,1,0]%%%}+%%%{-40,[2,2,1]%%%}+%%%{32,[0,3,2]%%%},%%%{-8,[5
,1,0]%%%}+%%%{56,[3,2,1]%%%}+%%%{-96,[1,3,2]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%
%%{4,[0,1,1]%%%}]%%},[1,0,1,1]%%%}+%%%{%%%{8,[2,4,0]%%%}+%%%{-32,[0,5,1]%%%
},[0,1,2,0]%%%}+%%%{%%{poly1[%%%{-4,[3,2,0]%%%}+%%%{16,[1,3,1]%%%},%%%{4,[4
,2,0]%%%}+%%%{-24,[2,3,1]%%%}+%%%{32,[0,4,2]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%
%%{4,[0,1,1]%%%}]%%},[0,0,2,1]%%%} / %%%{%%{[%%%{-4,[3,2,0]%%%}+%%%{16,[1,3
,1]%%%},%%%{4,[4,2,0]%%%}+%%%{-24,[2,3,1]%%%}+%%%{32,[0,4,2]%%%}]:[1,0,%%%{
-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%}]%%},[0,1,0,0]%%%}+%%%{%%{[%%%{-4,[5,0,0]%%
%}+%%%{24,[3,1,1]%%%}+%%%{-32,[1,2,2]%%%},%%%{4,[6,0,0]%%%}+%%%{-32,[4,1,1]
%%%}+%%%{72,[2,2,2]%%%}+%%%{-32,[0,3,3]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,
```

```
[0,1,1]%%%}]%%},[0,0,0,1]%%%} Error: Bad Argument ValueUnable to divide, pe
rhaps due to rounding error%%%{%%{[%%%{1,[3,0,0]%%%}+%%%{-4,[1,1,1]%%%},%%%
{1,[4,0,0]%%%}+%%%{-6,[2,1,1]%%%}+%%%{8,[0,2,2]%%%}]:[1,0,%%%{-1,[2,0,0]%%%
}+%%%{4,[0,1,1]%%%}]%%}/%%%{4,[0,0,2]%%%},[2,1,0,0]%%%}+%%%{%%%{1,[2,0,0]%%
%}+%%%{-4,[0,1,1]%%%}/2,[2,0,0,1]%%%}+%%%{%%{[%%%{-1,[4,0,0]%%%}+%%%{5,[2,1
,1]%%%}+%%%{-4,[0,2,2]%%%},%%%{-1,[5,0,0]%%%}+%%%{7,[3,1,1]%%%}+%%%{-12,[1,
2,2]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%}]%%}/%%%{2,[0,0,3]%%%},[
1,1,1,0]%%%}+%%%{%%{[%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%},%%%{-1,[3,0,0]%%%
}+%%%{4,[1,1,1]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%}]%%}/%%%{2,[0
,0,1]%%%},[1,0,1,1]%%%}+%%%{%%{[%%%{1,[5,0,0]%%%}+%%%{-6,[3,1,1]%%%}+%%%{8,
[1,2,2]%%%},%%%{1,[6,0,0]%%%}+%%%{-8,[4,1,1]%%%}+%%%{18,[2,2,2]%%%}+%%%{-8,
[0,3,3]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%}]%%}/%%%{4,[0,0,4]%%%
},[0,1,2,0]%%%}+%%%{%%{[%%%{1,[3,0,0]%%%}+%%%{-4,[1,1,1]%%%},%%%{1,[4,0,0]%
%%}+%%%{-6,[2,1,1]%%%}+%%%{8,[0,2,2]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,
1,1]%%%}]%%}/%%%{4,[0,0,2]%%%},[0,0,2,1]%%%} / %%%{%%{[%%%{1,[3,0,0]%%%}+%%
%{-4,[1,1,1]%%%},%%%{1,[4,0,0]%%%}+%%%{-6,[2,1,1]%%%}+%%%{8,[0,2,2]%%%}]:[1
,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%}]%%}/%%%{4,[0,0,4]%%%},[0,1,0,0]%%%}
+%%%{%%%{1,[2,0,0]%%%}+%%%{-4,[0,1,1]%%%}/%%%{2,[0,0,2]%%%},[0,0,0,1]%%%} E
rror: Bad Argument Value
```

**maple [B]** time = 0.03, size = 1771, normalized size = 5.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d)^(1/2),x)
```

```
[Out] 2/(-4*a*c+b^2)^(1/2)/(-2*(-(-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^
2)^(1/2)*ln((-(-(-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2-f*(b+(-4*
a*c+b^2)^(1/2))/c*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)+1/2*(-2*(-(-4*a*c+b^2)^(
1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2)*(4*(x+1/2*(b+(-4*a*c+b^2)^(1/2))
/c)^2*f-4*f*(b+(-4*a*c+b^2)^(1/2))/c*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)-2*(-(
-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2))/(x+1/2*(b+(-4*a*c+
b^2)^(1/2))/c))*A-1/c/(-2*(-(-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c
^2)^(1/2)*ln((-(-(-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2-f*(b+(-4
*a*c+b^2)^(1/2))/c*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)+1/2*(-2*(-(-4*a*c+b^2)^
(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2)*(4*(x+1/2*(b+(-4*a*c+b^2)^(1/2)
)/c)^2*f-4*f*(b+(-4*a*c+b^2)^(1/2))/c*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)-2*(-
(-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2))/(x+1/2*(b+(-4*a*c
+b^2)^(1/2))/c))*B-1/(-4*a*c+b^2)^(1/2)/c/(-2*(-(-4*a*c+b^2)^(1/2)*b*f+2*a*
c*f-b^2*f-2*c^2*d)/c^2)^(1/2)*ln((-(-(-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2
*c^2*d)/c^2-f*(b+(-4*a*c+b^2)^(1/2))/c*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)+1/2
*(-2*(-(-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2)*(4*(x+1/2*(
b+(-4*a*c+b^2)^(1/2))/c)^2*f-4*f*(b+(-4*a*c+b^2)^(1/2))/c*(x+1/2*(b+(-4*a*c
+b^2)^(1/2))/c)-2*(-(-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2
```

```
))/(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c))*b*B-2/(-4*a*c+b^2)^(1/2)/(-2*((-4*a*c+
b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2)*ln((-((-4*a*c+b^2)^(1/2)*b
*f+2*a*c*f-b^2*f-2*c^2*d)/c^2-f*(b-(-4*a*c+b^2)^(1/2))/c*(x-1/2*(-b+(-4*a*c
+b^2)^(1/2))/c)+1/2*(-2*((-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)
^(1/2)*(4*(x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)^2*f-4*f*(b-(-4*a*c+b^2)^(1/2))/
c*(x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)-2*((-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*f
-2*c^2*d)/c^2)^(1/2))/(x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c))*A-1/c/(-2*((-4*a*c
+b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2)*ln((-((-4*a*c+b^2)^(1/2)*
b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2-f*(b-(-4*a*c+b^2)^(1/2))/c*(x-1/2*(-b+(-4*a*
c+b^2)^(1/2))/c)+1/2*(-2*((-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2
)^(1/2)*(4*(x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)^2*f-4*f*(b-(-4*a*c+b^2)^(1/2))
/c*(x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)-2*((-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*
f-2*c^2*d)/c^2)^(1/2))/(x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c))*B+1/(-4*a*c+b^2)^
(1/2)/c/(-2*((-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2)*ln((-
((-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2-f*(b-(-4*a*c+b^2)^(1/2))
/c*(x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)+1/2*(-2*((-4*a*c+b^2)^(1/2)*b*f+2*a*c*
f-b^2*f-2*c^2*d)/c^2)^(1/2)*(4*(x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)^2*f-4*f*(b
-(-4*a*c+b^2)^(1/2))/c*(x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)-2*((-4*a*c+b^2)^(1
/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2))/(x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c
))*b*B
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,x}{\sqrt{f\,x^2+d}\,\left(c\,x^2+b\,x+a\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/((d + f*x^2)^(1/2)*(a + b*x + c*x^2)),x)
```

```
[Out] int((A + B*x)/((d + f*x^2)^(1/2)*(a + b*x + c*x^2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx}{\sqrt{d + fx^2}\,\left(a + bx + cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)/(f*x**2+d)**(1/2),x)
[Out] Integral((A + B*x)/(sqrt(d + f*x**2)*(a + b*x + c*x**2)), x)
```

## 3.24 $\int \frac{A+Bx}{\left(a+cx^2\right)\sqrt{d+fx^2}}\,dx$

**Optimal**. Leaf size=101

$$\frac{A\tan^{-1}\left(\frac{x\sqrt{cd-af}}{\sqrt{a}\sqrt{d+fx^2}}\right)}{\sqrt{a}\sqrt{cd-af}}-\frac{B\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+fx^2}}{\sqrt{cd-af}}\right)}{\sqrt{c}\sqrt{cd-af}}$$

**Rubi [A]** time = 0.13, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1010, 377, 205, 444, 63, 208}

$$\frac{A\tan^{-1}\left(\frac{x\sqrt{cd-af}}{\sqrt{a}\sqrt{d+fx^2}}\right)}{\sqrt{a}\sqrt{cd-af}}-\frac{B\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+fx^2}}{\sqrt{cd-af}}\right)}{\sqrt{c}\sqrt{cd-af}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/((a + c*x^2)*Sqrt[d + f*x^2]),x]
```

```
[Out] (A*ArcTan[(Sqrt[c*d - a*f]*x)/(Sqrt[a]*Sqrt[d + f*x^2])])/(Sqrt[a]*Sqrt[c*d
 - a*f]) - (B*ArcTanh[(Sqrt[c]*Sqrt[d + f*x^2])/Sqrt[c*d - a*f]])/(Sqrt[c]*
Sqrt[c*d - a*f])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1010

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx}{\left(a+cx^2\right)\sqrt{d+fx^2}}\,dx &= A\int \frac{1}{\left(a+cx^2\right)\sqrt{d+fx^2}}\,dx + B\int \frac{x}{\left(a+cx^2\right)\sqrt{d+fx^2}}\,dx \\
&= A\,\mathrm{Subst}\left(\int \frac{1}{a-(-cd+af)x^2}\,dx, x, \frac{x}{\sqrt{d+fx^2}}\right) + \frac{1}{2}B\,\mathrm{Subst}\left(\int \frac{1}{(a+cx)\sqrt{d+fx}}\,d\right. \\
&= \frac{A\tan^{-1}\left(\frac{\sqrt{cd-af}\,x}{\sqrt{a}\,\sqrt{d+fx^2}}\right)}{\sqrt{a}\,\sqrt{cd-af}} + \frac{B\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{cd}{f}+\frac{cx^2}{f}}\,dx, x, \sqrt{d+fx^2}\right)}{f} \\
&= \frac{A\tan^{-1}\left(\frac{\sqrt{cd-af}\,x}{\sqrt{a}\,\sqrt{d+fx^2}}\right)}{\sqrt{a}\,\sqrt{cd-af}} - \frac{B\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{d+fx^2}}{\sqrt{cd-af}}\right)}{\sqrt{c}\,\sqrt{cd-af}}
\end{aligned}$$

**Mathematica [A]** time = 0.17, size = 154, normalized size = 1.52

$$\frac{\left(A\sqrt{c}-\sqrt{-a}\,B\right)\tanh^{-1}\left(\frac{\sqrt{c}\,d-\sqrt{-a}\,fx}{\sqrt{d+fx^2}\,\sqrt{cd-af}}\right)-\left(\sqrt{-a}\,B+A\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{-a}\,fx+\sqrt{c}\,d}{\sqrt{d+fx^2}\,\sqrt{cd-af}}\right)}{2\sqrt{-a}\,\sqrt{c}\,\sqrt{cd-af}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/((a + c*x^2)*Sqrt[d + f*x^2]),x]
```

```
[Out] ((-(Sqrt[-a]*B) + A*Sqrt[c])*ArcTanh[(Sqrt[c]*d - Sqrt[-a]*f*x)/(Sqrt[c*d -
 a*f]*Sqrt[d + f*x^2])] - (Sqrt[-a]*B + A*Sqrt[c])*ArcTanh[(Sqrt[c]*d + Sqr
t[-a]*f*x)/(Sqrt[c*d - a*f]*Sqrt[d + f*x^2])])/(2*Sqrt[-a]*Sqrt[c]*Sqrt[c*d
 - a*f])
```

**IntegrateAlgebraic [B]** time = 2.27, size = 563, normalized size = 5.57

$$\frac{A\tanh^{-1}\left(\frac{c\sqrt{f}x^2}{\sqrt{a}\sqrt{af-cd}}-\frac{cx\sqrt{d+fx^2}}{\sqrt{a}\sqrt{af-cd}}+\frac{\sqrt{a}\sqrt{f}}{\sqrt{af-cd}}\right)}{\sqrt{a}\sqrt{af-cd}}+\frac{\left(Bcd\sqrt{-2\sqrt{a}\sqrt{f}\sqrt{af-cd}+2af-cd}-aBf\sqrt{-2\sqrt{a}\sqrt{f}\sqrt{af-cd}+2af-cd}-\sqrt{a}B\sqrt{f}\sqrt{af-cd}\sqrt{-2\sqrt{a}\sqrt{f}\sqrt{af-cd}+2af-cd}\right)\tan^{-1}\left(\frac{\sqrt{c}\left(\sqrt{f}x-\sqrt{d+fx^2}\right)}{\sqrt{-2\sqrt{a}\sqrt{f}\sqrt{af-cd}+2af-cd}}\right)}{c^{3/2}d(cd-af)}+\frac{\left(Bcd\sqrt{2\sqrt{a}\sqrt{f}\sqrt{af-cd}+2af-cd}-aBf\sqrt{2\sqrt{a}\sqrt{f}\sqrt{af-cd}+2af-cd}+\sqrt{a}B\sqrt{f}\sqrt{af-cd}\sqrt{2\sqrt{a}\sqrt{f}\sqrt{af-cd}+2af-cd}\right)\tan^{-1}\left(\frac{\sqrt{c}\left(\sqrt{f}x-\sqrt{d+fx^2}\right)}{\sqrt{2\sqrt{a}\sqrt{f}\sqrt{af-cd}+2af-cd}}\right)}{c^{3/2}d(cd-af)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x)/((a + c*x^2)*Sqrt[d + f*x^2]),x]
```

```
[Out] ((B*c*d*Sqrt[-(c*d) + 2*a*f - 2*Sqrt[a]*Sqrt[f]*Sqrt[-(c*d) + a*f]] - a*B*f
*Sqrt[-(c*d) + 2*a*f - 2*Sqrt[a]*Sqrt[f]*Sqrt[-(c*d) + a*f]] - Sqrt[a]*B*Sq
rt[f]*Sqrt[-(c*d) + a*f]*Sqrt[-(c*d) + 2*a*f - 2*Sqrt[a]*Sqrt[f]*Sqrt[-(c*d
) + a*f]])*ArcTan[(Sqrt[c]*(Sqrt[f]*x - Sqrt[d + f*x^2]))/Sqrt[-(c*d) + 2*a
*f - 2*Sqrt[a]*Sqrt[f]*Sqrt[-(c*d) + a*f]]])/(c^(3/2)*d*(c*d - a*f)) + ((B*
c*d*Sqrt[-(c*d) + 2*a*f + 2*Sqrt[a]*Sqrt[f]*Sqrt[-(c*d) + a*f]] - a*B*f*Sqr
t[-(c*d) + 2*a*f + 2*Sqrt[a]*Sqrt[f]*Sqrt[-(c*d) + a*f]] + Sqrt[a]*B*Sqrt[f
]*Sqrt[-(c*d) + a*f]*Sqrt[-(c*d) + 2*a*f + 2*Sqrt[a]*Sqrt[f]*Sqrt[-(c*d) +
a*f]])*ArcTan[(Sqrt[c]*(Sqrt[f]*x - Sqrt[d + f*x^2]))/Sqrt[-(c*d) + 2*a*f +
 2*Sqrt[a]*Sqrt[f]*Sqrt[-(c*d) + a*f]]])/(c^(3/2)*d*(c*d - a*f)) + (A*ArcTa
nh[(Sqrt[a]*Sqrt[f])/Sqrt[-(c*d) + a*f] + (c*Sqrt[f]*x^2)/(Sqrt[a]*Sqrt[-(c
*d) + a*f]) - (c*x*Sqrt[d + f*x^2])/(Sqrt[a]*Sqrt[-(c*d) + a*f])])/(Sqrt[a]
*Sqrt[-(c*d) + a*f])
```

**fricas [B]** time = 0.54, size = 1515, normalized size = 15.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+a)/(f*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt((B^2*a - A^2*c + 2*(a*c^2*d - a^2*c*f)*sqrt(-A^2*B^2/(a*c^3*d^2 -
 2*a^2*c^2*d*f + a^3*c*f^2)))/(a*c^2*d - a^2*c*f))*log(((A*B^3*a + A^3*B*c)
*f*x + (A^2*B*c^2*d - A^2*B*a*c*f + (B*a*c^3*d^2 - 2*B*a^2*c^2*d*f + B*a^3*
c*f^2)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))*sqrt(f*x^2 +
 d)*sqrt((B^2*a - A^2*c + 2*(a*c^2*d - a^2*c*f)*sqrt(-A^2*B^2/(a*c^3*d^2 -
2*a^2*c^2*d*f + a^3*c*f^2)))/(a*c^2*d - a^2*c*f)) + sqrt(-A^2*B^2/(a*c^3*d^
2 - 2*a^2*c^2*d*f + a^3*c*f^2))*((B^2*a*c^2 + A^2*c^3)*d^2 - (B^2*a^2*c + A
^2*a*c^2)*d*f))/x) + 1/4*sqrt((B^2*a - A^2*c + 2*(a*c^2*d - a^2*c*f)*sqrt(-
A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))/(a*c^2*d - a^2*c*f))*log(
((A*B^3*a + A^3*B*c)*f*x - (A^2*B*c^2*d - A^2*B*a*c*f + (B*a*c^3*d^2 - 2*B*
a^2*c^2*d*f + B*a^3*c*f^2)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c
```

```
*f^2)))*sqrt(f*x^2 + d)*sqrt((B^2*a - A^2*c + 2*(a*c^2*d - a^2*c*f)*sqrt(-A
^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))/(a*c^2*d - a^2*c*f)) + sqr
t(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2))*((B^2*a*c^2 + A^2*c^3)*
d^2 - (B^2*a^2*c + A^2*a*c^2)*d*f))/x) - 1/4*sqrt((B^2*a - A^2*c - 2*(a*c^2
*d - a^2*c*f)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))/(a*c^
2*d - a^2*c*f))*log(((A*B^3*a + A^3*B*c)*f*x + (A^2*B*c^2*d - A^2*B*a*c*f -
 (B*a*c^3*d^2 - 2*B*a^2*c^2*d*f + B*a^3*c*f^2)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2
*a^2*c^2*d*f + a^3*c*f^2)))*sqrt(f*x^2 + d)*sqrt((B^2*a - A^2*c - 2*(a*c^2*
d - a^2*c*f)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))/(a*c^2
*d - a^2*c*f)) - sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2))*((B
^2*a*c^2 + A^2*c^3)*d^2 - (B^2*a^2*c + A^2*a*c^2)*d*f))/x) + 1/4*sqrt((B^2*
a - A^2*c - 2*(a*c^2*d - a^2*c*f)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f
+ a^3*c*f^2)))/(a*c^2*d - a^2*c*f))*log(((A*B^3*a + A^3*B*c)*f*x - (A^2*B*c
^2*d - A^2*B*a*c*f - (B*a*c^3*d^2 - 2*B*a^2*c^2*d*f + B*a^3*c*f^2)*sqrt(-A^
2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))*sqrt(f*x^2 + d)*sqrt((B^2*a
 - A^2*c - 2*(a*c^2*d - a^2*c*f)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f +
 a^3*c*f^2)))/(a*c^2*d - a^2*c*f)) - sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d
*f + a^3*c*f^2))*((B^2*a*c^2 + A^2*c^3)*d^2 - (B^2*a^2*c + A^2*a*c^2)*d*f))
/x)
```

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+a)/(f*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple** [B] time = 0.02, size = 608, normalized size = 6.02

$$-\frac{A\ln\left(\frac{\frac{2\sqrt{-ac}\left(x-\frac{\sqrt{-ac}}{c}\right)f}{c}-\frac{2\left(af-cd\right)}{c}+2\sqrt{-\frac{af-cd}{c}}\,\sqrt{\left(x-\frac{\sqrt{-ac}}{c}\right)^{2}f+\frac{2\sqrt{-ac}\left(x-\frac{\sqrt{-ac}}{c}\right)f}{c}-\frac{af-cd}{c}}}{x-\frac{\sqrt{-ac}}{c}}\right)}{2\sqrt{-ac}\,\sqrt{-\frac{af-cd}{c}}}+\frac{A\ln\left(\frac{-\frac{2\sqrt{-ac}\left(x+\frac{\sqrt{-ac}}{c}\right)f}{c}-\frac{2\left(af-cd\right)}{c}+2\sqrt{-\frac{af-cd}{c}}\,\sqrt{\left(x+\frac{\sqrt{-ac}}{c}\right)^{2}f-\frac{2\sqrt{-ac}\left(x+\frac{\sqrt{-ac}}{c}\right)f}{c}-\frac{af-cd}{c}}}{x+\frac{\sqrt{-ac}}{c}}\right)}{2\sqrt{-ac}\,\sqrt{-\frac{af-cd}{c}}}-\frac{B\ln\left(\frac{\frac{2\sqrt{-ac}\left(x-\frac{\sqrt{-ac}}{c}\right)f}{c}-\frac{2\left(af-cd\right)}{c}+2\sqrt{-\frac{af-cd}{c}}\,\sqrt{\left(x-\frac{\sqrt{-ac}}{c}\right)^{2}f+\frac{2\sqrt{-ac}\left(x-\frac{\sqrt{-ac}}{c}\right)f}{c}-\frac{af-cd}{c}}}{x-\frac{\sqrt{-ac}}{c}}\right)}{2\sqrt{-\frac{af-cd}{c}}\,c}-\frac{B\ln\left(\frac{-\frac{2\sqrt{-ac}\left(x+\frac{\sqrt{-ac}}{c}\right)f}{c}-\frac{2\left(af-cd\right)}{c}+2\sqrt{-\frac{af-cd}{c}}\,\sqrt{\left(x+\frac{\sqrt{-ac}}{c}\right)^{2}f-\frac{2\sqrt{-ac}\left(x+\frac{\sqrt{-ac}}{c}\right)f}{c}-\frac{af-cd}{c}}}{x+\frac{\sqrt{-ac}}{c}}\right)}{2\sqrt{-\frac{af-cd}{c}}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(c*x^2+a)/(f*x^2+d)^(1/2),x)
```

```
[Out] -1/2/(-a*c)^(1/2)/(-(a*f-c*d)/c)^(1/2)*ln((-2*(a*f-c*d)/c+2*f*(-a*c)^(1/2)/
c*(x-(-a*c)^(1/2)/c)+2*(-(a*f-c*d)/c)^(1/2)*((x-(-a*c)^(1/2)/c)^2*f+2*f*(-a
*c)^(1/2)/c*(x-(-a*c)^(1/2)/c)-(a*f-c*d)/c)^(1/2))/(x-(-a*c)^(1/2)/c))*A-1/
2/c/(-(a*f-c*d)/c)^(1/2)*ln((-2*(a*f-c*d)/c+2*f*(-a*c)^(1/2)/c*(x-(-a*c)^(1
/2)/c)+2*(-(a*f-c*d)/c)^(1/2)*((x-(-a*c)^(1/2)/c)^2*f+2*f*(-a*c)^(1/2)/c*(x
-(-a*c)^(1/2)/c)-(a*f-c*d)/c)^(1/2))/(x-(-a*c)^(1/2)/c))*B+1/2/(-a*c)^(1/2)
/(-(a*f-c*d)/c)^(1/2)*ln((-2*(a*f-c*d)/c-2*f*(-a*c)^(1/2)/c*(x+(-a*c)^(1/2)
```

```
/c)+2*(-(a*f-c*d)/c)^(1/2)*((x+(-a*c)^(1/2)/c)^2*f-2*f*(-a*c)^(1/2)/c*(x+(-
a*c)^(1/2)/c)-(a*f-c*d)/c)^(1/2))/(x+(-a*c)^(1/2)/c))*A-1/2/c/(-(a*f-c*d)/c
)^(1/2)*ln((-2*(a*f-c*d)/c-2*f*(-a*c)^(1/2)/c*(x+(-a*c)^(1/2)/c)+2*(-(a*f-c
*d)/c)^(1/2)*((x+(-a*c)^(1/2)/c)^2*f-2*f*(-a*c)^(1/2)/c*(x+(-a*c)^(1/2)/c)-
(a*f-c*d)/c)^(1/2))/(x+(-a*c)^(1/2)/c))*B
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx + A}{\left(cx^2 + a\right)\sqrt{fx^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+a)/(f*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x + A)/((c*x^2 + a)*sqrt(f*x^2 + d)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\begin{cases} \frac{B\,\mathrm{atan}\left(\frac{c\,\sqrt{f\,x^2+d}}{\sqrt{a\,c\,f-c^2\,d}}\right)}{\sqrt{a\,c\,f-c^2\,d}}+\frac{A\,\mathrm{atan}\left(\frac{x\,\sqrt{c\,d-a\,f}}{\sqrt{a}\,\sqrt{f\,x^2+d}}\right)}{\sqrt{-a\,\left(a\,f-c\,d\right)}} & \text{if } 0<c\,d-a\,f \\ \frac{A\,\ln\left(\frac{\sqrt{a\,\left(f\,x^2+d\right)}+x\,\sqrt{a\,f-c\,d}}{\sqrt{a\,\left(f\,x^2+d\right)}-x\,\sqrt{a\,f-c\,d}}\right)}{2\,\sqrt{a\,\left(a\,f-c\,d\right)}}+\frac{B\,\mathrm{atan}\left(\frac{c\,\sqrt{f\,x^2+d}}{\sqrt{a\,c\,f-c^2\,d}}\right)}{\sqrt{a\,c\,f-c^2\,d}} & \text{if } c\,d-a\,f<0 \\ \int \frac{A+B\,x}{\left(c\,x^2+a\right)\,\sqrt{f\,x^2+d}}\, dx & \text{if } c\,d-a\,f\notin \mathbb{R}\vee a\,f=c\,d \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/((a + c*x^2)*(d + f*x^2)^(1/2)),x)
```

```
[Out] piecewise(0 < - a*f + c*d, (B*atan((c*(d + f*x^2)^(1/2))/(- c^2*d + a*c*f)^
(1/2)))/(- c^2*d + a*c*f)^(1/2) + (A*atan((x*(- a*f + c*d)^(1/2))/(a^(1/2)*
(d + f*x^2)^(1/2))))/(-a*(a*f - c*d))^(1/2), - a*f + c*d < 0, (A*log(((a*(d
 + f*x^2))^(1/2) + x*(a*f - c*d)^(1/2))/((a*(d + f*x^2))^(1/2) - x*(a*f - c
*d)^(1/2))))/(2*(a*(a*f - c*d))^(1/2)) + (B*atan((c*(d + f*x^2)^(1/2))/(- c
^2*d + a*c*f)^(1/2)))/(- c^2*d + a*c*f)^(1/2), ~in(- a*f + c*d, 'real') | a
*f == c*d, int((A + B*x)/((a + c*x^2)*(d + f*x^2)^(1/2)), x))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx}{\left(a + cx^2\right)\sqrt{d + fx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+a)/(f*x**2+d)**(1/2),x)
```

```
[Out] Integral((A + B*x)/((a + c*x**2)*sqrt(d + f*x**2)), x)
```

## 3.25 $\int \frac{2+x}{\left(2+4x-3x^2\right)\sqrt{1+3x-2x^2}}\,dx$

**Optimal**. Leaf size=139

$$\frac{1}{2}\sqrt{\sqrt{10}-\frac{13}{5}}\tan^{-1}\left(\frac{\left(1+4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)+\frac{1}{2}\sqrt{\frac{13}{5}+\sqrt{10}}\tanh^{-1}\left(\frac{\left(1-4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{2\sqrt{\sqrt{10}-1}\sqrt{-2x^2+3x+1}}\right)$$

**Rubi** [A] time = 0.22, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1032, 724, 204, 206}

$$\frac{1}{2}\sqrt{\sqrt{10}-\frac{13}{5}}\tan^{-1}\left(\frac{\left(1+4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)+\frac{1}{2}\sqrt{\frac{13}{5}+\sqrt{10}}\tanh^{-1}\left(\frac{\left(1-4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{2\sqrt{\sqrt{10}-1}\sqrt{-2x^2+3x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/((2 + 4*x - 3*x^2)*Sqrt[1 + 3*x - 2*x^2]),x]
```

```
[Out] (Sqrt[-13/5 + Sqrt[10]]*ArcTan[(3*(4 - Sqrt[10]) + (1 + 4*Sqrt[10])*x)/(2*S
qrt[1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/2 + (Sqrt[13/5 + Sqrt[10]]*ArcTa
nh[(3*(4 + Sqrt[10]) + (1 - 4*Sqrt[10])*x)/(2*Sqrt[-1 + Sqrt[10]]*Sqrt[1 +
3*x - 2*x^2])])/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{2+x}{\left(2+4x-3x^2\right)\sqrt{1+3x-2x^2}}\,dx = \frac{1}{5}\left(5-4\sqrt{10}\right)\int \frac{1}{\left(4-2\sqrt{10}-6x\right)\sqrt{1+3x-2x^2}}\,dx + \frac{1}{5}\left(5+4\sqrt{10}\right)\int$$

$$= -\left(\frac{1}{5}\left(2\left(5-4\sqrt{10}\right)\right)\text{Subst}\left(\int \frac{1}{144+72\left(4-2\sqrt{10}\right)-8\left(4-2\sqrt{10}\right)^2-x\right.\right.$$

$$= \frac{1}{10}\sqrt{-65+25\sqrt{10}}\,\tan^{-1}\left(\frac{3\left(4-\sqrt{10}\right)+\left(1+4\sqrt{10}\right)x}{2\sqrt{1+\sqrt{10}}\,\sqrt{1+3x-2x^2}}\right)+\frac{1}{10}\sqrt{65+25\sqrt{\phantom{1}}}$$

**Mathematica [A]** time = 0.30, size = 140, normalized size = 1.01

$$\frac{\left(4\sqrt{10}-5\right)\tan^{-1}\left(\frac{4\sqrt{10}x+x-3\sqrt{10}+12}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)+3\sqrt{5\left(7+2\sqrt{10}\right)}\tanh^{-1}\left(\frac{-4\sqrt{10}x+x+3\left(4+\sqrt{10}\right)}{2\sqrt{\sqrt{10}-1}\sqrt{-2x^2+3x+1}}\right)}{10\sqrt{1+\sqrt{10}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + x)/((2 + 4*x - 3*x^2)*Sqrt[1 + 3*x - 2*x^2]),x]
```

```
[Out] ((-5 + 4*Sqrt[10])*ArcTan[(12 - 3*Sqrt[10] + x + 4*Sqrt[10]*x)/(2*Sqrt[1 +
Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])] + 3*Sqrt[5*(7 + 2*Sqrt[10])]*ArcTanh[(3*(
4 + Sqrt[10]) + x - 4*Sqrt[10]*x)/(2*Sqrt[-1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x
^2])])/(10*Sqrt[1 + Sqrt[10]])
```

**IntegrateAlgebraic [C]** time = 0.34, size = 149, normalized size = 1.07

$$-\frac{1}{2}\text{RootSum}\left[2\#1^4-8\#1^3+8\#1^2+20\#1+5\&,\frac{2\#1^2\log\left(\#1(-x)+\sqrt{-2x^2+3x+1}-1\right)-2\#1^2\log(x)-2\#1\log\left(\#1(-x)+\sqrt{-2x^2+3x+1}-1\right)+7\log\left(\#1(-x)+\sqrt{-2x^2+3x+1}-1\right)+2\#1\log(x)-7\log(x)}{2\#1^3-6\#1^2+4\#1+5}\&\right]$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + x)/((2 + 4*x - 3*x^2)*Sqrt[1 + 3*x - 2*x^2]),x]
```

```
[Out] -1/2*RootSum[5 + 20*#1 + 8*#1^2 - 8*#1^3 + 2*#1^4 & , (-7*Log[x] + 7*Log[-1
 + Sqrt[1 + 3*x - 2*x^2] - x*#1] + 2*Log[x]*#1 - 2*Log[-1 + Sqrt[1 + 3*x -
2*x^2] - x*#1]*#1 - 2*Log[x]*#1^2 + 2*Log[-1 + Sqrt[1 + 3*x - 2*x^2] - x*#1
]*#1^2)/(5 + 4*#1 - 6*#1^2 + 2*#1^3) & ]
```

**fricas [B]** time = 0.44, size = 322, normalized size = 2.32

$$\frac{2}{5}\sqrt{5}\sqrt{5\sqrt{5}\sqrt{2}-13}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{5}x-\sqrt{2}x\right)\sqrt{5\sqrt{5}\sqrt{2}-13}\sqrt{\frac{\sqrt{5}\sqrt{2}\left(3x^2+2x\right)+6x^2-2\left(\sqrt{5}\sqrt{2}x+2x+2\right)\sqrt{-2x^2+3x+1}+10x+4}{x^2}}+2\left(\sqrt{2}(4x-1)+\sqrt{5}(x+2)-\sqrt{-2x^2+3x+1}\left(2\sqrt{5}-\sqrt{2}\right)\right)\sqrt{5\sqrt{5}\sqrt{2}-13}}{18x}\right)-\frac{1}{10}\sqrt{5}\sqrt{5\sqrt{5}\sqrt{2}+13}\log\left(\frac{9\sqrt{5}\sqrt{2}x+\left(4\sqrt{5}x-7\sqrt{2}x\right)\sqrt{5\sqrt{5}\sqrt{2}+13}-18x+18\sqrt{-2x^2+3x+1}-18}{x}\right)+\frac{1}{10}\sqrt{5}\sqrt{5\sqrt{5}\sqrt{2}+13}\log\left(\frac{9\sqrt{5}\sqrt{2}x-\left(4\sqrt{5}x-7\sqrt{2}x\right)\sqrt{5\sqrt{5}\sqrt{2}+13}-18x+18\sqrt{-2x^2+3x+1}-18}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(5)*sqrt(5*sqrt(5)*sqrt(2) - 13)*arctan(1/18*(sqrt(2)*(2*sqrt(5)*x
- sqrt(2)*x)*sqrt(5*sqrt(5)*sqrt(2) - 13)*sqrt((sqrt(5)*sqrt(2)*(3*x^2 + 2*
x) + 6*x^2 - 2*(sqrt(5)*sqrt(2)*x + 2*x + 2)*sqrt(-2*x^2 + 3*x + 1) + 10*x
+ 4)/x^2) + 2*(sqrt(2)*(4*x - 1) + sqrt(5)*(x + 2) - sqrt(-2*x^2 + 3*x + 1)
*(2*sqrt(5) - sqrt(2)))*sqrt(5*sqrt(5)*sqrt(2) - 13))/x) - 1/10*sqrt(5)*sqr
t(5*sqrt(5)*sqrt(2) + 13)*log((9*sqrt(5)*sqrt(2)*x + (4*sqrt(5)*x - 7*sqrt(
2)*x)*sqrt(5*sqrt(5)*sqrt(2) + 13) - 18*x + 18*sqrt(-2*x^2 + 3*x + 1) - 18)
/x) + 1/10*sqrt(5)*sqrt(5*sqrt(5)*sqrt(2) + 13)*log((9*sqrt(5)*sqrt(2)*x -
(4*sqrt(5)*x - 7*sqrt(2)*x)*sqrt(5*sqrt(5)*sqrt(2) + 13) - 18*x + 18*sqrt(-
2*x^2 + 3*x + 1) - 18)/x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.07, size = 324, normalized size = 2.33

$$\frac{2\sqrt{10}\,\operatorname{arctanh}\left(\frac{-1+\sqrt{10}+\frac{9\left(\frac{1}{3}-\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)}{2}}{\sqrt{-1+\sqrt{10}}\,\sqrt{-18\left(x-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)^2+9\left(\frac{1}{3}-\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)-1+\sqrt{10}}}\right)}{5\sqrt{-1+\sqrt{10}}}+\frac{\operatorname{arctanh}\left(\frac{-1+\sqrt{10}+\frac{9\left(\frac{1}{3}-\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)}{2}}{\sqrt{-1+\sqrt{10}}\,\sqrt{-18\left(x-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)^2+9\left(\frac{1}{3}-\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)-1+\sqrt{10}}}\right)}{2\sqrt{-1+\sqrt{10}}}+\frac{2\sqrt{10}\,\arctan\left(\frac{-1-\sqrt{10}+\frac{9\left(\frac{1}{3}+\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)}{2}}{\sqrt{1+\sqrt{10}}\,\sqrt{-18\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)^2+9\left(\frac{1}{3}+\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)-1-\sqrt{10}}}\right)}{5\sqrt{1+\sqrt{10}}}-\frac{\arctan\left(\frac{-1-\sqrt{10}+\frac{9\left(\frac{1}{3}+\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)}{2}}{\sqrt{1+\sqrt{10}}\,\sqrt{-18\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)^2+9\left(\frac{1}{3}+\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)-1-\sqrt{10}}}\right)}{2\sqrt{1+\sqrt{10}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(1/2),x)
```

```
[Out] 2/5*10^(1/2)/(1+10^(1/2))^(1/2)*arctan(9/2*(-2/9-2/9*10^(1/2)+(1/3+4/3*10^(
1/2))*(x-2/3+1/3*10^(1/2)))/(1+10^(1/2))^(1/2)/(-18*(x-2/3+1/3*10^(1/2))^2+
```

```
9*(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1-10^(1/2))^(1/2))-1/2/(1+10^(1/2
))^(1/2)*arctan(9/2*(-2/9-2/9*10^(1/2)+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/
2)))/(1+10^(1/2))^(1/2)/(-18*(x-2/3+1/3*10^(1/2))^2+9*(1/3+4/3*10^(1/2))*(x
-2/3+1/3*10^(1/2))-1-10^(1/2))^(1/2))+2/5*10^(1/2)/(-1+10^(1/2))^(1/2)*arct
anh(9/2*(-2/9+2/9*10^(1/2)+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2)))/(-1+10^
(1/2))^(1/2)/(-18*(x-2/3-1/3*10^(1/2))^2+9*(1/3-4/3*10^(1/2))*(x-2/3-1/3*10
^(1/2))-1+10^(1/2))^(1/2))+1/2/(-1+10^(1/2))^(1/2)*arctanh(9/2*(-2/9+2/9*10
^(1/2)+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2)))/(-1+10^(1/2))^(1/2)/(-18*(x
-2/3-1/3*10^(1/2))^2+9*(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1+10^(1/2))^
(1/2))
```

**maxima [B]** time = 1.07, size = 361, normalized size = 2.60

$$-\frac{1}{20}\sqrt{10}\left(\frac{\sqrt{10}\arcsin\left(\frac{8\sqrt{17}\sqrt{10}x}{17\left|6x+2\sqrt{10}-4\right|}+\frac{2\sqrt{17}x}{17\left|6x+2\sqrt{10}-4\right|}-\frac{6\sqrt{17}\sqrt{10}}{17\left|6x+2\sqrt{10}-4\right|}+\frac{24\sqrt{17}}{17\left|6x+2\sqrt{10}-4\right|}\right)}{\sqrt{\sqrt{10}+1}}-\frac{\sqrt{10}\log\left(-\frac{2}{9}\sqrt{10}+\frac{2\sqrt{-2x^2+3x+1}\sqrt{\sqrt{10}-1}}{3\left|6x-2\sqrt{10}-4\right|}+\frac{2\sqrt{10}}{9\left|6x-2\sqrt{10}-4\right|}-\frac{2}{9\left|6x-2\sqrt{10}-4\right|}+\frac{1}{18}\right)}{\sqrt{\sqrt{10}-1}}-\frac{8\arcsin\left(\frac{8\sqrt{17}\sqrt{10}x}{17\left|6x+2\sqrt{10}-4\right|}+\frac{2\sqrt{17}x}{17\left|6x+2\sqrt{10}-4\right|}-\frac{6\sqrt{17}\sqrt{10}}{17\left|6x+2\sqrt{10}-4\right|}+\frac{24\sqrt{17}}{17\left|6x+2\sqrt{10}-4\right|}\right)}{\sqrt{\sqrt{10}+1}}-\frac{8\log\left(-\frac{2}{9}\sqrt{10}+\frac{2\sqrt{-2x^2+3x+1}\sqrt{\sqrt{10}-1}}{3\left|6x-2\sqrt{10}-4\right|}+\frac{2\sqrt{10}}{9\left|6x-2\sqrt{10}-4\right|}-\frac{2}{9\left|6x-2\sqrt{10}-4\right|}+\frac{1}{18}\right)}{\sqrt{\sqrt{10}-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/20*sqrt(10)*(sqrt(10)*arcsin(8/17*sqrt(17)*sqrt(10)*x/abs(6*x + 2*sqrt(1
0) - 4) + 2/17*sqrt(17)*x/abs(6*x + 2*sqrt(10) - 4) - 6/17*sqrt(17)*sqrt(10
)/abs(6*x + 2*sqrt(10) - 4) + 24/17*sqrt(17)/abs(6*x + 2*sqrt(10) - 4))/sqr
t(sqrt(10) + 1) - sqrt(10)*log(-2/9*sqrt(10) + 2/3*sqrt(-2*x^2 + 3*x + 1)*s
qrt(sqrt(10) - 1)/abs(6*x - 2*sqrt(10) - 4) + 2/9*sqrt(10)/abs(6*x - 2*sqrt
(10) - 4) - 2/9/abs(6*x - 2*sqrt(10) - 4) + 1/18)/sqrt(sqrt(10) - 1) - 8*ar
csin(8/17*sqrt(17)*sqrt(10)*x/abs(6*x + 2*sqrt(10) - 4) + 2/17*sqrt(17)*x/a
bs(6*x + 2*sqrt(10) - 4) - 6/17*sqrt(17)*sqrt(10)/abs(6*x + 2*sqrt(10) - 4)
 + 24/17*sqrt(17)/abs(6*x + 2*sqrt(10) - 4))/sqrt(sqrt(10) + 1) - 8*log(-2/
9*sqrt(10) + 2/3*sqrt(-2*x^2 + 3*x + 1)*sqrt(sqrt(10) - 1)/abs(6*x - 2*sqrt
(10) - 4) + 2/9*sqrt(10)/abs(6*x - 2*sqrt(10) - 4) - 2/9/abs(6*x - 2*sqrt(1
0) - 4) + 1/18)/sqrt(sqrt(10) - 1))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+2}{\sqrt{-2\,x^2+3\,x+1}\,\left(-3\,x^2+4\,x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/((3*x - 2*x^2 + 1)^(1/2)*(4*x - 3*x^2 + 2)),x)
```

```
[Out] int((x + 2)/((3*x - 2*x^2 + 1)^(1/2)*(4*x - 3*x^2 + 2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{3x^2\sqrt{-2x^2+3x+1}-4x\sqrt{-2x^2+3x+1}-2\sqrt{-2x^2+3x+1}}\,dx-\int \frac{2}{3x^2\sqrt{-2x^2+3x+1}-4x\sqrt{-2x^2+3x+1}-2\sqrt{-2x^2+3x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x**2+4*x+2)/(-2*x**2+3*x+1)**(1/2),x)
```

```
[Out] -Integral(x/(3*x**2*sqrt(-2*x**2 + 3*x + 1) - 4*x*sqrt(-2*x**2 + 3*x + 1) -
 2*sqrt(-2*x**2 + 3*x + 1)), x) - Integral(2/(3*x**2*sqrt(-2*x**2 + 3*x + 1
) - 4*x*sqrt(-2*x**2 + 3*x + 1) - 2*sqrt(-2*x**2 + 3*x + 1)), x)
```

## 3.26 $\int \frac{2+x}{\left(2+4x-3x^2\right)\left(1+3x-2x^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=166

$$-\frac{2(14x+15)}{17\sqrt{-2x^2+3x+1}}-\frac{9}{2}\sqrt{\frac{1}{5}\left(\sqrt{10}-3\right)}\tan^{-1}\left(\frac{\left(1+4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)+\frac{9}{2}\sqrt{\frac{1}{5}\left(3+\sqrt{10}\right)}\tanh^{-1}\left(\frac{(1-4\cdots}{2\sqrt{\sqrt{10}\cdots}}\right.$$

**Rubi [A]** time = 0.22, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1016, 12, 1032, 724, 204, 206}

$$-\frac{2(14x+15)}{17\sqrt{-2x^2+3x+1}}-\frac{9}{2}\sqrt{\frac{1}{5}\left(\sqrt{10}-3\right)}\tan^{-1}\left(\frac{\left(1+4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)+\frac{9}{2}\sqrt{\frac{1}{5}\left(3+\sqrt{10}\right)}\tanh^{-1}\left(\frac{\left(1-4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{2\sqrt{\sqrt{10}-1}\sqrt{-2x^2+3x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x - 2*x^2)^(3/2)),x]
```

```
[Out] (-2*(15 + 14*x))/(17*Sqrt[1 + 3*x - 2*x^2]) - (9*Sqrt[(-3 + Sqrt[10])/5]*Ar
cTan[(3*(4 - Sqrt[10]) + (1 + 4*Sqrt[10])*x)/(2*Sqrt[1 + Sqrt[10]]*Sqrt[1 +
 3*x - 2*x^2])])/2 + (9*Sqrt[(3 + Sqrt[10])/5]*ArcTanh[(3*(4 + Sqrt[10]) +
(1 - 4*Sqrt[10])*x)/(2*Sqrt[-1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1016

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)
*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*
c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))
 - h*(b*c*d - 2*a*c*e + a*b*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d -
a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e
))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{2+x}{\left(2+4x-3x^2\right)\left(1+3x-2x^2\right)^{3/2}}\,dx = -\frac{2(15+14x)}{17\sqrt{1+3x-2x^2}} + \frac{2}{17}\int \frac{153x}{2\left(2+4x-3x^2\right)\sqrt{1+3x-2x^2}}\,dx$$

$$= -\frac{2(15+14x)}{17\sqrt{1+3x-2x^2}} + 9\int \frac{x}{\left(2+4x-3x^2\right)\sqrt{1+3x-2x^2}}\,dx$$

$$= -\frac{2(15+14x)}{17\sqrt{1+3x-2x^2}} + \frac{1}{5}\left(9\left(5-\sqrt{10}\right)\right)\int \frac{1}{\left(4-2\sqrt{10}-6x\right)\sqrt{1+3x-2\ldots}}$$

$$= -\frac{2(15+14x)}{17\sqrt{1+3x-2x^2}} - \frac{1}{5}\left(18\left(5-\sqrt{10}\right)\right)\mathrm{Subst}\left(\int \frac{\ldots}{144+72\left(4-2\sqrt{10}\right)\ldots}\right.$$

$$= -\frac{2(15+14x)}{17\sqrt{1+3x-2x^2}} - \frac{9}{2}\sqrt{\frac{1}{5}\left(-3+\sqrt{10}\right)}\tan^{-1}\left(\frac{3\left(4-\sqrt{10}\right)+\left(1+4\sqrt{\ldots}\right.}{2\sqrt{1+\sqrt{10}}\sqrt{1+3x-\ldots}}\right.$$

**Mathematica [A]** time = 0.38, size = 167, normalized size = 1.01

$$\frac{1}{170}\left(153\sqrt{5\left(3+\sqrt{10}\right)}\tanh^{-1}\left(\frac{-4\sqrt{10}x+x+3\left(4+\sqrt{10}\right)}{2\sqrt{\sqrt{10}-1}\sqrt{-2x^2+3x+1}}\right) - \frac{153\sqrt{5\left(\sqrt{10}-3\right)}\sqrt{-2x^2+3x+1}\tan^{-1}\left(\frac{4\sqrt{10}x+x-3\sqrt{10}+12}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)+280x+300}{\sqrt{-2x^2+3x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x - 2*x^2)^(3/2)),x]
```

```
[Out] (-((300 + 280*x + 153*Sqrt[5*(-3 + Sqrt[10])]*Sqrt[1 + 3*x - 2*x^2]*ArcTan[
(12 - 3*Sqrt[10] + x + 4*Sqrt[10]*x)/(2*Sqrt[1 + Sqrt[10]]*Sqrt[1 + 3*x - 2
*x^2])])/Sqrt[1 + 3*x - 2*x^2]) + 153*Sqrt[5*(3 + Sqrt[10])]*ArcTanh[(3*(4
+ Sqrt[10]) + x - 4*Sqrt[10]*x)/(2*Sqrt[-1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2
])])/170
```

**IntegrateAlgebraic [C]** time = 0.39, size = 149, normalized size = 0.90

$$\frac{9}{2}\mathrm{RootSum}\left[2\#1^4-8\#1^3+8\#1^2+20\#1+5\&,\frac{-3\log\left(\#1(-x)+\sqrt{-2x^2+3x+1}-1\right)+2\#1\log\left(\#1(-x)+\sqrt{-2x^2+3x+1}-1\right)-2\#1\log(x)+3\log(x)}{2\#1^3-6\#1^2+4\#1+5}\&\right]+\frac{2\sqrt{-2x^2+3x+1}\,(14x+15)}{17\left(2x^2-3x-1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x - 2*x^2)^(3/2)),x]
```

```
[Out] (2*(15 + 14*x)*Sqrt[1 + 3*x - 2*x^2])/(17*(-1 - 3*x + 2*x^2)) + (9*RootSum[
5 + 20*#1 + 8*#1^2 - 8*#1^3 + 2*#1^4 & , (3*Log[x] - 3*Log[-1 + Sqrt[1 + 3*
```

```
x - 2*x^2] - x*#1] - 2*Log[x]*#1 + 2*Log[-1 + Sqrt[1 + 3*x - 2*x^2] - x*#1]
*#1)/(5 + 4*#1 - 6*#1^2 + 2*#1^3) & ])/2
```

**fricas [B]** time = 0.44, size = 344, normalized size = 2.07

$$-\frac{612 \sqrt{5} \left(2 x^{2} - 3 x - 1\right) \sqrt{\sqrt{10} - 3} \arctan\left(\frac{\sqrt{10} \sqrt{5} \sqrt{2} x \sqrt{\sqrt{10} - 3} \sqrt{\frac{6 x^{2} + \sqrt{10} \left(3 x^{2} + 2 x\right) - 2 \sqrt{-2 x^{2} + 3 x + 1} \left(\sqrt{10} x + 2 x + 2\right) + 10 x + 4}{x^{2}}} + 2 \left(\sqrt{10} \sqrt{5} (x+1) - \sqrt{10} \sqrt{5} \sqrt{-2 x^{2} + 3 x + 1} + 5 \sqrt{5} x\right) \sqrt{\sqrt{10} - 3}}{10 x}\right) + 153 \sqrt{5} \left(2 x^{2} - 3 x - 1\right) \sqrt{\sqrt{10} + 3} \log\left(\frac{9 \left(5 \sqrt{10} x + \left(3 \sqrt{10} \sqrt{5} x - 10 \sqrt{5} x\right) \sqrt{\sqrt{10} + 3} - 10 x + 10 \sqrt{-2 x^{2} + 3 x + 1} - 10\right)}{x}\right) - 153 \sqrt{5} \left(2 x^{2} - 3 x - 1\right) \sqrt{\sqrt{10} + 3} \log\left(\frac{9 \left(5 \sqrt{10} x - \left(3 \sqrt{10} \sqrt{5} x - 10 \sqrt{5} x\right) \sqrt{\sqrt{10} + 3} - 10 x + 10 \sqrt{-2 x^{2} + 3 x + 1} - 10\right)}{x}\right) + 600 x^{2} - 20 \sqrt{-2 x^{2} + 3 x + 1} (14 x + 15) - 900 x - 300}{170 \left(2 x^{2} - 3 x - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/170*(612*sqrt(5)*(2*x^2 - 3*x - 1)*sqrt(sqrt(10) - 3)*arctan(1/10*(sqrt(
10)*sqrt(5)*sqrt(2)*x*sqrt(sqrt(10) - 3)*sqrt((6*x^2 + sqrt(10)*(3*x^2 + 2*
x) - 2*sqrt(-2*x^2 + 3*x + 1)*(sqrt(10)*x + 2*x + 2) + 10*x + 4)/x^2) + 2*(
sqrt(10)*sqrt(5)*(x + 1) - sqrt(10)*sqrt(5)*sqrt(-2*x^2 + 3*x + 1) + 5*sqrt
(5)*x)*sqrt(sqrt(10) - 3))/x) + 153*sqrt(5)*(2*x^2 - 3*x - 1)*sqrt(sqrt(10)
 + 3)*log(9*(5*sqrt(10)*x + (3*sqrt(10)*sqrt(5)*x - 10*sqrt(5)*x)*sqrt(sqrt
(10) + 3) - 10*x + 10*sqrt(-2*x^2 + 3*x + 1) - 10)/x) - 153*sqrt(5)*(2*x^2
- 3*x - 1)*sqrt(sqrt(10) + 3)*log(9*(5*sqrt(10)*x - (3*sqrt(10)*sqrt(5)*x -
 10*sqrt(5)*x)*sqrt(sqrt(10) + 3) - 10*x + 10*sqrt(-2*x^2 + 3*x + 1) - 10)/
x) + 600*x^2 - 20*sqrt(-2*x^2 + 3*x + 1)*(14*x + 15) - 900*x - 300)/(2*x^2
- 3*x - 1)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.03, size = 760, normalized size = 4.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(3/2),x)
```

```
[Out] 26/255*10^(1/2)/(-1/9-1/9*10^(1/2))/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^
(1/2))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^(1/2)+32/765/(-1/9-1/9*10^(1/
2))/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1/9-
1/9*10^(1/2))^(1/2)*10^(1/2)*x-62/153/(-1/9-1/9*10^(1/2))/(-2*(x-2/3+1/3*10
^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^(1/2)*x
+7/51/(-1/9-1/9*10^(1/2))/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-
2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^(1/2)+2/5*10^(1/2)/(-1/9-1/9*10^(1/2))/
```

```
(1+10^(1/2))^(1/2)*arctan(9/2*(-2/9-2/9*10^(1/2)+(1/3+4/3*10^(1/2))*(x-2/3+
1/3*10^(1/2)))/(1+10^(1/2))^(1/2)/(-18*(x-2/3+1/3*10^(1/2))^2+9*(1/3+4/3*10
^(1/2))*(x-2/3+1/3*10^(1/2))-1-10^(1/2))^(1/2))-1/2/(-1/9-1/9*10^(1/2))/(1+
10^(1/2))^(1/2)*arctan(9/2*(-2/9-2/9*10^(1/2)+(1/3+4/3*10^(1/2))*(x-2/3+1/3
*10^(1/2)))/(1+10^(1/2))^(1/2)/(-18*(x-2/3+1/3*10^(1/2))^2+9*(1/3+4/3*10^(1
/2))*(x-2/3+1/3*10^(1/2))-1-10^(1/2))^(1/2))-26/255*10^(1/2)/(-1/9+1/9*10^(
1/2))/(-2*(x-2/3-1/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/
9+1/9*10^(1/2))^(1/2)-32/765/(-1/9+1/9*10^(1/2))/(-2*(x-2/3-1/3*10^(1/2))^2
+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*10^(1/2))^(1/2)*10^(1/2)*x
-62/153/(-1/9+1/9*10^(1/2))/(-2*(x-2/3-1/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(
x-2/3-1/3*10^(1/2))-1/9+1/9*10^(1/2))^(1/2)*x+7/51/(-1/9+1/9*10^(1/2))/(-2*
(x-2/3-1/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*10^(
1/2))^(1/2)+2/5*10^(1/2)/(-1/9+1/9*10^(1/2))/(-1+10^(1/2))^(1/2)*arctanh(9/
2*(-2/9+2/9*10^(1/2)+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2)))/(-1+10^(1/2))
^(1/2)/(-18*(x-2/3-1/3*10^(1/2))^2+9*(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2)
)-1+10^(1/2))^(1/2))+1/2/(-1/9+1/9*10^(1/2))/(-1+10^(1/2))^(1/2)*arctanh(9/
2*(-2/9+2/9*10^(1/2)+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2)))/(-1+10^(1/2))
^(1/2)/(-18*(x-2/3-1/3*10^(1/2))^2+9*(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2)
)-1+10^(1/2))^(1/2))
```

**maxima [B]** time = 1.11, size = 678, normalized size = 4.08

$$\frac{1}{340}\sqrt{10}\left(\frac{124\sqrt{10}x}{\sqrt{10}\sqrt{-2x^2+3x+1}+\sqrt{-2x^2+3x+1}}-\frac{124\sqrt{10}x}{\sqrt{10}\sqrt{-2x^2+3x+1}-\sqrt{-2x^2+3x+1}}+\frac{153\sqrt{10}\arcsin\left(\frac{8\sqrt{17}\sqrt{10}x}{17|6x+2\sqrt{10}-4|}+\frac{2\sqrt{17}x}{17|6x+2\sqrt{10}-4|}-\frac{6\sqrt{17}\sqrt{10}}{17|6x+2\sqrt{10}-4|}+\frac{24\sqrt{17}}{17|6x+2\sqrt{10}-4|}\right)}{\sqrt{10}\sqrt{\sqrt{10}+1}+\sqrt{\sqrt{10}+1}}-\frac{128x}{\sqrt{10}\sqrt{-2x^2+3x+1}+\sqrt{-2x^2+3x+1}}-\frac{128x}{\sqrt{10}\sqrt{-2x^2+3x+1}-\sqrt{-2x^2+3x+1}}-\frac{1224\arcsin\left(\frac{8\sqrt{17}\sqrt{10}x}{17|6x+2\sqrt{10}-4|}+\frac{2\sqrt{17}x}{17|6x+2\sqrt{10}-4|}-\frac{6\sqrt{17}\sqrt{10}}{17|6x+2\sqrt{10}-4|}+\frac{24\sqrt{17}}{17|6x+2\sqrt{10}-4|}\right)}{\sqrt{10}\sqrt{\sqrt{10}+1}+\sqrt{\sqrt{10}+1}}+\frac{153\sqrt{10}\log\left(-\frac{2}{9}\sqrt{10}+\frac{2\sqrt{-2x^2+3x+1}\sqrt{\sqrt{10}-1}}{3|6x-2\sqrt{10}-4|}+\frac{2\sqrt{10}}{9|6x-2\sqrt{10}-4|}-\frac{2}{9|6x-2\sqrt{10}-4|}+\frac{1}{18}\right)}{\left(\sqrt{10}-1\right)^{\frac{3}{2}}}-\frac{42\sqrt{10}}{\sqrt{10}\sqrt{-2x^2+3x+1}+\sqrt{-2x^2+3x+1}}+\frac{42\sqrt{10}}{\sqrt{10}\sqrt{-2x^2+3x+1}-\sqrt{-2x^2+3x+1}}+\frac{1224\log\left(-\frac{2}{9}\sqrt{10}+\frac{2\sqrt{-2x^2+3x+1}\sqrt{\sqrt{10}-1}}{3|6x-2\sqrt{10}-4|}+\frac{2\sqrt{10}}{9|6x-2\sqrt{10}-4|}-\frac{2}{9|6x-2\sqrt{10}-4|}+\frac{1}{18}\right)}{\left(\sqrt{10}-1\right)^{\frac{3}{2}}}-\frac{312}{\sqrt{10}\sqrt{-2x^2+3x+1}+\sqrt{-2x^2+3x+1}}-\frac{312}{\sqrt{10}\sqrt{-2x^2+3x+1}-\sqrt{-2x^2+3x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/340*sqrt(10)*(124*sqrt(10)*x/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + sqrt(-2*x
^2 + 3*x + 1)) - 124*sqrt(10)*x/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - sqrt(-2*
x^2 + 3*x + 1)) + 153*sqrt(10)*arcsin(8/17*sqrt(17)*sqrt(10)*x/abs(6*x + 2*
sqrt(10) - 4) + 2/17*sqrt(17)*x/abs(6*x + 2*sqrt(10) - 4) - 6/17*sqrt(17)*s
qrt(10)/abs(6*x + 2*sqrt(10) - 4) + 24/17*sqrt(17)/abs(6*x + 2*sqrt(10) - 4
))/(sqrt(10)*sqrt(sqrt(10) + 1) + sqrt(sqrt(10) + 1)) - 128*x/(sqrt(10)*sqr
t(-2*x^2 + 3*x + 1) + sqrt(-2*x^2 + 3*x + 1)) - 128*x/(sqrt(10)*sqrt(-2*x^2
 + 3*x + 1) - sqrt(-2*x^2 + 3*x + 1)) - 1224*arcsin(8/17*sqrt(17)*sqrt(10)*
x/abs(6*x + 2*sqrt(10) - 4) + 2/17*sqrt(17)*x/abs(6*x + 2*sqrt(10) - 4) - 6
/17*sqrt(17)*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 24/17*sqrt(17)/abs(6*x +
2*sqrt(10) - 4))/(sqrt(10)*sqrt(sqrt(10) + 1) + sqrt(sqrt(10) + 1)) + 153*s
qrt(10)*log(-2/9*sqrt(10) + 2/3*sqrt(-2*x^2 + 3*x + 1)*sqrt(sqrt(10) - 1)/a
bs(6*x - 2*sqrt(10) - 4) + 2/9*sqrt(10)/abs(6*x - 2*sqrt(10) - 4) - 2/9/abs
(6*x - 2*sqrt(10) - 4) + 1/18)/(sqrt(10) - 1)^(3/2) - 42*sqrt(10)/(sqrt(10)
*sqrt(-2*x^2 + 3*x + 1) + sqrt(-2*x^2 + 3*x + 1)) + 42*sqrt(10)/(sqrt(10)*s
qrt(-2*x^2 + 3*x + 1) - sqrt(-2*x^2 + 3*x + 1)) + 1224*log(-2/9*sqrt(10) +
2/3*sqrt(-2*x^2 + 3*x + 1)*sqrt(sqrt(10) - 1)/abs(6*x - 2*sqrt(10) - 4) + 2
/9*sqrt(10)/abs(6*x - 2*sqrt(10) - 4) - 2/9/abs(6*x - 2*sqrt(10) - 4) + 1/1
```

```
8)/(sqrt(10) - 1)^(3/2) - 312/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + sqrt(-2*x^
2 + 3*x + 1)) - 312/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - sqrt(-2*x^2 + 3*x +
1)))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+2}{\left(-2\,x^2+3\,x+1\right)^{3/2}\,\left(-3\,x^2+4\,x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/((3*x - 2*x^2 + 1)^(3/2)*(4*x - 3*x^2 + 2)),x)
```

```
[Out] int((x + 2)/((3*x - 2*x^2 + 1)^(3/2)*(4*x - 3*x^2 + 2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-6x^4\sqrt{-2x^2+3x+1}+17x^3\sqrt{-2x^2+3x+1}-5x^2\sqrt{-2x^2+3x+1}-10x\sqrt{-2x^2+3x+1}-2\sqrt{-2x^2+3x+1}}\,dx-\int \frac{2}{-6x^4\sqrt{-2x^2+3x+1}+17x^3\sqrt{-2x^2+3x+1}-5x^2\sqrt{-2x^2+3x+1}-10x\sqrt{-2x^2+3x+1}-2\sqrt{-2x^2+3x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x**2+4*x+2)/(-2*x**2+3*x+1)**(3/2),x)
```

```
[Out] -Integral(x/(-6*x**4*sqrt(-2*x**2 + 3*x + 1) + 17*x**3*sqrt(-2*x**2 + 3*x +
 1) - 5*x**2*sqrt(-2*x**2 + 3*x + 1) - 10*x*sqrt(-2*x**2 + 3*x + 1) - 2*sqr
t(-2*x**2 + 3*x + 1)), x) - Integral(2/(-6*x**4*sqrt(-2*x**2 + 3*x + 1) + 1
7*x**3*sqrt(-2*x**2 + 3*x + 1) - 5*x**2*sqrt(-2*x**2 + 3*x + 1) - 10*x*sqrt
(-2*x**2 + 3*x + 1) - 2*sqrt(-2*x**2 + 3*x + 1)), x)
```

## 3.27 $\int \frac{2+x}{\left(2+4x-3x^2\right)\left(1+3x-2x^2\right)^{5/2}} \, dx$

**Optimal**. Leaf size=193

$$-\frac{2(14x+15)}{51\left(-2x^2+3x+1\right)^{3/2}}-\frac{2(4814x+291)}{867\sqrt{-2x^2+3x+1}}+\frac{9}{2}\sqrt{\frac{1}{5}\left(17\sqrt{10}-53\right)}\tan^{-1}\left(\frac{\left(1+4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)+\frac{9}{2}\sqrt{\frac{1}{5}}$$

**Rubi [A]** time = 0.27, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1016, 1060, 1032, 724, 204, 206}

$$-\frac{2(14x+15)}{51\left(-2x^2+3x+1\right)^{3/2}}-\frac{2(4814x+291)}{867\sqrt{-2x^2+3x+1}}+\frac{9}{2}\sqrt{\frac{1}{5}\left(17\sqrt{10}-53\right)}\tan^{-1}\left(\frac{\left(1+4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)+\frac{9}{2}\sqrt{\frac{1}{5}\left(53+17\sqrt{10}\right)}\tanh^{-1}\left(\frac{\left(1-4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{2\sqrt{\sqrt{10}-1}\sqrt{-2x^2+3x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x - 2*x^2)^(5/2)),x]
```

```
[Out] (-2*(15 + 14*x))/(51*(1 + 3*x - 2*x^2)^(3/2)) - (2*(291 + 4814*x))/(867*Sqr
t[1 + 3*x - 2*x^2]) + (9*Sqrt[(-53 + 17*Sqrt[10])/5]*ArcTan[(3*(4 - Sqrt[10
]) + (1 + 4*Sqrt[10])*x)/(2*Sqrt[1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/2 +
 (9*Sqrt[(53 + 17*Sqrt[10])/5]*ArcTanh[(3*(4 + Sqrt[10]) + (1 - 4*Sqrt[10])
*x)/(2*Sqrt[-1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1016

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)
*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*
c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))
 - h*(b*c*d - 2*a*c*e + a*b*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d -
a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e
))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
```

```
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{2+x}{\left(2+4x-3x^2\right)\left(1+3x-2x^2\right)^{5/2}}\,dx = -\frac{2(15+14x)}{51\left(1+3x-2x^2\right)^{3/2}} + \frac{2}{51}\int \frac{-56+\frac{235x}{2}+84x^2}{\left(2+4x-3x^2\right)\left(1+3x-2x^2\right)^{3/2}}\,dx$$

$$= -\frac{2(15+14x)}{51\left(1+3x-2x^2\right)^{3/2}} - \frac{2(291+4814x)}{867\sqrt{1+3x-2x^2}} + \frac{4}{867}\int \frac{\frac{7803}{2}+23}{\left(2+4x-3x^2\right)\sqrt{1}}$$

$$= -\frac{2(15+14x)}{51\left(1+3x-2x^2\right)^{3/2}} - \frac{2(291+4814x)}{867\sqrt{1+3x-2x^2}} + \frac{1}{5}\left(27\left(5-2\sqrt{10}\right)\right)\int \frac{}{(4}$$

$$= -\frac{2(15+14x)}{51\left(1+3x-2x^2\right)^{3/2}} - \frac{2(291+4814x)}{867\sqrt{1+3x-2x^2}} - \frac{1}{5}\left(54\left(5-2\sqrt{10}\right)\right)\text{Subst}$$

$$= -\frac{2(15+14x)}{51\left(1+3x-2x^2\right)^{3/2}} - \frac{2(291+4814x)}{867\sqrt{1+3x-2x^2}} + \frac{9}{2}\sqrt{\frac{1}{5}\left(-53+17\sqrt{10}\right)}\ \text{ta}$$

**Mathematica [A]** time = 0.60, size = 185, normalized size = 0.96

$$\frac{3}{10}\sqrt{1+\sqrt{10}}\left(7\sqrt{10}-25\right)\tan^{-1}\left(\frac{3\left(\sqrt{10}-4\right)-\left(1+4\sqrt{10}\right)x}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)-\frac{3}{10}\sqrt{\sqrt{10}-1}\left(25+7\sqrt{10}\right)\tanh^{-1}\left(\frac{\left(4\sqrt{10}-1\right)x-3\left(4+\sqrt{10}\right)}{2\sqrt{\sqrt{10}-1}\sqrt{-2x^2+3x+1}}\right)-\frac{2\left(-9628x^3+13860x^2+5925x+546\right)}{867\left(-2x^2+3x+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x - 2*x^2)^(5/2)),x]
```

```
[Out] (-2*(546 + 5925*x + 13860*x^2 - 9628*x^3))/(867*(1 + 3*x - 2*x^2)^(3/2)) +
(3*Sqrt[1 + Sqrt[10]]*(-25 + 7*Sqrt[10])*ArcTan[(3*(-4 + Sqrt[10]) - (1 + 4
*Sqrt[10])*x)/(2*Sqrt[1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/10 - (3*Sqrt[-
1 + Sqrt[10]]*(25 + 7*Sqrt[10])*ArcTanh[(-3*(4 + Sqrt[10]) + (-1 + 4*Sqrt[1
0])*x)/(2*Sqrt[-1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/10
```

**IntegrateAlgebraic [C]** time = 0.54, size = 195, normalized size = 1.01

$$\frac{2\sqrt{-2x^2+3x+1}\,(9628x^3-13860x^2-5925x-546)}{867\,(2x^2-3x-1)^2}-\frac{9}{2}\mathrm{RootSum}\left[2\#1^4-8\#1^3+8\#1^2+20\#1+5\&,\frac{2\#1^2\log\left(\#1(-x)+\sqrt{-2x^2+3x+1}-1\right)-2\#1^2\log(x)-6\#1\log\left(\#1(-x)+\sqrt{-2x^2+3x+1}-1\right)+13\log\left(\#1(-x)+\sqrt{-2x^2+3x+1}-1\right)+6\#1\log(x)-13\log(x)}{2\#1^3-6\#1^2+4\#1+5}\&\right]$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x - 2*x^2)^(5/2)),x]
```

```
[Out] (2*Sqrt[1 + 3*x - 2*x^2]*(-546 - 5925*x - 13860*x^2 + 9628*x^3))/(867*(-1 -
 3*x + 2*x^2)^2) - (9*RootSum[5 + 20*#1 + 8*#1^2 - 8*#1^3 + 2*#1^4 & , (-13
*Log[x] + 13*Log[-1 + Sqrt[1 + 3*x - 2*x^2] - x*#1] + 6*Log[x]*#1 - 6*Log[-
1 + Sqrt[1 + 3*x - 2*x^2] - x*#1]*#1 - 2*Log[x]*#1^2 + 2*Log[-1 + Sqrt[1 +
3*x - 2*x^2] - x*#1]*#1^2)/(5 + 4*#1 - 6*#1^2 + 2*#1^3) & ])/2
```

**fricas [B]** time = 0.44, size = 439, normalized size = 2.27

$$-\frac{43680x^4-131040x^3-31212\sqrt{5}\left(4x^4-12x^3+5x^2+6x+1\right)\sqrt{17\sqrt{10}-53}\arctan\left(\frac{\sqrt{2}\left(\sqrt{10}\sqrt{5}x+10\sqrt{5}x\right)\sqrt{17\sqrt{10}-53}\sqrt{\frac{6x^2+\sqrt{10}(3x^2+2x)-2\sqrt{-2x^2+3x+1}\left(\sqrt{10}x+2x+2\right)+10x+4}{x^2}}+2\left(\sqrt{10}\sqrt{5}(6x+1)-\sqrt{-2x^2+3x+1}\left(\sqrt{10}\sqrt{5}+10\sqrt{5}\right)+5\sqrt{5}(3x+2)\right)\sqrt{17\sqrt{10}-53}}{90x}\right)-7803\sqrt{5}\left(4x^4-12x^3+5x^2+6x+1\right)\sqrt{17\sqrt{10}+53}\log\left(\frac{9\left(45\sqrt{10}x+\left(13\sqrt{10}\sqrt{5}x-40\sqrt{5}x\right)\sqrt{17\sqrt{10}+53}-90x+90\sqrt{-2x^2+3x+1}-90\right)}{x}\right)+7803\sqrt{5}\left(4x^4-12x^3+5x^2+6x+1\right)\sqrt{17\sqrt{10}+53}\log\left(\frac{9\left(45\sqrt{10}x-\left(13\sqrt{10}\sqrt{5}x-40\sqrt{5}x\right)\sqrt{17\sqrt{10}+53}-90x+90\sqrt{-2x^2+3x+1}-90\right)}{x}\right)+54600x^2-20\left(9628x^3-13860x^2-5925x-546\right)\sqrt{-2x^2+3x+1}+65520x+10920}{8670\left(4x^4-12x^3+5x^2+6x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/8670*(43680*x^4 - 131040*x^3 - 31212*sqrt(5)*(4*x^4 - 12*x^3 + 5*x^2 + 6
*x + 1)*sqrt(17*sqrt(10) - 53)*arctan(1/90*(sqrt(2)*(sqrt(10)*sqrt(5)*x + 1
0*sqrt(5)*x)*sqrt(17*sqrt(10) - 53)*sqrt((6*x^2 + sqrt(10)*(3*x^2 + 2*x) -
2*sqrt(-2*x^2 + 3*x + 1)*(sqrt(10)*x + 2*x + 2) + 10*x + 4)/x^2) + 2*(sqrt(
10)*sqrt(5)*(6*x + 1) - sqrt(-2*x^2 + 3*x + 1)*(sqrt(10)*sqrt(5) + 10*sqrt(
5)) + 5*sqrt(5)*(3*x + 2))*sqrt(17*sqrt(10) - 53))/x) - 7803*sqrt(5)*(4*x^4
 - 12*x^3 + 5*x^2 + 6*x + 1)*sqrt(17*sqrt(10) + 53)*log(9*(45*sqrt(10)*x +
(13*sqrt(10)*sqrt(5)*x - 40*sqrt(5)*x)*sqrt(17*sqrt(10) + 53) - 90*x + 90*s
qrt(-2*x^2 + 3*x + 1) - 90)/x) + 7803*sqrt(5)*(4*x^4 - 12*x^3 + 5*x^2 + 6*x
 + 1)*sqrt(17*sqrt(10) + 53)*log(9*(45*sqrt(10)*x - (13*sqrt(10)*sqrt(5)*x
- 40*sqrt(5)*x)*sqrt(17*sqrt(10) + 53) - 90*x + 90*sqrt(-2*x^2 + 3*x + 1) -
 90)/x) + 54600*x^2 - 20*(9628*x^3 - 13860*x^2 - 5925*x - 546)*sqrt(-2*x^2
+ 3*x + 1) + 65520*x + 10920)/(4*x^4 - 12*x^3 + 5*x^2 + 6*x + 1)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.02, size = 1560, normalized size = 8.08

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(5/2),x)
```

```
[Out] 248/2601/(-1/9-1/9*10^(1/2))/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*
(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^(1/2)+248/2601/(-1/9+1/9*10^(1/2))/(
-2*(x-2/3-1/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*1
0^(1/2))^(1/2)+7/153/(-1/9+1/9*10^(1/2))/(-2*(x-2/3-1/3*10^(1/2))^2+(1/3-4/
3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*10^(1/2))^(3/2)+7/153/(-1/9-1/9*10
^(1/2))/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-
1/9-1/9*10^(1/2))^(3/2)+7/51/(-1/9+1/9*10^(1/2))^2/(-2*(x-2/3-1/3*10^(1/2))
^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*10^(1/2))^(1/2)+7/51/(-1
/9-1/9*10^(1/2))^2/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3
*10^(1/2))-1/9-1/9*10^(1/2))^(1/2)-32/765/(-1/9+1/9*10^(1/2))^2/(-2*(x-2/3-
1/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*10^(1/2))^(
1/2)*10^(1/2)*x-32/2295/(-1/9+1/9*10^(1/2))*10^(1/2)/(-2*(x-2/3-1/3*10^(1/2
))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*10^(1/2))^(3/2)*x+2/5*
10^(1/2)/(-1/9-1/9*10^(1/2))^2/(1+10^(1/2))^(1/2)*arctan(9/2*(-2/9-2/9*10^(
1/2)+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2)))/(1+10^(1/2))^(1/2)/(-18*(x-2/
3+1/3*10^(1/2))^2+9*(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1-10^(1/2))^(1/
2))+32/2295/(-1/9-1/9*10^(1/2))*10^(1/2)/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/
3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^(3/2)*x+32/765/(-1/9-1/9
*10^(1/2))^2/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1
/2))-1/9-1/9*10^(1/2))^(1/2)*10^(1/2)*x+2/5*10^(1/2)/(-1/9+1/9*10^(1/2))^2/
(-1+10^(1/2))^(1/2)*arctanh(9/2*(-2/9+2/9*10^(1/2)+(1/3-4/3*10^(1/2))*(x-2/
3-1/3*10^(1/2)))/(-1+10^(1/2))^(1/2)/(-18*(x-2/3-1/3*10^(1/2))^2+9*(1/3-4/3
*10^(1/2))*(x-2/3-1/3*10^(1/2))-1+10^(1/2))^(1/2))+512/39015/(-1/9-1/9*10^(
1/2))/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1/
9-1/9*10^(1/2))^(1/2)*10^(1/2)*x-512/39015/(-1/9+1/9*10^(1/2))/(-2*(x-2/3-1
/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*10^(1/2))^(1
/2)*10^(1/2)*x-62/153/(-1/9-1/9*10^(1/2))^2/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3
+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^(1/2)*x+26/255/(-1/9-
1/9*10^(1/2))^2/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10
^(1/2))-1/9-1/9*10^(1/2))^(1/2)*10^(1/2)-1/2/(-1/9-1/9*10^(1/2))^2/(1+10^(1
/2))^(1/2)*arctan(9/2*(-2/9-2/9*10^(1/2)+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(
1/2)))/(1+10^(1/2))^(1/2)/(-18*(x-2/3+1/3*10^(1/2))^2+9*(1/3+4/3*10^(1/2))*
(x-2/3+1/3*10^(1/2))-1-10^(1/2))^(1/2))-62/459/(-1/9-1/9*10^(1/2))/(-2*(x-2
/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2)
)^(3/2)*x+26/765/(-1/9-1/9*10^(1/2))*10^(1/2)/(-2*(x-2/3+1/3*10^(1/2))^2+(1
/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^(3/2)-26/255/(-1/9+
1/9*10^(1/2))^2/(-2*(x-2/3-1/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10
```

```
^(1/2))-1/9+1/9*10^(1/2))^(1/2)*10^(1/2)+1/2/(-1/9+1/9*10^(1/2))^2/(-1+10^(
1/2))^(1/2)*arctanh(9/2*(-2/9+2/9*10^(1/2)+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10
^(1/2)))/(-1+10^(1/2))^(1/2)/(-18*(x-2/3-1/3*10^(1/2))^2+9*(1/3-4/3*10^(1/2
))*(x-2/3-1/3*10^(1/2))-1+10^(1/2))^(1/2))-26/765/(-1/9+1/9*10^(1/2))*10^(1
/2)/(-2*(x-2/3-1/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+
1/9*10^(1/2))^(3/2)-62/153/(-1/9+1/9*10^(1/2))^2/(-2*(x-2/3-1/3*10^(1/2))^2
+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*10^(1/2))^(1/2)*x-62/459/(
-1/9+1/9*10^(1/2))/(-2*(x-2/3-1/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3
*10^(1/2))-1/9+1/9*10^(1/2))^(3/2)*x+128/13005*10^(1/2)/(-1/9+1/9*10^(1/2))
/(-2*(x-2/3-1/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9
*10^(1/2))^(1/2)-992/7803/(-1/9+1/9*10^(1/2))/(-2*(x-2/3-1/3*10^(1/2))^2+(1
/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*10^(1/2))^(1/2)*x-128/13005*1
0^(1/2)/(-1/9-1/9*10^(1/2))/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(
x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^(1/2)-992/7803/(-1/9-1/9*10^(1/2))/(-
2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10
^(1/2))^(1/2)*x
```

**maxima [B]** time = 1.26, size = 1276, normalized size = 6.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/17340*sqrt(10)*(2108*sqrt(10)*x/(sqrt(10)*(-2*x^2 + 3*x + 1)^(3/2) + (-2*
x^2 + 3*x + 1)^(3/2)) - 2108*sqrt(10)*x/(sqrt(10)*(-2*x^2 + 3*x + 1)^(3/2)
- (-2*x^2 + 3*x + 1)^(3/2)) - 56916*sqrt(10)*x/(2*sqrt(10)*sqrt(-2*x^2 + 3*
x + 1) + 11*sqrt(-2*x^2 + 3*x + 1)) + 56916*sqrt(10)*x/(2*sqrt(10)*sqrt(-2*
x^2 + 3*x + 1) - 11*sqrt(-2*x^2 + 3*x + 1)) + 1984*sqrt(10)*x/(sqrt(10)*sqr
t(-2*x^2 + 3*x + 1) + sqrt(-2*x^2 + 3*x + 1)) - 1984*sqrt(10)*x/(sqrt(10)*s
qrt(-2*x^2 + 3*x + 1) - sqrt(-2*x^2 + 3*x + 1)) - 70227*sqrt(10)*arcsin(8/1
7*sqrt(17)*sqrt(10)*x/abs(6*x + 2*sqrt(10) - 4) + 2/17*sqrt(17)*x/abs(6*x +
 2*sqrt(10) - 4) - 6/17*sqrt(17)*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 24/17
*sqrt(17)/abs(6*x + 2*sqrt(10) - 4))/(2*sqrt(10)*sqrt(sqrt(10) + 1) + 11*sq
rt(sqrt(10) + 1)) - 2176*x/(sqrt(10)*(-2*x^2 + 3*x + 1)^(3/2) + (-2*x^2 + 3
*x + 1)^(3/2)) - 2176*x/(sqrt(10)*(-2*x^2 + 3*x + 1)^(3/2) - (-2*x^2 + 3*x
+ 1)^(3/2)) + 58752*x/(2*sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + 11*sqrt(-2*x^2 +
 3*x + 1)) + 58752*x/(2*sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - 11*sqrt(-2*x^2 +
3*x + 1)) - 2048*x/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + sqrt(-2*x^2 + 3*x + 1
)) - 2048*x/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - sqrt(-2*x^2 + 3*x + 1)) + 56
1816*arcsin(8/17*sqrt(17)*sqrt(10)*x/abs(6*x + 2*sqrt(10) - 4) + 2/17*sqrt(
17)*x/abs(6*x + 2*sqrt(10) - 4) - 6/17*sqrt(17)*sqrt(10)/abs(6*x + 2*sqrt(1
0) - 4) + 24/17*sqrt(17)/abs(6*x + 2*sqrt(10) - 4))/(2*sqrt(10)*sqrt(sqrt(1
0) + 1) + 11*sqrt(sqrt(10) + 1)) - 714*sqrt(10)/(sqrt(10)*(-2*x^2 + 3*x + 1
)^(3/2) + (-2*x^2 + 3*x + 1)^(3/2)) + 714*sqrt(10)/(sqrt(10)*(-2*x^2 + 3*x
+ 1)^(3/2) - (-2*x^2 + 3*x + 1)^(3/2)) + 19278*sqrt(10)/(2*sqrt(10)*sqrt(-2
```

```
*x^2 + 3*x + 1) + 11*sqrt(-2*x^2 + 3*x + 1)) - 19278*sqrt(10)/(2*sqrt(10)*s
qrt(-2*x^2 + 3*x + 1) - 11*sqrt(-2*x^2 + 3*x + 1)) - 1488*sqrt(10)/(sqrt(10
)*sqrt(-2*x^2 + 3*x + 1) + sqrt(-2*x^2 + 3*x + 1)) + 1488*sqrt(10)/(sqrt(10
)*sqrt(-2*x^2 + 3*x + 1) - sqrt(-2*x^2 + 3*x + 1)) - 5304/(sqrt(10)*(-2*x^2
 + 3*x + 1)^(3/2) + (-2*x^2 + 3*x + 1)^(3/2)) - 5304/(sqrt(10)*(-2*x^2 + 3*
x + 1)^(3/2) - (-2*x^2 + 3*x + 1)^(3/2)) + 143208/(2*sqrt(10)*sqrt(-2*x^2 +
 3*x + 1) + 11*sqrt(-2*x^2 + 3*x + 1)) + 143208/(2*sqrt(10)*sqrt(-2*x^2 + 3
*x + 1) - 11*sqrt(-2*x^2 + 3*x + 1)) + 1536/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1
) + sqrt(-2*x^2 + 3*x + 1)) + 1536/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - sqrt(
-2*x^2 + 3*x + 1)) + 70227*sqrt(10)*log(-2/9*sqrt(10) + 2/3*sqrt(-2*x^2 + 3
*x + 1)*sqrt(sqrt(10) - 1)/abs(6*x - 2*sqrt(10) - 4) + 2/9*sqrt(10)/abs(6*x
 - 2*sqrt(10) - 4) - 2/9/abs(6*x - 2*sqrt(10) - 4) + 1/18)/(sqrt(10) - 1)^(
5/2) + 561816*log(-2/9*sqrt(10) + 2/3*sqrt(-2*x^2 + 3*x + 1)*sqrt(sqrt(10)
- 1)/abs(6*x - 2*sqrt(10) - 4) + 2/9*sqrt(10)/abs(6*x - 2*sqrt(10) - 4) - 2
/9/abs(6*x - 2*sqrt(10) - 4) + 1/18)/(sqrt(10) - 1)^(5/2))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+2}{\left(-2x^2+3x+1\right)^{5/2}\left(-3x^2+4x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/((3*x - 2*x^2 + 1)^(5/2)*(4*x - 3*x^2 + 2)),x)
```

```
[Out] int((x + 2)/((3*x - 2*x^2 + 1)^(5/2)*(4*x - 3*x^2 + 2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{12x^6\sqrt{-2x^2+3x+1}-52x^5\sqrt{-2x^2+3x+1}+55x^4\sqrt{-2x^2+3x+1}+22x^3\sqrt{-2x^2+3x+1}-31x^2\sqrt{-2x^2+3x+1}-16x\sqrt{-2x^2+3x+1}-2\sqrt{-2x^2+3x+1}}\,dx-\int \frac{2}{12x^6\sqrt{-2x^2+3x+1}-52x^5\sqrt{-2x^2+3x+1}+55x^4\sqrt{-2x^2+3x+1}+22x^3\sqrt{-2x^2+3x+1}-31x^2\sqrt{-2x^2+3x+1}-16x\sqrt{-2x^2+3x+1}-2\sqrt{-2x^2+3x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x**2+4*x+2)/(-2*x**2+3*x+1)**(5/2),x)
```

```
[Out] -Integral(x/(12*x**6*sqrt(-2*x**2 + 3*x + 1) - 52*x**5*sqrt(-2*x**2 + 3*x +
 1) + 55*x**4*sqrt(-2*x**2 + 3*x + 1) + 22*x**3*sqrt(-2*x**2 + 3*x + 1) - 3
1*x**2*sqrt(-2*x**2 + 3*x + 1) - 16*x*sqrt(-2*x**2 + 3*x + 1) - 2*sqrt(-2*x
**2 + 3*x + 1)), x) - Integral(2/(12*x**6*sqrt(-2*x**2 + 3*x + 1) - 52*x**5
*sqrt(-2*x**2 + 3*x + 1) + 55*x**4*sqrt(-2*x**2 + 3*x + 1) + 22*x**3*sqrt(-
2*x**2 + 3*x + 1) - 31*x**2*sqrt(-2*x**2 + 3*x + 1) - 16*x*sqrt(-2*x**2 + 3
*x + 1) - 2*sqrt(-2*x**2 + 3*x + 1)), x)
```

## 3.28 $\int \frac{2+x}{\left(2+4x-3x^2\right)\sqrt{1+3x+2x^2}}\,dx$

**Optimal**. Leaf size=151

$$\frac{1}{2}\sqrt{1-\frac{7\sqrt{\frac{2}{5}}}{5}}\tanh^{-1}\left(\frac{\left(17+4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{2\sqrt{55+17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)-\frac{1}{2}\sqrt{1+\frac{7\sqrt{\frac{2}{5}}}{5}}\tanh^{-1}\left(\frac{\left(17-4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)$$

**Rubi [A]** time = 0.23, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1032, 724, 206}

$$\frac{1}{2}\sqrt{1-\frac{7\sqrt{\frac{2}{5}}}{5}}\tanh^{-1}\left(\frac{\left(17+4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{2\sqrt{55+17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)-\frac{1}{2}\sqrt{1+\frac{7\sqrt{\frac{2}{5}}}{5}}\tanh^{-1}\left(\frac{\left(17-4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/((2 + 4*x - 3*x^2)*Sqrt[1 + 3*x + 2*x^2]),x]
```

```
[Out] -(Sqrt[1 + (7*Sqrt[2/5])/5]*ArcTanh[(3*(4 - Sqrt[10]) + (17 - 4*Sqrt[10])*x
)/(2*Sqrt[55 - 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/2 + (Sqrt[1 - (7*Sqrt[
2/5])/5]*ArcTanh[(3*(4 + Sqrt[10]) + (17 + 4*Sqrt[10])*x)/(2*Sqrt[55 + 17*S
qrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
```

```
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{2+x}{\left(2+4x-3x^2\right)\sqrt{1+3x+2x^2}}\,dx = \frac{1}{5}\left(5-4\sqrt{10}\right)\int \frac{1}{\left(4-2\sqrt{10}-6x\right)\sqrt{1+3x+2x^2}}\,dx + \frac{1}{5}\left(5+4\sqrt{10}\right)\int$$

$$= -\left(\frac{1}{5}\left(2\left(5-4\sqrt{10}\right)\right)\text{Subst}\left(\int \frac{1}{144+72\left(4-2\sqrt{10}\right)+8\left(4-2\sqrt{10}\right)^2-x\right.\right.$$

$$= -\frac{1}{10}\sqrt{25+7\sqrt{10}}\,\tanh^{-1}\left(\frac{3\left(4-\sqrt{10}\right)+\left(17-4\sqrt{10}\right)x}{2\sqrt{55-17\sqrt{10}}\,\sqrt{1+3x+2x^2}}\right) + \frac{1}{10}\sqrt{25-7}$$

**Mathematica [A]** time = 0.36, size = 148, normalized size = 0.98

$$\frac{\left(5-4\sqrt{10}\right)\tanh^{-1}\left(\frac{-4\sqrt{10}x+17x-3\sqrt{10}+12}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)+3\sqrt{285-90\sqrt{10}}\,\tanh^{-1}\left(\frac{\left(17+4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{2\sqrt{55+17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)}{10\sqrt{55-17\sqrt{10}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + x)/((2 + 4*x - 3*x^2)*Sqrt[1 + 3*x + 2*x^2]),x]
```

```
[Out] ((5 - 4*Sqrt[10])*ArcTanh[(12 - 3*Sqrt[10] + 17*x - 4*Sqrt[10]*x)/(2*Sqrt[5
5 - 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])] + 3*Sqrt[285 - 90*Sqrt[10]]*ArcTan
h[(3*(4 + Sqrt[10]) + (17 + 4*Sqrt[10])*x)/(2*Sqrt[55 + 17*Sqrt[10]]*Sqrt[1
 + 3*x + 2*x^2])])/(10*Sqrt[55 - 17*Sqrt[10]])
```

**IntegrateAlgebraic [A]** time = 0.78, size = 109, normalized size = 0.72

$$\frac{1}{5}\sqrt{25-7\sqrt{10}}\,\tanh^{-1}\left(\frac{\sqrt{1+\sqrt{\frac{2}{5}}}\,\sqrt{2x^2+3x+1}}{2x+1}\right)-\frac{1}{5}\sqrt{25+7\sqrt{10}}\,\tanh^{-1}\left(\frac{\sqrt{1-\sqrt{\frac{2}{5}}}\,\sqrt{2x^2+3x+1}}{2x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + x)/((2 + 4*x - 3*x^2)*Sqrt[1 + 3*x + 2*x^2]),x]
```

```
[Out] -1/5*(Sqrt[25 + 7*Sqrt[10]]*ArcTanh[(Sqrt[1 - Sqrt[2/5]]*Sqrt[1 + 3*x + 2*x
^2])/(1 + 2*x)]) + (Sqrt[25 - 7*Sqrt[10]]*ArcTanh[(Sqrt[1 + Sqrt[2/5]]*Sqrt
[1 + 3*x + 2*x^2])/(1 + 2*x)])/5
```

**fricas [B]** time = 0.43, size = 245, normalized size = 1.62

$$\frac{1}{10}\sqrt{7\sqrt{10}+25}\log\left(-\frac{3\sqrt{10}x+(\sqrt{10}x-4x)\sqrt{7\sqrt{10}+25}+6x-6\sqrt{2x^2+3x+1}+6}{x}\right)-\frac{1}{10}\sqrt{7\sqrt{10}+25}\log\left(-\frac{3\sqrt{10}x-(\sqrt{10}x-4x)\sqrt{7\sqrt{10}+25}+6x-6\sqrt{2x^2+3x+1}+6}{x}\right)+\frac{1}{10}\sqrt{-7\sqrt{10}+25}\log\left(\frac{3\sqrt{10}x+(\sqrt{10}x+4x)\sqrt{-7\sqrt{10}+25}-6x+6\sqrt{2x^2+3x+1}-6}{x}\right)-\frac{1}{10}\sqrt{-7\sqrt{10}+25}\log\left(\frac{3\sqrt{10}x-(\sqrt{10}x+4x)\sqrt{-7\sqrt{10}+25}-6x+6\sqrt{2x^2+3x+1}-6}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(7*sqrt(10) + 25)*log(-(3*sqrt(10)*x + (sqrt(10)*x - 4*x)*sqrt(7*s
qrt(10) + 25) + 6*x - 6*sqrt(2*x^2 + 3*x + 1) + 6)/x) - 1/10*sqrt(7*sqrt(10
) + 25)*log(-(3*sqrt(10)*x - (sqrt(10)*x - 4*x)*sqrt(7*sqrt(10) + 25) + 6*x
 - 6*sqrt(2*x^2 + 3*x + 1) + 6)/x) + 1/10*sqrt(-7*sqrt(10) + 25)*log((3*sqr
t(10)*x + (sqrt(10)*x + 4*x)*sqrt(-7*sqrt(10) + 25) - 6*x + 6*sqrt(2*x^2 +
3*x + 1) - 6)/x) - 1/10*sqrt(-7*sqrt(10) + 25)*log((3*sqrt(10)*x - (sqrt(10
)*x + 4*x)*sqrt(-7*sqrt(10) + 25) - 6*x + 6*sqrt(2*x^2 + 3*x + 1) - 6)/x)
```

**giac [A]** time = 0.48, size = 93, normalized size = 0.62

$$0.169235232112667\log\left(-\sqrt{2}x+\sqrt{2x^2+3x+1}+5.90976932712000\right)-0.686556214893333\log\left(-\sqrt{2}x+\sqrt{2x^2+3x+1}-0.176527156327000\right)+0.686556214893333\log\left(-\sqrt{2}x+\sqrt{2x^2+3x+1}-0.919278730509000\right)-0.169235232112667\log\left(-\sqrt{2}x+\sqrt{2x^2+3x+1}-1.04272727395000\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 0.169235232112667*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) + 5.90976932712000
) - 0.686556214893333*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) - 0.1765271563
27000) + 0.686556214893333*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) - 0.91927
8730509000) - 0.169235232112667*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) - 1.
04272727395000)
```

**maple [A]** time = 0.05, size = 186, normalized size = 1.23

$$\frac{\left(-8+\sqrt{10}\right)\sqrt{10}\,\operatorname{arctanh}\left(\frac{55-17\sqrt{10}+\frac{9\left(\frac{17}{3}-\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)}{2}}{\sqrt{55-17\sqrt{10}}\,\sqrt{18\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)^2+9\left(\frac{17}{3}-\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)+55-17\sqrt{10}}}\right)}{20\sqrt{55-17\sqrt{10}}}+\frac{\left(8+\sqrt{10}\right)\sqrt{10}\,\operatorname{arctanh}\left(\frac{55+17\sqrt{10}+\frac{9\left(\frac{17}{3}+\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)}{2}}{\sqrt{55+17\sqrt{10}}\,\sqrt{18\left(x-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)^2+9\left(\frac{17}{3}+\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)+55+17\sqrt{10}}}\right)}{20\sqrt{55+17\sqrt{10}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(1/2),x)
```

```
[Out] 1/20*(-8+10^(1/2))*10^(1/2)/(55-17*10^(1/2))^(1/2)*arctanh(9/2*(110/9-34/9*
10^(1/2)+(17/3-4/3*10^(1/2))*(x-2/3+1/3*10^(1/2)))/(55-17*10^(1/2))^(1/2)/(
18*(x-2/3+1/3*10^(1/2))^2+9*(17/3-4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))+55-17*
```

```
10^(1/2))^(1/2))+1/20*(8+10^(1/2))*10^(1/2)/(55+17*10^(1/2))^(1/2)*arctanh(
9/2*(110/9+34/9*10^(1/2)+(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2)))/(55+17*1
0^(1/2))^(1/2)/(18*(x-2/3-1/3*10^(1/2))^2+9*(17/3+4/3*10^(1/2))*(x-2/3-1/3*
10^(1/2))+55+17*10^(1/2))^(1/2))
```

**maxima [B]** time = 1.07, size = 363, normalized size = 2.40

$$\frac{1}{60}\sqrt{10}\left(\frac{3\sqrt{10}\log\left(\frac{2}{9}\sqrt{10}+\frac{2\sqrt{2x^2+3x+1}\sqrt{17\sqrt{10}+55}}{3\left|6x-2\sqrt{10}-4\right|}+\frac{34\sqrt{10}}{9\left|6x-2\sqrt{10}-4\right|}+\frac{110}{9\left|6x-2\sqrt{10}-4\right|}+\frac{17}{18}\right)}{\sqrt{17\sqrt{10}+55}}+\frac{\sqrt{10}\log\left(-\frac{2}{9}\sqrt{10}+\frac{2\sqrt{2x^2+3x+1}\sqrt{-\frac{17}{9}\sqrt{10}+\frac{55}{9}}}{\left|6x+2\sqrt{10}-4\right|}-\frac{34\sqrt{10}}{9\left|6x+2\sqrt{10}-4\right|}+\frac{110}{9\left|6x+2\sqrt{10}-4\right|}+\frac{17}{18}\right)}{\sqrt{-\frac{17}{9}\sqrt{10}+\frac{55}{9}}}+\frac{24\log\left(\frac{2}{9}\sqrt{10}+\frac{2\sqrt{2x^2+3x+1}\sqrt{17\sqrt{10}+55}}{3\left|6x-2\sqrt{10}-4\right|}+\frac{34\sqrt{10}}{9\left|6x-2\sqrt{10}-4\right|}+\frac{110}{9\left|6x-2\sqrt{10}-4\right|}+\frac{17}{18}\right)}{\sqrt{17\sqrt{10}+55}}-\frac{8\log\left(-\frac{2}{9}\sqrt{10}+\frac{2\sqrt{2x^2+3x+1}\sqrt{-\frac{17}{9}\sqrt{10}+\frac{55}{9}}}{\left|6x+2\sqrt{10}-4\right|}-\frac{34\sqrt{10}}{9\left|6x+2\sqrt{10}-4\right|}+\frac{110}{9\left|6x+2\sqrt{10}-4\right|}+\frac{17}{18}\right)}{\sqrt{-\frac{17}{9}\sqrt{10}+\frac{55}{9}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/60*sqrt(10)*(3*sqrt(10)*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x + 1)*sqrt
(17*sqrt(10) + 55)/abs(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/abs(6*x - 2*sq
rt(10) - 4) + 110/9/abs(6*x - 2*sqrt(10) - 4) + 17/18)/sqrt(17*sqrt(10) + 5
5) + sqrt(10)*log(-2/9*sqrt(10) + 2*sqrt(2*x^2 + 3*x + 1)*sqrt(-17/9*sqrt(1
0) + 55/9)/abs(6*x + 2*sqrt(10) - 4) - 34/9*sqrt(10)/abs(6*x + 2*sqrt(10) -
 4) + 110/9/abs(6*x + 2*sqrt(10) - 4) + 17/18)/sqrt(-17/9*sqrt(10) + 55/9)
+ 24*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x + 1)*sqrt(17*sqrt(10) + 55)/ab
s(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/abs(6*x - 2*sqrt(10) - 4) + 110/9/a
bs(6*x - 2*sqrt(10) - 4) + 17/18)/sqrt(17*sqrt(10) + 55) - 8*log(-2/9*sqrt(
10) + 2*sqrt(2*x^2 + 3*x + 1)*sqrt(-17/9*sqrt(10) + 55/9)/abs(6*x + 2*sqrt(
10) - 4) - 34/9*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 110/9/abs(6*x + 2*sqrt
(10) - 4) + 17/18)/sqrt(-17/9*sqrt(10) + 55/9))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+2}{\sqrt{2\,x^2+3\,x+1}\,\left(-3\,x^2+4\,x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/((3*x + 2*x^2 + 1)^(1/2)*(4*x - 3*x^2 + 2)),x)
```

```
[Out] int((x + 2)/((3*x + 2*x^2 + 1)^(1/2)*(4*x - 3*x^2 + 2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{3x^2\sqrt{2x^2+3x+1}-4x\sqrt{2x^2+3x+1}-2\sqrt{2x^2+3x+1}}\,dx-\int \frac{2}{3x^2\sqrt{2x^2+3x+1}-4x\sqrt{2x^2+3x+1}-2\sqrt{2x^2+3x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x**2+4*x+2)/(2*x**2+3*x+1)**(1/2),x)
```

```
[Out] -Integral(x/(3*x**2*sqrt(2*x**2 + 3*x + 1) - 4*x*sqrt(2*x**2 + 3*x + 1) - 2
*sqrt(2*x**2 + 3*x + 1)), x) - Integral(2/(3*x**2*sqrt(2*x**2 + 3*x + 1) -
4*x*sqrt(2*x**2 + 3*x + 1) - 2*sqrt(2*x**2 + 3*x + 1)), x)
```

## 3.29 $\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{3/2}}\,dx$

**Optimal**. Leaf size=174

$$\frac{2(22x+21)}{5\sqrt{2x^2+3x+1}}-\frac{1}{10}\sqrt{\frac{3}{5}\left(2065+653\sqrt{10}\right)}\tanh^{-1}\left(\frac{\left(17-4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)+\frac{1}{10}\sqrt{\frac{3}{5}\left(2065-653\sqrt{10}\right.}$$

**Rubi** [A] time = 0.25, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1016, 1032, 724, 206}

$$\frac{2(22x+21)}{5\sqrt{2x^2+3x+1}}-\frac{1}{10}\sqrt{\frac{3}{5}\left(2065+653\sqrt{10}\right)}\tanh^{-1}\left(\frac{\left(17-4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)+\frac{1}{10}\sqrt{\frac{3}{5}\left(2065-653\sqrt{10}\right)}\tanh^{-1}\left(\frac{\left(17+4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{2\sqrt{55+17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x + 2*x^2)^(3/2)),x]
```

```
[Out] (2*(21 + 22*x))/(5*Sqrt[1 + 3*x + 2*x^2]) - (Sqrt[(3*(2065 + 653*Sqrt[10]))
/5]*ArcTanh[(3*(4 - Sqrt[10]) + (17 - 4*Sqrt[10])*x)/(2*Sqrt[55 - 17*Sqrt[1
0]]*Sqrt[1 + 3*x + 2*x^2])])/10 + (Sqrt[(3*(2065 - 653*Sqrt[10]))/5]*ArcTan
h[(3*(4 + Sqrt[10]) + (17 + 4*Sqrt[10])*x)/(2*Sqrt[55 + 17*Sqrt[10]]*Sqrt[1
 + 3*x + 2*x^2])])/10
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1016

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)
*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*
c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))
```

```
 - h*(b*c*d - 2*a*c*e + a*b*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d -
a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e
))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{2+x}{\left(2+4x-3x^2\right)\left(1+3x+2x^2\right)^{3/2}}\,dx = \frac{2(21+22x)}{5\sqrt{1+3x+2x^2}} - \frac{2}{15}\int \frac{-72+\frac{81x}{2}}{\left(2+4x-3x^2\right)\sqrt{1+3x+2x^2}}\,dx$$

$$= \frac{2(21+22x)}{5\sqrt{1+3x+2x^2}} - \frac{1}{5}\left(9\left(3-\sqrt{10}\right)\right)\int \frac{1}{\left(4+2\sqrt{10}-6x\right)\sqrt{1+3x+2x^2}}$$

$$= \frac{2(21+22x)}{5\sqrt{1+3x+2x^2}} + \frac{1}{5}\left(18\left(3-\sqrt{10}\right)\right)\text{Subst}\left(\int \frac{1}{144+72\left(4+2\sqrt{10}\right)+\right.$$

$$= \frac{2(21+22x)}{5\sqrt{1+3x+2x^2}} - \frac{1}{10}\sqrt{\frac{3}{5}\left(2065+653\sqrt{10}\right)}\tanh^{-1}\left(\frac{3\left(4-\sqrt{10}\right)+\left(}{2\sqrt{55-17\sqrt{10}}}\right.$$

**Mathematica** [A] time = 0.57, size = 172, normalized size = 0.99

$$\frac{1}{50}\left(\frac{\sqrt{30975-9795\sqrt{10}}\,\sqrt{2x^2+3x+1}\,\tanh^{-1}\left(\frac{4\sqrt{10}\,x+17x+3\sqrt{10}+12}{2\sqrt{55+17\sqrt{10}}\,\sqrt{2x^2+3x+1}}\right)+440x+420}{\sqrt{2x^2+3x+1}}-\sqrt{30975+9795\sqrt{10}}\,\tanh^{-1}\left(\frac{-4\sqrt{10}\,x+17x-3\sqrt{10}+12}{2\sqrt{55-17\sqrt{10}}\,\sqrt{2x^2+3x+1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x + 2*x^2)^(3/2)),x]
```

```
[Out] (-(Sqrt[30975 + 9795*Sqrt[10]]*ArcTanh[(12 - 3*Sqrt[10] + 17*x - 4*Sqrt[10]
*x)/(2*Sqrt[55 - 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])]) + (420 + 440*x + Sqr
t[30975 - 9795*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2]*ArcTanh[(12 + 3*Sqrt[10] + 1
7*x + 4*Sqrt[10]*x)/(2*Sqrt[55 + 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/Sqrt
[1 + 3*x + 2*x^2])/50
```

**IntegrateAlgebraic** [A] time = 1.02, size = 148, normalized size = 0.85

$$\frac{2\sqrt{2x^2+3x+1}\,(22x+21)}{5(x+1)(2x+1)}-\frac{1}{5}\sqrt{\frac{1}{5}\left(6195+1959\sqrt{10}\right)}\,\tanh^{-1}\left(\frac{\sqrt{1-\sqrt{\frac{2}{5}}}\,\sqrt{2x^2+3x+1}}{2x+1}\right)+\frac{9\tanh^{-1}\left(\frac{\sqrt{1+\sqrt{\frac{2}{5}}}\,\sqrt{2x^2+3x+1}}{2x+1}\right)}{5\sqrt{2065+653\sqrt{10}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x + 2*x^2)^(3/2)),x]
```

```
[Out] (2*(21 + 22*x)*Sqrt[1 + 3*x + 2*x^2])/(5*(1 + x)*(1 + 2*x)) - (Sqrt[(6195 +
 1959*Sqrt[10])/5]*ArcTanh[(Sqrt[1 - Sqrt[2/5]]*Sqrt[1 + 3*x + 2*x^2])/(1 +
 2*x)])/5 + (9*ArcTanh[(Sqrt[1 + Sqrt[2/5]]*Sqrt[1 + 3*x + 2*x^2])/(1 + 2*x
)])/(5*Sqrt[2065 + 653*Sqrt[10]])
```

**fricas** [B] time = 0.45, size = 365, normalized size = 2.10

$$\frac{\sqrt{5}\left(2x^2+3x+1\right)\sqrt{1959\sqrt{10}+6195}\,\log\left(-\frac{45\sqrt{10}x+\left(41\sqrt{10}\sqrt{5}x-130\sqrt{5}x\right)\sqrt{1959\sqrt{10}+6195}+90x-90\sqrt{2x^2+3x+1}+90}{x}\right)-\sqrt{5}\left(2x^2+3x+1\right)\sqrt{1959\sqrt{10}+6195}\,\log\left(-\frac{45\sqrt{10}x-\left(41\sqrt{10}\sqrt{5}x-130\sqrt{5}x\right)\sqrt{1959\sqrt{10}+6195}+90x-90\sqrt{2x^2+3x+1}+90}{x}\right)+\sqrt{5}\left(2x^2+3x+1\right)\sqrt{-1959\sqrt{10}+6195}\,\log\left(\frac{45\sqrt{10}x+\left(41\sqrt{10}\sqrt{5}x+130\sqrt{5}x\right)\sqrt{-1959\sqrt{10}+6195}-90x+90\sqrt{2x^2+3x+1}-90}{x}\right)-\sqrt{5}\left(2x^2+3x+1\right)\sqrt{-1959\sqrt{10}+6195}\,\log\left(\frac{45\sqrt{10}x-\left(41\sqrt{10}\sqrt{5}x+130\sqrt{5}x\right)\sqrt{-1959\sqrt{10}+6195}-90x+90\sqrt{2x^2+3x+1}-90}{x}\right)+840x^2+20\sqrt{2x^2+3x+1}\,(22x+21)+1260x+420}{50\left(2x^2+3x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/50*(sqrt(5)*(2*x^2 + 3*x + 1)*sqrt(1959*sqrt(10) + 6195)*log(-(45*sqrt(10
)*x + (41*sqrt(10)*sqrt(5)*x - 130*sqrt(5)*x)*sqrt(1959*sqrt(10) + 6195) +
90*x - 90*sqrt(2*x^2 + 3*x + 1) + 90)/x) - sqrt(5)*(2*x^2 + 3*x + 1)*sqrt(1
959*sqrt(10) + 6195)*log(-(45*sqrt(10)*x - (41*sqrt(10)*sqrt(5)*x - 130*sqr
t(5)*x)*sqrt(1959*sqrt(10) + 6195) + 90*x - 90*sqrt(2*x^2 + 3*x + 1) + 90)/
x) + sqrt(5)*(2*x^2 + 3*x + 1)*sqrt(-1959*sqrt(10) + 6195)*log((45*sqrt(10)
```

```
*x + (41*sqrt(10)*sqrt(5)*x + 130*sqrt(5)*x)*sqrt(-1959*sqrt(10) + 6195) -
90*x + 90*sqrt(2*x^2 + 3*x + 1) - 90)/x) - sqrt(5)*(2*x^2 + 3*x + 1)*sqrt(-
1959*sqrt(10) + 6195)*log((45*sqrt(10)*x - (41*sqrt(10)*sqrt(5)*x + 130*sqr
t(5)*x)*sqrt(-1959*sqrt(10) + 6195) - 90*x + 90*sqrt(2*x^2 + 3*x + 1) - 90)
/x) + 840*x^2 + 20*sqrt(2*x^2 + 3*x + 1)*(22*x + 21) + 1260*x + 420)/(2*x^2
 + 3*x + 1)
```

**giac [A]** time = 0.50, size = 112, normalized size = 0.64

$$\frac{2\,(22\,x+21)}{5\,\sqrt{2\,x^{2}+3\,x+1}}+0.0140045514133333\,\log\left(-\sqrt{2}\,x+\sqrt{2\,x^{2}+3\,x+1}+5.90976932712000\right)-4.97793168620000\,\log\left(-\sqrt{2}\,x+\sqrt{2\,x^{2}+3\,x+1}-0.176527156327000\right)+4.97793168620000\,\log\left(-\sqrt{2}\,x+\sqrt{2\,x^{2}+3\,x+1}-0.919278730509000\right)-0.0140045514125333\,\log\left(-\sqrt{2}\,x+\sqrt{2\,x^{2}+3\,x+1}-1.04272727395000\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(3/2),x, algorithm="giac")
```

```
[Out] 2/5*(22*x + 21)/sqrt(2*x^2 + 3*x + 1) + 0.0140045514133333*log(-sqrt(2)*x +
 sqrt(2*x^2 + 3*x + 1) + 5.90976932712000) - 4.97793168620000*log(-sqrt(2)*
x + sqrt(2*x^2 + 3*x + 1) - 0.176527156327000) + 4.97793168620000*log(-sqrt
(2)*x + sqrt(2*x^2 + 3*x + 1) - 0.919278730509000) - 0.0140045514125333*log
(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) - 1.04272727395000)
```

**maple [B]** time = 0.02, size = 466, normalized size = 2.68

$$-\frac{\left(-8+\sqrt{10}\right)\sqrt{10}\left(-\frac{\operatorname{arctanh}\left(\frac{55-17\sqrt{10}+\frac{9\left(\frac{17}{3}-\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)}{2}}{\sqrt{55-17\sqrt{10}}\,\sqrt{18\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)^{2}+9\left(\frac{17}{3}-\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)+55-17\sqrt{10}}}\right)}{\left(\frac{55}{9}-\frac{17\sqrt{10}}{9}\right)\sqrt{55-17\sqrt{10}}}+\frac{1}{3\left(\frac{55}{9}-\frac{17\sqrt{10}}{9}\right)\sqrt{2\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)^{2}+\left(\frac{17}{3}-\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)+\frac{55}{9}-\frac{17\sqrt{10}}{9}}}-\frac{\left(\frac{17}{3}-\frac{4\sqrt{10}}{3}\right)(4x+3)}{3\left(\frac{55}{9}-\frac{17\sqrt{10}}{9}\right)\left(\frac{440}{9}-\frac{136\sqrt{10}}{9}-\left(\frac{17}{3}-\frac{4\sqrt{10}}{3}\right)^{2}\right)\sqrt{2\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)^{2}+\left(\frac{17}{3}-\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)+\frac{55}{9}-\frac{17\sqrt{10}}{9}}}\right)}{20}-\frac{\left(8+\sqrt{10}\right)\sqrt{10}\left(-\frac{\operatorname{arctanh}\left(\frac{55+17\sqrt{10}+\frac{9\left(\frac{17}{3}+\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)}{2}}{\sqrt{55+17\sqrt{10}}\,\sqrt{18\left(x-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)^{2}+9\left(\frac{17}{3}+\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)+55+17\sqrt{10}}}\right)}{\left(\frac{55}{9}+\frac{17\sqrt{10}}{9}\right)\sqrt{55+17\sqrt{10}}}+\frac{1}{3\left(\frac{55}{9}+\frac{17\sqrt{10}}{9}\right)\sqrt{2\left(x-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)^{2}+\left(\frac{17}{3}+\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)+\frac{55}{9}+\frac{17\sqrt{10}}{9}}}-\frac{\left(\frac{17}{3}+\frac{4\sqrt{10}}{3}\right)(4x+3)}{3\left(\frac{55}{9}+\frac{17\sqrt{10}}{9}\right)\left(\frac{440}{9}+\frac{136\sqrt{10}}{9}-\left(\frac{17}{3}+\frac{4\sqrt{10}}{3}\right)^{2}\right)\sqrt{2\left(x-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)^{2}+\left(\frac{17}{3}+\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)+\frac{55}{9}+\frac{17\sqrt{10}}{9}}}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(3/2),x)
```

```
[Out] -1/20*(-8+10^(1/2))*10^(1/2)*(1/3/(55/9-17/9*10^(1/2))/(2*(x-2/3+1/3*10^(1/
2))^2+(17/3-4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))+55/9-17/9*10^(1/2))^(1/2)-1/
3*(17/3-4/3*10^(1/2))/(55/9-17/9*10^(1/2))*(3+4*x)/(440/9-136/9*10^(1/2)-(1
7/3-4/3*10^(1/2))^2)/(2*(x-2/3+1/3*10^(1/2))^2+(17/3-4/3*10^(1/2))*(x-2/3+1
/3*10^(1/2))+55/9-17/9*10^(1/2))^(1/2)-1/(55/9-17/9*10^(1/2))/(55-17*10^(1/
2))^(1/2)*arctanh(9/2*(110/9-34/9*10^(1/2)+(17/3-4/3*10^(1/2))*(x-2/3+1/3*1
0^(1/2)))/(55-17*10^(1/2))^(1/2)/(18*(x-2/3+1/3*10^(1/2))^2+9*(17/3-4/3*10^
(1/2))*(x-2/3+1/3*10^(1/2))+55-17*10^(1/2))^(1/2)))-1/20*(8+10^(1/2))*10^(1
/2)*(1/3/(55/9+17/9*10^(1/2))/(2*(x-2/3-1/3*10^(1/2))^2+(17/3+4/3*10^(1/2))
*(x-2/3-1/3*10^(1/2))+55/9+17/9*10^(1/2))^(1/2)-1/3*(17/3+4/3*10^(1/2))/(55
/9+17/9*10^(1/2))*(3+4*x)/(440/9+136/9*10^(1/2)-(17/3+4/3*10^(1/2))^2)/(2*(
x-2/3-1/3*10^(1/2))^2+(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))+55/9+17/9*10
^(1/2))^(1/2)-1/(55/9+17/9*10^(1/2))/(55+17*10^(1/2))^(1/2)*arctanh(9/2*(11
0/9+34/9*10^(1/2)+(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2)))/(55+17*10^(1/2)
)^(1/2)/(18*(x-2/3-1/3*10^(1/2))^2+9*(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2
))+55+17*10^(1/2))^(1/2)))
```

**maxima [B]** time = 1.10, size = 668, normalized size = 3.84

$$-\frac{1}{60}\sqrt{10}\left(\frac{588\sqrt{10}x}{17\sqrt{10}\sqrt{2x^2+3x+1}+55\sqrt{2x^2+3x+1}}-\frac{588\sqrt{10}x}{17\sqrt{10}\sqrt{2x^2+3x+1}-55\sqrt{2x^2+3x+1}}+\frac{2112x}{17\sqrt{10}\sqrt{2x^2+3x+1}+55\sqrt{2x^2+3x+1}}+\frac{2112x}{17\sqrt{10}\sqrt{2x^2+3x+1}-55\sqrt{2x^2+3x+1}}-\frac{27\sqrt{10}\log\left(\frac{2}{9}\sqrt{10}+\frac{2\sqrt{2x^2+3x+1}\sqrt{17\sqrt{10}+55}}{3\left|6x-2\sqrt{10}-4\right|}+\frac{34\sqrt{10}}{9\left|6x-2\sqrt{10}-4\right|}+\frac{110}{9\left|6x-2\sqrt{10}-4\right|}+\frac{17}{18}\right)}{\left(17\sqrt{10}+55\right)^{\frac{3}{2}}}-\frac{\sqrt{10}\log\left(-\frac{2}{9}\sqrt{10}+\frac{2\sqrt{2x^2+3x+1}\sqrt{-\frac{17}{9}\sqrt{10}+\frac{55}{9}}}{\left|6x+2\sqrt{10}-4\right|}-\frac{34\sqrt{10}}{9\left|6x+2\sqrt{10}-4\right|}+\frac{110}{9\left|6x+2\sqrt{10}-4\right|}+\frac{17}{18}\right)}{\left(-\frac{17}{9}\sqrt{10}+\frac{55}{9}\right)^{\frac{3}{2}}}+\frac{450\sqrt{10}}{17\sqrt{10}\sqrt{2x^2+3x+1}+55\sqrt{2x^2+3x+1}}-\frac{450\sqrt{10}}{17\sqrt{10}\sqrt{2x^2+3x+1}-55\sqrt{2x^2+3x+1}}-\frac{216\log\left(\frac{2}{9}\sqrt{10}+\frac{2\sqrt{2x^2+3x+1}\sqrt{17\sqrt{10}+55}}{3\left|6x-2\sqrt{10}-4\right|}+\frac{34\sqrt{10}}{9\left|6x-2\sqrt{10}-4\right|}+\frac{110}{9\left|6x-2\sqrt{10}-4\right|}+\frac{17}{18}\right)}{\left(17\sqrt{10}+55\right)^{\frac{3}{2}}}+\frac{8\log\left(-\frac{2}{9}\sqrt{10}+\frac{2\sqrt{2x^2+3x+1}\sqrt{-\frac{17}{9}\sqrt{10}+\frac{55}{9}}}{\left|6x+2\sqrt{10}-4\right|}-\frac{34\sqrt{10}}{9\left|6x+2\sqrt{10}-4\right|}+\frac{110}{9\left|6x+2\sqrt{10}-4\right|}+\frac{17}{18}\right)}{\left(-\frac{17}{9}\sqrt{10}+\frac{55}{9}\right)^{\frac{3}{2}}}+\frac{1656}{17\sqrt{10}\sqrt{2x^2+3x+1}+55\sqrt{2x^2+3x+1}}+\frac{1656}{17\sqrt{10}\sqrt{2x^2+3x+1}-55\sqrt{2x^2+3x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/60*sqrt(10)*(588*sqrt(10)*x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sqrt
(2*x^2 + 3*x + 1)) - 588*sqrt(10)*x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 55
*sqrt(2*x^2 + 3*x + 1)) + 2112*x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sq
rt(2*x^2 + 3*x + 1)) + 2112*x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(
2*x^2 + 3*x + 1)) - 27*sqrt(10)*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x + 1
)*sqrt(17*sqrt(10) + 55)/abs(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/abs(6*x
- 2*sqrt(10) - 4) + 110/9/abs(6*x - 2*sqrt(10) - 4) + 17/18)/(17*sqrt(10) +
 55)^(3/2) - sqrt(10)*log(-2/9*sqrt(10) + 2*sqrt(2*x^2 + 3*x + 1)*sqrt(-17/
9*sqrt(10) + 55/9)/abs(6*x + 2*sqrt(10) - 4) - 34/9*sqrt(10)/abs(6*x + 2*sq
rt(10) - 4) + 110/9/abs(6*x + 2*sqrt(10) - 4) + 17/18)/(-17/9*sqrt(10) + 55
/9)^(3/2) + 450*sqrt(10)/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sqrt(2*x^2
 + 3*x + 1)) - 450*sqrt(10)/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(2*
x^2 + 3*x + 1)) - 216*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x + 1)*sqrt(17*
sqrt(10) + 55)/abs(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/abs(6*x - 2*sqrt(1
0) - 4) + 110/9/abs(6*x - 2*sqrt(10) - 4) + 17/18)/(17*sqrt(10) + 55)^(3/2)
 + 8*log(-2/9*sqrt(10) + 2*sqrt(2*x^2 + 3*x + 1)*sqrt(-17/9*sqrt(10) + 55/9
)/abs(6*x + 2*sqrt(10) - 4) - 34/9*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 110
/9/abs(6*x + 2*sqrt(10) - 4) + 17/18)/(-17/9*sqrt(10) + 55/9)^(3/2) + 1656/
(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sqrt(2*x^2 + 3*x + 1)) + 1656/(17*s
qrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(2*x^2 + 3*x + 1)))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+2}{\left(2\,x^2+3\,x+1\right)^{3/2}\,\left(-3\,x^2+4\,x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/((3*x + 2*x^2 + 1)^(3/2)*(4*x - 3*x^2 + 2)),x)
```

```
[Out] int((x + 2)/((3*x + 2*x^2 + 1)^(3/2)*(4*x - 3*x^2 + 2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{6x^4\sqrt{2x^2+3x+1}+x^3\sqrt{2x^2+3x+1}-13x^2\sqrt{2x^2+3x+1}-10x\sqrt{2x^2+3x+1}-2\sqrt{2x^2+3x+1}}\,dx-\int \frac{2}{6x^4\sqrt{2x^2+3x+1}+x^3\sqrt{2x^2+3x+1}-13x^2\sqrt{2x^2+3x+1}-10x\sqrt{2x^2+3x+1}-2\sqrt{2x^2+3x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x**2+4*x+2)/(2*x**2+3*x+1)**(3/2),x)
```

```
[Out] -Integral(x/(6*x**4*sqrt(2*x**2 + 3*x + 1) + x**3*sqrt(2*x**2 + 3*x + 1) -
13*x**2*sqrt(2*x**2 + 3*x + 1) - 10*x*sqrt(2*x**2 + 3*x + 1) - 2*sqrt(2*x**
2 + 3*x + 1)), x) - Integral(2/(6*x**4*sqrt(2*x**2 + 3*x + 1) + x**3*sqrt(2
*x**2 + 3*x + 1) - 13*x**2*sqrt(2*x**2 + 3*x + 1) - 10*x*sqrt(2*x**2 + 3*x
+ 1) - 2*sqrt(2*x**2 + 3*x + 1)), x)
```

## 3.30 $\int \frac{2+x}{\left(2+4x-3x^2\right)\left(1+3x+2x^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=197

$$\frac{2(22x+21)}{15\left(2x^2+3x+1\right)^{3/2}}+\frac{2(230x+273)}{15\sqrt{2x^2+3x+1}}-\frac{1}{50}\sqrt{\frac{1}{3}\left(4885115+1544809\sqrt{10}\right)}\tanh^{-1}\left(\frac{\left(17-4\sqrt{10}\right)x+3\left(4-\sqrt{\right.}}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x}}\right.$$

**Rubi** [A] time = 0.30, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1016, 1060, 1032, 724, 206}

$$\frac{2(22x+21)}{15\left(2x^2+3x+1\right)^{3/2}}+\frac{2(230x+273)}{15\sqrt{2x^2+3x+1}}-\frac{1}{50}\sqrt{\frac{1}{3}\left(4885115+1544809\sqrt{10}\right)}\tanh^{-1}\left(\frac{\left(17-4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)+\frac{1}{50}\sqrt{\frac{1}{3}\left(4885115-1544809\sqrt{10}\right)}\tanh^{-1}\left(\frac{\left(17+4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{2\sqrt{55+17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x + 2*x^2)^(5/2)),x]
```

```
[Out] (2*(21 + 22*x))/(15*(1 + 3*x + 2*x^2)^(3/2)) + (2*(273 + 230*x))/(15*Sqrt[1
 + 3*x + 2*x^2]) - (Sqrt[(4885115 + 1544809*Sqrt[10])/3]*ArcTanh[(3*(4 - Sq
rt[10]) + (17 - 4*Sqrt[10])*x)/(2*Sqrt[55 - 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x
^2])])/50 + (Sqrt[(4885115 - 1544809*Sqrt[10])/3]*ArcTanh[(3*(4 + Sqrt[10])
 + (17 + 4*Sqrt[10])*x)/(2*Sqrt[55 + 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/
50
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1016

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)
*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*
c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))
```

```
 - h*(b*c*d - 2*a*c*e + a*b*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d -
a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e
))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

<u>Rule 1032</u>

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

<u>Rule 1060</u>

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
```

```
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{5/2}}\,dx = \frac{2(21+22x)}{15(1+3x+2x^2)^{3/2}} - \frac{2}{45}\int \frac{-480-\frac{813x}{2}+396x^2}{(2+4x-3x^2)(1+3x+2x^2)^{3/2}}\,dx$$

$$= \frac{2(21+22x)}{15(1+3x+2x^2)^{3/2}} + \frac{2(273+230x)}{15\sqrt{1+3x+2x^2}} + \frac{4}{675}\int \frac{\frac{23355}{2} - 27\ldots}{(2+4x-3x^2)\sqrt{1\ldots}}$$

$$= \frac{2(21+22x)}{15(1+3x+2x^2)^{3/2}} + \frac{2(273+230x)}{15\sqrt{1+3x+2x^2}} - \frac{1}{25}\left(3\left(335-106\sqrt{10}\right)\right)\int \ldots$$

$$= \frac{2(21+22x)}{15(1+3x+2x^2)^{3/2}} + \frac{2(273+230x)}{15\sqrt{1+3x+2x^2}} + \frac{1}{25}\left(6\left(335-106\sqrt{10}\right)\right)\mathrm{Su}\ldots$$

$$= \frac{2(21+22x)}{15(1+3x+2x^2)^{3/2}} + \frac{2(273+230x)}{15\sqrt{1+3x+2x^2}} - \frac{1}{50}\sqrt{\frac{1}{3}\left(4885115+154480\ldots\right.}$$

**Mathematica [A]** time = 0.71, size = 190, normalized size = 0.96

$$\frac{1}{450}\left(\sqrt{55-17\sqrt{10}}\left(7289+2305\sqrt{10}\right)\tanh^{-1}\left(\frac{\left(4\sqrt{10}-17\right)x+3\left(\sqrt{10}-4\right)}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)-\sqrt{55+17\sqrt{10}}\left(2305\sqrt{10}-7289\right)\tanh^{-1}\left(\frac{-\left(\left(17+4\sqrt{10}\right)x\right)-3\left(4+\sqrt{10}\right)}{2\sqrt{55+17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)+\frac{60\left(460x^3+1236x^2+1071x+294\right)}{\left(2x^2+3x+1\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x + 2*x^2)^(5/2)),x]
```

```
[Out] ((60*(294 + 1071*x + 1236*x^2 + 460*x^3))/(1 + 3*x + 2*x^2)^(3/2) + Sqrt[55
 - 17*Sqrt[10]]*(7289 + 2305*Sqrt[10])*ArcTanh[(3*(-4 + Sqrt[10]) + (-17 +
4*Sqrt[10])*x)/(2*Sqrt[55 - 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])] - Sqrt[55
+ 17*Sqrt[10]]*(-7289 + 2305*Sqrt[10])*ArcTanh[(-3*(4 + Sqrt[10]) - (17 + 4
*Sqrt[10])*x)/(2*Sqrt[55 + 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/450
```

**IntegrateAlgebraic [A]** time = 0.97, size = 160, normalized size = 0.81

$$-\frac{1}{25}\sqrt{\frac{1}{3}\left(4885115+1544809\sqrt{10}\right)}\tanh^{-1}\left(\frac{\sqrt{1-\sqrt{\frac{2}{5}}}\sqrt{2x^2+3x+1}}{2x+1}\right)+\frac{81\tanh^{-1}\left(\frac{\sqrt{1+\sqrt{\frac{2}{5}}}\sqrt{2x^2+3x+1}}{2x+1}\right)}{5\sqrt{5\left(4885115+1544809\sqrt{10}\right)}}+\frac{2\sqrt{2x^2+3x+1}\left(460x^3+1236x^2+1071x+294\right)}{15(x+1)^2(2x+1)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x + 2*x^2)^(5/2)),x]
```

```
[Out] (2*Sqrt[1 + 3*x + 2*x^2]*(294 + 1071*x + 1236*x^2 + 460*x^3))/(15*(1 + x)^2
*(1 + 2*x)^2) - (Sqrt[(4885115 + 1544809*Sqrt[10])/3]*ArcTanh[(Sqrt[1 - Sqr
t[2/5]]*Sqrt[1 + 3*x + 2*x^2])/(1 + 2*x)])/25 + (81*ArcTanh[(Sqrt[1 + Sqrt[
2/5]]*Sqrt[1 + 3*x + 2*x^2])/(1 + 2*x)])/(5*Sqrt[5*(4885115 + 1544809*Sqrt[
10])])
```

**fricas [B]** time = 0.43, size = 435, normalized size = 2.21

$$\frac{23520 x^4 + 70560 x^3 + \sqrt{3}\left(4 x^4 + 12 x^3 + 13 x^2 + 6 x + 1\right)\sqrt{1544809 \sqrt{10} + 4885115}\log\left(-\frac{243 \sqrt{10} x + \left(893 \sqrt{10}\sqrt{3} x - 2824 \sqrt{3} x\right)\sqrt{1544809 \sqrt{10} + 4885115} + 486 x - 486 \sqrt{2 x^2 + 3 x + 1} + 486}{x}\right) - \sqrt{3}\left(4 x^4 + 12 x^3 + 13 x^2 + 6 x + 1\right)\sqrt{1544809 \sqrt{10} + 4885115}\log\left(-\frac{243 \sqrt{10} x - \left(893 \sqrt{10}\sqrt{3} x - 2824 \sqrt{3} x\right)\sqrt{1544809 \sqrt{10} + 4885115} + 486 x - 486 \sqrt{2 x^2 + 3 x + 1} + 486}{x}\right) + \sqrt{3}\left(4 x^4 + 12 x^3 + 13 x^2 + 6 x + 1\right)\sqrt{-1544809 \sqrt{10} + 4885115}\log\left(\frac{243 \sqrt{10} x + \left(893 \sqrt{10}\sqrt{3} x + 2824 \sqrt{3} x\right)\sqrt{-1544809 \sqrt{10} + 4885115} - 486 x + 486 \sqrt{2 x^2 + 3 x + 1} - 486}{x}\right) - \sqrt{3}\left(4 x^4 + 12 x^3 + 13 x^2 + 6 x + 1\right)\sqrt{-1544809 \sqrt{10} + 4885115}\log\left(\frac{243 \sqrt{10} x - \left(893 \sqrt{10}\sqrt{3} x + 2824 \sqrt{3} x\right)\sqrt{-1544809 \sqrt{10} + 4885115} - 486 x + 486 \sqrt{2 x^2 + 3 x + 1} - 486}{x}\right) + 76440 x^2 + 20\left(460 x^3 + 1236 x^2 + 1071 x + 294\right)\sqrt{2 x^2 + 3 x + 1} + 35280 x + 5880}{150\left(4 x^4 + 12 x^3 + 13 x^2 + 6 x + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/150*(23520*x^4 + 70560*x^3 + sqrt(3)*(4*x^4 + 12*x^3 + 13*x^2 + 6*x + 1)*
sqrt(1544809*sqrt(10) + 4885115)*log(-(243*sqrt(10)*x + (893*sqrt(10)*sqrt(
3)*x - 2824*sqrt(3)*x)*sqrt(1544809*sqrt(10) + 4885115) + 486*x - 486*sqrt(
2*x^2 + 3*x + 1) + 486)/x) - sqrt(3)*(4*x^4 + 12*x^3 + 13*x^2 + 6*x + 1)*sq
rt(1544809*sqrt(10) + 4885115)*log(-(243*sqrt(10)*x - (893*sqrt(10)*sqrt(3)
*x - 2824*sqrt(3)*x)*sqrt(1544809*sqrt(10) + 4885115) + 486*x - 486*sqrt(2*
x^2 + 3*x + 1) + 486)/x) + sqrt(3)*(4*x^4 + 12*x^3 + 13*x^2 + 6*x + 1)*sqrt
(-1544809*sqrt(10) + 4885115)*log((243*sqrt(10)*x + (893*sqrt(10)*sqrt(3)*x
 + 2824*sqrt(3)*x)*sqrt(-1544809*sqrt(10) + 4885115) - 486*x + 486*sqrt(2*x
^2 + 3*x + 1) - 486)/x) - sqrt(3)*(4*x^4 + 12*x^3 + 13*x^2 + 6*x + 1)*sqrt(
-1544809*sqrt(10) + 4885115)*log((243*sqrt(10)*x - (893*sqrt(10)*sqrt(3)*x
+ 2824*sqrt(3)*x)*sqrt(-1544809*sqrt(10) + 4885115) - 486*x + 486*sqrt(2*x^
2 + 3*x + 1) - 486)/x) + 76440*x^2 + 20*(460*x^3 + 1236*x^2 + 1071*x + 294)
*sqrt(2*x^2 + 3*x + 1) + 35280*x + 5880)/(4*x^4 + 12*x^3 + 13*x^2 + 6*x + 1
)
```

**giac [A]** time = 0.54, size = 121, normalized size = 0.61

$$\frac{2\left(\left(4\left(115 x + 309\right) x + 1071\right) x + 294\right)}{15\left(2 x^2 + 3 x + 1\right)^{\frac{3}{2}}} + 0.00115890443050800 \log\left(-\sqrt{2} x + \sqrt{2 x^2 + 3 x + 1} + 5.90976932712000\right) - 36.0928986365333 \log\left(-\sqrt{2} x + \sqrt{2 x^2 + 3 x + 1} - 0.176527156327000\right) + 36.0928986365333 \log\left(-\sqrt{2} x + \sqrt{2 x^2 + 3 x + 1} - 0.919278730509000\right) - 0.00115890442528267 \log\left(-\sqrt{2} x + \sqrt{2 x^2 + 3 x + 1} - 1.04272727395000\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(5/2),x, algorithm="giac")
```

```
[Out] 2/15*((4*(115*x + 309)*x + 1071)*x + 294)/(2*x^2 + 3*x + 1)^(3/2) + 0.00115
890443050800*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) + 5.90976932712000) - 3
6.0928986365333*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) - 0.176527156327000)
 + 36.0928986365333*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) - 0.919278730509
```

```
000) - 0.00115890442528267*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) - 1.04272
727395000)
```

**maple [B]** time = 0.02, size = 878, normalized size = 4.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(5/2),x)
```

```
[Out] -1/20*(-8+10^(1/2))*10^(1/2)*(1/9/(55/9-17/9*10^(1/2))/(2*(x-2/3+1/3*10^(1/
2))^2+(17/3-4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))+55/9-17/9*10^(1/2))^(3/2)-1/
6*(17/3-4/3*10^(1/2))/(55/9-17/9*10^(1/2))*(2/3*(4*x+3)/(440/9-136/9*10^(1/
2)-(17/3-4/3*10^(1/2))^2)/(2*(x-2/3+1/3*10^(1/2))^2+(17/3-4/3*10^(1/2))*(x-
2/3+1/3*10^(1/2))+55/9-17/9*10^(1/2))^(3/2)+32/3/(440/9-136/9*10^(1/2)-(17/
3-4/3*10^(1/2))^2)^2*(4*x+3)/(2*(x-2/3+1/3*10^(1/2))^2+(17/3-4/3*10^(1/2))*
(x-2/3+1/3*10^(1/2))+55/9-17/9*10^(1/2))^(1/2))+1/3/(55/9-17/9*10^(1/2))*(1
/(55/9-17/9*10^(1/2))/(2*(x-2/3+1/3*10^(1/2))^2+(17/3-4/3*10^(1/2))*(x-2/3+
1/3*10^(1/2))+55/9-17/9*10^(1/2))^(1/2)-(17/3-4/3*10^(1/2))/(55/9-17/9*10^(
1/2))*(4*x+3)/(440/9-136/9*10^(1/2)-(17/3-4/3*10^(1/2))^2)/(2*(x-2/3+1/3*10
^(1/2))^2+(17/3-4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))+55/9-17/9*10^(1/2))^(1/2
)-3/(55/9-17/9*10^(1/2))/(55-17*10^(1/2))^(1/2)*arctanh(9/2*(110/9-34/9*10^
(1/2)+(17/3-4/3*10^(1/2))*(x-2/3+1/3*10^(1/2)))/(55-17*10^(1/2))^(1/2)/(18*
(x-2/3+1/3*10^(1/2))^2+9*(17/3-4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))+55-17*10^
(1/2))^(1/2))))-1/20*(8+10^(1/2))*10^(1/2)*(1/9/(55/9+17/9*10^(1/2))/(2*(x-
2/3-1/3*10^(1/2))^2+(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))+55/9+17/9*10^(
1/2))^(3/2)-1/6*(17/3+4/3*10^(1/2))/(55/9+17/9*10^(1/2))*(2/3*(4*x+3)/(440/
9+136/9*10^(1/2)-(17/3+4/3*10^(1/2))^2)/(2*(x-2/3-1/3*10^(1/2))^2+(17/3+4/3
*10^(1/2))*(x-2/3-1/3*10^(1/2))+55/9+17/9*10^(1/2))^(3/2)+32/3/(440/9+136/9
*10^(1/2)-(17/3+4/3*10^(1/2))^2)^2*(4*x+3)/(2*(x-2/3-1/3*10^(1/2))^2+(17/3+
4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))+55/9+17/9*10^(1/2))^(1/2))+1/3/(55/9+17/
9*10^(1/2))*(1/(55/9+17/9*10^(1/2))/(2*(x-2/3-1/3*10^(1/2))^2+(17/3+4/3*10^
(1/2))*(x-2/3-1/3*10^(1/2))+55/9+17/9*10^(1/2))^(1/2)-(17/3+4/3*10^(1/2))/(
55/9+17/9*10^(1/2))*(4*x+3)/(440/9+136/9*10^(1/2)-(17/3+4/3*10^(1/2))^2)/(2
*(x-2/3-1/3*10^(1/2))^2+(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))+55/9+17/9*
10^(1/2))^(1/2)-3/(55/9+17/9*10^(1/2))/(55+17*10^(1/2))^(1/2)*arctanh(9/2*(
110/9+34/9*10^(1/2)+(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2)))/(55+17*10^(1/
2))^(1/2)/(18*(x-2/3-1/3*10^(1/2))^2+9*(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1
/2))+55+17*10^(1/2))^(1/2))))
```

**maxima [B]** time = 1.24, size = 1276, normalized size = 6.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/300*sqrt(10)*(980*sqrt(10)*x/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) + 55*(
2*x^2 + 3*x + 1)^(3/2)) - 980*sqrt(10)*x/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/
2) - 55*(2*x^2 + 3*x + 1)^(3/2)) + 5292*sqrt(10)*x/(374*sqrt(10)*sqrt(2*x^2
 + 3*x + 1) + 1183*sqrt(2*x^2 + 3*x + 1)) - 5292*sqrt(10)*x/(374*sqrt(10)*s
qrt(2*x^2 + 3*x + 1) - 1183*sqrt(2*x^2 + 3*x + 1)) - 15680*sqrt(10)*x/(17*s
qrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sqrt(2*x^2 + 3*x + 1)) + 15680*sqrt(10)*
x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(2*x^2 + 3*x + 1)) + 3520*x/(
17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) + 55*(2*x^2 + 3*x + 1)^(3/2)) + 3520*x/
(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) - 55*(2*x^2 + 3*x + 1)^(3/2)) + 19008*
x/(374*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 1183*sqrt(2*x^2 + 3*x + 1)) + 19008
*x/(374*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 1183*sqrt(2*x^2 + 3*x + 1)) - 5632
0*x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sqrt(2*x^2 + 3*x + 1)) - 56320*
x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(2*x^2 + 3*x + 1)) + 750*sqrt
(10)/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) + 55*(2*x^2 + 3*x + 1)^(3/2)) - 7
50*sqrt(10)/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) - 55*(2*x^2 + 3*x + 1)^(3/
2)) + 4050*sqrt(10)/(374*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 1183*sqrt(2*x^2 +
 3*x + 1)) - 4050*sqrt(10)/(374*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 1183*sqrt(
2*x^2 + 3*x + 1)) - 11760*sqrt(10)/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*
sqrt(2*x^2 + 3*x + 1)) + 11760*sqrt(10)/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1)
- 55*sqrt(2*x^2 + 3*x + 1)) + 2760/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) + 5
5*(2*x^2 + 3*x + 1)^(3/2)) + 2760/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) - 55
*(2*x^2 + 3*x + 1)^(3/2)) + 14904/(374*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 118
3*sqrt(2*x^2 + 3*x + 1)) + 14904/(374*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 1183
*sqrt(2*x^2 + 3*x + 1)) - 42240/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sqr
t(2*x^2 + 3*x + 1)) - 42240/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(2*
x^2 + 3*x + 1)) - 1215*sqrt(10)*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x + 1
)*sqrt(17*sqrt(10) + 55)/abs(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/abs(6*x
- 2*sqrt(10) - 4) + 110/9/abs(6*x - 2*sqrt(10) - 4) + 17/18)/(17*sqrt(10) +
 55)^(5/2) - 5*sqrt(10)*log(-2/9*sqrt(10) + 2*sqrt(2*x^2 + 3*x + 1)*sqrt(-1
7/9*sqrt(10) + 55/9)/abs(6*x + 2*sqrt(10) - 4) - 34/9*sqrt(10)/abs(6*x + 2*
sqrt(10) - 4) + 110/9/abs(6*x + 2*sqrt(10) - 4) + 17/18)/(-17/9*sqrt(10) +
55/9)^(5/2) - 9720*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x + 1)*sqrt(17*sqr
t(10) + 55)/abs(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/abs(6*x - 2*sqrt(10)
- 4) + 110/9/abs(6*x - 2*sqrt(10) - 4) + 17/18)/(17*sqrt(10) + 55)^(5/2) +
40*log(-2/9*sqrt(10) + 2*sqrt(2*x^2 + 3*x + 1)*sqrt(-17/9*sqrt(10) + 55/9)/
abs(6*x + 2*sqrt(10) - 4) - 34/9*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 110/9
/abs(6*x + 2*sqrt(10) - 4) + 17/18)/(-17/9*sqrt(10) + 55/9)^(5/2))
```

**mupad** **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+2}{\left(2\,x^2+3\,x+1\right)^{5/2}\,\left(-3\,x^2+4\,x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/((3*x + 2*x^2 + 1)^(5/2)*(4*x - 3*x^2 + 2)),x)
```

```
[Out] int((x + 2)/((3*x + 2*x^2 + 1)^(5/2)*(4*x - 3*x^2 + 2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{12x^6\sqrt{2x^2+3x+1}+20x^5\sqrt{2x^2+3x+1}-17x^4\sqrt{2x^2+3x+1}-58x^3\sqrt{2x^2+3x+1}-47x^2\sqrt{2x^2+3x+1}-16x\sqrt{2x^2+3x+1}-2\sqrt{2x^2+3x+1}}\,dx-\int \frac{2}{12x^6\sqrt{2x^2+3x+1}+20x^5\sqrt{2x^2+3x+1}-17x^4\sqrt{2x^2+3x+1}-58x^3\sqrt{2x^2+3x+1}-47x^2\sqrt{2x^2+3x+1}-16x\sqrt{2x^2+3x+1}-2\sqrt{2x^2+3x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x**2+4*x+2)/(2*x**2+3*x+1)**(5/2),x)
```

```
[Out] -Integral(x/(12*x**6*sqrt(2*x**2 + 3*x + 1) + 20*x**5*sqrt(2*x**2 + 3*x + 1
) - 17*x**4*sqrt(2*x**2 + 3*x + 1) - 58*x**3*sqrt(2*x**2 + 3*x + 1) - 47*x*
*2*sqrt(2*x**2 + 3*x + 1) - 16*x*sqrt(2*x**2 + 3*x + 1) - 2*sqrt(2*x**2 + 3
*x + 1)), x) - Integral(2/(12*x**6*sqrt(2*x**2 + 3*x + 1) + 20*x**5*sqrt(2*
x**2 + 3*x + 1) - 17*x**4*sqrt(2*x**2 + 3*x + 1) - 58*x**3*sqrt(2*x**2 + 3*
x + 1) - 47*x**2*sqrt(2*x**2 + 3*x + 1) - 16*x*sqrt(2*x**2 + 3*x + 1) - 2*s
qrt(2*x**2 + 3*x + 1)), x)
```

## 3.31 $\int \frac{1+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx$

**Optimal**. Leaf size=15

$$-\tanh^{-1}\left(\sqrt{x^2+2x+5}\right)$$

**Rubi** [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1024, 206}

$$-\tanh^{-1}\left(\sqrt{x^2+2x+5}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]
```

```
[Out] -ArcTanh[Sqrt[5 + 2*x + x^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e -
 b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &&
 EqQ[h*e - 2*g*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{2-2x^2}\,dx, x, \sqrt{5+2x+x^2}\right)\right)\\ &= -\tanh^{-1}\left(\sqrt{5+2x+x^2}\right)\end{aligned}$$

**Mathematica** [C] time = 0.04, size = 79, normalized size = 5.27

$$\frac{1}{2}\left(-\tanh^{-1}\left(\frac{-i\sqrt{3}\,x-i\sqrt{3}\,+4}{\sqrt{x^2+2x+5}}\right)-\tanh^{-1}\left(\frac{i\sqrt{3}\,x+i\sqrt{3}\,+4}{\sqrt{x^2+2x+5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]
```

```
[Out] (-ArcTanh[(4 - I*Sqrt[3] - I*Sqrt[3]*x)/Sqrt[5 + 2*x + x^2]] - ArcTanh[(4 +
 I*Sqrt[3] + I*Sqrt[3]*x)/Sqrt[5 + 2*x + x^2]])/2
```

**IntegrateAlgebraic [A]** time = 0.30, size = 15, normalized size = 1.00

$$-\tanh^{-1}\left(\sqrt{x^2+2x+5}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]
```

```
[Out] -ArcTanh[Sqrt[5 + 2*x + x^2]]
```

**fricas [B]** time = 0.41, size = 49, normalized size = 3.27

$$\frac{1}{2}\log\left(x^2-\sqrt{x^2+2\,x+5}\,(x+2)+3\,x+6\right)-\frac{1}{2}\log\left(x^2-\sqrt{x^2+2\,x+5}\,x+x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*log(x^2 - sqrt(x^2 + 2*x + 5)*(x + 2) + 3*x + 6) - 1/2*log(x^2 - sqrt(x
^2 + 2*x + 5)*x + x + 4)
```

**giac [B]** time = 0.35, size = 31, normalized size = 2.07

$$-\frac{1}{2}\log\left(\sqrt{x^2+2\,x+5}+1\right)+\frac{1}{2}\log\left(\sqrt{x^2+2\,x+5}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(sqrt(x^2 + 2*x + 5) + 1) + 1/2*log(sqrt(x^2 + 2*x + 5) - 1)
```

**maple [A]** time = 0.02, size = 14, normalized size = 0.93

$$-\operatorname{arctanh}\left(\sqrt{x^2+2x+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x)
```

```
[Out] -arctanh((x^2+2*x+5)^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{x^2+2x+5}\left(x^2+2x+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="maxima")
[Out] integrate((x + 1)/(sqrt(x^2 + 2*x + 5)*(x^2 + 2*x + 4)), x)
```

**mupad [B]** time = 3.76, size = 13, normalized size = 0.87

$$-\mathrm{atanh}\left(\sqrt{x^2+2x+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((2*x + x^2 + 4)*(2*x + x^2 + 5)^(1/2)),x)
[Out] -atanh((2*x + x^2 + 5)^(1/2))
```

**sympy [B]** time = 7.00, size = 36, normalized size = 2.40

$$\frac{\log\left(-1+\frac{1}{\sqrt{x^2+2x+5}}\right)}{2}-\frac{\log\left(1+\frac{1}{\sqrt{x^2+2x+5}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**2+2*x+4)/(x**2+2*x+5)**(1/2),x)
[Out] log(-1 + 1/sqrt(x**2 + 2*x + 5))/2 - log(1 + 1/sqrt(x**2 + 2*x + 5))/2
```

## 3.32 $\int \frac{4+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx$

**Optimal**. Leaf size=44

$$\sqrt{3}\tan^{-1}\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)-\tanh^{-1}\left(\sqrt{x^2+2x+5}\right)$$

**Rubi [A]** time = 0.05, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1025, 982, 204, 1024, 206}

$$\sqrt{3}\tan^{-1}\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)-\tanh^{-1}\left(\sqrt{x^2+2x+5}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x)/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]
```

```
[Out] Sqrt[3]*ArcTan[(1 + x)/(Sqrt[3]*Sqrt[5 + 2*x + x^2])] - ArcTanh[Sqrt[5 + 2*
x + x^2]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e -
```

```
b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &&
 EqQ[h*e - 2*g*f, 0]
```

Rule 1025

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(h*e - 2*g*f)/(2*f), Int[1/(
(a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/(2*f), Int[(e + 2
*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e
 - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{4+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx &= \frac{1}{2}\int \frac{2+2x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx + 3\int \frac{1}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx \\ &= -\left(2\,\mathrm{Subst}\left(\int \frac{1}{2-2x^2}\,dx, x, \sqrt{5+2x+x^2}\right)\right) - 12\,\mathrm{Subst}\left(\int \frac{1}{-24-2x^2}\,dx, \right. \\ &= \sqrt{3}\,\tan^{-1}\left(\frac{1+x}{\sqrt{3}\sqrt{5+2x+x^2}}\right) - \tanh^{-1}\left(\sqrt{5+2x+x^2}\right)\end{aligned}$$

**Mathematica [C]** time = 0.05, size = 101, normalized size = 2.30

$$-\frac{1}{2}\left(1+i\sqrt{3}\right)\tanh^{-1}\left(\frac{-i\sqrt{3}\,x-i\sqrt{3}+4}{\sqrt{x^2+2x+5}}\right)-\frac{1}{2}\left(1-i\sqrt{3}\right)\tanh^{-1}\left(\frac{i\sqrt{3}\,x+i\sqrt{3}+4}{\sqrt{x^2+2x+5}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x)/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]
```

```
[Out] -1/2*((1 + I*Sqrt[3])*ArcTanh[(4 - I*Sqrt[3] - I*Sqrt[3]*x)/Sqrt[5 + 2*x +
x^2]]) - ((1 - I*Sqrt[3])*ArcTanh[(4 + I*Sqrt[3] + I*Sqrt[3]*x)/Sqrt[5 + 2*
x + x^2]])/2
```

**IntegrateAlgebraic [A]** time = 0.35, size = 71, normalized size = 1.61

$$-\sqrt{3}\,\tan^{-1}\left(\frac{x^2}{\sqrt{3}}-\frac{(x+1)\sqrt{x^2+2x+5}}{\sqrt{3}}+\frac{2x}{\sqrt{3}}+\frac{4}{\sqrt{3}}\right)-\tanh^{-1}\left(\sqrt{x^2+2x+5}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(4 + x)/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]
```

```
[Out] -(Sqrt[3]*ArcTan[4/Sqrt[3] + (2*x)/Sqrt[3] + x^2/Sqrt[3] - ((1 + x)*Sqrt[5
+ 2*x + x^2])/Sqrt[3]]) - ArcTanh[Sqrt[5 + 2*x + x^2]]
```

**fricas [B]** time = 0.41, size = 106, normalized size = 2.41

$$-\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}(x+2)+\frac{1}{3}\sqrt{3}\sqrt{x^2+2x+5}\right)+\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}x+\frac{1}{3}\sqrt{3}\sqrt{x^2+2x+5}\right)+\frac{1}{2}\log\left(x^2-\sqrt{x^2+2x+5}(x+2)+3x+6\right)-\frac{1}{2}\log\left(x^2-\sqrt{x^2+2x+5}x+x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan(-1/3*sqrt(3)*(x + 2) + 1/3*sqrt(3)*sqrt(x^2 + 2*x + 5)) + s
qrt(3)*arctan(-1/3*sqrt(3)*x + 1/3*sqrt(3)*sqrt(x^2 + 2*x + 5)) + 1/2*log(x
^2 - sqrt(x^2 + 2*x + 5)*(x + 2) + 3*x + 6) - 1/2*log(x^2 - sqrt(x^2 + 2*x
+ 5)*x + x + 4)
```

**giac [B]** time = 0.37, size = 108, normalized size = 2.45

$$-\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2+2x+5}+2\right)\right)+\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2+2x+5}\right)\right)+\frac{1}{2}\log\left(\left(x-\sqrt{x^2+2x+5}\right)^2+4x-4\sqrt{x^2+2x+5}+7\right)-\frac{1}{2}\log\left(\left(x-\sqrt{x^2+2x+5}\right)^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5) + 2)) + sqrt(3)*arcta
n(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5))) + 1/2*log((x - sqrt(x^2 + 2*x + 5
))^2 + 4*x - 4*sqrt(x^2 + 2*x + 5) + 7) - 1/2*log((x - sqrt(x^2 + 2*x + 5))
^2 + 3)
```

**maple [A]** time = 0.01, size = 40, normalized size = 0.91

$$-\operatorname{arctanh}\left(\sqrt{x^2+2x+5}\right)+\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\ (2x+2)}{6\sqrt{x^2+2x+5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x)
```

```
[Out] -arctanh((x^2+2*x+5)^(1/2))+3^(1/2)*arctan(1/6*3^(1/2)/(x^2+2*x+5)^(1/2)*(2
*x+2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+4}{\sqrt{x^2+2x+5}\left(x^2+2x+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="maxima")
[Out] integrate((x + 4)/(sqrt(x^2 + 2*x + 5)*(x^2 + 2*x + 4)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x+4}{\left(x^2+2\,x+4\right)\,\sqrt{x^2+2\,x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 4)/((2*x + x^2 + 4)*(2*x + x^2 + 5)^(1/2)),x)
[Out] int((x + 4)/((2*x + x^2 + 4)*(2*x + x^2 + 5)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+4}{\left(x^2+2x+4\right)\sqrt{x^2+2x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(x**2+2*x+4)/(x**2+2*x+5)**(1/2),x)
[Out] Integral((x + 4)/((x**2 + 2*x + 4)*sqrt(x**2 + 2*x + 5)), x)
```

## 3.33 $\int \frac{1+2x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx$

**Optimal**. Leaf size=24

$$-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{x^2+x+5}}{\sqrt{2}}\right)$$

**Rubi [A]** time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1024, 206}

$$-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{x^2+x+5}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)/((3 + x + x^2)*Sqrt[5 + x + x^2]),x]
```

```
[Out] -(Sqrt[2]*ArcTanh[Sqrt[5 + x + x^2]/Sqrt[2]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e -
 b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &&
 EqQ[h*e - 2*g*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{5+x+x^2}\right)\right)\\ &= -\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{5+x+x^2}}{\sqrt{2}}\right)\end{aligned}$$

**Mathematica [C]** time = 0.06, size = 90, normalized size = 3.75

$$-\frac{\tanh^{-1}\left(\frac{-2i\sqrt{11}\,x-i\sqrt{11}+19}{4\sqrt{2}\,\sqrt{x^2+x+5}}\right)+\tanh^{-1}\left(\frac{2i\sqrt{11}\,x+i\sqrt{11}+19}{4\sqrt{2}\,\sqrt{x^2+x+5}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)/((3 + x + x^2)*Sqrt[5 + x + x^2]),x]
```

```
[Out] -((ArcTanh[(19 - I*Sqrt[11] - (2*I)*Sqrt[11]*x)/(4*Sqrt[2]*Sqrt[5 + x + x^2
])] + ArcTanh[(19 + I*Sqrt[11] + (2*I)*Sqrt[11]*x)/(4*Sqrt[2]*Sqrt[5 + x +
x^2])])/Sqrt[2])
```

**IntegrateAlgebraic [A]** time = 0.37, size = 24, normalized size = 1.00

$$-\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{x^2+x+5}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 2*x)/((3 + x + x^2)*Sqrt[5 + x + x^2]),x]
```

```
[Out] -(Sqrt[2]*ArcTanh[Sqrt[5 + x + x^2]/Sqrt[2]])
```

**fricas [A]** time = 0.42, size = 34, normalized size = 1.42

$$\frac{1}{2}\,\sqrt{2}\,\log\left(\frac{x^2-2\,\sqrt{2}\,\sqrt{x^2+x+5}+x+7}{x^2+x+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+x+3)/(x^2+x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log((x^2 - 2*sqrt(2)*sqrt(x^2 + x + 5) + x + 7)/(x^2 + x + 3))
```

**giac [B]** time = 0.36, size = 39, normalized size = 1.62

$$-\frac{1}{2}\,\sqrt{2}\,\log\left(\sqrt{2}+\sqrt{x^2+x+5}\right)+\frac{1}{2}\,\sqrt{2}\,\log\left(-\sqrt{2}+\sqrt{x^2+x+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+x+3)/(x^2+x+5)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(sqrt(2) + sqrt(x^2 + x + 5)) + 1/2*sqrt(2)*log(-sqrt(2) +
sqrt(x^2 + x + 5))
```

**maple [A]** time = 0.02, size = 20, normalized size = 0.83

$$-\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{x^2+x+5}\ \sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)/(x^2+x+3)/(x^2+x+5)^(1/2),x)
```

```
[Out] -arctanh(1/2*(x^2+x+5)^(1/2)*2^(1/2))*2^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+1}{\sqrt{x^2+x+5}\left(x^2+x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+x+3)/(x^2+x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x + 1)/(sqrt(x^2 + x + 5)*(x^2 + x + 3)), x)
```

**mupad [B]** time = 3.78, size = 19, normalized size = 0.79

$$-\sqrt{2}\ \operatorname{atanh}\left(\frac{\sqrt{2}\ \sqrt{x^2+x+5}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)/((x + x^2 + 3)*(x + x^2 + 5)^(1/2)),x)
```

```
[Out] -2^(1/2)*atanh((2^(1/2)*(x + x^2 + 5)^(1/2))/2)
```

**sympy [A]** time = 6.81, size = 68, normalized size = 2.83

$$2\left(\begin{cases} -\frac{\sqrt{2}\operatorname{acoth}\left(\frac{\sqrt{2}}{\sqrt{x^2+x+5}}\right)}{2} & \text{for}\ \frac{1}{x^2+x+5} > \frac{1}{2} \\ -\frac{\sqrt{2}\operatorname{atanh}\left(\frac{\sqrt{2}}{\sqrt{x^2+x+5}}\right)}{2} & \text{for}\ \frac{1}{x^2+x+5} < \frac{1}{2} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x**2+x+3)/(x**2+x+5)**(1/2),x)
```

```
[Out] 2*Piecewise((-sqrt(2)*acoth(sqrt(2)/sqrt(x**2 + x + 5))/2, 1/(x**2 + x + 5)
 > 1/2), (-sqrt(2)*atanh(sqrt(2)/sqrt(x**2 + x + 5))/2, 1/(x**2 + x + 5) <
1/2))
```

## 3.34 $\int \frac{x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx$

**Optimal**. Leaf size=56

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{2}{11}}(2x+1)}{\sqrt{x^2+x+5}}\right)}{\sqrt{22}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+x+5}}{\sqrt{2}}\right)}{\sqrt{2}}$$

**Rubi** [A] time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1025, 982, 204, 1024, 206}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{2}{11}}(2x+1)}{\sqrt{x^2+x+5}}\right)}{\sqrt{22}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+x+5}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((3 + x + x^2)*Sqrt[5 + x + x^2]),x]
```

```
[Out] -(ArcTan[(Sqrt[2/11]*(1 + 2*x))/Sqrt[5 + x + x^2]]/Sqrt[22]) - ArcTanh[Sqrt
[5 + x + x^2]/Sqrt[2]]/Sqrt[2]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e -
 b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &&
 EqQ[h*e - 2*g*f, 0]
```

Rule 1025

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(h*e - 2*g*f)/(2*f), Int[1/(
(a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/(2*f), Int[(e + 2
*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e
 - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(3+x+x^2\right)\sqrt{5+x+x^2}}\,dx &= -\left(\frac{1}{2}\int \frac{1}{\left(3+x+x^2\right)\sqrt{5+x+x^2}}\,dx\right)+\frac{1}{2}\int \frac{1+2x}{\left(3+x+x^2\right)\sqrt{5+x+x^2}}\,dx\\ &= \text{Subst}\left(\int \frac{1}{-11-2x^2}\,dx, x, \frac{1+2x}{\sqrt{5+x+x^2}}\right)-\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{5+x+\right.\\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{\frac{2}{11}}(1+2x)}{\sqrt{5+x+x^2}}\right)}{\sqrt{22}}-\frac{\tanh^{-1}\left(\frac{\sqrt{5+x+x^2}}{\sqrt{2}}\right)}{\sqrt{2}}\end{aligned}$$

**Mathematica [C]** time = 0.06, size = 114, normalized size = 2.04

$$\frac{-\left(\left(\sqrt{11}-i\right)\tanh^{-1}\left(\frac{-2i\sqrt{11}x-i\sqrt{11}+19}{4\sqrt{2}\sqrt{x^2+x+5}}\right)\right)-\left(\sqrt{11}+i\right)\tanh^{-1}\left(\frac{2i\sqrt{11}x+i\sqrt{11}+19}{4\sqrt{2}\sqrt{x^2+x+5}}\right)}{2\sqrt{22}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((3 + x + x^2)*Sqrt[5 + x + x^2]),x]
```

```
[Out] (-((-I + Sqrt[11])*ArcTanh[(19 - I*Sqrt[11] - (2*I)*Sqrt[11]*x)/(4*Sqrt[2]*
Sqrt[5 + x + x^2])]) - (I + Sqrt[11])*ArcTanh[(19 + I*Sqrt[11] + (2*I)*Sqrt
[11]*x)/(4*Sqrt[2]*Sqrt[5 + x + x^2])])/(2*Sqrt[22])
```

**IntegrateAlgebraic [C]** time = 0.24, size = 92, normalized size = 1.64

$$\text{RootSum}\left[\#1^4-2\#1^3+3\#1^2-2\#1+23\&,\frac{\#1^2\log\left(-\#1+\sqrt{x^2+x+5}-x\right)-5\log\left(-\#1+\sqrt{x^2+x+5}-x\right)}{2\#1^3-3\#1^2+3\#1-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((3 + x + x^2)*Sqrt[5 + x + x^2]),x]
```

```
[Out] RootSum[23 - 2*#1 + 3*#1^2 - 2*#1^3 + #1^4 & , (-5*Log[-x + Sqrt[5 + x + x^
2] - #1] + Log[-x + Sqrt[5 + x + x^2] - #1]*#1^2)/(-1 + 3*#1 - 3*#1^2 + 2*#
1^3) & ]
```

**fricas [B]** time = 0.45, size = 307, normalized size = 5.48

$$-\frac{1}{33}\sqrt{11}\sqrt{6}\sqrt{3}\arctan\left(\frac{2}{33}\sqrt{11}\sqrt{3}\sqrt{\sqrt{6}\sqrt{3}(2x+1)+6x^2-\sqrt{x^2+x+5}\left(2\sqrt{6}\sqrt{3}+6x+3\right)+6x+30}+\frac{1}{33}\sqrt{11}\left(2\sqrt{6}\sqrt{3}+6x+3\right)-\frac{2}{11}\sqrt{11}\sqrt{x^2+x+5}\right)+\frac{1}{33}\sqrt{11}\sqrt{6}\sqrt{3}\arctan\left(-\frac{1}{33}\sqrt{11}\left(2\sqrt{6}\sqrt{3}-6x-3\right)+\frac{1}{33}\sqrt{11}\sqrt{-12\sqrt{6}\sqrt{3}(2x+1)+72x^2+12\sqrt{x^2+x+5}\left(2\sqrt{6}\sqrt{3}-6x-3\right)+72x+360}-\frac{2}{11}\sqrt{11}\sqrt{x^2+x+5}\right)+\frac{1}{12}\sqrt{6}\sqrt{3}\log\left(12\sqrt{6}\sqrt{3}(2x+1)+72x^2-12\sqrt{x^2+x+5}\left(2\sqrt{6}\sqrt{3}+6x+3\right)+72x+360\right)-\frac{1}{12}\sqrt{6}\sqrt{3}\log\left(-12\sqrt{6}\sqrt{3}(2x+1)+72x^2+12\sqrt{x^2+x+5}\left(2\sqrt{6}\sqrt{3}-6x-3\right)+72x+360\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+3)/(x^2+x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/33*sqrt(11)*sqrt(6)*sqrt(3)*arctan(2/33*sqrt(11)*sqrt(3)*sqrt(sqrt(6)*sq
rt(3)*(2*x + 1) + 6*x^2 - sqrt(x^2 + x + 5)*(2*sqrt(6)*sqrt(3) + 6*x + 3) +
 6*x + 30) + 1/33*sqrt(11)*(2*sqrt(6)*sqrt(3) + 6*x + 3) - 2/11*sqrt(11)*sq
rt(x^2 + x + 5)) + 1/33*sqrt(11)*sqrt(6)*sqrt(3)*arctan(-1/33*sqrt(11)*(2*s
qrt(6)*sqrt(3) - 6*x - 3) + 1/33*sqrt(11)*sqrt(-12*sqrt(6)*sqrt(3)*(2*x + 1
) + 72*x^2 + 12*sqrt(x^2 + x + 5)*(2*sqrt(6)*sqrt(3) - 6*x - 3) + 72*x + 36
0) - 2/11*sqrt(11)*sqrt(x^2 + x + 5)) + 1/12*sqrt(6)*sqrt(3)*log(12*sqrt(6)
*sqrt(3)*(2*x + 1) + 72*x^2 - 12*sqrt(x^2 + x + 5)*(2*sqrt(6)*sqrt(3) + 6*x
 + 3) + 72*x + 360) - 1/12*sqrt(6)*sqrt(3)*log(-12*sqrt(6)*sqrt(3)*(2*x + 1
) + 72*x^2 + 12*sqrt(x^2 + x + 5)*(2*sqrt(6)*sqrt(3) - 6*x - 3) + 72*x + 36
0)
```

**giac [B]** time = 0.32, size = 133, normalized size = 2.38

$$\frac{1}{22}\sqrt{11}\sqrt{2}\arctan\left(-\frac{1}{11}\sqrt{11}\left(2x+2\sqrt{2}-2\sqrt{x^2+x+5}+1\right)\right)-\frac{1}{22}\sqrt{11}\sqrt{2}\arctan\left(-\frac{1}{11}\sqrt{11}\left(2x-2\sqrt{2}-2\sqrt{x^2+x+5}+1\right)\right)+\frac{1}{4}\sqrt{2}\log\left(324\left(2x+2\sqrt{2}-2\sqrt{x^2+x+5}+1\right)^2+3564\right)-\frac{1}{4}\sqrt{2}\log\left(324\left(2x-2\sqrt{2}-2\sqrt{x^2+x+5}+1\right)^2+3564\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+3)/(x^2+x+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/22*sqrt(11)*sqrt(2)*arctan(-1/11*sqrt(11)*(2*x + 2*sqrt(2) - 2*sqrt(x^2 +
 x + 5) + 1)) - 1/22*sqrt(11)*sqrt(2)*arctan(-1/11*sqrt(11)*(2*x - 2*sqrt(2
) - 2*sqrt(x^2 + x + 5) + 1)) + 1/4*sqrt(2)*log(324*(2*x + 2*sqrt(2) - 2*sq
rt(x^2 + x + 5) + 1)^2 + 3564) - 1/4*sqrt(2)*log(324*(2*x - 2*sqrt(2) - 2*s
qrt(x^2 + x + 5) + 1)^2 + 3564)
```

**maple [A]** time = 0.01, size = 45, normalized size = 0.80

$$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{x^2+x+5}\,\sqrt{2}}{2}\right)}{2}-\frac{\sqrt{22}\,\arctan\left(\frac{(2x+1)\sqrt{22}}{11\sqrt{x^2+x+5}}\right)}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+x+3)/(x^2+x+5)^(1/2),x)
```

```
[Out] -1/2*arctanh(1/2*(x^2+x+5)^(1/2)*2^(1/2))*2^(1/2)-1/22*arctan(1/11*(1+2*x)*
22^(1/2)/(x^2+x+5)^(1/2))*22^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^2+x+5}\,(x^2+x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+3)/(x^2+x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(x^2 + x + 5)*(x^2 + x + 3)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{(x^2+x+3)\,\sqrt{x^2+x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x + x^2 + 3)*(x + x^2 + 5)^(1/2)),x)
```

```
[Out] int(x/((x + x^2 + 3)*(x + x^2 + 5)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(x^2+x+3)\,\sqrt{x^2+x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+x+3)/(x**2+x+5)**(1/2),x)
```

```
[Out] Integral(x/((x**2 + x + 3)*sqrt(x**2 + x + 5)), x)
```

## 3.35 $\int \frac{A+Bx}{\sqrt{d+ex+fx^2}\left(ae+bex+bfx^2\right)^2}\,dx$

**Optimal**. Leaf size=249

$$\frac{(Be-2Af)\left(8aef-b\left(4df+e^2\right)\right)\tanh^{-1}\left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{2e^{3/2}f(bd-ae)^{3/2}(be-4af)^{3/2}}-\frac{\sqrt{d+ex+fx^2}\,(e(Ab-2aB)-bx(Be-2Af))}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)}+$$

**Rubi [A]** time = 0.91, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {1016, 1025, 982, 208, 1024}

$$\frac{(Be-2Af)\left(8aef-b\left(4df+e^2\right)\right)\tanh^{-1}\left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{2e^{3/2}f(bd-ae)^{3/2}(be-4af)^{3/2}}-\frac{\sqrt{d+ex+fx^2}\,(e(Ab-2aB)-bx(Be-2Af))}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex+fx^2}}{\sqrt{bd-ae}}\right)}{2\sqrt{b}f(bd-ae)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(Sqrt[d + e*x + f*x^2]*(a*e + b*e*x + b*f*x^2)^2),x]
```

```
[Out] -((((A*b - 2*a*B)*e - b*(B*e - 2*A*f)*x)*Sqrt[d + e*x + f*x^2])/(e*(b*d - a
*e)*(b*e - 4*a*f)*(a*e + b*e*x + b*f*x^2))) + ((B*e - 2*A*f)*(8*a*e*f - b*(
e^2 + 4*d*f))*ArcTanh[(Sqrt[b*d - a*e]*(e + 2*f*x))/(Sqrt[e]*Sqrt[b*e - 4*a
*f]*Sqrt[d + e*x + f*x^2])])/(2*e^(3/2)*(b*d - a*e)^(3/2)*f*(b*e - 4*a*f)^(
3/2)) + (B*ArcTanh[(Sqrt[b]*Sqrt[d + e*x + f*x^2])/Sqrt[b*d - a*e]])/(2*Sqr
t[b]*(b*d - a*e)^(3/2)*f)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 1016

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)
*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*
```

```
c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))
 - h*(b*c*d - 2*a*c*e + a*b*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d -
a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e
))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e -
 b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &&
 EqQ[h*e - 2*g*f, 0]
```

Rule 1025

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(h*e - 2*g*f)/(2*f), Int[1/(
(a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/(2*f), Int[(e + 2
*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e
 - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

Rubi steps

$$\int \frac{A+Bx}{\sqrt{d+ex+fx^2}\left(ae+bex+bfx^2\right)^2}dx = -\frac{((Ab-2aB)e-b(Be-2Af)x)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)} - \frac{\int \frac{-\frac{1}{2}b(bd-ae)f^2\left(2bBde-\right.}{}}{}$$

$$= -\frac{((Ab-2aB)e-b(Be-2Af)x)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)} - \frac{B\int \frac{e+2fx}{\sqrt{d+ex+fx^2}\left(ae+be\right.}}{4(bd-ae)}$$

$$= -\frac{((Ab-2aB)e-b(Be-2Af)x)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)} + \frac{(Be)\,\mathrm{Subst}\left(\int \frac{}{bde-a}\right.}{2}$$

$$= -\frac{((Ab-2aB)e-b(Be-2Af)x)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)} + \frac{(Be-2Af)\left(8aef\right.}{2e}$$

**Mathematica [B]** time = 1.49, size = 767, normalized size = 3.08

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(Sqrt[d + e*x + f*x^2]*(a*e + b*e*x + b*f*x^2)^2),x]
```

```
[Out] -1/4*(4*b*Sqrt[e]*Sqrt[b*d - a*e]*f*Sqrt[b*e - 4*a*f]*Sqrt[d + x*(e + f*x)]
*(-(B*e*(2*a + b*x)) + A*b*(e + 2*f*x)) - (-(b^(3/2)*B*e^(5/2)*Sqrt[b*e - 4
*a*f]) + 4*a*Sqrt[b]*B*e^(3/2)*f*Sqrt[b*e - 4*a*f] - 8*a*b*e*f*(B*e - 2*A*f
) + b^2*(B*e - 2*A*f)*(e^2 + 4*d*f))*(a*e + b*x*(e + f*x))*Log[-(Sqrt[b]*Sq
rt[e]*Sqrt[b*e - 4*a*f]) + b*(e + 2*f*x)] + (b^(3/2)*B*e^(5/2)*Sqrt[b*e - 4
*a*f] - 4*a*Sqrt[b]*B*e^(3/2)*f*Sqrt[b*e - 4*a*f] - 8*a*b*e*f*(B*e - 2*A*f)
 + b^2*(B*e - 2*A*f)*(e^2 + 4*d*f))*(a*e + b*x*(e + f*x))*Log[Sqrt[b]*Sqrt[
e]*Sqrt[b*e - 4*a*f] + b*(e + 2*f*x)] - (b^(3/2)*B*e^(5/2)*Sqrt[b*e - 4*a*f
] - 4*a*Sqrt[b]*B*e^(3/2)*f*Sqrt[b*e - 4*a*f] - 8*a*b*e*f*(B*e - 2*A*f) + b
^2*(B*e - 2*A*f)*(e^2 + 4*d*f))*(a*e + b*x*(e + f*x))*Log[Sqrt[b]*(e^(3/2)*
Sqrt[b*e - 4*a*f] + Sqrt[b]*(e^2 - 4*d*f) + 2*Sqrt[e]*f*Sqrt[b*e - 4*a*f]*x
 - 4*Sqrt[b*d - a*e]*f*Sqrt[d + x*(e + f*x)])] + (-(b^(3/2)*B*e^(5/2)*Sqrt[
b*e - 4*a*f]) + 4*a*Sqrt[b]*B*e^(3/2)*f*Sqrt[b*e - 4*a*f] - 8*a*b*e*f*(B*e
- 2*A*f) + b^2*(B*e - 2*A*f)*(e^2 + 4*d*f))*(a*e + b*x*(e + f*x))*Log[Sqrt[
b]*(e^(3/2)*Sqrt[b*e - 4*a*f] - Sqrt[b]*(e^2 - 4*d*f) + 2*Sqrt[e]*f*Sqrt[b*
e - 4*a*f]*x + 4*Sqrt[b*d - a*e]*f*Sqrt[d + x*(e + f*x)])])/(b*e^(3/2)*(b*d
 - a*e)^(3/2)*f*(b*e - 4*a*f)^(3/2)*(a*e + b*x*(e + f*x)))
```

**IntegrateAlgebraic [C]** time = 2.43, size = 1019, normalized size = 4.09

$$-\frac{\sqrt{fx^2+ex+d}\,(-Abe+2aBe+bBxe-2Abfx)}{e(ae-bd)(be-4af)\left(bfx^2+bex+ae\right)}-\frac{4B\mathrm{RootSum}\left[bf\#1^4-2be\sqrt{f}\#1^3+be^2\#1^2-2bdf\#1^2+4aef\#1^2-4ae^2\sqrt{f}\#1+2bde\sqrt{f}\#1+ae^3-bde^2+bd^2f\&,\frac{\log\left(-\sqrt{f}x-\#1+\sqrt{fx^2+ex+d}\right)}{-2bf\#1^3+3be\sqrt{f}\#1^2-be^2\#1+2bdf\#1-4aef\#1+2ae^2\sqrt{f}-bde\sqrt{f}}\&\right]}{b}-\frac{\mathrm{RootSum}\left[bf\#1^4-2be\sqrt{f}\#1^3+be^2\#1^2-2bdf\#1^2+4aef\#1^2-4ae^2\sqrt{f}\#1+2bde\sqrt{f}\#1+ae^3-bde^2+bd^2f\&,\frac{-Ab^2\log\left(-\sqrt{f}x-\#1+\sqrt{fx^2+ex+d}\right)e^3+6abB\log\left(-\sqrt{f}x-\#1+\sqrt{fx^2+ex+d}\right)e^3-b^2B\log\left(-\sqrt{f}x-\#1+\sqrt{fx^2+ex+d}\right)\#1^2e^2-5b^2Bd\log\left(-\sqrt{f}x-\#1+\sqrt{fx^2+ex+d}\right)e^2+8aAbf\log\left(-\sqrt{f}x-\#1+\sqrt{fx^2+ex+d}\right)e^2-32a^2Bf\log\left(-\sqrt{f}x-\#1+\sqrt{fx^2+ex+d}\right)e^2+2Ab^2\sqrt{f}\log\left(-\sqrt{f}x-\#1+\sqrt{fx^2+ex+d}\right)\#1e^2+4abB\sqrt{f}\log\left(-\sqrt{f}x-\#1+\sqrt{fx^2+ex+d}\right)\#1e^2+4abBf\log\left(-\sqrt{f}x-\#1+\sqrt{fx^2+ex+d}\right)\#1^2e-4Ab^2df\log\left(-\sqrt{f}x-\#1+\sqrt{fx^2+ex+d}\right)e+28abBdf\log\left(-\sqrt{f}x-\#1+\sqrt{fx^2+ex+d}\right)e-16aAbf^{3/2}\log\left(-\sqrt{f}x-\#1+\sqrt{fx^2+ex+d}\right)\#1e-4b^2Bd\sqrt{f}\log\left(-\sqrt{f}x-\#1+\sqrt{fx^2+ex+d}\right)\#1e+8Ab^2df^{3/2}\log\left(-\sqrt{f}x-\#1+\sqrt{fx^2+ex+d}\right)\#1}{-2bf\#1^3+3be\sqrt{f}\#1^2-be^2\#1+2bdf\#1-4aef\#1+2ae^2\sqrt{f}-bde\sqrt{f}}\&\right]}{2be(bd-ae)(be-4af)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x)/(Sqrt[d + e*x + f*x^2]*(a*e + b*e*x + b*f*x^2)
^2),x]
```

```
[Out] -(((-(A*b*e) + 2*a*B*e + b*B*e*x - 2*A*b*f*x)*Sqrt[d + e*x + f*x^2])/(e*(-(
b*d) + a*e)*(b*e - 4*a*f)*(a*e + b*e*x + b*f*x^2))) - (4*B*RootSum[-(b*d*e^
2) + a*e^3 + b*d^2*f + 2*b*d*e*Sqrt[f]*#1 - 4*a*e^2*Sqrt[f]*#1 + b*e^2*#1^2
 - 2*b*d*f*#1^2 + 4*a*e*f*#1^2 - 2*b*e*Sqrt[f]*#1^3 + b*f*#1^4 & , Log[-(Sq
rt[f]*x) + Sqrt[d + e*x + f*x^2] - #1]/(-(b*d*e*Sqrt[f]) + 2*a*e^2*Sqrt[f]
- b*e^2*#1 + 2*b*d*f*#1 - 4*a*e*f*#1 + 3*b*e*Sqrt[f]*#1^2 - 2*b*f*#1^3) & ]
)/b - RootSum[-(b*d*e^2) + a*e^3 + b*d^2*f + 2*b*d*e*Sqrt[f]*#1 - 4*a*e^2*S
qrt[f]*#1 + b*e^2*#1^2 - 2*b*d*f*#1^2 + 4*a*e*f*#1^2 - 2*b*e*Sqrt[f]*#1^3 +
 b*f*#1^4 & , (-5*b^2*B*d*e^2*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1
] - A*b^2*e^3*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1] + 6*a*b*B*e^3*
Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1] - 4*A*b^2*d*e*f*Log[-(Sqrt[f
]*x) + Sqrt[d + e*x + f*x^2] - #1] + 28*a*b*B*d*e*f*Log[-(Sqrt[f]*x) + Sqrt
[d + e*x + f*x^2] - #1] + 8*a*A*b*e^2*f*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f
*x^2] - #1] - 32*a^2*B*e^2*f*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1]
 - 4*b^2*B*d*e*Sqrt[f]*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1]*#1 +
2*A*b^2*e^2*Sqrt[f]*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1]*#1 + 4*a
*b*B*e^2*Sqrt[f]*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1]*#1 + 8*A*b^
2*d*f^(3/2)*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1]*#1 - 16*a*A*b*e*
f^(3/2)*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1]*#1 - b^2*B*e^2*Log[-
(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1]*#1^2 + 4*a*b*B*e*f*Log[-(Sqrt[f]*
x) + Sqrt[d + e*x + f*x^2] - #1]*#1^2)/(-(b*d*e*Sqrt[f]) + 2*a*e^2*Sqrt[f]
- b*e^2*#1 + 2*b*d*f*#1 - 4*a*e*f*#1 + 3*b*e*Sqrt[f]*#1^2 - 2*b*f*#1^3) & ]
/(2*b*e*(b*d - a*e)*(b*e - 4*a*f))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x, algorithm="f
ricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%%{1,[2]%%%},[8,2,0,0,0]%%%}+%%%{%%{[%%%{-4,[1]%%%},0]:[1,0,%%%{-1,
[1]%%%}]%%},[7,2,1,0,0]%%%}+%%%{%%%{6,[1]%%%},[6,2,2,0,0]%%%}+%%%{%%%{-4,[2
]%%%},[6,2,0,0,1]%%%}+%%%{%%%{8,[2]%%%},[6,1,1,1,0]%%%}+%%%{%%{[-4,0]:[1,0,
%%%{-1,[1]%%%}]%%},[5,2,3,0,0]%%%}+%%%{%%{[%%%{12,[1]%%%},0]:[1,0,%%%{-1,[1
]%%%}]%%},[5,2,1,0,1]%%%}+%%%{%%{[%%%{-24,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%
},[5,1,2,1,0]%%%}+%%%{1,[4,2,4,0,0]%%%}+%%%{%%%{-14,[1]%%%},[4,2,2,0,1]%%%}
+%%%{%%%{6,[2]%%%},[4,2,0,0,2]%%%}+%%%{%%%{26,[1]%%%},[4,1,3,1,0]%%%}+%%%{%
%%{-16,[2]%%%},[4,1,1,1,1]%%%}+%%%{%%%{16,[2]%%%},[4,0,2,2,0]%%%}+%%%{%%{[8
,0]:[1,0,%%%{-1,[1]%%%}]%%},[3,2,3,0,1]%%%}+%%%{%%{[%%%{-12,[1]%%%},0]:[1,0
,%%%{-1,[1]%%%}]%%},[3,2,1,0,2]%%%}+%%%{%%{[-12,0]:[1,0,%%%{-1,[1]%%%}]%%},
[3,1,4,1,0]%%%}+%%%{%%{[%%%{32,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,1,2,1,
1]%%%}+%%%{%%{[%%%{-32,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,0,3,2,0]%%%}+%
%%{-2,[2,2,4,0,1]%%%}+%%%{%%%{10,[1]%%%},[2,2,2,0,2]%%%}+%%%{%%%{-4,[2]%%%}
,[2,2,0,0,3]%%%}+%%%{2,[2,1,5,1,0]%%%}+%%%{%%%{-28,[1]%%%},[2,1,3,1,1]%%%}+
%%%{%%%{8,[2]%%%},[2,1,1,1,2]%%%}+%%%{%%%{24,[1]%%%},[2,0,4,2,0]%%%}+%%%{%%
{[-4,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,2,3,0,2]%%%}+%%%{%%{[%%%{4,[1]%%%},0]:[1
,0,%%%{-1,[1]%%%}]%%},[1,2,1,0,3]%%%}+%%%{%%{[12,0]:[1,0,%%%{-1,[1]%%%}]%%}
,[1,1,4,1,1]%%%}+%%%{%%{[%%%{-8,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,1,2,1
,2]%%%}+%%%{%%{[-8,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,0,5,2,0]%%%}+%%%{1,[0,2,4,
0,2]%%%}+%%%{%%%{-2,[1]%%%},[0,2,2,0,3]%%%}+%%%{%%%{1,[2]%%%},[0,2,0,0,4]%%
%}+%%%{-2,[0,1,5,1,1]%%%}+%%%{%%%{2,[1]%%%},[0,1,3,1,2]%%%}+%%%{1,[0,0,6,2,
0]%%%} / %%%{%%%{1,[3]%%%},[8,2,0,0,0]%%%}+%%%{%%{poly1[%%%{-4,[2]%%%},0]:[
1,0,%%%{-1,[1]%%%}]%%},[7,2,1,0,0]%%%}+%%%{%%%{6,[2]%%%},[6,2,2,0,0]%%%}+%%
%{%%%{-4,[3]%%%},[6,2,0,0,1]%%%}+%%%{%%%{8,[3]%%%},[6,1,1,1,0]%%%}+%%%{%%{p
oly1[%%%{-4,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,2,3,0,0]%%%}+%%%{%%{poly1
[%%%{12,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,2,1,0,1]%%%}+%%%{%%{poly1[%%%
{-24,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,1,2,1,0]%%%}+%%%{%%%{1,[1]%%%},[
4,2,4,0,0]%%%}+%%%{%%%{-14,[2]%%%},[4,2,2,0,1]%%%}+%%%{%%%{6,[3]%%%},[4,2,0
,0,2]%%%}+%%%{%%%{26,[2]%%%},[4,1,3,1,0]%%%}+%%%{%%%{-16,[3]%%%},[4,1,1,1,1
]%%%}+%%%{%%%{16,[3]%%%},[4,0,2,2,0]%%%}+%%%{%%{poly1[%%%{8,[1]%%%},0]:[1,0
,%%%{-1,[1]%%%}]%%},[3,2,3,0,1]%%%}+%%%{%%{poly1[%%%{-12,[2]%%%},0]:[1,0,%%
%{-1,[1]%%%}]%%},[3,2,1,0,2]%%%}+%%%{%%{poly1[%%%{-12,[1]%%%},0]:[1,0,%%%{-
1,[1]%%%}]%%},[3,1,4,1,0]%%%}+%%%{%%{poly1[%%%{32,[2]%%%},0]:[1,0,%%%{-1,[1
]%%%}]%%},[3,1,2,1,1]%%%}+%%%{%%{poly1[%%%{-32,[2]%%%},0]:[1,0,%%%{-1,[1]%%
%}]%%},[3,0,3,2,0]%%%}+%%%{%%%{-2,[1]%%%},[2,2,4,0,1]%%%}+%%%{%%%{10,[2]%%%
},[2,2,2,0,2]%%%}+%%%{%%%{-4,[3]%%%},[2,2,0,0,3]%%%}+%%%{%%%{2,[1]%%%},[2,1
,5,1,0]%%%}+%%%{%%%{-28,[2]%%%},[2,1,3,1,1]%%%}+%%%{%%%{8,[3]%%%},[2,1,1,1,
2]%%%}+%%%{%%%{24,[2]%%%},[2,0,4,2,0]%%%}+%%%{%%{poly1[%%%{-4,[1]%%%},0]:[1
```

```
,0,%%%{-1,[1]%%%}]%%},[1,2,3,0,2]%%%}+%%%{%%{poly1[%%%{4,[2]%%%},0]:[1,0,%%
%{-1,[1]%%%}]%%},[1,2,1,0,3]%%%}+%%%{%%{poly1[%%%{12,[1]%%%},0]:[1,0,%%%{-1
,[1]%%%}]%%},[1,1,4,1,1]%%%}+%%%{%%{poly1[%%%{-8,[2]%%%},0]:[1,0,%%%{-1,[1]
%%%}]%%},[1,1,2,1,2]%%%}+%%%{%%{poly1[%%%{-8,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}
]%%},[1,0,5,2,0]%%%}+%%%{%%%{1,[1]%%%},[0,2,4,0,2]%%%}+%%%{%%%{-2,[2]%%%},[
0,2,2,0,3]%%%}+%%%{%%%{1,[3]%%%},[0,2,0,0,4]%%%}+%%%{%%%{-2,[1]%%%},[0,1,5,
1,1]%%%}+%%%{%%%{2,[2]%%%},[0,1,3,1,2]%%%}+%%%{%%%{1,[1]%%%},[0,0,6,2,0]%%%
} Error: Bad Argument Value
```

**maple [B]** time = 0.05, size = 3606, normalized size = 14.48

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x)
```

```
[Out] -1/e/(4*a*f-b*e)/(a*e-b*d)/(x+1/2/f*e-1/2/b/f*(-b*e*(4*a*f-b*e))^(1/2))*((x
-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f+(-b*e*(4*a*f-b*e))^(1/2)/b*(x
-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d))^(1/2)*A+1/2/f/(4*a
*f-b*e)/(a*e-b*d)/(x+1/2/f*e-1/2/b/f*(-b*e*(4*a*f-b*e))^(1/2))*((x-1/2*(-b*
e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*
e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d))^(1/2)*B-1/2/f/e/(4*a*f-b*e)
/b/(a*e-b*d)/(x+1/2/f*e-1/2/b/f*(-b*e*(4*a*f-b*e))^(1/2))*((x-1/2*(-b*e+(-b
*e*(4*a*f-b*e))^(1/2))/b/f)^2*f+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b
*e*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d))^(1/2)*B*(-b*e*(4*a*f-b*e))^(1/2)
+1/2/e/(4*a*f-b*e)/b*(-b*e*(4*a*f-b*e))^(1/2)/(a*e-b*d)/(-1/b*(a*e-b*d))^(1
/2)*ln((-2/b*(a*e-b*d)+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f
-b*e))^(1/2))/b/f)+2*(-1/b*(a*e-b*d))^(1/2)*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e)
)^(1/2))/b/f)^2*f+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e)
)^(1/2))/b/f)-1/b*(a*e-b*d))^(1/2))/(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/
b/f))*A-1/4/f/(4*a*f-b*e)/b*(-b*e*(4*a*f-b*e))^(1/2)/(a*e-b*d)/(-1/b*(a*e-b
*d))^(1/2)*ln((-2/b*(a*e-b*d)+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e
*(4*a*f-b*e))^(1/2))/b/f)+2*(-1/b*(a*e-b*d))^(1/2)*((x-1/2*(-b*e+(-b*e*(4*a
*f-b*e))^(1/2))/b/f)^2*f+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a
*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d))^(1/2))/(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^
(1/2))/b/f))*B-1/4/f/b/(a*e-b*d)/(-1/b*(a*e-b*d))^(1/2)*ln((-2/b*(a*e-b*d)+
(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-
1/b*(a*e-b*d))^(1/2)*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f+(-b*e
*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-
b*d))^(1/2))/(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f))*B-2/e/(4*a*f-b*e)
/(-b*e*(4*a*f-b*e))^(1/2)/(-1/b*(a*e-b*d))^(1/2)*ln((-2/b*(a*e-b*d)+(-b*e*(
4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-1/b*(a*
e-b*d))^(1/2)*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f+(-b*e*(4*a*f
-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d))^(
1/2))/(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f))*A*f+1/(4*a*f-b*e)/(-b*e*
```

```
(4*a*f-b*e))^(1/2)/(-1/b*(a*e-b*d))^(1/2)*ln((-2/b*(a*e-b*d)+(-b*e*(4*a*f-b
*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-1/b*(a*e-b*d))
^(1/2)*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f+(-b*e*(4*a*f-b*e))^
(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d))^(1/2))/(
x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f))*B+2/e/(4*a*f-b*e)/(-b*e*(4*a*f-
b*e))^(1/2)/(-1/b*(a*e-b*d))^(1/2)*ln((-2/b*(a*e-b*d)-(-b*e*(4*a*f-b*e))^(1
/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-1/b*(a*e-b*d))^(1/2)*(
(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f-(-b*e*(4*a*f-b*e))^(1/2)/b*(
x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d))^(1/2))/(x+1/2*(b*e
+(-b*e*(4*a*f-b*e))^(1/2))/b/f))*A*f-1/(4*a*f-b*e)/(-b*e*(4*a*f-b*e))^(1/2)
/(-1/b*(a*e-b*d))^(1/2)*ln((-2/b*(a*e-b*d)-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/
2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-1/b*(a*e-b*d))^(1/2)*((x+1/2*(b*e
+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+
(-b*e*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d))^(1/2))/(x+1/2*(b*e+(-b*e*(4*a
*f-b*e))^(1/2))/b/f))*B-1/e/(4*a*f-b*e)/(a*e-b*d)/(x+1/2/f*e+1/2/b/f*(-b*e*
(4*a*f-b*e))^(1/2))*((x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f-(-b*e*(
4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d
))^(1/2)*A+1/2/f/(4*a*f-b*e)/(a*e-b*d)/(x+1/2/f*e+1/2/b/f*(-b*e*(4*a*f-b*e)
)^(1/2))*((x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f-(-b*e*(4*a*f-b*e))
^(1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d))^(1/2)*B+
1/2/f/e/(4*a*f-b*e)/b/(a*e-b*d)/(x+1/2/f*e+1/2/b/f*(-b*e*(4*a*f-b*e))^(1/2)
)*((x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f-(-b*e*(4*a*f-b*e))^(1/2)/
b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d))^(1/2)*B*(-b*e*(
4*a*f-b*e))^(1/2)-1/2/e/(4*a*f-b*e)/b*(-b*e*(4*a*f-b*e))^(1/2)/(a*e-b*d)/(-
1/b*(a*e-b*d))^(1/2)*ln((-2/b*(a*e-b*d)-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(
b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-1/b*(a*e-b*d))^(1/2)*((x+1/2*(b*e+(-
b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+(-b
*e*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d))^(1/2))/(x+1/2*(b*e+(-b*e*(4*a*f-
b*e))^(1/2))/b/f))*A+1/4/f/(4*a*f-b*e)/b*(-b*e*(4*a*f-b*e))^(1/2)/(a*e-b*d)
/(-1/b*(a*e-b*d))^(1/2)*ln((-2/b*(a*e-b*d)-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/
2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-1/b*(a*e-b*d))^(1/2)*((x+1/2*(b*e
+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+
(-b*e*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d))^(1/2))/(x+1/2*(b*e+(-b*e*(4*a
*f-b*e))^(1/2))/b/f))*B-1/4/f/b/(a*e-b*d)/(-1/b*(a*e-b*d))^(1/2)*ln((-2/b*(
a*e-b*d)-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b
/f)+2*(-1/b*(a*e-b*d))^(1/2)*((x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*
f-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-1/b
*(a*e-b*d))^(1/2))/(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f))*B
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx + A}{\left(bfx^2 + bex + ae\right)^2 \sqrt{fx^2 + ex + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate((B*x + A)/((b*f*x^2 + b*e*x + a*e)^2*sqrt(f*x^2 + e*x + d)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,x}{{\left(b\,f\,x^2+b\,e\,x+a\,e\right)}^2\,\sqrt{f\,x^2+e\,x+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/((a*e + b*e*x + b*f*x^2)^2*(d + e*x + f*x^2)^(1/2)),x)
```

```
[Out] int((A + B*x)/((a*e + b*e*x + b*f*x^2)^2*(d + e*x + f*x^2)^(1/2)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*f*x**2+b*e*x+a*e)**2/(f*x**2+e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

## 3.36 $\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{\left(ad+bdx+cdx^2\right)^2}\,dx$

**Optimal**. Leaf size=48

$$-\frac{2(-2ah+x(2cg-bh)+bg)}{d^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

**Rubi** [A] time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {998, 636}

$$-\frac{2(-2ah+x(2cg-bh)+bg)}{d^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)*Sqrt[a + b*x + c*x^2])/(a*d + b*d*x + c*d*x^2)^2,x]
```

```
[Out] (-2*(b*g - 2*a*h + (2*c*g - b*h)*x))/((b^2 - 4*a*c)*d^2*Sqrt[a + b*x + c*x^
2])
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 998

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(c/f)^p, Int[(g + h*
x)^m*(d + e*x + f*x^2)^(p + q), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p,
 q}, x] && EqQ[c*d - a*f, 0] && EqQ[b*d - a*e, 0] && (IntegerQ[p] || GtQ[c/
f, 0]) && ( !IntegerQ[q] || LeafCount[d + e*x + f*x^2] <= LeafCount[a + b*x
 + c*x^2])
```

Rubi steps

$$\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{\left(ad+bdx+cdx^2\right)^2}\,dx = \frac{\int \frac{g+hx}{\left(a+bx+cx^2\right)^{3/2}}\,dx}{d^2}$$
$$= -\frac{2(bg-2ah+(2cg-bh)x)}{\left(b^2-4ac\right)d^2\sqrt{a+bx+cx^2}}$$

**Mathematica** [A] time = 0.19, size = 46, normalized size = 0.96

$$\frac{4ah-2bg+2bhx-4cgx}{d^2\left(b^2-4ac\right)\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)*Sqrt[a + b*x + c*x^2])/(a*d + b*d*x + c*d*x^2)^2,x]
```

```
[Out] (-2*b*g + 4*a*h - 4*c*g*x + 2*b*h*x)/((b^2 - 4*a*c)*d^2*Sqrt[a + x*(b + c*x
)])
```

**IntegrateAlgebraic** [A] time = 0.42, size = 47, normalized size = 0.98

$$-\frac{2(-2ah+bg-bhx+2cgx)}{d^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((g + h*x)*Sqrt[a + b*x + c*x^2])/(a*d + b*d*x + c*d*x^2
)^2,x]
```

```
[Out] (-2*(b*g - 2*a*h + 2*c*g*x - b*h*x))/((b^2 - 4*a*c)*d^2*Sqrt[a + b*x + c*x^
2])
```

**fricas** [A] time = 0.63, size = 85, normalized size = 1.77

$$-\frac{2\sqrt{cx^2+bx+a}\left(bg-2\,ah+\left(2\,cg-bh\right)x\right)}{\left(b^2c-4\,ac^2\right)d^2x^2+\left(b^3-4\,abc\right)d^2x+\left(ab^2-4\,a^2c\right)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="f
ricas")
```

```
[Out] -2*sqrt(c*x^2 + b*x + a)*(b*g - 2*a*h + (2*c*g - b*h)*x)/((b^2*c - 4*a*c^2)
*d^2*x^2 + (b^3 - 4*a*b*c)*d^2*x + (a*b^2 - 4*a^2*c)*d^2)
```

**giac [A]** time = 0.27, size = 81, normalized size = 1.69

$$-\frac{2\left(\frac{\left(2\,cd^2g-bd^2h\right)x}{b^2d^4-4\,acd^4}+\frac{bd^2g-2\,ad^2h}{b^2d^4-4\,acd^4}\right)}{\sqrt{cx^2+bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="g
iac")
```

```
[Out] -2*((2*c*d^2*g - b*d^2*h)*x/(b^2*d^4 - 4*a*c*d^4) + (b*d^2*g - 2*a*d^2*h)/(
b^2*d^4 - 4*a*c*d^4))/sqrt(c*x^2 + b*x + a)
```

**maple [A]** time = 0.00, size = 48, normalized size = 1.00

$$-\frac{2\left(bhx-2cgx+2ah-bg\right)}{\sqrt{c\,x^2+bx+a}\,\left(4ac-b^2\right)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^2,x)
```

```
[Out] -2/(c*x^2+b*x+a)^(1/2)*(b*h*x-2*c*g*x+2*a*h-b*g)/d^2/(4*a*c-b^2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{cx^2+bx+a}\left(hx+g\right)}{\left(cdx^2+bdx+ad\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(h*x + g)/(c*d*x^2 + b*d*x + a*d)^2, x)
```

**mupad [B]** time = 3.75, size = 49, normalized size = 1.02

$$\frac{4\,a\,h-2\,b\,g+2\,b\,h\,x-4\,c\,g\,x}{\left(b^2\,d^2-4\,a\,c\,d^2\right)\,\sqrt{c\,x^2+b\,x+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)*(a + b*x + c*x^2)^(1/2))/(a*d + b*d*x + c*d*x^2)^2,x)
```

```
[Out] (4*a*h - 2*b*g + 2*b*h*x - 4*c*g*x)/((b^2*d^2 - 4*a*c*d^2)*(a + b*x + c*x^2
)^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{g}{a\sqrt{a+bx+cx^2}+bx\sqrt{a+bx+cx^2}+cx^2\sqrt{a+bx+cx^2}}\,dx+\int \frac{hx}{a\sqrt{a+bx+cx^2}+bx\sqrt{a+bx+cx^2}+cx^2\sqrt{a+bx+cx^2}}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x**2+b*x+a)**(1/2)/(c*d*x**2+b*d*x+a*d)**2,x)
```

```
[Out] (Integral(g/(a*sqrt(a + b*x + c*x**2) + b*x*sqrt(a + b*x + c*x**2) + c*x**2
*sqrt(a + b*x + c*x**2)), x) + Integral(h*x/(a*sqrt(a + b*x + c*x**2) + b*x
*sqrt(a + b*x + c*x**2) + c*x**2*sqrt(a + b*x + c*x**2)), x))/d**2
```

## 3.37 $\int \frac{3+2x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx$

**Optimal**. Leaf size=17

$$\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

**Rubi** **[A]** time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1027, 206}

$$\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x)/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] ArcTanh[x/Sqrt[-3 - 4*x - x^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1027

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[g, Subst[Int[1/(a + (c*d - a*f)
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
 EqQ[2*h*d - g*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{3+2x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx &= 3\,\text{Subst}\left(\int \frac{1}{3-3x^2}\,dx, x, \frac{x}{\sqrt{-3-4x-x^2}}\right)\\ &= \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)\end{aligned}$$

**Mathematica [C]** time = 0.29, size = 165, normalized size = 9.71

$$\frac{1}{6}\left(\sqrt{1-2i\sqrt{2}}\left(\sqrt{2}+i\right)\tanh^{-1}\left(\frac{\left(2-i\sqrt{2}\right)x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right)+\sqrt{1+2i\sqrt{2}}\left(\sqrt{2}-i\right)\tanh^{-1}\left(\frac{\left(2+i\sqrt{2}\right)x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x)/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] (Sqrt[1 - (2*I)*Sqrt[2]]*(I + Sqrt[2])*ArcTanh[(2 - (2*I)*Sqrt[2] + (2 - I*
Sqrt[2])*x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])] + Sqrt[1 + (2*I
)*Sqrt[2]]*(-I + Sqrt[2])*ArcTanh[(2 + (2*I)*Sqrt[2] + (2 + I*Sqrt[2])*x)/(
Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])])/6
```

**IntegrateAlgebraic [A]** time = 0.30, size = 17, normalized size = 1.00

$$\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 + 2*x)/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] ArcTanh[x/Sqrt[-3 - 4*x - x^2]]
```

**fricas [B]** time = 0.42, size = 56, normalized size = 3.29

$$-\frac{1}{4}\log\left(-\frac{2\sqrt{-x^2-4x-3}\,x+4x+3}{x^2}\right)+\frac{1}{4}\log\left(\frac{2\sqrt{-x^2-4x-3}\,x-4x-3}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) + 1/4*log((2*sqrt(-x^2
- 4*x - 3)*x - 4*x - 3)/x^2)
```

**giac [B]** time = 0.23, size = 98, normalized size = 5.76

$$\frac{1}{2}\log\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{3\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+1\right)-\frac{1}{2}\log\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)
^2/(x + 2)^2 + 1) - 1/2*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x
^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)
```

**maple [B]** time = 0.02, size = 94, normalized size = 5.53

$$-\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\,\operatorname{arctanh}\left(\frac{3x}{\left(-x-\frac{3}{2}\right)\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}}\right)}{6\sqrt{\frac{\frac{x^2}{\left(-x-\frac{3}{2}\right)^2}-4}{\left(\frac{x}{-x-\frac{3}{2}}+1\right)^2}}\,\left(\frac{x}{-x-\frac{3}{2}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+2*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)
```

```
[Out] -1/6*3^(1/2)*4^(1/2)/((x^2/(-3/2-x)^2-4)/(x/(-3/2-x)+1)^2)^(1/2)/(x/(-3/2-x
)+1)*(3*x^2/(-3/2-x)^2-12)^(1/2)*arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)
^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x+3}{\left(2\,x^2+4\,x+3\right)\sqrt{-x^2-4\,x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x + 3)/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{2\,x+3}{\sqrt{-x^2-4\,x-3}\,\left(2\,x^2+4\,x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 3)/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)
```

```
[Out] int((2*x + 3)/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+3}{\sqrt{-(x+1)(x+3)}\,\left(2x^2+4x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] Integral((2*x + 3)/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

## 3.38 $\int \frac{3+4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx$

**Optimal**. Leaf size=86

$$\sqrt{2}\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)-\sqrt{2}\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)+\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

**Rubi** [A] time = 0.18, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {1028, 986, 12, 1026, 1161, 618, 204, 1027, 206}

$$\sqrt{2}\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)-\sqrt{2}\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)+\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x)/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] Sqrt[2]*ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]] - Sqrt[2]*ArcTan
[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]] + ArcTanh[x/Sqrt[-3 - 4*x - x^
2]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 986

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
 b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1026

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 -
 4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1027

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[g, Subst[Int[1/(a + (c*d - a*f)
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
 EqQ[2*h*d - g*e, 0]
```

Rule 1028

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(2*h*d - g*e)/e, Int[1/((a + b
*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g
, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
 && NeQ[2*h*d - g*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
```

```
0]))
```

Rubi steps

$$\int \frac{3+4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx = -\left(3\int \frac{1}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx\right) - \int \frac{-6-4x}{\sqrt{-3-4x-x^2}\left(3+4x+\right.}$$

$$= \frac{1}{2}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx - \frac{1}{2}\int -\frac{4x}{\sqrt{-3-4x-x^2}\left(3+4x+\right.}$$

$$= 2\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + 2\int \frac{x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx - 3\,\mathrm{S}$$

$$= \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + 16\,\mathrm{Subst}\left(\int \frac{1+3x^2}{-4-8x^2-36x^4}\,dx, x, \frac{1+}{\sqrt{-3-4.}}\right.$$

$$= \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) - \frac{2}{3}\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2}\,dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right.$$

$$= \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{4}{3}\,\mathrm{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx, x, \frac{2}{3}\left(-1+\frac{3+}{\sqrt{-3-4x}}\right.\right.$$

$$= \sqrt{2}\,\tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) - \sqrt{2}\,\tan^{-1}\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) + \tanh^{-1}\left(\frac{}{\sqrt{-3-}}\right.$$

**Mathematica [C]** time = 0.11, size = 150, normalized size = 1.74

$$-\frac{1}{2}i\left(\sqrt{1+2i\sqrt{2}}\,\tanh^{-1}\left(\frac{\left(2-i\sqrt{2}\right)x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}}\,\sqrt{-x^2-4x-3}}\right)-\sqrt{1-2i\sqrt{2}}\,\tanh^{-1}\left(\frac{\left(2+i\sqrt{2}\right)x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}}\,\sqrt{-x^2-4x-3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x)/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] (-1/2*I)*(Sqrt[1 + (2*I)*Sqrt[2]]*ArcTanh[(2 - (2*I)*Sqrt[2] + (2 - I*Sqrt[
2])*x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])] - Sqrt[1 - (2*I)*Sqr
t[2]]*ArcTanh[(2 + (2*I)*Sqrt[2] + (2 + I*Sqrt[2])*x)/(Sqrt[2 - (4*I)*Sqrt[
2]]*Sqrt[-3 - 4*x - x^2])])
```

**IntegrateAlgebraic** [A] time = 0.35, size = 56, normalized size = 0.65

$$\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}x+\frac{3}{\sqrt{2}}}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 + 4*x)/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] -(Sqrt[2]*ArcTan[(3/Sqrt[2] + Sqrt[2]*x)/Sqrt[-3 - 4*x - x^2]]) + ArcTanh[x
/Sqrt[-3 - 4*x - x^2]]
```

**fricas** [A] time = 0.43, size = 132, normalized size = 1.53

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x-3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)-\frac{1}{4}\log\left(-\frac{2\sqrt{-x^2-4x-3}x+4x+3}{x^2}\right)+\frac{1}{4}\log\left(\frac{2\sqrt{-x^2-4x-3}x-4x-3}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x +
3)) + 1/2*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/
(2*x + 3)) - 1/4*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) + 1/4*log((
2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)
```

**giac** [B] time = 0.27, size = 163, normalized size = 1.90

$$\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)+\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right)+\frac{1}{2}\log\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{3\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+1\right)-\frac{1}{2}\log\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + sq
rt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/2*lo
g(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x
+ 2)^2 + 1) - 1/2*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x^2 - 4
*x - 3) - 1)^2/(x + 2)^2 + 3)
```

**maple [A]** time = 0.01, size = 123, normalized size = 1.43

$$\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\,\left(-\operatorname{arctanh}\left(\frac{3x}{\left(-x-\frac{3}{2}\right)\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}}\right)+\sqrt{2}\,\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\,\sqrt{2}}{6}\right)\right)}{6\sqrt{\frac{\frac{x^2}{\left(-x-\frac{3}{2}\right)^2}-4}{\left(\frac{x}{-x-\frac{3}{2}}+1\right)^2}}\,\left(\frac{x}{-x-\frac{3}{2}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x+3)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)
```

```
[Out] 1/6*3^(1/2)*4^(1/2)*(3/(-x-3/2)^2*x^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3/(-x-
3/2)^2*x^2-12)^(1/2)*2^(1/2))-arctanh(3/(-x-3/2)/(3/(-x-3/2)^2*x^2-12)^(1/2
)*x))/((1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)*x+1)^2)^(1/2)/(1/(-x-3/2)*x+1)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4\,x+3}{\left(2\,x^2+4\,x+3\right)\sqrt{-x^2-4\,x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((4*x + 3)/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{4\,x+3}{\sqrt{-x^2-4\,x-3}\,\left(2\,x^2+4\,x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 3)/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)
```

```
[Out] int((4*x + 3)/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x+3}{\sqrt{-(x+1)(x+3)}\,\left(2x^2+4x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] Integral((4*x + 3)/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

## 3.39 $\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{\left(ad+bdx+cdx^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=136

$$\frac{h\sqrt{a+bx+cx^2}\,\log\left(a+bx+cx^2\right)}{2cd\sqrt{ad+bdx+cdx^2}}-\frac{\sqrt{a+bx+cx^2}\,(2cg-bh)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{cd\sqrt{b^2-4ac}\,\sqrt{ad+bdx+cdx^2}}$$

**Rubi [A]** time = 0.11, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {999, 634, 618, 206, 628}

$$\frac{h\sqrt{a+bx+cx^2}\,\log\left(a+bx+cx^2\right)}{2cd\sqrt{ad+bdx+cdx^2}}-\frac{\sqrt{a+bx+cx^2}\,(2cg-bh)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{cd\sqrt{b^2-4ac}\,\sqrt{ad+bdx+cdx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)*Sqrt[a + b*x + c*x^2])/(a*d + b*d*x + c*d*x^2)^(3/2),x]
```

```
[Out] -(((2*c*g - b*h)*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c
]])/(c*Sqrt[b^2 - 4*a*c]*d*Sqrt[a*d + b*d*x + c*d*x^2])) + (h*Sqrt[a + b*x
+ c*x^2]*Log[a + b*x + c*x^2])/(2*c*d*Sqrt[a*d + b*d*x + c*d*x^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 999

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b
*x + c*x^2)^FracPart[p])/(d^IntPart[p]*(d + e*x + f*x^2)^FracPart[p]), Int[
(g + h*x)^m*(d + e*x + f*x^2)^(p + q), x], x] /; FreeQ[{a, b, c, d, e, f, g
, h, p, q}, x] && EqQ[c*d - a*f, 0] && EqQ[b*d - a*e, 0] &&  !IntegerQ[p] &
&  !IntegerQ[q] &&  !GtQ[c/f, 0]
```

Rubi steps

$$\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{\left(ad+bdx+cdx^2\right)^{3/2}}\,dx = \frac{\sqrt{a+bx+cx^2}\int \frac{g+hx}{ad+bdx+cdx^2}\,dx}{\sqrt{ad+bdx+cdx^2}}$$
$$= \frac{\left(h\sqrt{a+bx+cx^2}\right)\int \frac{bd+2cdx}{ad+bdx+cdx^2}\,dx}{2cd\sqrt{ad+bdx+cdx^2}} + \frac{\left((2cdg-bdh)\sqrt{a+bx+cx^2}\right)\int \frac{1}{ad+bdx+cd}$$
$$= \frac{h\sqrt{a+bx+cx^2}\,\log\left(a+bx+cx^2\right)}{2cd\sqrt{ad+bdx+cdx^2}} - \frac{\left((2cdg-bdh)\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int \frac{}{(}\right.}{cd\sqrt{ad+bdx+c}}$$
$$= -\frac{(2cg-bh)\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\,d\sqrt{ad+bdx+cdx^2}} + \frac{h\sqrt{a+bx+cx^2}\,\log\left(a+bx+c\right.}{2cd\sqrt{ad+bdx+cdx^2}}$$

**Mathematica [A]** time = 0.09, size = 108, normalized size = 0.79

$$\frac{(a+x(b+cx))^{3/2}\left((4cg-2bh)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)+h\sqrt{4ac-b^2}\,\log(a+x(b+cx))\right)}{2c\sqrt{4ac-b^2}\,(d(a+x(b+cx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)*Sqrt[a + b*x + c*x^2])/(a*d + b*d*x + c*d*x^2)^(3/2),x
]
```

```
[Out] ((a + x*(b + c*x))^(3/2)*((4*c*g - 2*b*h)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*
a*c]] + Sqrt[-b^2 + 4*a*c]*h*Log[a + x*(b + c*x)]))/(2*c*Sqrt[-b^2 + 4*a*c]
*(d*(a + x*(b + c*x)))^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.28, size = 127, normalized size = 0.93

$$\frac{d^{3/2}(a+x(b+cx))^{3/2}\left(\frac{(2cg-bh)\tan^{-1}\left(\frac{2cx}{\sqrt{4ac-b^2}}+\frac{b}{\sqrt{4ac-b^2}}\right)}{cd^{3/2}\sqrt{4ac-b^2}}+\frac{h\log\left(a+bx+cx^2\right)}{2cd^{3/2}}\right)}{(d(a+x(b+cx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((g + h*x)*Sqrt[a + b*x + c*x^2])/(a*d + b*d*x + c*d*x^2
)^(3/2),x]
```

```
[Out] (d^(3/2)*(a + x*(b + c*x))^(3/2)*(((2*c*g - b*h)*ArcTan[b/Sqrt[-b^2 + 4*a*c
] + (2*c*x)/Sqrt[-b^2 + 4*a*c]])/(c*Sqrt[-b^2 + 4*a*c]*d^(3/2)) + (h*Log[a
+ b*x + c*x^2])/(2*c*d^(3/2))))/(d*(a + x*(b + c*x)))^(3/2)
```

**fricas [F]** time = 50.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cdx^2+bdx+ad}\,\sqrt{cx^2+bx+a}\,(hx+g)}{c^2d^2x^4+2\,bcd^2x^3+2\,abd^2x+\left(b^2+2\,ac\right)d^2x^2+a^2d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^(3/2),x, algorith
m="fricas")
```

```
[Out] integral(sqrt(c*d*x^2 + b*d*x + a*d)*sqrt(c*x^2 + b*x + a)*(h*x + g)/(c^2*d
^2*x^4 + 2*b*c*d^2*x^3 + 2*a*b*d^2*x + (b^2 + 2*a*c)*d^2*x^2 + a^2*d^2), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2+bx+a}\,(hx+g)}{\left(cdx^2+bdx+ad\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^(3/2),x, algorith
m="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(h*x + g)/(c*d*x^2 + b*d*x + a*d)^(3/2), x)
```

**maple [A]** time = 0.03, size = 121, normalized size = 0.89

$$\frac{\sqrt{\left(c\,x^{2}+bx+a\right)d}\,\left(-2bh\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)+4cg\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)+\sqrt{4ac-b^{2}}\,h\ln\left(c\,x^{2}+bx+a\right)\right)}{2\sqrt{c\,x^{2}+bx+a}\,\sqrt{4ac-b^{2}}\,c\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^(3/2),x)
```

```
[Out] 1/2/(c*x^2+b*x+a)^(1/2)*(d*(c*x^2+b*x+a))^(1/2)*(h*ln(c*x^2+b*x+a)*(4*a*c-b
^2)^(1/2)-2*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*h+4*arctan((2*c*x+b)/(4*a
*c-b^2)^(1/2))*c*g)/d^2/c/(4*a*c-b^2)^(1/2)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^(3/2),x, algorith
m="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(g+h\,x\right)\,\sqrt{c\,x^2+b\,x+a}}{{\left(c\,d\,x^2+b\,d\,x+a\,d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)*(a + b*x + c*x^2)^(1/2))/(a*d + b*d*x + c*d*x^2)^(3/2),x)
```

```
[Out] int(((g + h*x)*(a + b*x + c*x^2)^(1/2))/(a*d + b*d*x + c*d*x^2)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g + hx\right) \sqrt{a + bx + cx^{2}}}{\left(d \left(a + bx + cx^{2}\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x**2+b*x+a)**(1/2)/(c*d*x**2+b*d*x+a*d)**(3/2),x)
```

```
[Out] Integral((g + h*x)*sqrt(a + b*x + c*x**2)/(d*(a + b*x + c*x**2))**(3/2), x)
```

## 3.40 $\int x^2\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}\,dx$

**Optimal**. Leaf size=212

$$-\frac{ac^2\sqrt{a^2+2abx+b^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}\,\left(c+dx^2\right)^{3/2}(8bc-15adx)}{60d^2(a+bx)}-\frac{acx\sqrt{a^2+2abx+b^2x^2}}{8d(a+bx)}$$

**Rubi [A]** time = 0.12, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {1001, 833, 780, 195, 217, 206}

$$-\frac{ac^2\sqrt{a^2+2abx+b^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}\,\left(c+dx^2\right)^{3/2}(8bc-15adx)}{60d^2(a+bx)}-\frac{acx\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{8d(a+bx)}+\frac{bx^2\sqrt{a^2+2abx+b^2x^2}\,\left(c+dx^2\right)^{3/2}}{5d(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] -(a*c*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(8*d*(a + b*x)) + (b
*x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + d*x^2)^(3/2))/(5*d*(a + b*x)) - ((8
*b*c - 15*a*d*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + d*x^2)^(3/2))/(60*d^2*(
a + b*x)) - (a*c^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sqrt[c
 + d*x^2]])/(8*d^(3/2)*(a + b*x))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1001

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4
*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(b + 2*c*x)^(2
*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, g, h, m, p, q}, x] && Eq
Q[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int x^2\left(2ab+2b^2x\right)\sqrt{c+dx^2}\,dx}{2ab+2b^2x}$$
$$= \frac{bx^2\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{5d(a+bx)} + \frac{\sqrt{a^2+2abx+b^2x^2}\int x\left(-4b^2c+1\right.}{5d\left(2ab+2b^2x\right.}$$
$$= \frac{bx^2\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{5d(a+bx)} - \frac{(8bc-15adx)\sqrt{a^2+2abx+b^2x^2}\,(}{60d^2(a+bx)}$$
$$= -\frac{acx\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{8d(a+bx)} + \frac{bx^2\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{5d(a+bx)}$$
$$= -\frac{acx\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{8d(a+bx)} + \frac{bx^2\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{5d(a+bx)}$$
$$= -\frac{acx\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{8d(a+bx)} + \frac{bx^2\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{5d(a+bx)}$$

**Mathematica** [A] time = 0.15, size = 129, normalized size = 0.61

$$\frac{\sqrt{(a+bx)^2}\sqrt{c+dx^2}\left(\sqrt{\frac{dx^2}{c}+1}\left(15adx\left(c+2dx^2\right)+8b\left(-2c^2+cdx^2+3d^2x^4\right)\right)-15ac^{3/2}\sqrt{d}\sinh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right)}{120d^2(a+bx)\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[(a + b*x)^2]*Sqrt[c + d*x^2]*(Sqrt[1 + (d*x^2)/c]*(15*a*d*x*(c + 2*d*
x^2) + 8*b*(-2*c^2 + c*d*x^2 + 3*d^2*x^4)) - 15*a*c^(3/2)*Sqrt[d]*ArcSinh[(
Sqrt[d]*x)/Sqrt[c]]))/(120*d^2*(a + b*x)*Sqrt[1 + (d*x^2)/c])
```

**IntegrateAlgebraic** [A] time = 0.29, size = 111, normalized size = 0.52

$$\frac{\sqrt{(a+bx)^2}\left(\frac{\sqrt{c+dx^2}\left(15acdx+30ad^2x^3-16bc^2+8bcdx^2+24bd^2x^4\right)}{120d^2}+\frac{ac^2\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{8d^{3/2}}\right)}{a+bx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[(a + b*x)^2]*((Sqrt[c + d*x^2]*(-16*b*c^2 + 15*a*c*d*x + 8*b*c*d*x^2
+ 30*a*d^2*x^3 + 24*b*d^2*x^4))/(120*d^2) + (a*c^2*Log[-(Sqrt[d]*x) + Sqrt[
c + d*x^2]])/(8*d^(3/2))))/(a + b*x)
```

**fricas [A]** time = 0.43, size = 175, normalized size = 0.83

$$\left[\frac{15\,ac^2\sqrt{d}\,\log\left(-2\,dx^2+2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+2\left(24\,bd^2x^4+30\,ad^2x^3+8\,bcdx^2+15\,acdx-16\,bc^2\right)\sqrt{dx^2+c}}{240\,d^2},\ \frac{15\,ac^2\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)+\left(24\,bd^2x^4+30\,ad^2x^3+8\,bcdx^2+15\,acdx-16\,bc^2\right)\sqrt{dx^2+c}}{120\,d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/240*(15*a*c^2*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) +
2*(24*b*d^2*x^4 + 30*a*d^2*x^3 + 8*b*c*d*x^2 + 15*a*c*d*x - 16*b*c^2)*sqrt(
d*x^2 + c))/d^2, 1/120*(15*a*c^2*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)
) + (24*b*d^2*x^4 + 30*a*d^2*x^3 + 8*b*c*d*x^2 + 15*a*c*d*x - 16*b*c^2)*sqr
t(d*x^2 + c))/d^2]
```

**giac [A]** time = 0.25, size = 117, normalized size = 0.55

$$\frac{ac^2\log\left(\left|-\sqrt{d}\,x+\sqrt{dx^2+c}\right|\right)\operatorname{sgn}(bx+a)}{8\,d^{\frac{3}{2}}}+\frac{1}{120}\,\sqrt{dx^2+c}\left(\left(2\left(3\left(4\,bx\operatorname{sgn}(bx+a)+5\,a\operatorname{sgn}(bx+a)\right)x+\frac{4\,bc\operatorname{sgn}(bx+a)}{d}\right)x+\frac{15\,ac\operatorname{sgn}(bx+a)}{d}\right)x-\frac{16\,bc^2\operatorname{sgn}(bx+a)}{d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*a*c^2*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sgn(b*x + a)/d^(3/2) + 1/1
20*sqrt(d*x^2 + c)*((2*(3*(4*b*x*sgn(b*x + a) + 5*a*sgn(b*x + a))*x + 4*b*c
*sgn(b*x + a)/d)*x + 15*a*c*sgn(b*x + a)/d)*x - 16*b*c^2*sgn(b*x + a)/d^2)
```

**maple [C]** time = 0.05, size = 103, normalized size = 0.49

$$-\frac{\left(15a\,c^2d\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)+15\sqrt{d\,x^2+c}\,ac\,d^{\frac{3}{2}}x-24\left(d\,x^2+c\right)^{\frac{3}{2}}b\,d^{\frac{3}{2}}x^2-30\left(d\,x^2+c\right)^{\frac{3}{2}}a\,d^{\frac{3}{2}}x+16\left(d\,x^2+c\right)^{\frac{3}{2}}bc\sqrt{d}\right)\operatorname{csgn}(bx+a)}{120d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x)
```

```
[Out] -1/120*csgn(b*x+a)*(-24*d^(3/2)*(d*x^2+c)^(3/2)*x^2*b-30*d^(3/2)*(d*x^2+c)^
(3/2)*x*a+16*d^(1/2)*(d*x^2+c)^(3/2)*b*c+15*d^(3/2)*(d*x^2+c)^(1/2)*x*a*c+1
5*ln(x*d^(1/2)+(d*x^2+c)^(1/2))*a*c^2*d)/d^(5/2)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx^2 + c}\sqrt{(bx + a)^2}\,x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)*x^2, x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\sqrt{(a + b\,x)^2}\,\sqrt{d\,x^2 + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2),x)
```

```
[Out] int(x^2*((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{c + dx^2}\,\sqrt{(a + bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(c + d*x**2)*sqrt((a + b*x)**2), x)
```

## 3.41 $\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx$

**Optimal**. Leaf size=161

$$-\frac{bc^2\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}(a+bx)}-\frac{bcx\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{8d(a+bx)}+\frac{(4a+3bx)\sqrt{a^2+2abx+b^2x^2}\left(c+\right.}{12d(a+bx)}$$

**Rubi** [A] time = 0.07, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1001, 780, 195, 217, 206}

$$-\frac{bc^2\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}(a+bx)}-\frac{bcx\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{8d(a+bx)}+\frac{(4a+3bx)\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{12d(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] -(b*c*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(8*d*(a + b*x)) + ((
4*a + 3*b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + d*x^2)^(3/2))/(12*d*(a + b*
x)) - (b*c^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x
^2]])/(8*d^(3/2)*(a + b*x))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1001

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4
*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(b + 2*c*x)^(2
*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, g, h, m, p, q}, x] && Eq
Q[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int x\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\,\int x\left(2ab+2b^2x\right)\sqrt{c+dx^2}\,dx}{2ab+2b^2x}$$

$$= \frac{(4a+3bx)\sqrt{a^2+2abx+b^2x^2}\,\left(c+dx^2\right)^{3/2}}{12d(a+bx)} - \frac{\left(b^2c\sqrt{a^2+2abx+b^2x^2}\right)\int \sqrt{}}{2d\left(2ab+2b^2x\right)}$$

$$= -\frac{bcx\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{8d(a+bx)} + \frac{(4a+3bx)\sqrt{a^2+2abx+b^2x^2}\,\left(c+d\right.}{12d(a+bx)}$$

$$= -\frac{bcx\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{8d(a+bx)} + \frac{(4a+3bx)\sqrt{a^2+2abx+b^2x^2}\,\left(c+d\right.}{12d(a+bx)}$$

$$= -\frac{bcx\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{8d(a+bx)} + \frac{(4a+3bx)\sqrt{a^2+2abx+b^2x^2}\,\left(c+d\right.}{12d(a+bx)}$$

**Mathematica [A]** time = 0.12, size = 117, normalized size = 0.73

$$\frac{\sqrt{(a+bx)^2}\,\sqrt{c+dx^2}\left(\sqrt{d}\sqrt{\frac{dx^2}{c}+1}\left(8a\left(c+dx^2\right)+3bx\left(c+2dx^2\right)\right)-3bc^{3/2}\sinh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\right)}{24d^{3/2}(a+bx)\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[(a + b*x)^2]*Sqrt[c + d*x^2]*(Sqrt[d]*Sqrt[1 + (d*x^2)/c]*(8*a*(c + d
*x^2) + 3*b*x*(c + 2*d*x^2)) - 3*b*c^(3/2)*ArcSinh[(Sqrt[d]*x)/Sqrt[c]]))/(
24*d^(3/2)*(a + b*x)*Sqrt[1 + (d*x^2)/c])
```

**IntegrateAlgebraic [A]** time = 0.26, size = 96, normalized size = 0.60

$$\frac{\sqrt{(a+bx)^2}\left(\frac{\sqrt{c+dx^2}\left(8ac+8adx^2+3bcx+6bdx^3\right)}{24d}+\frac{bc^2\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{8d^{3/2}}\right)}{a+bx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[(a + b*x)^2]*((Sqrt[c + d*x^2]*(8*a*c + 3*b*c*x + 8*a*d*x^2 + 6*b*d*x
^3))/(24*d) + (b*c^2*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(8*d^(3/2))))/(a
+ b*x)
```

**fricas [A]** time = 0.43, size = 157, normalized size = 0.98

$$\left[\frac{3\,bc^2\sqrt{d}\,\log\left(-2\,dx^2+2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+2\left(6\,bd^2x^3+8\,ad^2x^2+3\,bcdx+8\,acd\right)\sqrt{dx^2+c}}{48\,d^2},\ \frac{3\,bc^2\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)+\left(6\,bd^2x^3+8\,ad^2x^2+3\,bcdx+8\,acd\right)\sqrt{dx^2+c}}{24\,d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*b*c^2*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*
(6*b*d^2*x^3 + 8*a*d^2*x^2 + 3*b*c*d*x + 8*a*c*d)*sqrt(d*x^2 + c))/d^2, 1/2
4*(3*b*c^2*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (6*b*d^2*x^3 + 8*a
*d^2*x^2 + 3*b*c*d*x + 8*a*c*d)*sqrt(d*x^2 + c))/d^2]
```

**giac [A]** time = 0.21, size = 98, normalized size = 0.61

$$\frac{bc^2\log\left(\left|-\sqrt{d}\,x+\sqrt{dx^2+c}\right|\right)\mathrm{sgn}\left(bx+a\right)}{8\,d^{\frac{3}{2}}}+\frac{1}{24}\,\sqrt{dx^2+c}\left(\left(2\left(3\,bx\mathrm{sgn}\left(bx+a\right)+4\,a\mathrm{sgn}\left(bx+a\right)\right)x+\frac{3\,bc\mathrm{sgn}\left(bx+a\right)}{d}\right)x+\frac{8\,ac\mathrm{sgn}\left(bx+a\right)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*b*c^2*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sgn(b*x + a)/d^(3/2) + 1/2
4*sqrt(d*x^2 + c)*((2*(3*b*x*sgn(b*x + a) + 4*a*sgn(b*x + a))*x + 3*b*c*sgn
(b*x + a)/d)*x + 8*a*c*sgn(b*x + a)/d)
```

**maple [C]** time = 0.01, size = 83, normalized size = 0.52

$$\frac{\left(-3b\,c^{2}\ln\left(\sqrt{d}\;x+\sqrt{d\,x^{2}+c}\right)-3\sqrt{d\,x^{2}+c}\;bc\sqrt{d}\;x+6\left(d\,x^{2}+c\right)^{\frac{3}{2}}b\sqrt{d}\;x+8\left(d\,x^{2}+c\right)^{\frac{3}{2}}a\sqrt{d}\right)\mathrm{csgn}\left(bx+a\right)}{24d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x)
```

```
[Out] 1/24*csgn(b*x+a)*(6*d^(1/2)*(d*x^2+c)^(3/2)*x*b+8*a*(d*x^2+c)^(3/2)*d^(1/2)
-3*d^(1/2)*(d*x^2+c)^(1/2)*x*b*c-3*ln(d^(1/2)*x+(d*x^2+c)^(1/2))*b*c^2)/d^(
3/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{dx^{2}+c}\,\sqrt{(bx+a)^{2}}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)*x, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sqrt{(a+b\,x)^{2}}\;\sqrt{d\,x^{2}+c}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2),x)
```

```
[Out] int(x*((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{c+dx^{2}}\,\sqrt{(a+bx)^{2}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*sqrt(c + d*x**2)*sqrt((a + b*x)**2), x)
```

## 3.42 $\int \sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}\,dx$

**Optimal**. Leaf size=148

$$\frac{b\sqrt{a^2+2abx+b^2x^2}\,\left(c+dx^2\right)^{3/2}}{3d(a+bx)}+\frac{ax\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{2(a+bx)}+\frac{ac\sqrt{a^2+2abx+b^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}\,(a+bx)}$$

**Rubi** [A] time = 0.06, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {970, 641, 195, 217, 206}

$$\frac{b\sqrt{a^2+2abx+b^2x^2}\,\left(c+dx^2\right)^{3/2}}{3d(a+bx)}+\frac{ax\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{2(a+bx)}+\frac{ac\sqrt{a^2+2abx+b^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] (a*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(2*(a + b*x)) + (b*Sqrt
[a^2 + 2*a*b*x + b^2*x^2]*(c + d*x^2)^(3/2))/(3*d*(a + b*x)) + (a*c*Sqrt[a^
2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*Sqrt[d]*(a
+ b*x))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 970

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
 b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int\left(2ab+2b^2x\right)\sqrt{c+dx^2}\,dx}{2ab+2b^2x}$$

$$= \frac{b\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{3d(a+bx)} + \frac{\left(2ab\sqrt{a^2+2abx+b^2x^2}\right)\int\sqrt{c+dx^2}\,d}{2ab+2b^2x}$$

$$= \frac{ax\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{2(a+bx)} + \frac{b\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{3d(a+bx)} + \frac{(abc}{}$$

$$= \frac{ax\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{2(a+bx)} + \frac{b\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{3d(a+bx)} + \frac{(abc}{}$$

$$= \frac{ax\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{2(a+bx)} + \frac{b\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{3d(a+bx)} + \frac{ac\sqrt{}}{}$$

**Mathematica [A]** time = 0.06, size = 85, normalized size = 0.57

$$\frac{\sqrt{(a+bx)^2}\left(\sqrt{c+dx^2}\left(3adx+2b\left(c+dx^2\right)\right)+3ac\sqrt{d}\log\left(\sqrt{d}\,\sqrt{c+dx^2}+dx\right)\right)}{6d(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(Sqrt[c + d*x^2]*(3*a*d*x + 2*b*(c + d*x^2)) + 3*a*c*Sqr
t[d]*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]]))/(6*d*(a + b*x))
```

**IntegrateAlgebraic [A]** time = 0.29, size = 87, normalized size = 0.59

$$\frac{\sqrt{(a+bx)^2}\left(\frac{\sqrt{c+dx^2}\,(3adx+2bc+2bdx^2)}{6d}-\frac{ac\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{2\sqrt{d}}\right)}{a+bx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[(a + b*x)^2]*((Sqrt[c + d*x^2]*(2*b*c + 3*a*d*x + 2*b*d*x^2))/(6*d) -
 (a*c*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(2*Sqrt[d])))/(a + b*x)
```

**fricas [A]** time = 0.43, size = 128, normalized size = 0.86

$$\left[\frac{3\,ac\sqrt{d}\,\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+2\left(2\,bdx^2+3\,adx+2\,bc\right)\sqrt{dx^2+c}}{12\,d},\,-\frac{3\,ac\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)-\left(2\,bdx^2+3\,adx+2\,bc\right)\sqrt{dx^2+c}}{6\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*a*c*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(2
*b*d*x^2 + 3*a*d*x + 2*b*c)*sqrt(d*x^2 + c))/d, -1/6*(3*a*c*sqrt(-d)*arctan
(sqrt(-d)*x/sqrt(d*x^2 + c)) - (2*b*d*x^2 + 3*a*d*x + 2*b*c)*sqrt(d*x^2 + c
))/d]
```

**giac [A]** time = 0.27, size = 79, normalized size = 0.53

$$-\frac{ac\log\left(\left|-\sqrt{d}\,x+\sqrt{dx^2+c}\right|\right)\mathrm{sgn}\left(bx+a\right)}{2\,\sqrt{d}}+\frac{1}{6}\,\sqrt{dx^2+c}\left(\left(2\,bx\mathrm{sgn}\left(bx+a\right)+3\,a\mathrm{sgn}\left(bx+a\right)\right)x+\frac{2\,bc\mathrm{sgn}\left(bx+a\right)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*a*c*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sgn(b*x + a)/sqrt(d) + 1/6*
sqrt(d*x^2 + c)*((2*b*x*sgn(b*x + a) + 3*a*sgn(b*x + a))*x + 2*b*c*sgn(b*x
+ a)/d)
```

**maple [C]** time = 0.01, size = 65, normalized size = 0.44

$$\frac{\left(3acd\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)+3\sqrt{d\,x^2+c}\,a\,d^{\frac{3}{2}}x+2\left(d\,x^2+c\right)^{\frac{3}{2}}b\sqrt{d}\right)\mathrm{csgn}\left(bx+a\right)}{6d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x)
```

```
[Out] 1/6*csgn(b*x+a)*(2*b*(d*x^2+c)^(3/2)*d^(1/2)+3*d^(3/2)*(d*x^2+c)^(1/2)*x*a+
3*ln(d^(1/2)*x+(d*x^2+c)^(1/2))*a*c*d)/d^(3/2)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx^2 + c}\sqrt{(bx + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{(a + b\,x)^2}\,\sqrt{d\,x^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2),x)
```

```
[Out] int(((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c + dx^2}\sqrt{(a + bx)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x)**2), x)
```

## 3.43 $\int \frac{\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{x}\,dx$

**Optimal**. Leaf size=160

$$\frac{\sqrt{a^2+2abx+b^2x^2}\,(2a+bx)\sqrt{c+dx^2}}{2(a+bx)}+\frac{bc\sqrt{a^2+2abx+b^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}\,(a+bx)}-\frac{a\sqrt{c}\,\sqrt{a^2+2abx+b^2x^2}\,\tanh^{-}}{a+bx}$$

**Rubi [A]** time = 0.12, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {1001, 815, 844, 217, 206, 266, 63, 208}

$$\frac{\sqrt{a^2+2abx+b^2x^2}\,(2a+bx)\sqrt{c+dx^2}}{2(a+bx)}+\frac{bc\sqrt{a^2+2abx+b^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}\,(a+bx)}-\frac{a\sqrt{c}\,\sqrt{a^2+2abx+b^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x,x]
```

```
[Out] ((2*a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(2*(a + b*x)) +
 (b*c*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(
2*Sqrt[d]*(a + b*x)) - (a*Sqrt[c]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[Sqr
t[c + d*x^2]/Sqrt[c]])/(a + b*x)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1001

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4
*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(b + 2*c*x)^(2
*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, g, h, m, p, q}, x] && Eq
Q[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{x}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\,\int \frac{\left(2ab+2b^2x\right)\sqrt{c+dx^2}}{x}\,dx}{2ab+2b^2x}$$

$$= \frac{(2a+bx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{2(a+bx)} + \frac{\sqrt{a^2+2abx+b^2x^2}\,\int \frac{4abcd+2b^2c\cdots}{x\sqrt{c+dx^2}}\cdots}{2d\left(2ab+2b^2x\right)}$$

$$= \frac{(2a+bx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{2(a+bx)} + \frac{\left(2abc\sqrt{a^2+2abx+b^2x^2}\right)\int \frac{\cdots}{x\sqrt{\cdots}}\cdots}{2ab+2b^2x}$$

$$= \frac{(2a+bx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{2(a+bx)} + \frac{\left(abc\sqrt{a^2+2abx+b^2x^2}\right)\text{Subst}\cdots}{2ab+2b^2x\cdots}$$

$$= \frac{(2a+bx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{2(a+bx)} + \frac{bc\sqrt{a^2+2abx+b^2x^2}\,\tanh^{-1}\left(\cdots\right.}{2\sqrt{d}\,(a+bx)}$$

$$= \frac{(2a+bx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{2(a+bx)} + \frac{bc\sqrt{a^2+2abx+b^2x^2}\,\tanh^{-1}\left(\cdots\right.}{2\sqrt{d}\,(a+bx)}$$

**Mathematica [A]** time = 0.14, size = 139, normalized size = 0.87

$$\frac{\sqrt{(a+bx)^2}\left(\sqrt{d}\sqrt{\frac{dx^2}{c}+1}\left((2a+bx)\sqrt{c+dx^2}-2a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)\right)+b\sqrt{c}\,\sqrt{c+dx^2}\,\sinh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\right)}{2\sqrt{d}\,(a+bx)\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(b*Sqrt[c]*Sqrt[c + d*x^2]*ArcSinh[(Sqrt[d]*x)/Sqrt[c]]
+ Sqrt[d]*Sqrt[1 + (d*x^2)/c]*((2*a + b*x)*Sqrt[c + d*x^2] - 2*a*Sqrt[c]*Ar
cTanh[Sqrt[c + d*x^2]/Sqrt[c]])))/(2*Sqrt[d]*(a + b*x)*Sqrt[1 + (d*x^2)/c])
```

**IntegrateAlgebraic [A]** time = 0.31, size = 114, normalized size = 0.71

$$\frac{\sqrt{(a+bx)^2}\left(\frac{1}{2}(2a+bx)\sqrt{c+dx^2}+2a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}-\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)-\frac{bc\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{2\sqrt{d}}\right)}{a+bx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(((2*a + b*x)*Sqrt[c + d*x^2])/2 + 2*a*Sqrt[c]*ArcTanh[(
Sqrt[d]*x)/Sqrt[c] - Sqrt[c + d*x^2]/Sqrt[c]] - (b*c*Log[-(Sqrt[d]*x) + Sqr
t[c + d*x^2]])/(2*Sqrt[d])))/(a + b*x)
```

**fricas [A]** time = 0.45, size = 341, normalized size = 2.13

$$\left[\frac{bc\sqrt{d}\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+2\,a\sqrt{c}\,d\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\,\sqrt{c}+2\,c}{x^2}\right)+2\,(bdx+2\,ad)\sqrt{dx^2+c}}{4\,d}, -\frac{bc\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)-a\sqrt{c}\,d\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\,\sqrt{c}+2\,c}{x^2}\right)-(bdx+2\,ad)\sqrt{dx^2+c}}{2\,d}, \frac{4\,a\sqrt{-c}\,d\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+bc\sqrt{d}\,\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+2\,(bdx+2\,ad)\sqrt{dx^2+c}}{4\,d}, -\frac{bc\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)-2\,a\sqrt{-c}\,d\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)-(bdx+2\,ad)\sqrt{dx^2+c}}{2\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*(b*c*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*a*sqr
t(c)*d*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(b*d*x + 2*a
*d)*sqrt(d*x^2 + c))/d, -1/2*(b*c*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c
)) - a*sqrt(c)*d*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - (b*d
*x + 2*a*d)*sqrt(d*x^2 + c))/d, 1/4*(4*a*sqrt(-c)*d*arctan(sqrt(-c)/sqrt(d*
x^2 + c)) + b*c*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2
*(b*d*x + 2*a*d)*sqrt(d*x^2 + c))/d, -1/2*(b*c*sqrt(-d)*arctan(sqrt(-d)*x/s
qrt(d*x^2 + c)) - 2*a*sqrt(-c)*d*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - (b*d*x
+ 2*a*d)*sqrt(d*x^2 + c))/d]
```

**giac [A]** time = 0.25, size = 102, normalized size = 0.64

$$\frac{2\,ac\arctan\left(-\frac{\sqrt{d}\,x-\sqrt{dx^2+c}}{\sqrt{-c}}\right)\mathrm{sgn}\left(bx+a\right)}{\sqrt{-c}}-\frac{bc\log\left(\left|-\sqrt{d}\,x+\sqrt{dx^2+c}\right|\right)\mathrm{sgn}\left(bx+a\right)}{2\,\sqrt{d}}+\frac{1}{2}\,\sqrt{dx^2+c}\left(bx\mathrm{sgn}\left(bx+a\right)+2\,a\mathrm{sgn}\left(bx+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a*c*arctan(-(sqrt(d)*x - sqrt(d*x^2 + c))/sqrt(-c))*sgn(b*x + a)/sqrt(-c)
 - 1/2*b*c*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sgn(b*x + a)/sqrt(d) + 1/
2*sqrt(d*x^2 + c)*(b*x*sgn(b*x + a) + 2*a*sgn(b*x + a))
```

**maple [C]** time = 0.01, size = 94, normalized size = 0.59

$$-\frac{\left(2a\sqrt{c}\,\sqrt{d}\,\ln\left(\frac{2c+2\sqrt{d\,x^2+c}\,\sqrt{c}}{x}\right)-bc\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)-\sqrt{d\,x^2+c}\,b\sqrt{d}\,x-2\sqrt{d\,x^2+c}\,a\sqrt{d}\right)\mathrm{csgn}\left(bx+a\right)}{2\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x,x)
```

```
[Out] -1/2*csgn(b*x+a)*(2*c^(1/2)*d^(1/2)*ln(2*(c^(1/2)*(d*x^2+c)^(1/2)+c)/x)*a-d
^(1/2)*(d*x^2+c)^(1/2)*x*b-2*d^(1/2)*(d*x^2+c)^(1/2)*a-ln(d^(1/2)*x+(d*x^2+
c)^(1/2))*b*c)/d^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^2+c}\sqrt{(bx+a)^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)/x, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{(a+b\,x)^2}\,\sqrt{d\,x^2+c}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2))/x,x)
[Out] int((((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2))/x, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^2}\sqrt{(a+bx)^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2)/x,x)
[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x)**2)/x, x)
```

## 3.44 $\int \frac{\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{x^2}\,dx$

**Optimal**. Leaf size=156

$$-\frac{\sqrt{a^2+2abx+b^2x^2}\,(a-bx)\sqrt{c+dx^2}}{x(a+bx)}+\frac{a\sqrt{d}\,\sqrt{a^2+2abx+b^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{a+bx}-\frac{b\sqrt{c}\,\sqrt{a^2+2abx+b^2x^2}\,\tanh}{a+bx}$$

**Rubi [A]** time = 0.11, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {1001, 813, 844, 217, 206, 266, 63, 208}

$$-\frac{\sqrt{a^2+2abx+b^2x^2}\,(a-bx)\sqrt{c+dx^2}}{x(a+bx)}+\frac{a\sqrt{d}\,\sqrt{a^2+2abx+b^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{a+bx}-\frac{b\sqrt{c}\,\sqrt{a^2+2abx+b^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x^2,x]
```

```
[Out] -(((a - b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(x*(a + b*x)))
+ (a*Sqrt[d]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x
^2]])/(a + b*x) - (b*Sqrt[c]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[Sqrt[c +
 d*x^2]/Sqrt[c]])/(a + b*x)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] && !Rational
Q[m])) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1001

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4
*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(b + 2*c*x)^(2
*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, g, h, m, p, q}, x] && Eq
Q[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{x^2}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(2ab+2b^2x)\sqrt{c+dx^2}}{x^2}\,dx}{2ab+2b^2x}$$

$$= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{x(a+bx)} - \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{-4b^2c-4abdx}{x\sqrt{c+dx^2}}}{2\left(2ab+2b^2x\right)}$$

$$= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{x(a+bx)} + \frac{\left(2b^2c\sqrt{a^2+2abx+b^2x^2}\right)\int \frac{1}{x\sqrt{c+}}}{2ab+2b^2x}$$

$$= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{x(a+bx)} + \frac{\left(b^2c\sqrt{a^2+2abx+b^2x^2}\right)\text{Subst}\Big(}{2ab+2b^2x}$$

$$= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{x(a+bx)} + \frac{a\sqrt{d}\,\sqrt{a^2+2abx+b^2x^2}\,\tanh^{-1}\Big(}{a+bx}$$

$$= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{x(a+bx)} + \frac{a\sqrt{d}\,\sqrt{a^2+2abx+b^2x^2}\,\tanh^{-1}\Big(}{a+bx}$$

**Mathematica [A]** time = 0.28, size = 118, normalized size = 0.76

$$\frac{\sqrt{(a+bx)^2}\left(\frac{(bx-a)\sqrt{c+dx^2}}{x} + \frac{a\sqrt{c}\,\sqrt{d}\,\sqrt{\frac{dx^2}{c}+1}\,\sinh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c+dx^2}} - b\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)\right)}{a+bx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x^2,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(((-a + b*x)*Sqrt[c + d*x^2])/x + (a*Sqrt[c]*Sqrt[d]*Sqr
t[1 + (d*x^2)/c]*ArcSinh[(Sqrt[d]*x)/Sqrt[c]])/Sqrt[c + d*x^2] - b*Sqrt[c]*
ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]))/(a + b*x)
```

**IntegrateAlgebraic [A]** time = 0.31, size = 111, normalized size = 0.71

$$\frac{\sqrt{(a+bx)^2}\left(\frac{(bx-a)\sqrt{c+dx^2}}{x} - a\sqrt{d}\,\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right) + 2b\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}} - \frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)\right)}{a+bx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x^2,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(((-a + b*x)*Sqrt[c + d*x^2])/x + 2*b*Sqrt[c]*ArcTanh[(S
qrt[d]*x)/Sqrt[c] - Sqrt[c + d*x^2]/Sqrt[c]] - a*Sqrt[d]*Log[-(Sqrt[d]*x) +
 Sqrt[c + d*x^2]]))/(a + b*x)
```

**fricas [A]** time = 0.44, size = 333, normalized size = 2.13

$$\left[\frac{a\sqrt{d}\,x\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+b\sqrt{c}\,x\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\,\sqrt{c}+2\,c}{x^2}\right)+2\,\sqrt{dx^2+c}\,(bx-a)}{2\,x}, -\frac{2\,a\sqrt{-d}\,x\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)-b\sqrt{c}\,x\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\,\sqrt{c}+2\,c}{x^2}\right)-2\,\sqrt{dx^2+c}\,(bx-a)}{2\,x}, \frac{2\,b\sqrt{-c}\,x\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+a\sqrt{d}\,x\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+2\,\sqrt{dx^2+c}\,(bx-a)}{2\,x}, -\frac{a\sqrt{-d}\,x\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)-b\sqrt{-c}\,x\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)-\sqrt{dx^2+c}\,(bx-a)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(a*sqrt(d)*x*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + b*sqrt(
c)*x*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*sqrt(d*x^2 + c
)*(b*x - a))/x, -1/2*(2*a*sqrt(-d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - b
*sqrt(c)*x*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*sqrt(d*x
^2 + c)*(b*x - a))/x, 1/2*(2*b*sqrt(-c)*x*arctan(sqrt(-c)/sqrt(d*x^2 + c))
+ a*sqrt(d)*x*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*sqrt(d*x^
2 + c)*(b*x - a))/x, -(a*sqrt(-d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - b*
sqrt(-c)*x*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - sqrt(d*x^2 + c)*(b*x - a))/x]
```

**giac [A]** time = 0.26, size = 126, normalized size = 0.81

$$\frac{2\,bc\arctan\left(-\frac{\sqrt{d}\,x-\sqrt{dx^2+c}}{\sqrt{-c}}\right)\mathrm{sgn}\left(bx+a\right)}{\sqrt{-c}}-a\sqrt{d}\,\log\left(\left|-\sqrt{d}\,x+\sqrt{dx^2+c}\right|\right)\mathrm{sgn}\left(bx+a\right)+\sqrt{dx^2+c}\,b\mathrm{sgn}\left(bx+a\right)+\frac{2\,ac\sqrt{d}\,\mathrm{sgn}\left(bx+a\right)}{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*b*c*arctan(-(sqrt(d)*x - sqrt(d*x^2 + c))/sqrt(-c))*sgn(b*x + a)/sqrt(-c)
 - a*sqrt(d)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sgn(b*x + a) + sqrt(d*x
^2 + c)*b*sgn(b*x + a) + 2*a*c*sqrt(d)*sgn(b*x + a)/((sqrt(d)*x - sqrt(d*x^
2 + c))^2 - c)
```

**maple [C]** time = 0.01, size = 118, normalized size = 0.76

$$\frac{\left(acdx\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)-b\,c^{\frac{3}{2}}\sqrt{d}\,x\ln\left(\frac{2c+2\sqrt{d\,x^2+c}\,\sqrt{c}}{x}\right)+\sqrt{d\,x^2+c}\,a\,d^{\frac{3}{2}}x^2+\sqrt{d\,x^2+c}\,bc\sqrt{d}\,x-\left(d\,x^2+c\right)^{\frac{3}{2}}a\sqrt{d}\right)\mathrm{csgn}\left(bx+a\right)}{c\sqrt{d}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^2,x)
```

```
[Out] csgn(b*x+a)*(d^(3/2)*(d*x^2+c)^(1/2)*x^2*a-d^(1/2)*c^(3/2)*ln(2*(c+(d*x^2+c
)^(1/2)*c^(1/2))/x)*x*b-(d*x^2+c)^(3/2)*a*d^(1/2)+(d*x^2+c)^(1/2)*b*c*d^(1/
2)*x+ln(d^(1/2)*x+(d*x^2+c)^(1/2))*x*a*c*d)/c/x/d^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^2+c}\,\sqrt{(bx+a)^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)/x^2, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{(a+b\,x)^2}\,\sqrt{d\,x^2+c}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2))/x^2,x)
```

```
[Out] int((((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2))/x^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^2}\,\sqrt{(a+bx)^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x)**2)/x**2, x)
```

## 3.45 $\int \frac{\sqrt{a^2+2abx+b^2x^2}\ \sqrt{c+dx^2}}{x^3}\,dx$

**Optimal**. Leaf size=161

$$-\frac{\sqrt{a^2+2abx+b^2x^2}\,(a+2bx)\sqrt{c+dx^2}}{2x^2(a+bx)}+\frac{b\sqrt{d}\ \sqrt{a^2+2abx+b^2x^2}\ \tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{a+bx}-\frac{ad\sqrt{a^2+2abx+b^2x^2}\ \text{tanh}}{2\sqrt{c}\,(a+bx)}$$

**Rubi [A]** time = 0.12, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {1001, 811, 844, 217, 206, 266, 63, 208}

$$-\frac{\sqrt{a^2+2abx+b^2x^2}\,(a+2bx)\sqrt{c+dx^2}}{2x^2(a+bx)}+\frac{b\sqrt{d}\ \sqrt{a^2+2abx+b^2x^2}\ \tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{a+bx}-\frac{ad\sqrt{a^2+2abx+b^2x^2}\ \tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2\sqrt{c}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x^3,x]
```

```
[Out] -((a + 2*b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(2*x^2*(a + b*
x)) + (b*Sqrt[d]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sqrt[c +
 d*x^2]])/(a + b*x) - (a*d*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[Sqrt[c + d
*x^2]/Sqrt[c]])/(2*Sqrt[c]*(a + b*x))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1001

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4
*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(b + 2*c*x)^(2
*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, g, h, m, p, q}, x] && Eq
Q[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{x^3}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\,\int \frac{(2ab+2b^2x)\sqrt{c+dx^2}}{x^3}\,dx}{2ab+2b^2x}$$

$$= -\frac{(a+2bx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{2x^2(a+bx)} - \frac{\sqrt{a^2+2abx+b^2x^2}\,\int \frac{-4abcd-8b}{x\sqrt{c+d}}}{4c\left(2ab+2b^2x\right)}$$

$$= -\frac{(a+2bx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{2x^2(a+bx)} + \frac{\left(abd\sqrt{a^2+2abx+b^2x^2}\right)\int \frac{}{x\sqrt{}}}{2ab+2b^2x}$$

$$= -\frac{(a+2bx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{2x^2(a+bx)} + \frac{\left(abd\sqrt{a^2+2abx+b^2x^2}\right)\text{Sub}}{2\left(2ab+2b\right.}$$

$$= -\frac{(a+2bx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{2x^2(a+bx)} + \frac{b\sqrt{d}\,\sqrt{a^2+2abx+b^2x^2}\,\tanh}{a+bx}$$

$$= -\frac{(a+2bx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{2x^2(a+bx)} + \frac{b\sqrt{d}\,\sqrt{a^2+2abx+b^2x^2}\,\tanh}{a+bx}$$

**Mathematica** [A] time = 0.12, size = 126, normalized size = 0.78

$$-\frac{\sqrt{(a+bx)^2}\,\sqrt{c+dx^2}\left(c(a+2bx)\sqrt{\frac{dx^2}{c}+1}+adx^2\tanh^{-1}\left(\sqrt{\frac{dx^2}{c}+1}\right)-2b\sqrt{c}\,\sqrt{d}\,x^2\sinh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\right)}{2cx^2(a+bx)\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x^3,x]
```

```
[Out] -1/2*(Sqrt[(a + b*x)^2]*Sqrt[c + d*x^2]*(c*(a + 2*b*x)*Sqrt[1 + (d*x^2)/c]
- 2*b*Sqrt[c]*Sqrt[d]*x^2*ArcSinh[(Sqrt[d]*x)/Sqrt[c]] + a*d*x^2*ArcTanh[Sq
rt[1 + (d*x^2)/c]]))/(c*x^2*(a + b*x)*Sqrt[1 + (d*x^2)/c])
```

**IntegrateAlgebraic** [A] time = 0.41, size = 115, normalized size = 0.71

$$\frac{\sqrt{(a+bx)^2}\left(\frac{(-a-2bx)\sqrt{c+dx^2}}{2x^2}+\frac{ad\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}-\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}-b\sqrt{d}\,\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)\right)}{a+bx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x^3,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(((-a - 2*b*x)*Sqrt[c + d*x^2])/(2*x^2) + (a*d*ArcTanh[(
Sqrt[d]*x)/Sqrt[c] - Sqrt[c + d*x^2]/Sqrt[c]])/Sqrt[c] - b*Sqrt[d]*Log[-(Sq
rt[d]*x) + Sqrt[c + d*x^2]]))/(a + b*x)
```

**fricas [A]** time = 0.44, size = 377, normalized size = 2.34

$$\left[\frac{2\,bc\sqrt{d}\,x^2\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+a\sqrt{c}\,dx^2\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\,\sqrt{c}+2\,c}{x^2}\right)-2\,(2\,bcx+ac)\sqrt{dx^2+c}}{4\,cx^2},\ -\frac{4\,bc\sqrt{-d}\,x^2\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)-a\sqrt{c}\,dx^2\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\,\sqrt{c}+2\,c}{x^2}\right)+2\,(2\,bcx+ac)\sqrt{dx^2+c}}{4\,cx^2},\ \frac{a\sqrt{-c}\,dx^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+bc\sqrt{d}\,x^2\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)-(2\,bcx+ac)\sqrt{dx^2+c}}{2\,cx^2},\ -\frac{2\,bc\sqrt{-d}\,x^2\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)-a\sqrt{-c}\,dx^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+(2\,bcx+ac)\sqrt{dx^2+c}}{2\,cx^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*b*c*sqrt(d)*x^2*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + a
*sqrt(c)*d*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(2*b
*c*x + a*c)*sqrt(d*x^2 + c))/(c*x^2), -1/4*(4*b*c*sqrt(-d)*x^2*arctan(sqrt(
-d)*x/sqrt(d*x^2 + c)) - a*sqrt(c)*d*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sq
rt(c) + 2*c)/x^2) + 2*(2*b*c*x + a*c)*sqrt(d*x^2 + c))/(c*x^2), 1/2*(a*sqrt
(-c)*d*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + b*c*sqrt(d)*x^2*log(-2*d*x^2
- 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - (2*b*c*x + a*c)*sqrt(d*x^2 + c))/(c*x^
2), -1/2*(2*b*c*sqrt(-d)*x^2*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - a*sqrt(-c
)*d*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (2*b*c*x + a*c)*sqrt(d*x^2 + c))
/(c*x^2)]
```

**giac [A]** time = 0.28, size = 199, normalized size = 1.24

$$\frac{ad\arctan\left(-\frac{\sqrt{d}x-\sqrt{dx^2+c}}{\sqrt{-c}}\right)\operatorname{sgn}(bx+a)}{\sqrt{-c}}-b\sqrt{d}\,\log\left(\left|-\sqrt{d}\,x+\sqrt{dx^2+c}\right|\right)\operatorname{sgn}(bx+a)+\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^3 ad\operatorname{sgn}(bx+a)+2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2 bc\sqrt{d}\operatorname{sgn}(bx+a)+\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)acd\operatorname{sgn}(bx+a)-2\,bc^2\sqrt{d}\operatorname{sgn}(bx+a)}{\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2-c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] a*d*arctan(-(sqrt(d)*x - sqrt(d*x^2 + c))/sqrt(-c))*sgn(b*x + a)/sqrt(-c) -
 b*sqrt(d)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sgn(b*x + a) + ((sqrt(d)*
x - sqrt(d*x^2 + c))^3*a*d*sgn(b*x + a) + 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2
*b*c*sqrt(d)*sgn(b*x + a) + (sqrt(d)*x - sqrt(d*x^2 + c))*a*c*d*sgn(b*x + a
) - 2*b*c^2*sqrt(d)*sgn(b*x + a))/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^2
```

**maple [C]** time = 0.02, size = 141, normalized size = 0.88

$$-\frac{\left(a\sqrt{c}\,d^{\frac{3}{2}}x^2\ln\left(\frac{2c+2\sqrt{d\,x^2+c}\,\sqrt{c}}{x}\right)-2bcd\,x^2\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)-2\sqrt{d\,x^2+c}\,b\,d^{\frac{3}{2}}x^3-\sqrt{d\,x^2+c}\,a\,d^{\frac{3}{2}}x^2+2\left(d\,x^2+c\right)^{\frac{3}{2}}b\sqrt{d}\,x+\left(d\,x^2+c\right)^{\frac{3}{2}}a\sqrt{d}\right)\operatorname{csgn}(bx+a)}{2c\sqrt{d}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^3,x)
```

```
[Out] -1/2*csgn(b*x+a)*(c^(1/2)*ln(2*(c+(d*x^2+c)^(1/2)*c^(1/2))/x)*d^(3/2)*x^2*a
-2*d^(3/2)*(d*x^2+c)^(1/2)*x^3*b+2*(d*x^2+c)^(3/2)*b*d^(1/2)*x-(d*x^2+c)^(1
/2)*a*d^(3/2)*x^2-2*ln(d^(1/2)*x+(d*x^2+c)^(1/2))*x^2*b*c*d+(d*x^2+c)^(3/2)
*a*d^(1/2))/x^2/c/d^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^2+c}\,\sqrt{(bx+a)^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)/x^3, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{(a+b\,x)^2}\,\sqrt{d\,x^2+c}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2))/x^3,x)
```

```
[Out] int((((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2))/x^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^2}\,\sqrt{(a+bx)^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x)**2)/x**3, x)
```

## 3.46 $\int x^2\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+ex+dx^2}\,dx$

**Optimal**. Leaf size=317

$$-\frac{\sqrt{a^2+2abx+b^2x^2}\,(2dx+e)\sqrt{c+dx^2+ex}\,\left(2ad\left(4cd-5e^2\right)-b\left(12cde-7e^3\right)\right)}{128d^4(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}\,\left(c+dx^2+\cdots\right.}{}$$

**Rubi** [A] time = 0.33, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1000, 832, 779, 612, 621, 206}

$$-\frac{\sqrt{a^2+2abx+b^2x^2}\,\left(c+dx^2+ex\right)^{3/2}\left(-6dx(10ad-7be)+50ade+32bcd-35be^2\right)}{240d^3(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}\,(2dx+e)\sqrt{c+dx^2+ex}\,\left(2ad\left(4cd-5e^2\right)-b\left(12cde-7e^3\right)\right)}{128d^4(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}\,\left(4cd-e^2\right)\left(8acd^2-10ade^2-12bcde+7be^3\right)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{256d^{9/2}(a+bx)}+\frac{bx^2\sqrt{a^2+2abx+b^2x^2}\,\left(c+dx^2+ex\right)^{3/2}}{5d(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]
```

```
[Out] -((2*a*d*(4*c*d - 5*e^2) - b*(12*c*d*e - 7*e^3))*(e + 2*d*x)*Sqrt[a^2 + 2*a
*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/(128*d^4*(a + b*x)) + (b*x^2*Sqrt[a^
2 + 2*a*b*x + b^2*x^2]*(c + e*x + d*x^2)^(3/2))/(5*d*(a + b*x)) - ((32*b*c*
d + 50*a*d*e - 35*b*e^2 - 6*d*(10*a*d - 7*b*e)*x)*Sqrt[a^2 + 2*a*b*x + b^2*
x^2]*(c + e*x + d*x^2)^(3/2))/(240*d^3*(a + b*x)) - ((4*c*d - e^2)*(8*a*c*d
^2 - 12*b*c*d*e - 10*a*d*e^2 + 7*b*e^3)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTa
nh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(256*d^(9/2)*(a + b*x))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1000

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^Fr
acPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(
b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 h, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+ex+dx^2}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\,\int x^2\left(2ab+2b^2x\right)\sqrt{c+ex+dx^2}\,dx}{2ab+2b^2x}$$

$$= \frac{bx^2\sqrt{a^2+2abx+b^2x^2}\,\left(c+ex+dx^2\right)^{3/2}}{5d(a+bx)} + \frac{\sqrt{a^2+2abx+b^2x^2}\,\int x\,(\ldots}{\ldots}$$

$$= \frac{bx^2\sqrt{a^2+2abx+b^2x^2}\,\left(c+ex+dx^2\right)^{3/2}}{5d(a+bx)} - \frac{\left(32bcd+50ade-35be^2 \ldots\right.}{\ldots}$$

$$= -\frac{\left(8acd^2-12bcde-10ade^2+7be^3\right)(e+2dx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c \ldots}}{128d^4(a+bx)}$$

$$= -\frac{\left(8acd^2-12bcde-10ade^2+7be^3\right)(e+2dx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c \ldots}}{128d^4(a+bx)}$$

$$= -\frac{\left(8acd^2-12bcde-10ade^2+7be^3\right)(e+2dx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c \ldots}}{128d^4(a+bx)}$$

**Mathematica** [**A**]  time = 0.27, size = 198, normalized size = 0.62

$$\frac{\sqrt{(a+bx)^2}\left(-\frac{5\left(2ad\left(4cd-5e^2\right)+b\left(7e^3-12cde\right)\right)\left(\left(4cd-e^2\right)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\,\sqrt{c+x(dx+e)}}\right)+2\sqrt{d}\,(2dx+e)\sqrt{c+x(dx+e)}\right)}{256d^{7/2}}+\frac{(c+x(dx+e))^{3/2}(10ad(6dx-5e)-32bcd+7be(5e-6dx))}{48d^2}+bx^2(c+x(dx+e))^{3/2}\right)}{5d(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(b*x^2*(c + x*(e + d*x))^(3/2) + ((c + x*(e + d*x))^(3/2
)*(-32*b*c*d + 7*b*e*(5*e - 6*d*x) + 10*a*d*(-5*e + 6*d*x)))/(48*d^2) - (5*
(2*a*d*(4*c*d - 5*e^2) + b*(-12*c*d*e + 7*e^3))*(2*Sqrt[d]*(e + 2*d*x)*Sqrt
[c + x*(e + d*x)] + (4*c*d - e^2)*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + x
*(e + d*x)])]))/(256*d^(7/2))))/(5*d*(a + b*x))
```

**IntegrateAlgebraic** [**A**]  time = 1.00, size = 263, normalized size = 0.83

$$\frac{\sqrt{(a+bx)^2}\left(\frac{\left(32ac^2d^3-48acd^2e^2+10ade^4-48bc^2d^2e+40bcde^3-7be^5\right)\log\left(-2\sqrt{d}\,\sqrt{c+dx^2+ex}+2dx+e\right)}{256d^{9/2}}+\frac{\sqrt{c+dx^2+ex}\left(240acd^3x-520acd^2e+480ad^4x^3+80ad^3ex^2-100ad^2e^2x+150ade^3-256bc^2d^2+128bcd^3x^2-232bcd^2ex+460bcde^2+384bd^4x^4+48bd^3ex^3-56bd^2e^2x^2+70bde^3x-105be^4\right)}{1920d^4}\right)}{a+bx}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2], x]

[Out] (Sqrt[(a + b*x)^2]*((Sqrt[c + e*x + d*x^2]*(-256*b*c^2*d^2 - 520*a*c*d^2*e + 460*b*c*d*e^2 + 150*a*d*e^3 - 105*b*e^4 + 240*a*c*d^3*x - 232*b*c*d^2*e*x - 100*a*d^2*e^2*x + 70*b*d*e^3*x + 128*b*c*d^3*x^2 + 80*a*d^3*e*x^2 - 56*b *d^2*e^2*x^2 + 480*a*d^4*x^3 + 48*b*d^3*e*x^3 + 384*b*d^4*x^4))/(1920*d^4) + ((32*a*c^2*d^3 - 48*b*c^2*d^2*e - 48*a*c*d^2*e^2 + 40*b*c*d*e^3 + 10*a*d* e^4 - 7*b*e^5)*Log[e + 2*d*x - 2*Sqrt[d]*Sqrt[c + e*x + d*x^2]])/(256*d^(9/ 2))))/(a + b*x)

**fricas [A]**  time = 0.47, size = 517, normalized size = 1.63

$$\left[-\frac{15\left(32ac^2d^3-48bc^2d^2e-48acd^2e^2+40bcde^3+10ade^4-7be^5\right)\sqrt{d}\log\left(8d^2x^2+8dex+4\sqrt{dx^2+ex+c}(2dx+e)\sqrt{d}+4cd+e^2\right)-4\left(384bd^5x^4-256bc^2d^3-520acd^3e+460bcd^2e^2+150ad^2e^3-105bde^4+48\left(10ad^5+bd^4e\right)x^3+8\left(16bcd^4+10ad^4e-7bd^3e^2\right)x^2+2\left(120acd^4-116bcd^3e-50ad^3e^2+35bd^2e^3\right)x\right)\sqrt{dx^2+ex+c}}{7680d^5}, \frac{15\left(32ac^2d^3-48bc^2d^2e-48acd^2e^2+40bcde^3+10ade^4-7be^5\right)\sqrt{-d}\arctan\left(\frac{\sqrt{dx^2+ex+c}(2dx+e)\sqrt{-d}}{2\left(d^2x^2+dex+cd\right)}\right)+2\left(384bd^5x^4-256bc^2d^3-520acd^3e+460bcd^2e^2+150ad^2e^3-105bde^4+48\left(10ad^5+bd^4e\right)x^3+8\left(16bcd^4+10ad^4e-7bd^3e^2\right)x^2+2\left(120acd^4-116bcd^3e-50ad^3e^2+35bd^2e^3\right)x\right)\sqrt{dx^2+ex+c}}{3840d^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="fricas")

[Out] [-1/7680*(15*(32*a*c^2*d^3 - 48*b*c^2*d^2*e - 48*a*c*d^2*e^2 + 40*b*c*d*e^3 + 10*a*d*e^4 - 7*b*e^5)*sqrt(d)*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 + e *x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) - 4*(384*b*d^5*x^4 - 256*b*c^2*d ^3 - 520*a*c*d^3*e + 460*b*c*d^2*e^2 + 150*a*d^2*e^3 - 105*b*d*e^4 + 48*(10 *a*d^5 + b*d^4*e)*x^3 + 8*(16*b*c*d^4 + 10*a*d^4*e - 7*b*d^3*e^2)*x^2 + 2*( 120*a*c*d^4 - 116*b*c*d^3*e - 50*a*d^3*e^2 + 35*b*d^2*e^3)*x)*sqrt(d*x^2 + e*x + c))/d^5, 1/3840*(15*(32*a*c^2*d^3 - 48*b*c^2*d^2*e - 48*a*c*d^2*e^2 + 40*b*c*d*e^3 + 10*a*d*e^4 - 7*b*e^5)*sqrt(-d)*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(384*b*d^5*x^4 - 256 *b*c^2*d^3 - 520*a*c*d^3*e + 460*b*c*d^2*e^2 + 150*a*d^2*e^3 - 105*b*d*e^4 + 48*(10*a*d^5 + b*d^4*e)*x^3 + 8*(16*b*c*d^4 + 10*a*d^4*e - 7*b*d^3*e^2)*x ^2 + 2*(120*a*c*d^4 - 116*b*c*d^3*e - 50*a*d^3*e^2 + 35*b*d^2*e^3)*x)*sqrt( d*x^2 + e*x + c))/d^5]

**giac [A]**  time = 0.31, size = 368, normalized size = 1.16

$$\frac{1}{1920}\sqrt{dx^2+xe+c}\left(2\left(4\left(6\left(8bx\,\mathrm{sgn}(bx+a)+\frac{10ad^4\mathrm{sgn}(bx+a)+bd^3e\,\mathrm{sgn}(bx+a)}{d^4}\right)x+\frac{16bcd^3\mathrm{sgn}(bx+a)+10ad^3e\,\mathrm{sgn}(bx+a)-7bd^2e^2\mathrm{sgn}(bx+a)}{d^4}\right)x+\frac{120acd^3\mathrm{sgn}(bx+a)-116bcd^2e\,\mathrm{sgn}(bx+a)-50ad^2e^2\mathrm{sgn}(bx+a)+35bde^3\mathrm{sgn}(bx+a)}{d^4}\right)x-\frac{256bc^2d^2\mathrm{sgn}(bx+a)+520acd^2e\,\mathrm{sgn}(bx+a)-460bcde^2\mathrm{sgn}(bx+a)-150ade^3\mathrm{sgn}(bx+a)+105be^4\mathrm{sgn}(bx+a)}{d^4}\right)+\frac{\left(32ac^2d^3\mathrm{sgn}(bx+a)-48bc^2d^2e\,\mathrm{sgn}(bx+a)-48acd^2e^2\mathrm{sgn}(bx+a)+40bcde^3\mathrm{sgn}(bx+a)+10ade^4\mathrm{sgn}(bx+a)-7be^5\mathrm{sgn}(bx+a)\right)\log\left(\left|-2\left(\sqrt{d}x-\sqrt{dx^2+xe+c}\right)\sqrt{d}-e\right|\right)}{256d^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="giac")

[Out] 1/1920*sqrt(d*x^2 + x*e + c)*(2*(4*(6*(8*b*x*sgn(b*x + a) + (10*a*d^4*sgn(b *x + a) + b*d^3*e*sgn(b*x + a))/d^4)*x + (16*b*c*d^3*sgn(b*x + a) + 10*a*d^ 3*e*sgn(b*x + a) - 7*b*d^2*e^2*sgn(b*x + a))/d^4)*x + (120*a*c*d^3*sgn(b*x + a) - 116*b*c*d^2*e*sgn(b*x + a) - 50*a*d^2*e^2*sgn(b*x + a) + 35*b*d*e^3* sgn(b*x + a))/d^4)*x - (256*b*c^2*d^2*sgn(b*x + a) + 520*a*c*d^2*e*sgn(b*x + a) - 460*b*c*d*e^2*sgn(b*x + a) - 150*a*d*e^3*sgn(b*x + a) + 105*b*e^4*sg

```
n(b*x + a))/d^4) + 1/256*(32*a*c^2*d^3*sgn(b*x + a) - 48*b*c^2*d^2*e*sgn(b*
x + a) - 48*a*c*d^2*e^2*sgn(b*x + a) + 40*b*c*d*e^3*sgn(b*x + a) + 10*a*d*e
^4*sgn(b*x + a) - 7*b*e^5*sgn(b*x + a))*log(abs(-2*(sqrt(d)*x - sqrt(d*x^2
+ x*e + c))*sqrt(d) - e))/d^(9/2)
```

**maple [C]** time = 0.02, size = 530, normalized size = 1.67

$$\frac{\left(-480 a c^2 d^4 \ln\left(\frac{2dx+e+2\sqrt{d x^2+ex+c}\,\sqrt{d}}{2\sqrt{d}}\right)+720 a c d^3 e^2 \ln\left(\frac{2dx+e+2\sqrt{d x^2+ex+c}\,\sqrt{d}}{2\sqrt{d}}\right)-150 a d^2 e^4 \ln\left(\frac{2dx+e+2\sqrt{d x^2+ex+c}\,\sqrt{d}}{2\sqrt{d}}\right)+720 b c^2 d^3 e \ln\left(\frac{2dx+e+2\sqrt{d x^2+ex+c}\,\sqrt{d}}{2\sqrt{d}}\right)-600 b c d^2 e^3 \ln\left(\frac{2dx+e+2\sqrt{d x^2+ex+c}\,\sqrt{d}}{2\sqrt{d}}\right)+105 b d e^5 \ln\left(\frac{2dx+e+2\sqrt{d x^2+ex+c}\,\sqrt{d}}{2\sqrt{d}}\right)-480\sqrt{d x^2+ex+c}\, a c d^{\frac{9}{2}} x+600\sqrt{d x^2+ex+c}\, a d^{\frac{7}{2}} e^2 x+720\sqrt{d x^2+ex+c}\, b c d^{\frac{7}{2}} e x+768\left(d x^2+ex+c\right)^{\frac{3}{2}} b d^{\frac{9}{2}} x^2-420\sqrt{d x^2+ex+c}\, b d^{\frac{5}{2}} e^3 x-240\sqrt{d x^2+ex+c}\, a c d^{\frac{7}{2}} e+960\left(d x^2+ex+c\right)^{\frac{3}{2}} a d^{\frac{9}{2}} x+300\sqrt{d x^2+ex+c}\, a d^{\frac{5}{2}} e^3+360\sqrt{d x^2+ex+c}\, b c d^{\frac{5}{2}} e^2-672\left(d x^2+ex+c\right)^{\frac{3}{2}} b d^{\frac{7}{2}} e x-210\sqrt{d x^2+ex+c}\, b d^{\frac{3}{2}} e^4-800\left(d x^2+ex+c\right)^{\frac{3}{2}} a d^{\frac{7}{2}} e-512\left(d x^2+ex+c\right)^{\frac{3}{2}} b c d^{\frac{7}{2}}+560\left(d x^2+ex+c\right)^{\frac{3}{2}} b d^{\frac{5}{2}} e^2\right)\mathrm{csgn}\left(bx+a\right)}{3840 d^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x)
```

```
[Out] 1/3840*csgn(b*x+a)*(768*(d*x^2+e*x+c)^(3/2)*d^(9/2)*x^2*b+960*(d*x^2+e*x+c)
^(3/2)*d^(9/2)*x*a-672*(d*x^2+e*x+c)^(3/2)*d^(7/2)*x*b*e-800*(d*x^2+e*x+c)^
(3/2)*d^(7/2)*a*e-512*(d*x^2+e*x+c)^(3/2)*d^(7/2)*b*c+560*(d*x^2+e*x+c)^(3/
2)*d^(5/2)*b*e^2-480*(d*x^2+e*x+c)^(1/2)*d^(9/2)*x*a*c+600*(d*x^2+e*x+c)^(1
/2)*d^(7/2)*x*a*e^2+720*(d*x^2+e*x+c)^(1/2)*d^(7/2)*x*b*c*e-420*(d*x^2+e*x+
c)^(1/2)*d^(5/2)*x*b*e^3-240*(d*x^2+e*x+c)^(1/2)*d^(7/2)*a*c*e+300*(d*x^2+e
*x+c)^(1/2)*d^(5/2)*a*e^3+360*(d*x^2+e*x+c)^(1/2)*d^(5/2)*b*c*e^2-210*(d*x^
2+e*x+c)^(1/2)*d^(3/2)*b*e^4-480*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*
x+e)/d^(1/2))*a*c^2*d^4+720*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/
d^(1/2))*a*c*d^3*e^2-150*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(
1/2))*a*d^2*e^4+720*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))
*b*c^2*d^3*e-600*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*b*
c*d^2*e^3+105*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*b*d*e
^5)/d^(11/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx^2 + ex + c}\,\sqrt{(bx + a)^2}\, x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)*x^2, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\sqrt{{\left(a+b\,x\right)}^2}\,\sqrt{d\,x^2+e\,x+c} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2),x)
```

```
[Out] int(x^2*((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{c + dx^2 + ex} \sqrt{(a + bx)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2),x)
[Out] Integral(x**2*sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2), x)
```

## 3.47 $\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx$

**Optimal**. Leaf size=227

$$-\frac{\sqrt{a^2+2abx+b^2x^2}\left(4cd-e^2\right)\left(8ade+4bcd-5be^2\right)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{128d^{7/2}(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}\,(2dx+e)\sqrt{c+}}{64d^3(a+}$$

**Rubi [A]** time = 0.13, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {1000, 779, 612, 621, 206}

$$-\frac{\sqrt{a^2+2abx+b^2x^2}\,(2dx+e)\sqrt{c+dx^2+ex}\left(8ade+4bcd-5be^2\right)}{64d^3(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}\left(4cd-e^2\right)\left(8ade+4bcd-5be^2\right)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{128d^{7/2}(a+bx)}+\frac{\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2+ex\right)^{3/2}(8ad+6bdx-5be)}{24d^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]
```

```
[Out] -((4*b*c*d + 8*a*d*e - 5*b*e^2)*(e + 2*d*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*S
qrt[c + e*x + d*x^2])/(64*d^3*(a + b*x)) + ((8*a*d - 5*b*e + 6*b*d*x)*Sqrt[
a^2 + 2*a*b*x + b^2*x^2]*(c + e*x + d*x^2)^(3/2))/(24*d^2*(a + b*x)) - ((4*
c*d - e^2)*(4*b*c*d + 8*a*d*e - 5*b*e^2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcT
anh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(128*d^(7/2)*(a + b*x))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1000

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^Fr
acPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(
b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 h, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int x\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+ex+dx^2}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\,\int x\left(2ab+2b^2x\right)\sqrt{c+ex+dx^2}\,dx}{2ab+2b^2x}$$

$$= \frac{(8ad-5be+6bdx)\sqrt{a^2+2abx+b^2x^2}\,\left(c+ex+dx^2\right)^{3/2}}{24d^2(a+bx)} - \frac{\left(b\left(4bcd\right.\right.}{}$$

$$= -\frac{\left(4bcd+8ade-5be^2\right)(e+2dx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+ex+dx^2}}{64d^3(a+bx)}$$

$$= -\frac{\left(4bcd+8ade-5be^2\right)(e+2dx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+ex+dx^2}}{64d^3(a+bx)}$$

$$= -\frac{\left(4bcd+8ade-5be^2\right)(e+2dx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+ex+dx^2}}{64d^3(a+bx)}$$

**Mathematica [A]** time = 0.13, size = 147, normalized size = 0.65

$$\frac{\sqrt{(a+bx)^2}\left((c+x(dx+e))^{3/2}(8ad+6bdx-5be)-\frac{3\left(8ade+4bcd-5be^2\right)\left(\left(4cd-e^2\right)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\,\sqrt{c+x(dx+e)}}\right)+2\sqrt{d}\,(2dx+e)\sqrt{c+x(dx+e)}\right)}{16d^{3/2}}\right)}{24d^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]
```

```
[Out] (Sqrt[(a + b*x)^2]*((8*a*d - 5*b*e + 6*b*d*x)*(c + x*(e + d*x))^(3/2) - (3*
(4*b*c*d + 8*a*d*e - 5*b*e^2)*(2*Sqrt[d]*(e + 2*d*x)*Sqrt[c + x*(e + d*x)]
+ (4*c*d - e^2)*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + x*(e + d*x)])]))/(1
6*d^(3/2))))/(24*d^2*(a + b*x))
```

**IntegrateAlgebraic [A]** time = 0.70, size = 196, normalized size = 0.86

$$\frac{\sqrt{(a+bx)^2}\left(\frac{\left(32acd^2e-8ade^3+16bc^2d^2-24bcde^2+5be^4\right)\log\left(-2\sqrt{d}\sqrt{c+dx^2+ex}+2dx+e\right)}{128d^{7/2}}+\frac{\sqrt{c+dx^2+ex}\left(64acd^2+64ad^3x^2+16ad^2ex-24ade^2+24bcd^2x-52bcde+48bd^3x^3+8bd^2ex^2-10bde^2x+15be^3\right)}{192d^3}\right)}{a+bx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]
```

```
[Out] (Sqrt[(a + b*x)^2]*((Sqrt[c + e*x + d*x^2]*(64*a*c*d^2 - 52*b*c*d*e - 24*a*
d*e^2 + 15*b*e^3 + 24*b*c*d^2*x + 16*a*d^2*e*x - 10*b*d*e^2*x + 64*a*d^3*x^
2 + 8*b*d^2*e*x^2 + 48*b*d^3*x^3))/(192*d^3) + ((16*b*c^2*d^2 + 32*a*c*d^2*
e - 24*b*c*d*e^2 - 8*a*d*e^3 + 5*b*e^4)*Log[e + 2*d*x - 2*Sqrt[d]*Sqrt[c +
e*x + d*x^2]])/(128*d^(7/2))))/(a + b*x)
```

**fricas [A]** time = 0.45, size = 391, normalized size = 1.72

$$\left[\frac{3\left(16bc^2d^2+32acd^2e-24bcde^2-8ade^3+5be^4\right)\sqrt{d}\log\left(8d^2x^2+8dex-4\sqrt{dx^2+ex+c}\,(2dx+e)\sqrt{d}+4cd+e^2\right)+4\left(48bd^4x^3+64acd^3-52bcd^2e-24ad^2e^2+15bde^3+8\left(8ad^4+bd^3e\right)x^2+2\left(12bcd^3+8ad^3e-5bd^2e^2\right)x\right)\sqrt{dx^2+ex+c}}{768d^4}, \frac{3\left(16bc^2d^2+32acd^2e-24bcde^2-8ade^3+5be^4\right)\sqrt{-d}\arctan\left(\frac{\sqrt{dx^2+ex+c}\,(2dx+e)\sqrt{-d}}{2\left(d^2x^2+dex+cd\right)}\right)+2\left(48bd^4x^3+64acd^3-52bcd^2e-24ad^2e^2+15bde^3+8\left(8ad^4+bd^3e\right)x^2+2\left(12bcd^3+8ad^3e-5bd^2e^2\right)x\right)\sqrt{dx^2+ex+c}}{384d^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(16*b*c^2*d^2 + 32*a*c*d^2*e - 24*b*c*d*e^2 - 8*a*d*e^3 + 5*b*e^4
)*sqrt(d)*log(8*d^2*x^2 + 8*d*e*x - 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqr
t(d) + 4*c*d + e^2) + 4*(48*b*d^4*x^3 + 64*a*c*d^3 - 52*b*c*d^2*e - 24*a*d^
2*e^2 + 15*b*d*e^3 + 8*(8*a*d^4 + b*d^3*e)*x^2 + 2*(12*b*c*d^3 + 8*a*d^3*e
- 5*b*d^2*e^2)*x)*sqrt(d*x^2 + e*x + c))/d^4, 1/384*(3*(16*b*c^2*d^2 + 32*a
*c*d^2*e - 24*b*c*d*e^2 - 8*a*d*e^3 + 5*b*e^4)*sqrt(-d)*arctan(1/2*sqrt(d*x
^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(48*b*d^4*x
^3 + 64*a*c*d^3 - 52*b*c*d^2*e - 24*a*d^2*e^2 + 15*b*d*e^3 + 8*(8*a*d^4 + b
*d^3*e)*x^2 + 2*(12*b*c*d^3 + 8*a*d^3*e - 5*b*d^2*e^2)*x)*sqrt(d*x^2 + e*x
+ c))/d^4]
```

**giac [A]** time = 0.28, size = 268, normalized size = 1.18

$$\frac{1}{192}\sqrt{dx^2+xe+c}\left(2\left(4\left(6bx\mathrm{sgn}(bx+a)+\frac{8ad^3\mathrm{sgn}(bx+a)+bd^2e\mathrm{sgn}(bx+a)}{d^3}\right)x+\frac{12bcd^2\mathrm{sgn}(bx+a)+8ad^2e\mathrm{sgn}(bx+a)-5bde^2\mathrm{sgn}(bx+a)}{d^3}\right)x+\frac{64acd^2\mathrm{sgn}(bx+a)-52bcde\mathrm{sgn}(bx+a)-24ade^2\mathrm{sgn}(bx+a)+15be^3\mathrm{sgn}(bx+a)}{d^3}\right)+\frac{\left(16bc^2d^2\mathrm{sgn}(bx+a)+32acd^2e\mathrm{sgn}(bx+a)-24bcde^2\mathrm{sgn}(bx+a)-8ade^3\mathrm{sgn}(bx+a)+5be^4\mathrm{sgn}(bx+a)\right)\log\left(\left|-2\left(\sqrt{d}x-\sqrt{dx^2+xe+c}\right)\sqrt{d}-e\right|\right)}{128d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(d*x^2 + x*e + c)*(2*(4*(6*b*x*sgn(b*x + a) + (8*a*d^3*sgn(b*x +
a) + b*d^2*e*sgn(b*x + a))/d^3)*x + (12*b*c*d^2*sgn(b*x + a) + 8*a*d^2*e*sg
n(b*x + a) - 5*b*d*e^2*sgn(b*x + a))/d^3)*x + (64*a*c*d^2*sgn(b*x + a) - 52
*b*c*d*e*sgn(b*x + a) - 24*a*d*e^2*sgn(b*x + a) + 15*b*e^3*sgn(b*x + a))/d^
3) + 1/128*(16*b*c^2*d^2*sgn(b*x + a) + 32*a*c*d^2*e*sgn(b*x + a) - 24*b*c*
d*e^2*sgn(b*x + a) - 8*a*d*e^3*sgn(b*x + a) + 5*b*e^4*sgn(b*x + a))*log(abs
(-2*(sqrt(d)*x - sqrt(d*x^2 + x*e + c))*sqrt(d) - e))/d^(7/2)
```

**maple [C]** time = 0.01, size = 381, normalized size = 1.68

$$\frac{\left(-96ac\,d^{3}e\ln\left(\frac{2dx+e+2\sqrt{d\,x^{2}+ex+c}\,\sqrt{d}}{2\sqrt{d}}\right)+24a\,d^{2}e^{3}\ln\left(\frac{2dx+e+2\sqrt{d\,x^{2}+ex+c}\,\sqrt{d}}{2\sqrt{d}}\right)-48b\,c^{2}d^{3}\ln\left(\frac{2dx+e+2\sqrt{d\,x^{2}+ex+c}\,\sqrt{d}}{2\sqrt{d}}\right)+72bc\,d^{2}e^{2}\ln\left(\frac{2dx+e+2\sqrt{d\,x^{2}+ex+c}\,\sqrt{d}}{2\sqrt{d}}\right)-15bd\,e^{4}\ln\left(\frac{2dx+e+2\sqrt{d\,x^{2}+ex+c}\,\sqrt{d}}{2\sqrt{d}}\right)-96\sqrt{d\,x^{2}+ex+c}\,a\,d^{\frac{7}{2}}ex-48\sqrt{d\,x^{2}+ex+c}\,bc\,d^{\frac{7}{2}}x+60\sqrt{d\,x^{2}+ex+c}\,b\,d^{\frac{5}{2}}e^{2}x-48\sqrt{d\,x^{2}+ex+c}\,a\,d^{\frac{5}{2}}e^{2}-24\sqrt{d\,x^{2}+ex+c}\,bc\,d^{\frac{5}{2}}e+96\left(d\,x^{2}+ex+c\right)^{\frac{3}{2}}b\,d^{\frac{7}{2}}x+30\sqrt{d\,x^{2}+ex+c}\,b\,d^{\frac{3}{2}}e^{3}+128\left(d\,x^{2}+ex+c\right)^{\frac{3}{2}}a\,d^{\frac{7}{2}}-80\left(d\,x^{2}+ex+c\right)^{\frac{3}{2}}b\,d^{\frac{5}{2}}e\right)\mathrm{csgn}\left(bx+a\right)}{384d^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x)
```

```
[Out] 1/384*csgn(b*x+a)*(96*d^(7/2)*(d*x^2+e*x+c)^(3/2)*x*b+128*d^(7/2)*(d*x^2+e*
x+c)^(3/2)*a-80*d^(5/2)*(d*x^2+e*x+c)^(3/2)*b*e-96*d^(7/2)*(d*x^2+e*x+c)^(1
/2)*x*a*e-48*d^(7/2)*(d*x^2+e*x+c)^(1/2)*x*b*c+60*d^(5/2)*(d*x^2+e*x+c)^(1/
2)*x*b*e^2-48*d^(5/2)*(d*x^2+e*x+c)^(1/2)*a*e^2-24*d^(5/2)*(d*x^2+e*x+c)^(1
/2)*b*c*e+30*d^(3/2)*(d*x^2+e*x+c)^(1/2)*b*e^3-96*ln(1/2*(2*d*x+e+2*(d*x^2+
e*x+c)^(1/2)*d^(1/2))/d^(1/2))*a*c*d^3*e+24*ln(1/2*(2*d*x+e+2*(d*x^2+e*x+c)
^(1/2)*d^(1/2))/d^(1/2))*a*d^2*e^3-48*ln(1/2*(2*d*x+e+2*(d*x^2+e*x+c)^(1/2)
*d^(1/2))/d^(1/2))*b*c^2*d^3+72*ln(1/2*(2*d*x+e+2*(d*x^2+e*x+c)^(1/2)*d^(1/
2))/d^(1/2))*b*c*d^2*e^2-15*ln(1/2*(2*d*x+e+2*(d*x^2+e*x+c)^(1/2)*d^(1/2))/
d^(1/2))*b*d*e^4)/d^(9/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx^{2} + ex + c}\,\sqrt{(bx + a)^{2}}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)*x, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\sqrt{{\left(a+b\,x\right)}^2}\,\sqrt{d\,x^2+e\,x+c} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2),x)
```

```
[Out] int(x*((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{c + dx^2 + ex}\,\sqrt{(a + bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2),x)
```

```
[Out] Integral(x*sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2), x)
```

## 3.48 $\int \sqrt{a^2 + 2abx + b^2x^2}\,\sqrt{c + ex + dx^2}\,dx$

**Optimal**. Leaf size=198

$$\frac{\sqrt{a^2 + 2abx + b^2x^2}\,(4cd - e^2)(2ad - be)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\,\sqrt{c+dx^2+ex}}\right)}{16d^{5/2}(a + bx)} + \frac{\sqrt{a^2 + 2abx + b^2x^2}\,(2dx + e)(2ad - be)\sqrt{c + d}}{8d^2(a + bx)}$$

**Rubi** [A] time = 0.10, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {969, 640, 612, 621, 206}

$$\frac{\sqrt{a^2 + 2abx + b^2x^2}\,(4cd - e^2)(2ad - be)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\,\sqrt{c+dx^2+ex}}\right)}{16d^{5/2}(a + bx)} + \frac{\sqrt{a^2 + 2abx + b^2x^2}\,(2dx + e)(2ad - be)\sqrt{c + dx^2 + ex}}{8d^2(a + bx)} + \frac{b\sqrt{a^2 + 2abx + b^2x^2}\,(c + dx^2 + ex)^{3/2}}{3d(a + bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]
```

```
[Out] ((2*a*d - b*e)*(e + 2*d*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x
^2])/(8*d^2*(a + b*x)) + (b*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + e*x + d*x^2)
^(3/2))/(3*d*(a + b*x)) + ((2*a*d - b*e)*(4*c*d - e^2)*Sqrt[a^2 + 2*a*b*x +
 b^2*x^2]*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(16*d^(5/
2)*(a + b*x))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 969

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)
^2)^(q_.), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p
]*(b + 2*c*x)^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !In
tegerQ[p]
```

Rubi steps

$$\int \sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+ex+dx^2}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(2ab+2b^2x\right)\sqrt{c+ex+dx^2}\,dx}{2ab+2b^2x}$$

$$= \frac{b\sqrt{a^2+2abx+b^2x^2}\left(c+ex+dx^2\right)^{3/2}}{3d(a+bx)} + \frac{\left(b(2ad-be)\sqrt{a^2+2abx+b^2\right.}{d\left(2ab+2\right.}$$

$$= \frac{(2ad-be)(e+2dx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+ex+dx^2}}{8d^2(a+bx)} + \frac{b\sqrt{a^2+2abx}}{}$$

$$= \frac{(2ad-be)(e+2dx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+ex+dx^2}}{8d^2(a+bx)} + \frac{b\sqrt{a^2+2abx}}{}$$

$$= \frac{(2ad-be)(e+2dx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+ex+dx^2}}{8d^2(a+bx)} + \frac{b\sqrt{a^2+2abx}}{}$$

**Mathematica [A]** time = 0.13, size = 134, normalized size = 0.68

$$\frac{\sqrt{(a+bx)^2}\left(2\sqrt{d}\,\sqrt{c+x(dx+e)}\left(6ad(2dx+e)+b\left(8cd+8d^2x^2+2dex-3e^2\right)\right)+3\left(4cd-e^2\right)(2ad-be)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\,\sqrt{c+x(dx+e)}}\right)\right)}{48d^{5/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(2*Sqrt[d]*Sqrt[c + x*(e + d*x)]*(6*a*d*(e + 2*d*x) + b*
(8*c*d - 3*e^2 + 2*d*e*x + 8*d^2*x^2)) + 3*(2*a*d - b*e)*(4*c*d - e^2)*ArcT
anh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + x*(e + d*x)])]))/(48*d^(5/2)*(a + b*x))
```

**IntegrateAlgebraic [A]** time = 0.60, size = 150, normalized size = 0.76

$$\frac{\sqrt{(a+bx)^2}\left(\frac{\sqrt{c+dx^2+ex}\left(12ad^2x+6ade+8bcd+8bd^2x^2+2bdex-3be^2\right)}{24d^2}+\frac{\left(-8acd^2+2ade^2+4bcde-be^3\right)\log\left(-2d^{5/2}\sqrt{c+dx^2+ex}+2d^3x+d^2e\right)}{16d^{5/2}}\right)}{a+bx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]
```

```
[Out] (Sqrt[(a + b*x)^2]*((Sqrt[c + e*x + d*x^2]*(8*b*c*d + 6*a*d*e - 3*b*e^2 + 1
2*a*d^2*x + 2*b*d*e*x + 8*b*d^2*x^2))/(24*d^2) + ((-8*a*c*d^2 + 4*b*c*d*e +
 2*a*d*e^2 - b*e^3)*Log[d^2*e + 2*d^3*x - 2*d^(5/2)*Sqrt[c + e*x + d*x^2]])
/(16*d^(5/2))))/(a + b*x)
```

**fricas [A]** time = 0.45, size = 287, normalized size = 1.45

$$\left[\frac{3\left(8acd^2-4bcde-2ade^2+be^3\right)\sqrt{d}\log\left(8d^2x^2+8dex+4\sqrt{dx^2+ex+c}(2dx+e)\sqrt{d}+4cd+e^2\right)+4\left(8bd^3x^2+8bcd^2+6ad^2e-3bde^2+2\left(6ad^3+bd^2e\right)x\right)\sqrt{dx^2+ex+c}}{96d^3}, -\frac{3\left(8acd^2-4bcde-2ade^2+be^3\right)\sqrt{-d}\arctan\left(\frac{\sqrt{dx^2+ex+c}(2dx+e)\sqrt{-d}}{2\left(d^2x^2+dex+cd\right)}\right)-2\left(8bd^3x^2+8bcd^2+6ad^2e-3bde^2+2\left(6ad^3+bd^2e\right)x\right)\sqrt{dx^2+ex+c}}{48d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(8*a*c*d^2 - 4*b*c*d*e - 2*a*d*e^2 + b*e^3)*sqrt(d)*log(8*d^2*x^2
+ 8*d*e*x + 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + 4*
(8*b*d^3*x^2 + 8*b*c*d^2 + 6*a*d^2*e - 3*b*d*e^2 + 2*(6*a*d^3 + b*d^2*e)*x)
*sqrt(d*x^2 + e*x + c))/d^3, -1/48*(3*(8*a*c*d^2 - 4*b*c*d*e - 2*a*d*e^2 +
b*e^3)*sqrt(-d)*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*
x^2 + d*e*x + c*d)) - 2*(8*b*d^3*x^2 + 8*b*c*d^2 + 6*a*d^2*e - 3*b*d*e^2 +
2*(6*a*d^3 + b*d^2*e)*x)*sqrt(d*x^2 + e*x + c))/d^3]
```

**giac [A]** time = 0.30, size = 185, normalized size = 0.93

$$\frac{1}{24}\sqrt{dx^2+xe+c}\left(2\left(4bx\operatorname{sgn}(bx+a)+\frac{6ad^2\operatorname{sgn}(bx+a)+bde\operatorname{sgn}(bx+a)}{d^2}\right)x+\frac{8bcd\operatorname{sgn}(bx+a)+6ade\operatorname{sgn}(bx+a)-3be^2\operatorname{sgn}(bx+a)}{d^2}\right)-\frac{\left(8acd^2\operatorname{sgn}(bx+a)-4bcde\operatorname{sgn}(bx+a)-2ade^2\operatorname{sgn}(bx+a)+be^3\operatorname{sgn}(bx+a)\right)\log\left(\left|-2\left(\sqrt{d}x-\sqrt{dx^2+xe+c}\right)\sqrt{d}-e\right|\right)}{16d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(d*x^2 + x*e + c)*(2*(4*b*x*sgn(b*x + a) + (6*a*d^2*sgn(b*x + a) +
 b*d*e*sgn(b*x + a))/d^2)*x + (8*b*c*d*sgn(b*x + a) + 6*a*d*e*sgn(b*x + a)
- 3*b*e^2*sgn(b*x + a))/d^2) - 1/16*(8*a*c*d^2*sgn(b*x + a) - 4*b*c*d*e*sgn
(b*x + a) - 2*a*d*e^2*sgn(b*x + a) + b*e^3*sgn(b*x + a))*log(abs(-2*(sqrt(d
)*x - sqrt(d*x^2 + x*e + c))*sqrt(d) - e))/d^(5/2)
```

**maple [C]** time = 0.01, size = 257, normalized size = 1.30

$$\frac{\left(24acd^3\ln\left(\frac{2dx+e+2\sqrt{dx^2+ex+c}\,\sqrt{d}}{2\sqrt{d}}\right)-6ad^2e^2\ln\left(\frac{2dx+e+2\sqrt{dx^2+ex+c}\,\sqrt{d}}{2\sqrt{d}}\right)-12bcd^2e\ln\left(\frac{2dx+e+2\sqrt{dx^2+ex+c}\,\sqrt{d}}{2\sqrt{d}}\right)+3bde^3\ln\left(\frac{2dx+e+2\sqrt{dx^2+ex+c}\,\sqrt{d}}{2\sqrt{d}}\right)+24\sqrt{dx^2+ex+c}\,ad^{\frac{7}{2}}x-12\sqrt{dx^2+ex+c}\,bd^{\frac{5}{2}}ex+12\sqrt{dx^2+ex+c}\,ad^{\frac{5}{2}}e-6\sqrt{dx^2+ex+c}\,bd^{\frac{3}{2}}e^2+16\left(dx^2+ex+c\right)^{\frac{3}{2}}bd^{\frac{5}{2}}\right)\operatorname{csgn}(bx+a)}{48d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x)
```

```
[Out] 1/48*csgn(b*x+a)*(16*d^(5/2)*(d*x^2+e*x+c)^(3/2)*b+24*d^(7/2)*(d*x^2+e*x+c)
^(1/2)*x*a-12*d^(5/2)*(d*x^2+e*x+c)^(1/2)*x*b*e+12*d^(5/2)*(d*x^2+e*x+c)^(1
/2)*a*e-6*d^(3/2)*(d*x^2+e*x+c)^(1/2)*b*e^2+24*ln(1/2*(2*d*x+e+2*(d*x^2+e*x
+c)^(1/2)*d^(1/2))/d^(1/2))*a*c*d^3-6*ln(1/2*(2*d*x+e+2*(d*x^2+e*x+c)^(1/2)
*d^(1/2))/d^(1/2))*a*d^2*e^2-12*ln(1/2*(2*d*x+e+2*(d*x^2+e*x+c)^(1/2)*d^(1/
2))/d^(1/2))*b*c*d^2*e+3*ln(1/2*(2*d*x+e+2*(d*x^2+e*x+c)^(1/2)*d^(1/2))/d^(
1/2))*b*d*e^3)/d^(7/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx^2 + ex + c}\,\sqrt{(bx + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{(a + b\,x)^2}\,\sqrt{d\,x^2 + e\,x + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2),x)
```

```
[Out] int(((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c + dx^2 + ex}\,\sqrt{(a + bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2), x)
```

## 3.49 $\int \frac{\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+ex+dx^2}}{x}\,dx$

**Optimal**. Leaf size=211

$$\frac{\sqrt{a^2+2abx+b^2x^2}\,\left(4ade+4bcd-be^2\right)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\,\sqrt{c+dx^2+ex}}\right)}{8d^{3/2}(a+bx)}+\frac{\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2+ex}\,(4ad+2bdx}{4d(a+bx)}$$

**Rubi** [A] time = 0.22, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1000, 814, 843, 621, 206, 724}

$$\frac{\sqrt{a^2+2abx+b^2x^2}\,\left(4ade+4bcd-be^2\right)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\,\sqrt{c+dx^2+ex}}\right)}{8d^{3/2}(a+bx)}+\frac{\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2+ex}\,(4ad+2bdx+be)}{4d(a+bx)}-\frac{a\sqrt{c}\,\sqrt{a^2+2abx+b^2x^2}\,\tanh^{-1}\left(\frac{2c+ex}{2\sqrt{c}\,\sqrt{c+dx^2+ex}}\right)}{a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x,x]
```

```
[Out] ((4*a*d + b*e + 2*b*d*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2
])/(4*d*(a + b*x)) + ((4*b*c*d + 4*a*d*e - b*e^2)*Sqrt[a^2 + 2*a*b*x + b^2*
x^2]*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(8*d^(3/2)*(a
+ b*x)) - (a*Sqrt[c]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(2*c + e*x)/(2*S
qrt[c]*Sqrt[c + e*x + d*x^2])])/(a + b*x)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1000

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^Fr
acPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(
b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 h, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+ex+dx^2}}{x}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\,\int \frac{(2ab+2b^2x)\sqrt{c+ex+dx^2}}{x}\,dx}{2ab+2b^2x}$$

$$= \frac{(4ad+be+2bdx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+ex+dx^2}}{4d(a+bx)} - \frac{\sqrt{a^2+2abx\cdots}}{}$$

$$= \frac{(4ad+be+2bdx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+ex+dx^2}}{4d(a+bx)} + \frac{\left(2abc\sqrt{a^2+\cdots}\right.}{}$$

$$= \frac{(4ad+be+2bdx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+ex+dx^2}}{4d(a+bx)} - \frac{\left(4abc\sqrt{a^2+\cdots}\right.}{}$$

$$= \frac{(4ad+be+2bdx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+ex+dx^2}}{4d(a+bx)} + \frac{\left(4bcd+4ade\cdots\right.}{}$$

**Mathematica** [A] time = 0.20, size = 149, normalized size = 0.71

$$\frac{\sqrt{(a+bx)^2}\left(\left(4ade+4bcd-be^2\right)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\,\sqrt{c+x(dx+e)}}\right)+2\sqrt{d}\left(\sqrt{c+x(dx+e)}\,(4ad+b(2dx+e))-4a\sqrt{c}\,d\tanh^{-1}\left(\frac{2c+ex}{2\sqrt{c}\,\sqrt{c+x(dx+e)}}\right)\right)\right)}{8d^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*((4*b*c*d + 4*a*d*e - b*e^2)*ArcTanh[(e + 2*d*x)/(2*Sqrt
[d]*Sqrt[c + x*(e + d*x)])] + 2*Sqrt[d]*(Sqrt[c + x*(e + d*x)]*(4*a*d + b*(
e + 2*d*x)) - 4*a*Sqrt[c]*d*ArcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + x*(e +
d*x)])])))/(8*d^(3/2)*(a + b*x))
```

**IntegrateAlgebraic** [A] time = 0.64, size = 154, normalized size = 0.73

$$\frac{\sqrt{(a+bx)^2}\left(\frac{\left(-4ade-4bcd+be^2\right)\log\left(-2d^{3/2}\sqrt{c+dx^2+ex}+2d^2x+de\right)}{8d^{3/2}}+\frac{\sqrt{c+dx^2+ex}\,(4ad+2bdx+be)}{4d}+2a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}-\frac{\sqrt{c+dx^2+ex}}{\sqrt{c}}\right)\right)}{a+bx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x,
x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(((4*a*d + b*e + 2*b*d*x)*Sqrt[c + e*x + d*x^2])/(4*d) +
 2*a*Sqrt[c]*ArcTanh[(Sqrt[d]*x)/Sqrt[c] - Sqrt[c + e*x + d*x^2]/Sqrt[c]] +
 ((-4*b*c*d - 4*a*d*e + b*e^2)*Log[d*e + 2*d^2*x - 2*d^(3/2)*Sqrt[c + e*x +
 d*x^2]])/(8*d^(3/2))))/(a + b*x)
```

**fricas** [**A**] time = 1.14, size = 651, normalized size = 3.09

$$\left[\frac{8\,a\sqrt{c}\,d^2\log\left(\frac{8\,cex+\left(4\,cd+e^2\right)x^2-4\,\sqrt{dx^2+ex+c}\,(ex+2\,c)\sqrt{c}+8\,c^2}{x^2}\right)-\left(4\,bcd+4\,ade-be^2\right)\sqrt{d}\,\log\left(8\,d^2x^2+8\,dex-4\,\sqrt{dx^2+ex+c}\,(2\,dx+e)\sqrt{d}+4\,cd+e^2\right)+4\left(2\,bd^2x+4\,ad^2+bde\right)\sqrt{dx^2+ex+c}}{16\,d^2}, \frac{4\,a\sqrt{c}\,d^2\log\left(\frac{8\,cex+\left(4\,cd+e^2\right)x^2-4\,\sqrt{dx^2+ex+c}\,(ex+2\,c)\sqrt{c}+8\,c^2}{x^2}\right)-\left(4\,bcd+4\,ade-be^2\right)\sqrt{-d}\,\arctan\left(\frac{\sqrt{dx^2+ex+c}\,(2\,dx+e)\sqrt{-d}}{2\left(d^2x^2+dex+cd\right)}\right)+2\left(2\,bd^2x+4\,ad^2+bde\right)\sqrt{dx^2+ex+c}}{8\,d^2}, \frac{16\,a\sqrt{-c}\,d^2\arctan\left(\frac{\sqrt{dx^2+ex+c}\,(ex+2\,c)\sqrt{-c}}{2\left(cdx^2+cex+c^2\right)}\right)-\left(4\,bcd+4\,ade-be^2\right)\sqrt{d}\,\log\left(8\,d^2x^2+8\,dex-4\,\sqrt{dx^2+ex+c}\,(2\,dx+e)\sqrt{d}+4\,cd+e^2\right)+4\left(2\,bd^2x+4\,ad^2+bde\right)\sqrt{dx^2+ex+c}}{16\,d^2}, \frac{8\,a\sqrt{-c}\,d^2\arctan\left(\frac{\sqrt{dx^2+ex+c}\,(ex+2\,c)\sqrt{-c}}{2\left(cdx^2+cex+c^2\right)}\right)-\left(4\,bcd+4\,ade-be^2\right)\sqrt{-d}\,\arctan\left(\frac{\sqrt{dx^2+ex+c}\,(2\,dx+e)\sqrt{-d}}{2\left(d^2x^2+dex+cd\right)}\right)+2\left(2\,bd^2x+4\,ad^2+bde\right)\sqrt{dx^2+ex+c}}{8\,d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/16*(8*a*sqrt(c)*d^2*log((8*c*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e*
x + c)*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) - (4*b*c*d + 4*a*d*e - b*e^2)*sqrt
(d)*log(8*d^2*x^2 + 8*d*e*x - 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) +
 4*c*d + e^2) + 4*(2*b*d^2*x + 4*a*d^2 + b*d*e)*sqrt(d*x^2 + e*x + c))/d^2,
 1/8*(4*a*sqrt(c)*d^2*log((8*c*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e*x
 + c)*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) - (4*b*c*d + 4*a*d*e - b*e^2)*sqrt(
-d)*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x
+ c*d)) + 2*(2*b*d^2*x + 4*a*d^2 + b*d*e)*sqrt(d*x^2 + e*x + c))/d^2, 1/16*
(16*a*sqrt(-c)*d^2*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(-c)/(c
*d*x^2 + c*e*x + c^2)) - (4*b*c*d + 4*a*d*e - b*e^2)*sqrt(d)*log(8*d^2*x^2
+ 8*d*e*x - 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + 4*
(2*b*d^2*x + 4*a*d^2 + b*d*e)*sqrt(d*x^2 + e*x + c))/d^2, 1/8*(8*a*sqrt(-c)
*d^2*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(-c)/(c*d*x^2 + c*e*x
 + c^2)) - (4*b*c*d + 4*a*d*e - b*e^2)*sqrt(-d)*arctan(1/2*sqrt(d*x^2 + e*x
 + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(2*b*d^2*x + 4*a*d^
2 + b*d*e)*sqrt(d*x^2 + e*x + c))/d^2]
```

**giac** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(b*x
+a)]Warning, replacing 0 by ` u`, a substitution variable should perhaps be
 purged.Warning, replacing 0 by ` u`, a substitution variable should perhap
s be purged.Warning, replacing 0 by ` u`, a substitution variable should pe
rhaps be purged.index.cc index_m operator + Error: Bad Argument Value
```

**maple [C]** time = 0.01, size = 214, normalized size = 1.01

$$-\frac{\left(8a\sqrt{c}\, d^{\frac{5}{2}} \ln\left(\frac{ex+2c+2\sqrt{d\,x^2+ex+c}\,\sqrt{c}}{x}\right) - 4a\,d^2 e \ln\left(\frac{2dx+e+2\sqrt{d\,x^2+ex+c}\,\sqrt{d}}{2\sqrt{d}}\right) - 4bc\,d^2 \ln\left(\frac{2dx+e+2\sqrt{d\,x^2+ex+c}\,\sqrt{d}}{2\sqrt{d}}\right) + bd\,e^2 \ln\left(\frac{2dx+e+2\sqrt{d\,x^2+ex+c}\,\sqrt{d}}{2\sqrt{d}}\right) - 4\sqrt{d\,x^2+ex+c}\, b\,d^{\frac{5}{2}} x - 8\sqrt{d\,x^2+ex+c}\, a\,d^{\frac{5}{2}} - 2\sqrt{d\,x^2+ex+c}\, b\,d^{\frac{3}{2}} e\right) \operatorname{csgn}(bx+a)}{8d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x,x)
```

```
[Out] -1/8*csgn(b*x+a)*(8*c^(1/2)*d^(5/2)*ln((2*c+e*x+2*c^(1/2)*(d*x^2+e*x+c)^(1/
2))/x)*a-4*d^(5/2)*(d*x^2+e*x+c)^(1/2)*x*b-8*d^(5/2)*(d*x^2+e*x+c)^(1/2)*a-
2*d^(3/2)*(d*x^2+e*x+c)^(1/2)*b*e-4*d^2*ln(1/2*(2*d*x+e+2*(d*x^2+e*x+c)^(1/
2)*d^(1/2))/d^(1/2))*a*e-4*ln(1/2*(2*d*x+e+2*(d*x^2+e*x+c)^(1/2)*d^(1/2))/d
^(1/2))*b*c*d^2+ln(1/2*(2*d*x+e+2*(d*x^2+e*x+c)^(1/2)*d^(1/2))/d^(1/2))*b*d
*e^2)/d^(5/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^2 + ex + c}\,\sqrt{(bx+a)^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)/x, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{(a+b\,x)^2}\,\sqrt{d\,x^2+e\,x+c}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x,x)
```

```
[Out] int((((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^2 + ex}\,\sqrt{(a+bx)^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2)/x, x)
```

## 3.50 $\int \frac{\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+ex+dx^2}}{x^2}\,dx$

**Optimal**. Leaf size=202

$$-\frac{\sqrt{a^2+2abx+b^2x^2}\,(a-bx)\sqrt{c+dx^2+ex}}{x(a+bx)}+\frac{\sqrt{a^2+2abx+b^2x^2}\,(2ad+be)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\,\sqrt{c+dx^2+ex}}\right)}{2\sqrt{d}\,(a+bx)}-\frac{\sqrt{a^2+2abx}}{}$$

**Rubi [A]** time = 0.19, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1000, 812, 843, 621, 206, 724}

$$-\frac{\sqrt{a^2+2abx+b^2x^2}\,(a-bx)\sqrt{c+dx^2+ex}}{x(a+bx)}+\frac{\sqrt{a^2+2abx+b^2x^2}\,(2ad+be)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\,\sqrt{c+dx^2+ex}}\right)}{2\sqrt{d}\,(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}\,(ae+2bc)\tanh^{-1}\left(\frac{2c+ex}{2\sqrt{c}\,\sqrt{c+dx^2+ex}}\right)}{2\sqrt{c}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x^2,x]
```

```
[Out] -(((a - b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/(x*(a + b
*x))) + ((2*a*d + b*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(e + 2*d*x)/(2
*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(2*Sqrt[d]*(a + b*x)) - ((2*b*c + a*e)*Sq
rt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + e*x + d
*x^2])])/(2*Sqrt[c]*(a + b*x))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1000

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^Fr
acPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(
b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 h, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^2}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(2ab+2b^2x)\sqrt{c+ex+dx^2}}{x^2}\,dx}{2ab+2b^2x}$$

$$= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x(a+bx)} - \frac{\sqrt{a^2+2abx+b^2x^2}\int -}{2\left(2ab\right.}$$

$$= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x(a+bx)} + \frac{\left(b(2bc+ae)\sqrt{a^2+2ab\right.}{2ab}$$

$$= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x(a+bx)} - \frac{\left(2b(2bc+ae)\sqrt{a^2+2a\right.}{}$$

$$= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x(a+bx)} + \frac{(2ad+be)\sqrt{a^2+2abx}}{2\sqrt{}}$$

**Mathematica [A]** time = 0.19, size = 155, normalized size = 0.77

$$\frac{\sqrt{(a+bx)^2}\left(\sqrt{c}\,x(2ad+be)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+x(dx+e)}}\right)+\sqrt{d}\left(2\sqrt{c}(bx-a)\sqrt{c+x(dx+e)}-x(ae+2bc)\tanh^{-1}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+x(dx+e)}}\right)\right)\right)}{2\sqrt{c}\sqrt{d}\,x(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x^2,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(Sqrt[c]*(2*a*d + b*e)*x*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*
Sqrt[c + x*(e + d*x)])] + Sqrt[d]*(2*Sqrt[c]*(-a + b*x)*Sqrt[c + x*(e + d*x
)] - (2*b*c + a*e)*x*ArcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + x*(e + d*x)])]
)))/(2*Sqrt[c]*Sqrt[d]*x*(a + b*x))
```

**IntegrateAlgebraic [A]** time = 0.62, size = 136, normalized size = 0.67

$$\frac{\sqrt{(a+bx)^2}\left(\frac{(bx-a)\sqrt{c+dx^2+ex}}{x}+\frac{(-2ad-be)\log\left(-2\sqrt{d}\sqrt{c+dx^2+ex}+2dx+e\right)}{2\sqrt{d}}+\frac{(-ae-2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2+ex}-\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}}\right)}{a+bx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x^
2,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(((-a + b*x)*Sqrt[c + e*x + d*x^2])/x + ((-2*b*c - a*e)*
ArcTanh[(-(Sqrt[d]*x) + Sqrt[c + e*x + d*x^2])/Sqrt[c]])/Sqrt[c] + ((-2*a*d
 - b*e)*Log[e + 2*d*x - 2*Sqrt[d]*Sqrt[c + e*x + d*x^2]])/(2*Sqrt[d])))/(a
+ b*x)
```

**fricas** [**A**] time = 0.69, size = 647, normalized size = 3.20

$$\left[\frac{(2\,acd + bce)\sqrt{d}\,x\log\left(8\,d^2x^2 + 8\,dex + 4\,\sqrt{dx^2 + ex + c}\,(2\,dx + e)\sqrt{d} + 4\,cd + e^2\right) + (2\,bcd + ade)\sqrt{c}\,x\log\left(\frac{8\,cex + (4\,cd + e^2)x^2 - 4\,\sqrt{dx^2 + ex + c}\,(ex + 2\,c)\sqrt{c} + 8\,c^2}{x^2}\right) + 4\,(bcdx - acd)\sqrt{dx^2 + ex + c}}{4\,cdx}, -\frac{2\,(2\,acd + bce)\sqrt{-d}\,x\arctan\left(\frac{\sqrt{dx^2 + ex + c}\,(2\,dx + e)\sqrt{-d}}{2\,(d^2x^2 + dex + cd)}\right) - (2\,bcd + ade)\sqrt{c}\,x\log\left(\frac{8\,cex + (4\,cd + e^2)x^2 - 4\,\sqrt{dx^2 + ex + c}\,(ex + 2\,c)\sqrt{c} + 8\,c^2}{x^2}\right) - 4\,(bcdx - acd)\sqrt{dx^2 + ex + c}}{4\,cdx}, \frac{2\,(2\,bcd + ade)\sqrt{-c}\,x\arctan\left(\frac{\sqrt{dx^2 + ex + c}\,(ex + 2\,c)\sqrt{-c}}{2\,(cdx^2 + cex + c^2)}\right) + (2\,acd + bce)\sqrt{d}\,x\log\left(8\,d^2x^2 + 8\,dex + 4\,\sqrt{dx^2 + ex + c}\,(2\,dx + e)\sqrt{d} + 4\,cd + e^2\right) + 4\,(bcdx - acd)\sqrt{dx^2 + ex + c}}{4\,cdx}, \frac{(2\,bcd + ade)\sqrt{-c}\,x\arctan\left(\frac{\sqrt{dx^2 + ex + c}\,(ex + 2\,c)\sqrt{-c}}{2\,(cdx^2 + cex + c^2)}\right) - (2\,acd + bce)\sqrt{-d}\,x\arctan\left(\frac{\sqrt{dx^2 + ex + c}\,(2\,dx + e)\sqrt{-d}}{2\,(d^2x^2 + dex + cd)}\right) + 2\,(bcdx - acd)\sqrt{dx^2 + ex + c}}{2\,cdx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/4*((2*a*c*d + b*c*e)*sqrt(d)*x*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 +
e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + (2*b*c*d + a*d*e)*sqrt(c)*x*l
og((8*c*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(
c) + 8*c^2)/x^2) + 4*(b*c*d*x - a*c*d)*sqrt(d*x^2 + e*x + c))/(c*d*x), -1/4
*(2*(2*a*c*d + b*c*e)*sqrt(-d)*x*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*d*x +
e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) - (2*b*c*d + a*d*e)*sqrt(c)*x*log((8*c
*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(c) + 8*
c^2)/x^2) - 4*(b*c*d*x - a*c*d)*sqrt(d*x^2 + e*x + c))/(c*d*x), 1/4*(2*(2*b
*c*d + a*d*e)*sqrt(-c)*x*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(
-c)/(c*d*x^2 + c*e*x + c^2)) + (2*a*c*d + b*c*e)*sqrt(d)*x*log(8*d^2*x^2 +
8*d*e*x + 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + 4*(b
*c*d*x - a*c*d)*sqrt(d*x^2 + e*x + c))/(c*d*x), 1/2*((2*b*c*d + a*d*e)*sqrt
(-c)*x*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(-c)/(c*d*x^2 + c*e
*x + c^2)) - (2*a*c*d + b*c*e)*sqrt(-d)*x*arctan(1/2*sqrt(d*x^2 + e*x + c)*
(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(b*c*d*x - a*c*d)*sqrt(d*
x^2 + e*x + c))/(c*d*x)]
```

**giac** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(b*x
+a)]Unable to divide, perhaps due to rounding error%%%{2,[4,4,0]%%%}+%%%{%%
%{-16,[1]%%%},[4,2,1]%%%}+%%%{%%%{32,[2]%%%},[4,0,2]%%%}+%%%{-4,[2,4,1]%%%}
+%%%{%%%{32,[1]%%%},[2,2,2]%%%}+%%%{%%%{-64,[2]%%%},[2,0,3]%%%}+%%%{2,[0,4,
2]%%%}+%%%{%%%{-16,[1]%%%},[0,2,3]%%%}+%%%{%%%{32,[2]%%%},[0,0,4]%%%} / %%%
{4,[4,0,0]%%%}+%%%{-8,[2,0,1]%%%}+%%%{4,[0,0,2]%%%} Error: Bad Argument Val
ue
```

**maple [C]** time = 0.01, size = 249, normalized size = 1.23

$$\frac{\left(2ac\,d^{2}x\ln\left(\frac{2dx+e+2\sqrt{d\,x^{2}+ex+c}\,\sqrt{d}}{2\sqrt{d}}\right)-a\sqrt{c}\,d^{\frac{3}{2}}ex\ln\left(\frac{ex+2c+2\sqrt{d\,x^{2}+ex+c}\,\sqrt{c}}{x}\right)-2b\,c^{\frac{3}{2}}d^{\frac{3}{2}}x\ln\left(\frac{ex+2c+2\sqrt{d\,x^{2}+ex+c}\,\sqrt{c}}{x}\right)+bcdex\ln\left(\frac{2dx+e+2\sqrt{d\,x^{2}+ex+c}\,\sqrt{d}}{2\sqrt{d}}\right)+2\sqrt{d\,x^{2}+ex+c}\,a\,d^{\frac{5}{2}}x^{2}+2\sqrt{d\,x^{2}+ex+c}\,a\,d^{\frac{3}{2}}ex+2\sqrt{d\,x^{2}+ex+c}\,bc\,d^{\frac{3}{2}}x-2\left(d\,x^{2}+ex+c\right)^{\frac{3}{2}}a\,d^{\frac{3}{2}}\right)\operatorname{csgn}\left(bx+a\right)}{2c\,d^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^2,x)
```

```
[Out] 1/2*csgn(b*x+a)*(2*d^(5/2)*(d*x^2+e*x+c)^(1/2)*x^2*a-2*d^(3/2)*c^(3/2)*ln((
e*x+2*c+2*(d*x^2+e*x+c)^(1/2)*c^(1/2))/x)*x*b-d^(3/2)*c^(1/2)*ln((e*x+2*c+2
*(d*x^2+e*x+c)^(1/2)*c^(1/2))/x)*x*a*e-2*d^(3/2)*(d*x^2+e*x+c)^(3/2)*a+2*d^
(3/2)*(d*x^2+e*x+c)^(1/2)*x*a*e+2*d^(3/2)*(d*x^2+e*x+c)^(1/2)*x*b*c+2*ln(1/
2*(2*d*x+e+2*(d*x^2+e*x+c)^(1/2)*d^(1/2))/d^(1/2))*x*a*c*d^2+ln(1/2*(2*d*x+
e+2*(d*x^2+e*x+c)^(1/2)*d^(1/2))/d^(1/2))*d*x*b*c*e)/c/x/d^(3/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^2 + ex + c}\,\sqrt{(bx + a)^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)/x^2, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{(a + b\,x)^2}\,\sqrt{d\,x^2 + e\,x + c}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^2,x)
```

```
[Out] int((((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^2 + ex}\,\sqrt{(a + bx)^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2)/x**2, x)
```

## 3.51 $\int \frac{\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+ex+dx^2}}{x^3}\,dx$

**Optimal**. Leaf size=215

$$-\frac{\sqrt{a^2+2abx+b^2x^2}\,\left(4acd-ae^2+4bce\right)\tanh^{-1}\left(\frac{2c+ex}{2\sqrt{c}\,\sqrt{c+dx^2+ex}}\right)}{8c^{3/2}(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2+ex}\,(x(ae+4bc}{4cx^2(a+bx)}$$

**Rubi** [A] time = 0.18, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1000, 810, 843, 621, 206, 724}

$$-\frac{\sqrt{a^2+2abx+b^2x^2}\,\left(4acd-ae^2+4bce\right)\tanh^{-1}\left(\frac{2c+ex}{2\sqrt{c}\,\sqrt{c+dx^2+ex}}\right)}{8c^{3/2}(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2+ex}\,(x(ae+4bc)+2ac)}{4cx^2(a+bx)}+\frac{b\sqrt{d}\,\sqrt{a^2+2abx+b^2x^2}\,\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\,\sqrt{c+dx^2+ex}}\right)}{a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x^3,x]
```

```
[Out] -((2*a*c + (4*b*c + a*e)*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*
x^2])/(4*c*x^2*(a + b*x)) + (b*Sqrt[d]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTan
h[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(a + b*x) - ((4*a*c*d + 4
*b*c*e - a*e^2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(2*c + e*x)/(2*Sqrt[c
]*Sqrt[c + e*x + d*x^2])])/(8*c^(3/2)*(a + b*x))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1000

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^Fr
acPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(
b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 h, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^3}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(2ab+2b^2x)\sqrt{c+ex+dx^2}}{x^3}\,dx}{2ab+2b^2x}$$

$$= -\frac{(2ac+(4bc+ae)x)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{4cx^2(a+bx)} - \frac{\sqrt{a^2+2ab}}{}$$

$$= -\frac{(2ac+(4bc+ae)x)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{4cx^2(a+bx)} + \frac{\left(2b^2d\sqrt{a^2}\right.}{}$$

$$= -\frac{(2ac+(4bc+ae)x)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{4cx^2(a+bx)} + \frac{\left(4b^2d\sqrt{a^2}\right.}{}$$

$$= -\frac{(2ac+(4bc+ae)x)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{4cx^2(a+bx)} + \frac{b\sqrt{d}\sqrt{a^2}}{}$$

**Mathematica [A]** time = 0.22, size = 161, normalized size = 0.75

$$-\frac{\sqrt{(a+bx)^2}\left(x^2\left(4acd-ae^2+4bce\right)\tanh^{-1}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+x(dx+e)}}\right)+2\sqrt{c}\sqrt{c+x(dx+e)}\,(2ac+aex+4bcx)-8bc^{3/2}\sqrt{d}\,x^2\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+x(dx+e)}}\right)\right)}{8c^{3/2}x^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x^3,x]
```

```
[Out] -1/8*(Sqrt[(a + b*x)^2]*(2*Sqrt[c]*(2*a*c + 4*b*c*x + a*e*x)*Sqrt[c + x*(e
+ d*x)] - 8*b*c^(3/2)*Sqrt[d]*x^2*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + x
*(e + d*x)])] + (4*a*c*d + 4*b*c*e - a*e^2)*x^2*ArcTanh[(2*c + e*x)/(2*Sqrt
[c]*Sqrt[c + x*(e + d*x)])]))/(c^(3/2)*x^2*(a + b*x))
```

**IntegrateAlgebraic [A]** time = 0.81, size = 150, normalized size = 0.70

$$\frac{\sqrt{(a+bx)^2}\left(\frac{\left(-4acd+ae^2-4bce\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2+ex}-\sqrt{d}\,x}{\sqrt{c}}\right)}{4c^{3/2}}+\frac{\sqrt{c+dx^2+ex}\,(-2ac-aex-4bcx)}{4cx^2}-b\sqrt{d}\log\left(-2\sqrt{d}\sqrt{c+dx^2+ex}+2dx+e\right)\right)}{a+bx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x^
3,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(((-2*a*c - 4*b*c*x - a*e*x)*Sqrt[c + e*x + d*x^2])/(4*c
*x^2) + ((-4*a*c*d - 4*b*c*e + a*e^2)*ArcTanh[(-(Sqrt[d]*x) + Sqrt[c + e*x
+ d*x^2])/Sqrt[c]])/(4*c^(3/2)) - b*Sqrt[d]*Log[e + 2*d*x - 2*Sqrt[d]*Sqrt[
c + e*x + d*x^2]]))/(a + b*x)
```

**fricas [A]** time = 0.86, size = 693, normalized size = 3.22

$$\left[\frac{8\,bc^{2}\sqrt{d}x^{2}\log\left(8\,d^{2}x^{2}+8\,dex+4\,\sqrt{dx^{2}+ex+c}\,(2\,dx+e)\sqrt{d}+4\,cd+e^{2}\right)-\left(4\,acd+4\,bce-ae^{2}\right)\sqrt{c}x^{2}\log\left(\frac{8\,cex+\left(4\,cd+e^{2}\right)x^{2}+4\,\sqrt{dx^{2}+ex+c}\,(ex+2\,c)\sqrt{c}+8\,c^{2}}{x^{2}}\right)-4\,\left(2\,ac^{2}+\left(4\,bc^{2}+ace\right)x\right)\sqrt{dx^{2}+ex+c}}{16\,c^{2}x^{2}},\ -\frac{16\,bc^{2}\sqrt{-d}x^{2}\arctan\left(\frac{\sqrt{dx^{2}+ex+c}\,(2\,dx+e)\sqrt{-d}}{2\,\left(d^{2}x^{2}+dex+cd\right)}\right)+\left(4\,acd+4\,bce-ae^{2}\right)\sqrt{c}x^{2}\log\left(\frac{8\,cex+\left(4\,cd+e^{2}\right)x^{2}+4\,\sqrt{dx^{2}+ex+c}\,(ex+2\,c)\sqrt{c}+8\,c^{2}}{x^{2}}\right)+4\,\left(2\,ac^{2}+\left(4\,bc^{2}+ace\right)x\right)\sqrt{dx^{2}+ex+c}}{16\,c^{2}x^{2}},\ \frac{4\,bc^{2}\sqrt{d}x^{2}\log\left(8\,d^{2}x^{2}+8\,dex+4\,\sqrt{dx^{2}+ex+c}\,(2\,dx+e)\sqrt{d}+4\,cd+e^{2}\right)+\left(4\,acd+4\,bce-ae^{2}\right)\sqrt{-c}x^{2}\arctan\left(\frac{\sqrt{dx^{2}+ex+c}\,(ex+2\,c)\sqrt{-c}}{2\,\left(cdx^{2}+cex+c^{2}\right)}\right)-2\,\left(2\,ac^{2}+\left(4\,bc^{2}+ace\right)x\right)\sqrt{dx^{2}+ex+c}}{8\,c^{2}x^{2}},\ -\frac{8\,bc^{2}\sqrt{-d}x^{2}\arctan\left(\frac{\sqrt{dx^{2}+ex+c}\,(2\,dx+e)\sqrt{-d}}{2\,\left(d^{2}x^{2}+dex+cd\right)}\right)-\left(4\,acd+4\,bce-ae^{2}\right)\sqrt{-c}x^{2}\arctan\left(\frac{\sqrt{dx^{2}+ex+c}\,(ex+2\,c)\sqrt{-c}}{2\,\left(cdx^{2}+cex+c^{2}\right)}\right)+2\,\left(2\,ac^{2}+\left(4\,bc^{2}+ace\right)x\right)\sqrt{dx^{2}+ex+c}}{8\,c^{2}x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/16*(8*b*c^2*sqrt(d)*x^2*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 + e*x + c
)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) - (4*a*c*d + 4*b*c*e - a*e^2)*sqrt(c)*
x^2*log((8*c*e*x + (4*c*d + e^2)*x^2 + 4*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*
sqrt(c) + 8*c^2)/x^2) - 4*(2*a*c^2 + (4*b*c^2 + a*c*e)*x)*sqrt(d*x^2 + e*x
+ c))/(c^2*x^2), -1/16*(16*b*c^2*sqrt(-d)*x^2*arctan(1/2*sqrt(d*x^2 + e*x +
 c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + (4*a*c*d + 4*b*c*e - a*
e^2)*sqrt(c)*x^2*log((8*c*e*x + (4*c*d + e^2)*x^2 + 4*sqrt(d*x^2 + e*x + c)
*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) + 4*(2*a*c^2 + (4*b*c^2 + a*c*e)*x)*sqrt
(d*x^2 + e*x + c))/(c^2*x^2), 1/8*(4*b*c^2*sqrt(d)*x^2*log(8*d^2*x^2 + 8*d*
e*x + 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + (4*a*c*d
 + 4*b*c*e - a*e^2)*sqrt(-c)*x^2*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*
c)*sqrt(-c)/(c*d*x^2 + c*e*x + c^2)) - 2*(2*a*c^2 + (4*b*c^2 + a*c*e)*x)*sq
rt(d*x^2 + e*x + c))/(c^2*x^2), -1/8*(8*b*c^2*sqrt(-d)*x^2*arctan(1/2*sqrt(
d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) - (4*a*c*d +
 4*b*c*e - a*e^2)*sqrt(-c)*x^2*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)
*sqrt(-c)/(c*d*x^2 + c*e*x + c^2)) + 2*(2*a*c^2 + (4*b*c^2 + a*c*e)*x)*sqrt
(d*x^2 + e*x + c))/(c^2*x^2)]
```

**giac [B]** time = 0.41, size = 450, normalized size = 2.09

$$-b\sqrt{d}\log\left(\left|-2\left(\sqrt{d}x-\sqrt{dx^{2}+xe+c}\right)d-\sqrt{d}e\right|\right)\mathrm{sgn}\left(bx+a\right)+\frac{\left(4\,acd\mathrm{sgn}\left(bx+a\right)+4\,bce\mathrm{sgn}\left(bx+a\right)-ae^{2}\mathrm{sgn}\left(bx+a\right)\right)\arctan\left(-\frac{\sqrt{d}x-\sqrt{dx^{2}+xe+c}}{\sqrt{-c}}\right)}{4\,\sqrt{-c}c}+\frac{4\left(\sqrt{d}x-\sqrt{dx^{2}+xe+c}\right)^{3}acd\mathrm{sgn}\left(bx+a\right)+4\left(\sqrt{d}x-\sqrt{dx^{2}+xe+c}\right)^{3}bce\mathrm{sgn}\left(bx+a\right)+8\left(\sqrt{d}x-\sqrt{dx^{2}+xe+c}\right)^{2}bc^{2}\sqrt{d}\mathrm{sgn}\left(bx+a\right)+8\left(\sqrt{d}x-\sqrt{dx^{2}+xe+c}\right)^{2}ac\sqrt{d}e\mathrm{sgn}\left(bx+a\right)+4\left(\sqrt{d}x-\sqrt{dx^{2}+xe+c}\right)ac^{2}d\mathrm{sgn}\left(bx+a\right)+\left(\sqrt{d}x-\sqrt{dx^{2}+xe+c}\right)^{3}ae^{2}\mathrm{sgn}\left(bx+a\right)-4\left(\sqrt{d}x-\sqrt{dx^{2}+xe+c}\right)bc^{2}e\mathrm{sgn}\left(bx+a\right)-8\,bc^{3}\sqrt{d}\mathrm{sgn}\left(bx+a\right)+\left(\sqrt{d}x-\sqrt{dx^{2}+xe+c}\right)ace^{2}\mathrm{sgn}\left(bx+a\right)}{4\left(\left(\sqrt{d}x-\sqrt{dx^{2}+xe+c}\right)^{2}-c\right)^{2}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -b*sqrt(d)*log(abs(-2*(sqrt(d)*x - sqrt(d*x^2 + x*e + c))*d - sqrt(d)*e))*s
gn(b*x + a) + 1/4*(4*a*c*d*sgn(b*x + a) + 4*b*c*e*sgn(b*x + a) - a*e^2*sgn(
b*x + a))*arctan(-(sqrt(d)*x - sqrt(d*x^2 + x*e + c))/sqrt(-c))/(sqrt(-c)*c
) + 1/4*(4*(sqrt(d)*x - sqrt(d*x^2 + x*e + c))^3*a*c*d*sgn(b*x + a) + 4*(sq
rt(d)*x - sqrt(d*x^2 + x*e + c))^3*b*c*e*sgn(b*x + a) + 8*(sqrt(d)*x - sqrt
(d*x^2 + x*e + c))^2*b*c^2*sqrt(d)*sgn(b*x + a) + 8*(sqrt(d)*x - sqrt(d*x^2
 + x*e + c))^2*a*c*sqrt(d)*e*sgn(b*x + a) + 4*(sqrt(d)*x - sqrt(d*x^2 + x*e
```

```
 + c))*a*c^2*d*sgn(b*x + a) + (sqrt(d)*x - sqrt(d*x^2 + x*e + c))^3*a*e^2*s
gn(b*x + a) - 4*(sqrt(d)*x - sqrt(d*x^2 + x*e + c))*b*c^2*e*sgn(b*x + a) -
8*b*c^3*sqrt(d)*sgn(b*x + a) + (sqrt(d)*x - sqrt(d*x^2 + x*e + c))*a*c*e^2*
sgn(b*x + a))/(((sqrt(d)*x - sqrt(d*x^2 + x*e + c))^2 - c)^2*c)
```

**maple** [C] time = 0.02, size = 358, normalized size = 1.67

$$\frac{\left(-4a\,c^{\frac{3}{2}}d^{\frac{5}{2}}x^2\ln\left(\frac{ex+2c+2\sqrt{d\,x^2+ex+c}\,\sqrt{c}}{x}\right)+a\sqrt{c}\,d^{\frac{3}{2}}e^2x^2\ln\left(\frac{ex+2c+2\sqrt{d\,x^2+ex+c}\,\sqrt{c}}{x}\right)+8b\,c^2d^2x^2\ln\left(\frac{2dx+e+2\sqrt{d\,x^2+ex+c}\,\sqrt{d}}{2\sqrt{d}}\right)-4b\,c^{\frac{3}{2}}d^{\frac{3}{2}}e\,x^2\ln\left(\frac{ex+2c+2\sqrt{d\,x^2+ex+c}\,\sqrt{c}}{x}\right)-2\sqrt{d\,x^2+ex+c}\,a\,d^{\frac{5}{2}}e\,x^3+8\sqrt{d\,x^2+ex+c}\,bc\,d^{\frac{5}{2}}x^3+4\sqrt{d\,x^2+ex+c}\,ac\,d^{\frac{5}{2}}x^2-2\sqrt{d\,x^2+ex+c}\,a\,d^{\frac{3}{2}}e^2x^2+8\sqrt{d\,x^2+ex+c}\,bc\,d^{\frac{3}{2}}e\,x^2+2\left(d\,x^2+ex+c\right)^{\frac{3}{2}}a\,d^{\frac{3}{2}}ex-8\left(d\,x^2+ex+c\right)^{\frac{3}{2}}bc\,d^{\frac{3}{2}}x-4\left(d\,x^2+ex+c\right)^{\frac{3}{2}}ac\,d^{\frac{3}{2}}\right)\mathrm{csgn}\left(bx+a\right)}{8c^2d^{\frac{3}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^3,x)
```

```
[Out] 1/8*csgn(b*x+a)*(-4*d^(5/2)*c^(3/2)*ln((e*x+2*c+2*(d*x^2+e*x+c)^(1/2)*c^(1/
2))/x)*x^2*a-2*d^(5/2)*(d*x^2+e*x+c)^(1/2)*x^3*a*e+8*d^(5/2)*(d*x^2+e*x+c)^
(1/2)*x^3*b*c-4*d^(3/2)*c^(3/2)*ln((e*x+2*c+2*(d*x^2+e*x+c)^(1/2)*c^(1/2))/
x)*x^2*b*e+4*d^(5/2)*(d*x^2+e*x+c)^(1/2)*x^2*a*c+d^(3/2)*c^(1/2)*ln((e*x+2*
c+2*(d*x^2+e*x+c)^(1/2)*c^(1/2))/x)*x^2*a*e^2+2*d^(3/2)*(d*x^2+e*x+c)^(3/2)
*x*a*e-8*d^(3/2)*(d*x^2+e*x+c)^(3/2)*x*b*c-2*d^(3/2)*(d*x^2+e*x+c)^(1/2)*x^
2*a*e^2+8*d^(3/2)*(d*x^2+e*x+c)^(1/2)*x^2*b*c*e+8*ln(1/2*(2*d*x+e+2*(d*x^2+
e*x+c)^(1/2)*d^(1/2))/d^(1/2))*x^2*b*c^2*d^2-4*d^(3/2)*(d*x^2+e*x+c)^(3/2)*
a*c)/x^2/c^2/d^(3/2)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^2 + ex + c}\,\sqrt{(bx + a)^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)/x^3, x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{{(a+b\,x)}^2}\,\sqrt{d\,x^2+e\,x+c}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^3,x)
```

```
[Out] int((((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^2 + ex}\,\sqrt{(a + bx)^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2)/x**3, x)
```

## 3.52 $\int \frac{x^2\sqrt{a+cx^2}}{d+ex+fx^2}\,dx$

**Optimal**. Leaf size=452

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(af^2+2c\left(e^2-df\right)\right)}{2\sqrt{c}f^3}-\frac{\left(e\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-2df\right)\right)-2df\left(af^2+c\left(e^2-df\right)\right)\right)\tanh}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}\right.}}$$

**Rubi** [A] time = 1.97, antiderivative size = 452, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1069, 1080, 217, 206, 1034, 725}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(af^2+2c\left(e^2-df\right)\right)}{2\sqrt{c}f^3}-\frac{\left(e\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-2df\right)\right)-2df\left(af^2+c\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(e\left(\sqrt{e^2-4df}+e\right)\left(af^2+c\left(e^2-2df\right)\right)-2df\left(af^2+c\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\sqrt{a+cx^2}(2e-fx)}{2f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[a + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] -((2*e - f*x)*Sqrt[a + c*x^2])/(2*f^2) + ((a*f^2 + 2*c*(e^2 - d*f))*ArcTanh
[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c]*f^3) - ((e*(e - Sqrt[e^2 - 4*d*f]
)*(a*f^2 + c*(e^2 - 2*d*f)) - 2*d*f*(a*f^2 + c*(e^2 - d*f)))*ArcTanh[(2*a*f
 - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*
Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[
2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + ((e*(e + Sqrt[e^2 - 4*d
*f])*(a*f^2 + c*(e^2 - 2*d*f)) - 2*d*f*(a*f^2 + c*(e^2 - d*f)))*ArcTanh[(2*
a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f +
 e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sq
rt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1069

```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) + (e_.)*(x_) +
(f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((C*(-(c*e*(2*p + q + 2))) + 2*c*C*f*
(p + q + 1)*x)*(a + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1
)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[
(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(-(a*e))*(C*(c*e)*(q + 1) -
c*(C*e)*(2*p + 2*q + 3)) + (p + q + 1)*(a*c*(C*(2*d*f - e^2*(2*p + q + 2))
+ f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(c*e)*(q + 1) - c*(C*e
)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(-4*a*c)))*x + (p*(c*e)*(C*(c*e)*
(q + 1) - c*(C*e)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(-4*a*c) - c^2*(C
*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3))))*x^2, x]
, x], x] /; FreeQ[{a, c, d, e, f, A, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && Gt
Q[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !
IGtQ[q, 0]
```

Rule 1080

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rubi steps

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = -\frac{(2e-fx)\sqrt{a+cx^2}}{2f^2} - \frac{\int \frac{acdf-ce(2cd-af)x-c\left(af^2+2c\left(e^2-df\right)\right)x^2}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{2cf^2}$$

$$= -\frac{(2e-fx)\sqrt{a+cx^2}}{2f^2} - \frac{\int \frac{acdf^2+cd\left(af^2+2c\left(e^2-df\right)\right)+\left(-cef(2cd-af)+ce\left(af^2+2c\left(e^2-df\right)\right)\right)x}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{2cf^3} + \frac{\left(af^2 + \right.}{}$$

$$= -\frac{(2e-fx)\sqrt{a+cx^2}}{2f^2} + \frac{\left(af^2+2c\left(e^2-df\right)\right)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{2f^3} + \frac{\left(e\left(e-\sqrt{e^2-\right.\right.}{}$$

$$= -\frac{(2e-fx)\sqrt{a+cx^2}}{2f^2} + \frac{\left(af^2+2c\left(e^2-df\right)\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}\,f^3} - \frac{\left(e\left(e-\sqrt{e^2-4df}\right)\left(af^2\right.\right.}{}$$

$$= -\frac{(2e-fx)\sqrt{a+cx^2}}{2f^2} + \frac{\left(af^2+2c\left(e^2-df\right)\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}\,f^3} - \frac{\left(e\left(e-\sqrt{e^2-4df}\right)\left(af^2\right.\right.}{}$$

**Mathematica** [A] time = 2.50, size = 516, normalized size = 1.14

$$\frac{\frac{2f\left(\frac{a^{3/2}\sqrt{\frac{cx^2}{a}+1}\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{c}}+ax+cx^3\right)}{\sqrt{a+cx^2}} + \frac{\left(\frac{2df-e^2}{\sqrt{e^2-4df}}+e\right)\left(\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)-\sqrt{c}\left(\sqrt{e^2-4df}-e\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)\right)}{f} + \frac{\left(e\sqrt{e^2-4df}-2df+e^2\right)\left(\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{a+cx^2}\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)+\sqrt{c}\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)\right)}{f\sqrt{e^2-4df}} - 2\sqrt{a+cx^2}\left(\frac{e^2-2df}{\sqrt{e^2-4df}}+e\right) - 2\sqrt{a+cx^2}\left(\frac{2df-e^2}{\sqrt{e^2-4df}}+e\right)}{4f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[a + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] (-2*(e + (e^2 - 2*d*f)/Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2] - 2*(e + (-e^2 +
2*d*f)/Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2] + (2*f*(a*x + c*x^3 + (a^(3/2)*Sq
rt[1 + (c*x^2)/a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])/Sqrt[c]))/Sqrt[a + c*x^2] +
 ((e + (-e^2 + 2*d*f)/Sqrt[e^2 - 4*d*f])*(-(Sqrt[c]*(-e + Sqrt[e^2 - 4*d*f]
)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]) + Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f
+ e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqr
t[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]))/f
 + ((e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*(Sqrt[c]*(e + Sqrt[e^2 - 4*d*f])*Ar
cTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] + Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sq
rt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f
^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]))/(f*Sqrt[e
^2 - 4*d*f]))/(4*f^2)
```

**IntegrateAlgebraic [C]** time = 0.67, size = 496, normalized size = 1.10

$$\frac{\text{RootSum}\left[\#1^4 f - 2\#1^3\sqrt{c}e - 2\#1^2 af + 4\#1^2 cd + 2\#1a\sqrt{c}e + a^2 f\&, \frac{2\#1^2 cdef\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+\#1^2(-c)e^3\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)-\#1^2aef^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+a^2ef^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)-2\#1c^{3/2}d^2f\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+2\#1c^{3/2}de^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)-2acdef\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+2\#1a\sqrt{c}df^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+ace^3\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)}{2\#1^3f-3\#1^2\sqrt{c}e-2\#1af+4\#1cd+a\sqrt{c}e}\&\right]}{f^3}+\frac{\log\left(\sqrt{a+cx^2}-\sqrt{c}x\right)\left(-af^2+2cdf-2ce^2\right)}{2\sqrt{c}f^3}+\frac{\sqrt{a+cx^2}(fx-2e)}{2f^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*Sqrt[a + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] ((-2*e + f*x)*Sqrt[a + c*x^2])/(2*f^2) + ((-2*c*e^2 + 2*c*d*f - a*f^2)*Log[
-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(2*Sqrt[c]*f^3) + RootSum[a^2*f + 2*a*Sqrt
[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (a*c*e^3
*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - 2*a*c*d*e*f*Log[-(Sqrt[c]*x) +
Sqrt[a + c*x^2] - #1] + a^2*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]
+ 2*c^(3/2)*d*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - 2*c^(3/2)*d
^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + 2*a*Sqrt[c]*d*f^2*Log[-(
Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - c*e^3*Log[-(Sqrt[c]*x) + Sqrt[a + c
*x^2] - #1]*#1^2 + 2*c*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2
- a*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c
*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]/f^3
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.04, size = 7739, normalized size = 17.12

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \sqrt{c\,x^2+a}}{f\,x^2+e\,x+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + c*x^2)^(1/2))/(d + e*x + f*x^2),x)
```

```
[Out] int((x^2*(a + c*x^2)^(1/2))/(d + e*x + f*x^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x**2*sqrt(a + c*x**2)/(d + e*x + f*x**2), x)
```

## 3.53 $\int \frac{x\sqrt{a+cx^2}}{d+ex+fx^2}\,dx$

**Optimal**. Leaf size=395

$$-\frac{\left(2cdef-\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(2cdef-\left(\sqrt{e^2-4\right.\right.}{}$$

**Rubi** [A] time = 0.93, antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1020, 1080, 217, 206, 1034, 725}

$$-\frac{\left(2cdef-\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(2cdef-\left(\sqrt{e^2-4df}+e\right)\left(af^2+c\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f^2}+\frac{\sqrt{a+cx^2}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[a + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] Sqrt[a + c*x^2]/f - (Sqrt[c]*e*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/f^2 -
((2*c*d*e*f - (e - Sqrt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - d*f)))*ArcTanh[(2*a
*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f -
e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqr
t[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + ((2*c*d*e*f - (e + Sq
rt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2
- 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]
*Sqrt[a + c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 -
2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 1020

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
eQ[p + q + 1, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1080

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rubi steps

$$\int \frac{x\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = \frac{\sqrt{a+cx^2}}{f} + \frac{\int \frac{-((cd-af)x)-cex^2}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{f}$$

$$= \frac{\sqrt{a+cx^2}}{f} + \frac{\int \frac{cde+\left(ce^2+f(-cd+af)\right)x}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{f^2} - \frac{(ce)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{f^2}$$

$$= \frac{\sqrt{a+cx^2}}{f} - \frac{(ce)\,\mathrm{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{f^2} + \frac{\left(2cdef - \left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-d\right.\right.\right.}{f^2\sqrt{e^2-4d}}$$

$$= \frac{\sqrt{a+cx^2}}{f} - \frac{\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{f^2} - \frac{\left(2cdef - \left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-df\right)\right)\right)\mathrm{Subst}}{f^2\sqrt{e^2-4}}$$

$$= \frac{\sqrt{a+cx^2}}{f} - \frac{\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{f^2} - \frac{\left(2cdef - \left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-df\right)\right)\right)\tanh^{-1}}{\sqrt{2}\,f^2\sqrt{e^2-4df}\,\sqrt{2af^2+c\left(e^2-2d\right.}}$$

**Mathematica [A]** time = 1.52, size = 422, normalized size = 1.07

$$-\frac{\frac{\left(\sqrt{e^2-4df}-e\right)\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)}{\sqrt{e^2-4df}} + \frac{e\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{a+cx^2}\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}} + \sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{a+cx^2}\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right) + 4\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right) - 4f\sqrt{a+cx^2}}{4f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[a + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] -1/4*(-4*f*Sqrt[a + c*x^2] + 4*Sqrt[c]*e*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2
]] + ((-e + Sqrt[e^2 - 4*d*f])*Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^
2 - 4*d*f])]*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 -
 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/Sqrt[e^2 - 4*
d*f] + Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a
*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt
[e^2 - 4*d*f])]*Sqrt[a + c*x^2])] + (e*Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*
Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a
*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/Sqrt[e^2
 - 4*d*f])/f^2
```

**IntegrateAlgebraic [C]** time = 0.53, size = 383, normalized size = 0.97

$$-\frac{\mathrm{RootSum}\left[\#1^4 f - 2\#1^3\sqrt{c}\,e - 2\#1^2 af + 4\#1^2 cd + 2\#1 a\sqrt{c}\,e + a^2 f\&, \frac{\#1^2 cdf\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right) + \#1^2(-c)e^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right) - \#1^2 af^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right) + a^2f^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right) + 2\#1c^{3/2}de\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right) - acdf\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right) + ace^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)}{2\#1^3 f - 3\#1^2\sqrt{c}\,e - 2\#1af + 4\#1cd + a\sqrt{c}\,e}\&\right]}{f^2} + \frac{\sqrt{c}\,e\log\left(\sqrt{a+cx^2}-\sqrt{c}\,x\right)}{f^2} + \frac{\sqrt{a+cx^2}}{f}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*Sqrt[a + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] Sqrt[a + c*x^2]/f + (Sqrt[c]*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/f^2 - R
ootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^
3 + f*#1^4 & , (a*c*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - a*c*d*f*
Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + a^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[
a + c*x^2] - #1] + 2*c^(3/2)*d*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#
1 - c*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 + c*d*f*Log[-(Sqrt[
c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 - a*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^
2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*
#1^3) & ]/f^2
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.02, size = 5581, normalized size = 14.13

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\sqrt{c\,x^2+a}}{f\,x^2+e\,x+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + c*x^2)^(1/2))/(d + e*x + f*x^2),x)
```

```
[Out] int((x*(a + c*x^2)^(1/2))/(d + e*x + f*x^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{a+cx^2}}{d+ex+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x*sqrt(a + c*x**2)/(d + e*x + f*x**2), x)
```

## 3.54 $\int \frac{\sqrt{a+cx^2}}{d+ex+fx^2}\,dx$

**Optimal**. Leaf size=298

$$-\frac{\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f\sqrt{e^2-4df}}+\frac{\sqrt{2af^2+c\left(e\sqrt{e^2-4df}\right.}}{}$$

**Rubi** [A] time = 0.38, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {991, 217, 206, 1034, 725}

$$-\frac{\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f\sqrt{e^2-4df}}+\frac{\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f\sqrt{e^2-4df}}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[c]*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/f - (Sqrt[2*a*f^2 + c*(e^2 -
 2*d*f - e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x
)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c
*x^2])])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f]) + (Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e
*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]
*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(
Sqrt[2]*f*Sqrt[e^2 - 4*d*f])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 991

```
Int[Sqrt[(a_) + (c_.)*(x_)^2]/((d_) + (e_.)*(x_) + (f_.)*(x_)^2), x_Symbol]
 :> Dist[c/f, Int[1/Sqrt[a + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*f + c*
e*x)/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)), x], x] /; FreeQ[{a, c, d, e, f},
x] && NeQ[e^2 - 4*d*f, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = -\frac{\int \frac{cd-af+cex}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{f} + \frac{c\int \frac{1}{\sqrt{a+cx^2}}\,dx}{f}$$

$$= \frac{c\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{f} + \frac{\left(2f(cd-af)-ce\left(e+\sqrt{e^2-4df}\right)\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+c}}}{f\sqrt{e^2-4df}}$$

$$= \frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{f} - \frac{\left(2f(cd-af)-ce\left(e+\sqrt{e^2-4df}\right)\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e+\sqrt{e^2-4df}\right)^2-x^2}\,d\right.}{f\sqrt{e^2-4df}}$$

$$= \frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{f} - \frac{\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\,\tanh^{-1}\left(\frac{2af-c\left(e-\sqrt{e^2-4d}\right.}{\sqrt{2}\,\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2}\right.}}\right.}{\sqrt{2}\,f\sqrt{e^2-4df}}$$

**Mathematica [A]** time = 0.36, size = 282, normalized size = 0.95

$$\frac{-\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}\,\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)+\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}\,\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{a+cx^2}\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)+2\sqrt{c}\sqrt{e^2-4df}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2f\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(d + e*x + f*x^2),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[e^2 - 4*d*f]*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] - Sqrt[4*
a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f + c*(-e +
```

```
Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*
f])]*Sqrt[a + c*x^2])] + Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d
*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^
2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(2*f*Sqrt[e^2 - 4*d*f]
)
```

**IntegrateAlgebraic [C]** time = 0.45, size = 267, normalized size = 0.90

$$\frac{\text{RootSum}\left[\#1^4 f-2\#1^3\sqrt{c}\,e-2\#1^2 af+4\#1^2 cd+2\#1 a\sqrt{c}\,e+a^2 f\&,\frac{\#1^2(-c)e\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)+2\#1c^{3/2}d\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)+ace\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)-2\#1a\sqrt{c}\,f\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)}{2\#1^3 f-3\#1^2\sqrt{c}\,e-2\#1af+4\#1cd+a\sqrt{c}\,e}\&\right]}{f}-\frac{\sqrt{c}\log\left(\sqrt{a+cx^2}-\sqrt{c}\,x\right)}{f}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + c*x^2]/(d + e*x + f*x^2),x]
```

```
[Out] -((Sqrt[c]*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/f) + RootSum[a^2*f + 2*a*Sq
rt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (a*c*e
*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + 2*c^(3/2)*d*Log[-(Sqrt[c]*x) +
Sqrt[a + c*x^2] - #1]*#1 - 2*a*Sqrt[c]*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]
 - #1]*#1 - c*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e
 + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]/f
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.01, size = 3249, normalized size = 10.90

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/(f*x^2+e*x+d),x)

[Out] -1/2/(-4*d*f+e^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*
d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)+1/2/(-4*d*f+e^2)^(1/2)*c^(1/2)/f*ln((-1
/2*c*(e+(-4*d*f+e^2)^(1/2))/f+c*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))/c^(1/2)+(
(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(
-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^
2)^(1/2))*e+1/2*c^(1/2)/f*ln((-1/2*c*(e+(-4*d*f+e^2)^(1/2))/f+c*(x+1/2*(e+(
-4*d*f+e^2)^(1/2))/f))/c^(1/2)+((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(
-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))+1/2/f^2*2^(1/2)/(((-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f
^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/
2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/
2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+
1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e
^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*c*e+1/(-4*d*f+e^2)^(1/2)*
2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4
*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(
x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^
2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+
(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*
a-1/(-4*d*f+e^2)^(1/2)/f*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c
*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(
e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*
d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e
^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2)
)/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e
+(-4*d*f+e^2)^(1/2))/f))*c*d+1/2/(-4*d*f+e^2)^(1/2)/f^2*2^(1/2)/(((-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^
2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^
2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))
/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d
*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*c*e^2+1/2/(-4*d*f+e
^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2
))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2
*c*d*f+c*e^2)/f^2)^(1/2)+1/2*c^(1/2)/f*ln((-1/2*c*(e-(-4*d*f+e^2)^(1/2))/f+
c*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))/c^(1/2)+((x-1/2*(-e+(-4*d*f+e^2)^(1/2)
)/f)^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(
-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))-1/2/(-4*d*f+e^2)
^(1/2)*c^(1/2)/f*ln((-1/2*c*(e-(-4*d*f+e^2)^(1/2))/f+c*(x-1/2*(-e+(-4*d*f+e
^2)^(1/2))/f))/c^(1/2)+((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e-(-4*d*f+
```

```
e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))*e+1/2/f^2*2^(1/2)/((-(-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^
2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/
2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1
/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(
x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f
+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*c*e-1/(-4*d*f+e^2)^(
1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln
(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/
2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2
*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d
*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2
)^(1/2))/f))*a+1/(-4*d*f+e^2)^(1/2)/f*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f
+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/
2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x
-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e
+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f
^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*c*d-1/2/(-4*d*f+e^2)^(1/2)/f^
2*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((
-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))
/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-
4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(
1/2))/f))*c*e^2
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+a}}{f\,x^2+e\,x+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(1/2)/(d + e*x + f*x^2),x)
```

```
[Out] int((a + c*x^2)^(1/2)/(d + e*x + f*x^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + cx^2}}{d + ex + fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(d + e*x + f*x**2), x)
```

## 3.55 $\int \frac{\sqrt{a+cx^2}}{x\left(d+ex+fx^2\right)}\,dx$

**Optimal**. Leaf size=358

$$\frac{\left(\left(e-\sqrt{e^2-4df}\right)(cd-af)+2aef\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\,d\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(\left(\sqrt{e^2-4df}+e\right)(cd-af)+\right.}{\sqrt{2}\,d\sqrt{e^2-4df}}$$

**Rubi [A]** time = 1.31, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6728, 266, 50, 63, 208, 1020, 1034, 725, 206}

$$\frac{\left(\left(e-\sqrt{e^2-4df}\right)(cd-af)+2aef\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\,d\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(\left(\sqrt{e^2-4df}+e\right)(cd-af)+2aef\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\,d\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(x*(d + e*x + f*x^2)),x]
```

```
[Out] ((2*a*e*f + (c*d - a*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sq
rt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4
*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e
^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - ((2*a*e*f + (c*d - a*f)*(e + Sqrt[e^2
 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a
*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d
*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) -
 (Sqrt[a]*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1020

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
eQ[p + q + 1, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+cx^2}}{x\left(d+ex+fx^2\right)}\,dx &= \int \left(\frac{\sqrt{a+cx^2}}{dx}+\frac{(-e-fx)\sqrt{a+cx^2}}{d\left(d+ex+fx^2\right)}\right)dx \\
&= \frac{\int \frac{\sqrt{a+cx^2}}{x}\,dx}{d}+\frac{\int \frac{(-e-fx)\sqrt{a+cx^2}}{d+ex+fx^2}\,dx}{d} \\
&= -\frac{\sqrt{a+cx^2}}{d}+\frac{\operatorname{Subst}\left(\int \frac{\sqrt{a+cx}}{x}\,dx,x,x^2\right)}{2d}+\frac{\int \frac{-aef+f(cd-af)x}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{df} \\
&= \frac{a\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx,x,x^2\right)}{2d}-\frac{\left(2aef+(cd-af)\left(e-\sqrt{e^2-4df}\right)\right)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{\cdots}}\cdots}{d\sqrt{e^2-4df}\cdots} \\
&= \frac{a\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx,x,\sqrt{a+cx^2}\right)}{cd}+\frac{\left(2aef+(cd-af)\left(e-\sqrt{e^2-4df}\right)\right)\operatorname{Subst}\left(\int \frac{\cdots}{4a\cdots}\cdots\right.}{d\sqrt{e^2-4\cdots}} \\
&= \frac{\left(2aef+(cd-af)\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}\,d\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}-\frac{(2a\cdots}{\cdots}
\end{aligned}
$$

**Mathematica [A]** time = 0.65, size = 314, normalized size = 0.88

$$
\frac{\left(\sqrt{e^2-4df}+e\right)\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)+\left(\sqrt{e^2-4df}-e\right)\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{a+cx^2}\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)-4\sqrt{a}\,f\sqrt{e^2-4df}\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{4df\sqrt{e^2-4df}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(x*(d + e*x + f*x^2)),x]
```

```
[Out] ((e + Sqrt[e^2 - 4*d*f])*Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*
d*f])]*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(
-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])] + (-e + Sqrt[e^2 - 4
*d*f])*Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a
*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt
```

```
[e^2 - 4*d*f])]*Sqrt[a + c*x^2])] - 4*Sqrt[a]*f*Sqrt[e^2 - 4*d*f]*ArcTanh[S
qrt[a + c*x^2]/Sqrt[a]])/(4*d*f*Sqrt[e^2 - 4*d*f])
```

**IntegrateAlgebraic [C]** time = 0.49, size = 306, normalized size = 0.85

$$\frac{\text{RootSum}\left[\#1^4 f-2\#1^3\sqrt{c}\,e-2\#1^2 af+4\#1^2 cd+2\#1 a\sqrt{c}\,e+a^2 f\&,\frac{\#1^2 cd\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)-\#1^2 af\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)+a^2 f\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)-acd\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)+2\#1 a\sqrt{c}\,e\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)}{2\#1^3 f-3\#1^2\sqrt{c}\,e-2\#1 af+4\#1 cd+a\sqrt{c}\,e}\&\right]}{d}+\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}-\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + c*x^2]/(x*(d + e*x + f*x^2)),x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[(Sqrt[c]*x)/Sqrt[a] - Sqrt[a + c*x^2]/Sqrt[a]])/d + Root
Sum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 +
 f*#1^4 & , (-(a*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]) + a^2*f*Log[
-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + 2*a*Sqrt[c]*e*Log[-(Sqrt[c]*x) + Sqr
t[a + c*x^2] - #1]*#1 + c*d*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 -
 a*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1
 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]/d
```

**fricas [B]** time = 38.14, size = 2266, normalized size = 6.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(2)*d*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f + (d^2*e^2 - 4*d^3*f)*sqrt
(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*f))*log((2*a*c*d*e*x - a^2*
e^2 + sqrt(2)*(d^3*e^2 - 4*d^4*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))*sqrt(c*
x^2 + a)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f + (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2
/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*f)) - (a*d^2*e^2 - 4*a*d^3*f)*sqrt(
a^2*e^2/(d^4*e^2 - 4*d^5*f)))/x) - sqrt(2)*d*sqrt((2*c*d^2 + a*e^2 - 2*a*d*
f + (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3
*f))*log((2*a*c*d*e*x - a^2*e^2 - sqrt(2)*(d^3*e^2 - 4*d^4*f)*sqrt(a^2*e^2/
(d^4*e^2 - 4*d^5*f))*sqrt(c*x^2 + a)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f + (d^2
*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*f)) - (
a*d^2*e^2 - 4*a*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/x) - sqrt(2)*d*sq
rt((2*c*d^2 + a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 -
 4*d^5*f)))/(d^2*e^2 - 4*d^3*f))*log((2*a*c*d*e*x - a^2*e^2 + sqrt(2)*(d^3*
e^2 - 4*d^4*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))*sqrt(c*x^2 + a)*sqrt((2*c*
d^2 + a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f
)))/(d^2*e^2 - 4*d^3*f)) + (a*d^2*e^2 - 4*a*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 -
4*d^5*f)))/x) + sqrt(2)*d*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^
3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*f))*log((2*a*c*d*e
*x - a^2*e^2 - sqrt(2)*(d^3*e^2 - 4*d^4*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)
)*sqrt(c*x^2 + a)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^3*f)*sqr
```

```
t(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*f)) + (a*d^2*e^2 - 4*a*d^3
*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/x) - 2*sqrt(a)*log(-(c*x^2 - 2*sqrt(
c*x^2 + a)*sqrt(a) + 2*a)/x^2))/d, -1/4*(sqrt(2)*d*sqrt((2*c*d^2 + a*e^2 -
2*a*d*f + (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 -
 4*d^3*f))*log((2*a*c*d*e*x - a^2*e^2 + sqrt(2)*(d^3*e^2 - 4*d^4*f)*sqrt(a^
2*e^2/(d^4*e^2 - 4*d^5*f))*sqrt(c*x^2 + a)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f
+ (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*f
)) - (a*d^2*e^2 - 4*a*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/x) - sqrt(2
)*d*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f + (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4
*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*f))*log((2*a*c*d*e*x - a^2*e^2 - sqrt(2)
*(d^3*e^2 - 4*d^4*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))*sqrt(c*x^2 + a)*sqrt
((2*c*d^2 + a*e^2 - 2*a*d*f + (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4
*d^5*f)))/(d^2*e^2 - 4*d^3*f)) - (a*d^2*e^2 - 4*a*d^3*f)*sqrt(a^2*e^2/(d^4*
e^2 - 4*d^5*f)))/x) - sqrt(2)*d*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f - (d^2*e^2
- 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*f))*log((2*a
*c*d*e*x - a^2*e^2 + sqrt(2)*(d^3*e^2 - 4*d^4*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*
d^5*f))*sqrt(c*x^2 + a)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^3*
f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*f)) + (a*d^2*e^2 - 4
*a*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/x) + sqrt(2)*d*sqrt((2*c*d^2 +
 a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(
d^2*e^2 - 4*d^3*f))*log((2*a*c*d*e*x - a^2*e^2 - sqrt(2)*(d^3*e^2 - 4*d^4*f
)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))*sqrt(c*x^2 + a)*sqrt((2*c*d^2 + a*e^2 -
 2*a*d*f - (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2
- 4*d^3*f)) + (a*d^2*e^2 - 4*a*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/x)
 - 4*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)))/d]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 2Error index.cc index_gcd E
rror: Bad Argument Value
```

**maple [B]** time = 0.02, size = 3544, normalized size = 9.90

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/x/(f*x^2+e*x+d),x)
```

```
[Out] f/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2)
)/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)-1/(e+(-4*d*f+e^2)^(
1/2))*c^(1/2)*ln((-1/2*c*(e+(-4*d*f+e^2)^(1/2))/f+c*(x+1/2*(e+(-4*d*f+e^2)^
(1/2))/f))/c^(1/2)+((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^
(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f
^2-2*c*d*f+c*e^2)/f^2)^(1/2))-1/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*c
^(1/2)*ln((-1/2*c*(e+(-4*d*f+e^2)^(1/2))/f+c*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/
f))/c^(1/2)+((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/
f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*
d*f+c*e^2)/f^2)^(1/2))*e-1/f/(e+(-4*d*f+e^2)^(1/2))*2^(1/2)/(((-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(
1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(
1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(
x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c
*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*c*e-2*f/(e+(-4*d*f+e^2)
^(1/2))/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f
+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c
*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-
4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f
+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/
2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*
(e+(-4*d*f+e^2)^(1/2))/f))*a+2/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*2^
(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d
*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+
1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-
4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*c*
d-1/f/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2
-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2)
)/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)
*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2
)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*c*e^2+f/(-e+(-4*d*f+e^2)^(1
/2))/(-4*d*f+e^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4
*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)
*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)+1/(-e+(-4*d*f+e^2)^(1/2))*c^(1/2)*ln
((-1/2*c*(e-(-4*d*f+e^2)^(1/2))/f+c*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))/c^(1
/2)+((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c
*e^2)/f^2)^(1/2))-1/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*c^(1/2)*ln((
-1/2*c*(e-(-4*d*f+e^2)^(1/2))/f+c*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))/c^(1/2
```

```
)+((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*
(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e
^2)/f^2)^(1/2))*e+1/f/(-e+(-4*d*f+e^2)^(1/2))*2^(1/2)/((-(-4*d*f+e^2)^(1/2)
*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2
-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2
))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/
2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x
-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+
c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*c*e-2*f/(-e+(-4*d*f+e
^2)^(1/2))/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c
*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/
f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2
)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e
+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f
+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2
))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*a+2/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e
^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/
2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2
)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))
/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*
f+e^2)^(1/2))/f))*c*d-1/f/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*2^(1/2
)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f
+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(
-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)
)*c*e^2+4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*a^(1/2)*ln((2*a+
2*a^(1/2)*(c*x^2+a)^(1/2))/x)-4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(
1/2))*(c*x^2+a)^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2+a}}{\left(fx^2+ex+d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)/((f*x^2 + e*x + d)*x), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+a}}{x\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(1/2)/(x*(d + e*x + f*x^2)),x)
[Out] int((a + c*x^2)^(1/2)/(x*(d + e*x + f*x^2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+cx^2}}{x\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/x/(f*x**2+e*x+d),x)
[Out] Integral(sqrt(a + c*x**2)/(x*(d + e*x + f*x**2)), x)
```

## 3.56 $\int \frac{\sqrt{a+cx^2}}{x^2(d+ex+fx^2)}\,dx$

**Optimal**. Leaf size=382

$$-\frac{f\left(a\left(e\sqrt{e^2-4df}-2df+e^2\right)+2cd^2\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\,d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(a\left(-e\sqrt{e^2-4df}-2df\right.\right.}{\sqrt{2}\,d^2\sqrt{e^2}}$$

**Rubi** [A] time = 1.42, antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6728, 277, 217, 206, 266, 50, 63, 208, 1020, 1080, 1034, 725}

$$-\frac{f\left(a\left(e\sqrt{e^2-4df}-2df+e^2\right)+2cd^2\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\,d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(a\left(-e\sqrt{e^2-4df}-2df+e^2\right)+2cd^2\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\,d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}-\frac{\sqrt{a+cx^2}}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(x^2*(d + e*x + f*x^2)),x]
```

```
[Out] -(Sqrt[a + c*x^2]/(d*x)) - (f*(2*c*d^2 + a*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*
f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 +
c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[2]*d^2*Sqrt
[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (f*(
2*c*d^2 + a*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sq
rt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4
*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*
(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) + (Sqrt[a]*e*ArcTanh[Sqrt[a + c*x^2]/
Sqrt[a]])/d^2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1020

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
```

```
 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
eQ[p + q + 1, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1080

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{x^2\left(d+ex+fx^2\right)}\,dx = \int \left(\frac{\sqrt{a+cx^2}}{dx^2} - \frac{e\sqrt{a+cx^2}}{d^2x} + \frac{\left(e^2-df+efx\right)\sqrt{a+cx^2}}{d^2\left(d+ex+fx^2\right)}\right) dx$$

$$= \frac{\int \frac{\left(e^2-df+efx\right)\sqrt{a+cx^2}}{d+ex+fx^2}\,dx}{d^2} + \frac{\int \frac{\sqrt{a+cx^2}}{x^2}\,dx}{d} - \frac{e\int \frac{\sqrt{a+cx^2}}{x}\,dx}{d^2}$$

$$= \frac{e\sqrt{a+cx^2}}{d^2} - \frac{\sqrt{a+cx^2}}{dx} + \frac{c\int \frac{1}{\sqrt{a+cx^2}}\,dx}{d} - \frac{e\,\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x}\,dx, x, x^2\right)}{2d^2} + \frac{\int \frac{af\left(e^2-df\right)-ef}{\sqrt{a+cx^2}}}{}$$

$$= -\frac{\sqrt{a+cx^2}}{dx} - \frac{c\int \frac{1}{\sqrt{a+cx^2}}\,dx}{d} + \frac{c\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{d} - \frac{(ae)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\right.}{2d^2}$$

$$= -\frac{\sqrt{a+cx^2}}{dx} + \frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{d} - \frac{c\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{d} - \frac{(ae)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\sqrt{a+cx^2}}{dx} + \frac{\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2} + \frac{\left(f\left(2cd^2+a\left(e^2-2df-e\sqrt{e^2-4df}\right)\right)\right)\text{Subst}}{d^2\sqrt{e^2}}$$

$$= -\frac{\sqrt{a+cx^2}}{dx} - \frac{f\left(2cd^2+a\left(e^2-2df+e\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right.}}\right.}{\sqrt{2}\,d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}$$

**Mathematica** [A]   time = 3.03, size = 569, normalized size = 1.49

$$\frac{\frac{\left(e\sqrt{e^2-4df}-2df+e^2\right)\left(\sqrt{c}\left(\sqrt{e^2-4df}-e\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)-\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)\right)}{f\sqrt{e^2-4df}}+\frac{\left(-e\sqrt{e^2-4df}-2df+e^2\right)\left(\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{a+cx^2}\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)+\sqrt{c}\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\right)}{f\sqrt{e^2-4df}}+2\sqrt{a+cx^2}\left(\frac{e^2-2df}{\sqrt{e^2-4df}}+e\right)+2\sqrt{a+cx^2}\left(\frac{2df-e^2}{\sqrt{e^2-4df}}+e\right)-\frac{4d\left(-\sqrt{a}\sqrt{c}x\sqrt{\frac{cx^2}{a}+1}\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)+a+cx^2\right)}{x\sqrt{a+cx^2}}-4e\left(\sqrt{a+cx^2}-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)\right)}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(x^2*(d + e*x + f*x^2)),x]
```

```
[Out] (2*(e + (e^2 - 2*d*f)/Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2] + 2*(e + (-e^2 + 2
*d*f)/Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2] - (4*d*(a + c*x^2 - Sqrt[a]*Sqrt[c
]*x*Sqrt[1 + (c*x^2)/a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]]))/(x*Sqrt[a + c*x^2])
+ ((e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*(Sqrt[c]*(-e + Sqrt[e^2 - 4*d*f])*Ar
cTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] - Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*S
qrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a
*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]))/(f*Sqr
```

```
t[e^2 - 4*d*f]) + ((e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])*(Sqrt[c]*(e + Sqrt[e
^2 - 4*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] + Sqrt[4*a*f^2 + 2*c*(e^2
 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])
*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2
])]))/(f*Sqrt[e^2 - 4*d*f]) - 4*e*(Sqrt[a + c*x^2] - Sqrt[a]*ArcTanh[Sqrt[a
 + c*x^2]/Sqrt[a]]))/(4*d^2)
```

**IntegrateAlgebraic [C]** time = 0.58, size = 343, normalized size = 0.90

$$-\frac{\text{RootSum}\left[\#1^4 f-2\#1^3\sqrt{c}e-2\#1^2af+4\#1^2cd+2\#1a\sqrt{c}e+a^2f\&,\frac{-\#1^2aef\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+a^2ef\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+2\#1c^{3/2}d^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)-2\#1a\sqrt{c}df\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+2\#1a\sqrt{c}e^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)}{2\#1^3f-3\#1^2\sqrt{c}e-2\#1af+4\#1cd+a\sqrt{c}e}\&\right]}{d^2}-\frac{2\sqrt{a}e\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}-\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}-\frac{\sqrt{a+cx^2}}{dx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + c*x^2]/(x^2*(d + e*x + f*x^2)),x]
```

```
[Out] -(Sqrt[a + c*x^2]/(d*x)) - (2*Sqrt[a]*e*ArcTanh[(Sqrt[c]*x)/Sqrt[a] - Sqrt[
a + c*x^2]/Sqrt[a]])/d^2 - RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 -
2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (a^2*e*f*Log[-(Sqrt[c]*x) + Sqrt
[a + c*x^2] - #1] + 2*c^(3/2)*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*
#1 + 2*a*Sqrt[c]*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - 2*a*Sqrt
[c]*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - a*e*f*Log[-(Sqrt[c]*x
) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt
[c]*e*#1^2 + 2*f*#1^3) & ]/d^2
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 5.11Error index.cc index_gc
d Error: Bad Argument Value
```

**maple [B]** time = 0.02, size = 3703, normalized size = 9.69

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/x^2/(f*x^2+e*x+d),x)
```

```
[Out] -2*f^2/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2
)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/
f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)+2*f/(e+(-4*d
*f+e^2)^(1/2))^2*c^(1/2)*ln((-1/2*c*(e+(-4*d*f+e^2)^(1/2))/f+c*(x+1/2*(e+(-
4*d*f+e^2)^(1/2))/f))/c^(1/2)+((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-
4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))+2*f/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d
*f+e^2)^(1/2)*c^(1/2)*ln((-1/2*c*(e+(-4*d*f+e^2)^(1/2))/f+c*(x+1/2*(e+(-4*d
*f+e^2)^(1/2))/f))/c^(1/2)+((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d
*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))*e+2/(e+(-4*d*f+e^2)^(1/2))^2*2^(1/2)/
(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2
)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e
+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*
f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+
e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*c*e+4*f^
2/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2
*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/
f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(
4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*
(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/
f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*a-4*f/(e+(-4*d*f+e^2)^(1/2))^
2/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2
)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-
4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^
(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)
+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4
*d*f+e^2)^(1/2))/f))*c*d+2/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*2^(1
/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f
+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*
c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*
d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*c*e^
```

```
2+4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a/x*(c*x^2+a)^(3/2)-4*
f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*c/a*x*(c*x^2+a)^(1/2)-4*f/
(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*c^(1/2)*ln(c^(1/2)*x+(c*x^2+
a)^(1/2))+2*f^2/(-e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*(4*(x-1/2*(-e+
(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+
e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)
+2*f/(-e+(-4*d*f+e^2)^(1/2))^2*c^(1/2)*ln((-1/2*c*(e-(-4*d*f+e^2)^(1/2))/f+
c*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))/c^(1/2)+((x-1/2*(-e+(-4*d*f+e^2)^(1/2)
)/f)^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(
-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))-2*f/(-e+(-4*d*f+
e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*c^(1/2)*ln((-1/2*c*(e-(-4*d*f+e^2)^(1/2))/
f+c*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))/c^(1/2)+((x-1/2*(-e+(-4*d*f+e^2)^(1/
2))/f)^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2
*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))*e+2/(-e+(-4*d*
f+e^2)^(1/2))^2*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^
2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d
*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^
(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f
)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+
(-4*d*f+e^2)^(1/2))/f))*c*e-4*f^2/(-e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1
/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(
((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2
))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*
c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*
f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)
^(1/2))/f))*a+4*f/(-e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+
(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*
f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f
+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*c*d-
2/(-e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^
2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/
2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1
/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(
x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f
+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*c*e^2+16*f^2*e/(-e+(
-4*d*f+e^2)^(1/2))^2/(e+(-4*d*f+e^2)^(1/2))^2*a^(1/2)*ln((2*a+2*a^(1/2)*(c*
x^2+a)^(1/2))/x)-16*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+(-4*d*f+e^2)^(1/2))^
2*(c*x^2+a)^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2+a}}{(fx^2+ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)/((f*x^2 + e*x + d)*x^2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+a}}{x^2\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(1/2)/(x^2*(d + e*x + f*x^2)),x)
[Out] int((a + c*x^2)^(1/2)/(x^2*(d + e*x + f*x^2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+cx^2}}{x^2\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/x**2/(f*x**2+e*x+d),x)
[Out] Integral(sqrt(a + c*x**2)/(x**2*(d + e*x + f*x**2)), x)
```

## 3.57 $\int \frac{\sqrt{a+cx^2}}{x^3\left(d+ex+fx^2\right)} \, dx$

**Optimal**. Leaf size=507

$$-\frac{\sqrt{a}\left(e^2-df\right)\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3}+\frac{e\sqrt{a+cx^2}}{d^2x}+\frac{f\left(a\left(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3\right)+cd^2\left(\sqrt{e^2-4d\ldots\right.\right.}{\sqrt{2}\,d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e\ldots\right.}}$$

**Rubi [A]** time = 1.88, antiderivative size = 507, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {6728, 266, 47, 63, 208, 277, 217, 206, 50, 1020, 1080, 1034, 725}

$$\frac{f\left(a\left(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3\right)+cd^2\left(\sqrt{e^2-4df}+e\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\,d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{f\left(a\left(-e^2\sqrt{e^2-4df}+df\sqrt{e^2-4df}-3def+e^3\right)+cd^2\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\,d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\sqrt{a}\left(e^2-df\right)\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3}+\frac{e\sqrt{a+cx^2}}{d^2x}-\frac{\sqrt{a+cx^2}}{2dx^2}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(x^3*(d + e*x + f*x^2)),x]
```

```
[Out] -Sqrt[a + c*x^2]/(2*d*x^2) + (e*Sqrt[a + c*x^2])/(d^2*x) + (f*(c*d^2*(e + S
qrt[e^2 - 4*d*f]) + a*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2
 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a
*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d
^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])])
 - (f*(c*d^2*(e - Sqrt[e^2 - 4*d*f]) + a*(e^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*
d*f] + d*f*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x
)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c
*x^2])])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*S
qrt[e^2 - 4*d*f])]) - (c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*Sqrt[a]*d) -
(Sqrt[a]*(e^2 - d*f)*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d^3
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
```

```
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1020

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
eQ[p + q + 1, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1080

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{x^3\left(d+ex+fx^2\right)}\,dx = \int\left(\frac{\sqrt{a+cx^2}}{dx^3} - \frac{e\sqrt{a+cx^2}}{d^2x^2} + \frac{\left(e^2-df\right)\sqrt{a+cx^2}}{d^3x} + \frac{\left(-e\left(e^2-2df\right)-f\left(e^2-df\right)x\right)\sqrt{}}{d^3\left(d+ex+fx^2\right)}\right.$$

$$= \frac{\int \frac{\left(-e\left(e^2-2df\right)-f\left(e^2-df\right)x\right)\sqrt{a+cx^2}}{d+ex+fx^2}\,dx}{d^3} + \frac{\int \frac{\sqrt{a+cx^2}}{x^3}\,dx}{d} - \frac{e\int \frac{\sqrt{a+cx^2}}{x^2}\,dx}{d^2} + \frac{\left(e^2-df\right)\int \frac{\sqrt{a+cx^2}}{x}\,d}{d^3}$$

$$= -\frac{\left(e^2-df\right)\sqrt{a+cx^2}}{d^3} + \frac{e\sqrt{a+cx^2}}{d^2x} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x^2}\,dx, x, x^2\right)}{2d} - \frac{(ce)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{d^2} + $$

$$= -\frac{\sqrt{a+cx^2}}{2dx^2} + \frac{e\sqrt{a+cx^2}}{d^2x} + \frac{c\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{4d} + \frac{(ce)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{d^2} - \frac{(ce)\,\text{Sub}}{}$$

$$= -\frac{\sqrt{a+cx^2}}{2dx^2} + \frac{e\sqrt{a+cx^2}}{d^2x} - \frac{\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{d^2} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{2d}$$

$$= -\frac{\sqrt{a+cx^2}}{2dx^2} + \frac{e\sqrt{a+cx^2}}{d^2x} - \frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}\,d} - \frac{\sqrt{a}\left(e^2-df\right)\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3} + \frac{\left(f\right.}{}$$

$$= -\frac{\sqrt{a+cx^2}}{2dx^2} + \frac{e\sqrt{a+cx^2}}{d^2x} + \frac{f\left(cd^2\left(e+\sqrt{e^2-4df}\right)+a\left(e^3-3def+e^2\sqrt{e^2-4df}-d\right.\right.}{\sqrt{2}\,d^3\sqrt{e^2-4df}\,\sqrt{2af^2+c}}$$

**Mathematica** [A]   time = 2.61, size = 642, normalized size = 1.27

$$\frac{-\frac{2d^2\left(cx^2\sqrt{\frac{cx^2}{a}+1}\tanh^{-1}\left(\sqrt{\frac{cx^2}{a}+1}\right)+a+cx^2\right)}{x^2\sqrt{a+cx^2}} + \frac{\left(\frac{e\left(e^2-3df\right)}{\sqrt{e^2-4df}}-df+e^2\right)\left(\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)-\sqrt{c}\left(\sqrt{e^2-4df}-e\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\right)}{f} + \frac{\left(-\frac{e\left(e^2-3df\right)}{\sqrt{e^2-4df}}-df+e^2\right)\left(\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{a+cx^2}\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)+\sqrt{c}\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\right)}{f} - 2\sqrt{a+cx^2}\left(-\frac{e\left(e^2-3df\right)}{\sqrt{e^2-4df}}-df+e^2\right) - 2\sqrt{a+cx^2}\left(\frac{e\left(e^2-3df\right)}{\sqrt{e^2-4df}}-df+e^2\right) + 4\left(e^2-df\right)\left(\sqrt{a+cx^2}-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)\right) + \frac{4de\left(-\sqrt{a}\sqrt{c}x\sqrt{\frac{cx^2}{a}+1}\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)+a+cx^2\right)}{x\sqrt{a+cx^2}}}{4d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(x^3*(d + e*x + f*x^2)),x]
```

```
[Out] (-2*(e^2 - d*f - (e*(e^2 - 3*d*f))/Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2] - 2*(
e^2 - d*f + (e*(e^2 - 3*d*f))/Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2] + (4*d*e*(
a + c*x^2 - Sqrt[a]*Sqrt[c]*x*Sqrt[1 + (c*x^2)/a]*ArcSinh[(Sqrt[c]*x)/Sqrt[
a]]))/(x*Sqrt[a + c*x^2]) + ((e^2 - d*f + (e*(e^2 - 3*d*f))/Sqrt[e^2 - 4*d*
f])*(-(Sqrt[c]*(-e + Sqrt[e^2 - 4*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]
]) + Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*
f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqr
```

```
t[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]))/f + ((e^2 - d*f - (e*(e^2 - 3*d*f))/Sq
rt[e^2 - 4*d*f])*(Sqrt[c]*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[
a + c*x^2]] + Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTa
nh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f +
 e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]))/f + 4*(e^2 - d*f)*(Sqrt[a + c*x^
2] - Sqrt[a]*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]) - (2*d^2*(a + c*x^2 + c*x^2*
Sqrt[1 + (c*x^2)/a]*ArcTanh[Sqrt[1 + (c*x^2)/a]]))/(x^2*Sqrt[a + c*x^2]))/(
4*d^3)
```

**IntegrateAlgebraic [C]** time = 0.99, size = 541, normalized size = 1.07

$$-\frac{\text{RootSum}\left[\#1^4 f-2\#1^3\sqrt{c}\,e-2\#1^2 a f+4\#1^2 c d+2\#1 a\sqrt{c}\,e+a^2 f\&,\frac{\#1^2 c d^2 f\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)-\#1^2 a d f^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)+\#1^2 a e^2 f\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)+a^2 d f^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)-a^2 e^2 f\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)-2\#1 c^{3/2} d^2 e\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)-a c d^2 f\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)+4\#1 a\sqrt{c}\,d e f\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)-2\#1 a\sqrt{c}\,e^3\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)}{2\#1^3 f-3\#1^2\sqrt{c}\,e-2\#1 a f+4\#1 c d+a\sqrt{c}\,e}\&\right]}{d^3}+\frac{2\sqrt{a}\left(d f-e^2\right)\tanh^{-1}\left(\frac{\sqrt{a+cx^2}-\sqrt{c}\,x}{\sqrt{a}}\right)}{d^3}+\frac{\sqrt{a+cx^2}\,(2ex-d)}{2d^2x^2}+\frac{c\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}-\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + c*x^2]/(x^3*(d + e*x + f*x^2)),x]
```

```
[Out] ((-d + 2*e*x)*Sqrt[a + c*x^2])/(2*d^2*x^2) + (2*Sqrt[a]*(-e^2 + d*f)*ArcTan
h[(-(Sqrt[c]*x) + Sqrt[a + c*x^2])/Sqrt[a]])/d^3 + (c*ArcTanh[(Sqrt[c]*x)/S
qrt[a] - Sqrt[a + c*x^2]/Sqrt[a]])/(Sqrt[a]*d) - RootSum[a^2*f + 2*a*Sqrt[c
]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (-(a*c*d^2
*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]) - a^2*e^2*f*Log[-(Sqrt[c]*x) +
 Sqrt[a + c*x^2] - #1] + a^2*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]
 - 2*c^(3/2)*d^2*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - 2*a*Sqrt[c
]*e^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + 4*a*Sqrt[c]*d*e*f*Log[-
(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + c*d^2*f*Log[-(Sqrt[c]*x) + Sqrt[a
+ c*x^2] - #1]*#1^2 + a*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2
 - a*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*
c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]/d^3
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^3/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^3/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

**maple [B]** time = 0.02, size = 3993, normalized size = 7.88

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/x^3/(f*x^2+e*x+d),x)
```

```
[Out] 4*f^3/(e+(-4*d*f+e^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)
^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f
)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)-4*f^2/(e+(-4*
d*f+e^2)^(1/2))^3*c^(1/2)*ln((-1/2*c*(e+(-4*d*f+e^2)^(1/2))/f+c*(x+1/2*(e+(
-4*d*f+e^2)^(1/2))/f))/c^(1/2)+((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(
-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))-4*f^2/(e+(-4*d*f+e^2)^(1/2))^3/(-
4*d*f+e^2)^(1/2)*c^(1/2)*ln((-1/2*c*(e+(-4*d*f+e^2)^(1/2))/f+c*(x+1/2*(e+(-
4*d*f+e^2)^(1/2))/f))/c^(1/2)+((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-
4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))*e-4*f/(e+(-4*d*f+e^2)^(1/2))^3*2^(
1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*
f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1
/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2
*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4
*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*c*e
-8*f^3/(e+(-4*d*f+e^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*
f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1
/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1
/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x
+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*
e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*a+8*f^2/(e+(-4*d*f+e^2)^
(1/2))^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*
f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-
c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*
f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1
/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2
*(e+(-4*d*f+e^2)^(1/2))/f))*c*d-4*f/(e+(-4*d*f+e^2)^(1/2))^3/(-4*d*f+e^2)^(
1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(
(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2)
)/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2
```

*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*
c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)
^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))
/f))*c*e^2+16*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+(-4*d*f+e^2)^(1/2))^2/a/x*
(c*x^2+a)^(3/2)-16*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+(-4*d*f+e^2)^(1/2))^2
*c/a*x*(c*x^2+a)^(1/2)-16*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+(-4*d*f+e^2)^(
1/2))^2*c^(1/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))+2*f/(-e+(-4*d*f+e^2)^(1/2))/(
e+(-4*d*f+e^2)^(1/2))/a/x^2*(c*x^2+a)^(3/2)+2*f/(-e+(-4*d*f+e^2)^(1/2))/(e+
(-4*d*f+e^2)^(1/2))*c/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)-2*f/(-e
+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*c/a*(c*x^2+a)^(1/2)+4*f^3/(-e+(
-4*d*f+e^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f
)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-
4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)+4*f^2/(-e+(-4*d*f+e^
2)^(1/2))^3*c^(1/2)*ln((-1/2*c*(e-(-4*d*f+e^2)^(1/2))/f+c*(x-1/2*(-e+(-4*d*
f+e^2)^(1/2))/f))/c^(1/2)+((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e-(-4*d
*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)
*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))-4*f^2/(-e+(-4*d*f+e^2)^(1/2))^3/(-4
*d*f+e^2)^(1/2)*c^(1/2)*ln((-1/2*c*(e-(-4*d*f+e^2)^(1/2))/f+c*(x-1/2*(-e+(-
4*d*f+e^2)^(1/2))/f))/c^(1/2)+((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e-(
-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))*e+4*f/(-e+(-4*d*f+e^2)^(1/2))^3
*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/
f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2
*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4
*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1
/2))/f))*c*e-8*f^3/(-e+(-4*d*f+e^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)
^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e
+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d
*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*
f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*a+8
*f^2/(-e+(-4*d*f+e^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^
(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)
^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/
f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*
d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*c*d-4*f/(-e+(-4*d
*f+e^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f
^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c
*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*
2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1

```
/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(
-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2
)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*c*e^2-64*f^4/(-e+(-4*d*f+e^2)^(
1/2))^3/(e+(-4*d*f+e^2)^(1/2))^3*a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))
/x)*d+64*f^3/(-e+(-4*d*f+e^2)^(1/2))^3/(e+(-4*d*f+e^2)^(1/2))^3*a^(1/2)*ln(
(2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)*e^2+64*f^4/(-e+(-4*d*f+e^2)^(1/2))^3/(e+
(-4*d*f+e^2)^(1/2))^3*(c*x^2+a)^(1/2)*d-64*f^3/(-e+(-4*d*f+e^2)^(1/2))^3/(e
+(-4*d*f+e^2)^(1/2))^3*(c*x^2+a)^(1/2)*e^2
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + a}}{\left(fx^2 + ex + d\right)x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^3/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)/((f*x^2 + e*x + d)*x^3), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2 + a}}{x^3\,\left(f\,x^2 + e\,x + d\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(1/2)/(x^3*(d + e*x + f*x^2)),x)
```

```
[Out] int((a + c*x^2)^(1/2)/(x^3*(d + e*x + f*x^2)), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + cx^2}}{x^3\left(d + ex + fx^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/x**3/(f*x**2+e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(x**3*(d + e*x + f*x**2)), x)
```

## 3.58 $\int \frac{x^2\left(a+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx$

**Optimal**. Leaf size=795

$$-\frac{(4e-3fx)\left(cx^2+a\right)^{3/2}}{12f^2}-\frac{\left(8e\left(af^2+c\left(e^2-2df\right)\right)-f\left(3af^2+4c\left(e^2-df\right)\right)x\right)\sqrt{cx^2+a}}{8f^4}+\frac{\left(3a^2f^4+12ac\left(e^2-\right.\right.}{}$$

**Rubi [A]** time = 4.26, antiderivative size = 795, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1069, 1068, 1080, 217, 206, 1034, 725}

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
```

```
[Out] -((8*e*(a*f^2 + c*(e^2 - 2*d*f)) - f*(3*a*f^2 + 4*c*(e^2 - d*f))*x)*Sqrt[a
+ c*x^2])/(8*f^4) - ((4*e - 3*f*x)*(a + c*x^2)^(3/2))/(12*f^2) + ((3*a^2*f^
4 + 12*a*c*f^2*(e^2 - d*f) + 8*c^2*(e^4 - 3*d*e^2*f + d^2*f^2))*ArcTanh[(Sq
rt[c]*x)/Sqrt[a + c*x^2]])/(8*Sqrt[c]*f^5) - ((a^2*f^4*(e^2 - 2*d*f - e*Sqr
t[e^2 - 4*d*f]) + 2*a*c*f^2*(e^4 - 4*d*e^2*f + 2*d^2*f^2 - e^3*Sqrt[e^2 - 4
*d*f] + 2*d*e*f*Sqrt[e^2 - 4*d*f]) + c^2*(e^6 - 6*d*e^4*f + 9*d^2*e^2*f^2 -
 2*d^3*f^3 - e^5*Sqrt[e^2 - 4*d*f] + 4*d*e^3*f*Sqrt[e^2 - 4*d*f] - 3*d^2*e*
f^2*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt
[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]
)/(Sqrt[2]*f^5*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2
 - 4*d*f])]) + ((a^2*f^4*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + 2*a*c*f^2*(e
^4 - 4*d*e^2*f + 2*d^2*f^2 + e^3*Sqrt[e^2 - 4*d*f] - 2*d*e*f*Sqrt[e^2 - 4*d
*f]) + c^2*(e^6 - 6*d*e^4*f + 9*d^2*e^2*f^2 - 2*d^3*f^3 + e^5*Sqrt[e^2 - 4*
d*f] - 4*d*e^3*f*Sqrt[e^2 - 4*d*f] + 3*d^2*e*f^2*Sqrt[e^2 - 4*d*f]))*ArcTan
h[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*
d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*f^5*Sqrt[e^2 - 4*d*
f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1068

```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) +
(e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*p + 2*q + 3)
+ C*(-(c*e*(2*p + q + 2))) + 2*c*C*f*(p + q + 1)*x)*(a + c*x^2)^p*(d + e*x
+ f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f
^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^
q*Simp[p*(-(a*e))*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p +
q + 1)*(a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3
))) + (2*p*(c*d - a*f)*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) +
(p + q + 1)*(C*e*f*p*(-4*a*c)))*x + (p*(c*e)*(C*(c*e)*(q + 1) - c*(C*e - B*
f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(-4*a*c) - c^2*(C*(e^2 - 4*d*f)*
(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]
/; FreeQ[{a, c, d, e, f, A, B, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0
] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q
, 0]
```

Rule 1069

```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) + (e_.)*(x_) +
(f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((C*(-(c*e*(2*p + q + 2))) + 2*c*C*f*
(p + q + 1)*x)*(a + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1
)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[
(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(-(a*e))*(C*(c*e)*(q + 1) -
c*(C*e)*(2*p + 2*q + 3)) + (p + q + 1)*(a*c*(C*(2*d*f - e^2*(2*p + q + 2))
+ f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(c*e)*(q + 1) - c*(C*e
)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(-4*a*c)))*x + (p*(c*e)*(C*(c*e)*
(q + 1) - c*(C*e)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(-4*a*c) - c^2*(C
```

```
*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3))))*x^2, x]
, x], x] /; FreeQ[{a, c, d, e, f, A, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && Gt
Q[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !
IGtQ[q, 0]
```

Rule 1080

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rubi steps

$$\int \frac{x^2\left(a+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx = -\frac{(4e-3fx)\left(a+cx^2\right)^{3/2}}{12f^2} - \frac{\int \frac{\sqrt{a+cx^2}\left(3acdf-3ce(4cd-af)x-3c\left(3af^2+4c\left(e^2-df\right)\right)x^2\right)}{d+ex+fx^2}\,dx}{12cf^2}$$

$$= -\frac{\left(8e\left(af^2+c\left(e^2-2df\right)\right)-f\left(3af^2+4c\left(e^2-df\right)\right)x\right)\sqrt{a+cx^2}}{8f^4} - \frac{(4e-3fx)\left(a+cx^2\right)}{12f^2}$$

$$= -\frac{\left(8e\left(af^2+c\left(e^2-2df\right)\right)-f\left(3af^2+4c\left(e^2-df\right)\right)x\right)\sqrt{a+cx^2}}{8f^4} - \frac{(4e-3fx)\left(a+cx^2\right)}{12f^2}$$

$$= -\frac{\left(8e\left(af^2+c\left(e^2-2df\right)\right)-f\left(3af^2+4c\left(e^2-df\right)\right)x\right)\sqrt{a+cx^2}}{8f^4} - \frac{(4e-3fx)\left(a+cx^2\right)}{12f^2}$$

$$= -\frac{\left(8e\left(af^2+c\left(e^2-2df\right)\right)-f\left(3af^2+4c\left(e^2-df\right)\right)x\right)\sqrt{a+cx^2}}{8f^4} - \frac{(4e-3fx)\left(a+cx^2\right)}{12f^2}$$

$$= -\frac{\left(8e\left(af^2+c\left(e^2-2df\right)\right)-f\left(3af^2+4c\left(e^2-df\right)\right)x\right)\sqrt{a+cx^2}}{8f^4} - \frac{(4e-3fx)\left(a+cx^2\right)}{12f^2}$$

**Mathematica** [A] time = 3.45, size = 793, normalized size = 1.00

$$\frac{3f\sqrt{a+cx^2}\left(\frac{3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{c}\sqrt{\frac{cx^2}{a}+1}}+5ax+2cx^3\right)-\frac{3\left(\frac{2df-e^2}{\sqrt{e^2-4df}}+e\right)\left(\frac{2\left(2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)\left(-\sqrt{4af^2-2ce\sqrt{e^2-4df}-4cdf+2ce^2}\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)+\sqrt{c}\left(\sqrt{e^2-4df}-e\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)+2f\sqrt{a+cx^2}\right)}{f^2}+\frac{2\sqrt{c}\sqrt{a+cx^2}\left(\sqrt{e^2-4df}-e\right)\left(\sqrt{c}x\sqrt{\frac{cx^2}{a}+1}+\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\right)}{\sqrt{\frac{cx^2}{a}+1}}\right)}{2f}+\frac{3\left(\frac{e^2-2df}{\sqrt{e^2-4df}}+e\right)\left(\frac{2\left(2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)\right)\left(\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{a+cx^2}\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)+\sqrt{c}\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)-2f\sqrt{a+cx^2}\right)}{f^2}+\frac{2\sqrt{c}\sqrt{a+cx^2}\left(\sqrt{e^2-4df}+e\right)\left(\sqrt{c}x\sqrt{\frac{cx^2}{a}+1}+\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\right)}{\sqrt{\frac{cx^2}{a}+1}}\right)}{2f}-4\left(a+cx^2\right)^{3/2}\left(\frac{e^2-2df}{\sqrt{e^2-4df}}+e\right)-4\left(a+cx^2\right)^{3/2}\left(\frac{2df-e^2}{\sqrt{e^2-4df}}+e\right)}{24f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
```

```
[Out] (-4*(e + (e^2 - 2*d*f)/Sqrt[e^2 - 4*d*f])*(a + c*x^2)^(3/2) - 4*(e + (-e^2
+ 2*d*f)/Sqrt[e^2 - 4*d*f])*(a + c*x^2)^(3/2) + 3*f*Sqrt[a + c*x^2]*(5*a*x
+ 2*c*x^3 + (3*a^(3/2)*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[c]*Sqrt[1 + (c*x
^2)/a])) - (3*(e + (-e^2 + 2*d*f)/Sqrt[e^2 - 4*d*f])*((2*Sqrt[c]*(-e + Sqrt
[e^2 - 4*d*f])*Sqrt[a + c*x^2]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/a] + Sqrt[a]*Arc
Sinh[(Sqrt[c]*x)/Sqrt[a]]))/Sqrt[1 + (c*x^2)/a] + (2*(2*a*f^2 + c*(e^2 - 2*
d*f - e*Sqrt[e^2 - 4*d*f]))*(2*f*Sqrt[a + c*x^2] + Sqrt[c]*(-e + Sqrt[e^2 -
 4*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] - Sqrt[2*c*e^2 - 4*c*d*f + 4*
a*f^2 - 2*c*e*Sqrt[e^2 - 4*d*f]]*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f]
)*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x
^2])]))/f^2))/(2*f) + (3*(e + (e^2 - 2*d*f)/Sqrt[e^2 - 4*d*f])*((2*Sqrt[c]*
(e + Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/a] + Sq
rt[a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]]))/Sqrt[1 + (c*x^2)/a] + (2*(2*a*f^2 + c*
(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*(-2*f*Sqrt[a + c*x^2] + Sqrt[c]*(e + S
qrt[e^2 - 4*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] + Sqrt[4*a*f^2 + 2*c
*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*
d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a +
c*x^2])]))/f^2))/(2*f))/(24*f^2)
```

**IntegrateAlgebraic [C]** time = 1.22, size = 960, normalized size = 1.21

$$\frac{\sqrt{cx^2+a}\left(-24ce^3+12cfxe^2-32af^2e-8cf^2x^2e+48cdfe+6cf^3x^3+15af^3x-12cdf^2x\right)}{24f^4}+\frac{\left(-8c^2e^4-12acf^2e^2+24c^2dfe^2-3a^2f^4+12acdf^3-8c^2d^2f^2\right)\log\left(\sqrt{cx^2+a}-\sqrt{c}x\right)}{8\sqrt{c}f^5}+\text{RootSum}\left[f\#1^4-2\sqrt{c}e\#1^3+4cd\#1^2-2af\#1^2+2a\sqrt{c}e\#1+a^2f\&,\ \text{[illegible]}\right]$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(a + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(-24*c*e^3 + 48*c*d*e*f - 32*a*e*f^2 + 12*c*e^2*f*x - 12*c
*d*f^2*x + 15*a*f^3*x - 8*c*e*f^2*x^2 + 6*c*f^3*x^3))/(24*f^4) + ((-8*c^2*e
^4 + 24*c^2*d*e^2*f - 8*c^2*d^2*f^2 - 12*a*c*e^2*f^2 + 12*a*c*d*f^3 - 3*a^2
*f^4)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(8*Sqrt[c]*f^5) + RootSum[a^2*f
+ 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 &
, (a*c^2*e^5*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - 4*a*c^2*d*e^3*f*Log
[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + 3*a*c^2*d^2*e*f^2*Log[-(Sqrt[c]*x)
+ Sqrt[a + c*x^2] - #1] + 2*a^2*c*e^3*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2
] - #1] - 4*a^2*c*d*e*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + a^3*e*
f^4*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + 2*c^(5/2)*d*e^4*Log[-(Sqrt[c
]*x) + Sqrt[a + c*x^2] - #1]*#1 - 6*c^(5/2)*d^2*e^2*f*Log[-(Sqrt[c]*x) + Sq
rt[a + c*x^2] - #1]*#1 + 2*c^(5/2)*d^3*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^
2] - #1]*#1 + 4*a*c^(3/2)*d*e^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1
]*#1 - 4*a*c^(3/2)*d^2*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + 2*
a^2*Sqrt[c]*d*f^4*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - c^2*e^5*Log
```

```
[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 + 4*c^2*d*e^3*f*Log[-(Sqrt[c]*x)
 + Sqrt[a + c*x^2] - #1]*#1^2 - 3*c^2*d^2*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a +
 c*x^2] - #1]*#1^2 - 2*a*c*e^3*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]
*#1^2 + 4*a*c*d*e*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 - a^2*e
*f^4*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1
 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]/f^5
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$sage_2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage2
```

**maple [B]** time = 0.03, size = 19148, normalized size = 24.09

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\left(c\,x^2+a\right)^{3/2}}{f\,x^2+e\,x+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + c*x^2)^(3/2))/(d + e*x + f*x^2),x)
[Out] int((x^2*(a + c*x^2)^(3/2))/(d + e*x + f*x^2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
[Out] Timed out
```

## 3.59 $\int \frac{x\left(a+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx$

**Optimal**. Leaf size=553

$$-\frac{\left(2cdef\left(2af^2+c\left(e^2-2df\right)\right)-\left(e-\sqrt{e^2-4df}\right)\left(a^2f^4+2acf^2\left(e^2-df\right)+c^2\left(d^2f^2-3de^2f+e^4\right)\right)\right)\tanh^{-1}\left(\cdots\right.}{\sqrt{2}\,f^4\sqrt{e^2-4df}\,\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

**Rubi [A]** time = 2.43, antiderivative size = 553, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1020, 1068, 1080, 217, 206, 1034, 725}

$$-\frac{\left(2cdef\left(2af^2+c\left(e^2-2df\right)\right)-\left(e-\sqrt{e^2-4df}\right)\left(a^2f^4+2acf^2\left(e^2-df\right)+c^2\left(d^2f^2-3de^2f+e^4\right)\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\,f^4\sqrt{e^2-4df}\,\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(2cdef\left(2af^2+c\left(e^2-2df\right)\right)-\left(\sqrt{e^2-4df}+e\right)\left(a^2f^4+2acf^2\left(e^2-df\right)+c^2\left(d^2f^2-3de^2f+e^4\right)\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\,f^4\sqrt{e^2-4df}\,\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\sqrt{a+cx^2}\left(2\left(af^2+c\left(e^2-df\right)\right)-cefx\right)}{2f^3}-\frac{\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)\left(3af^2+2c\left(e^2-2df\right)\right)}{2f^4}+\frac{\left(a+cx^2\right)^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
```

```
[Out] ((2*(a*f^2 + c*(e^2 - d*f)) - c*e*f*x)*Sqrt[a + c*x^2])/(2*f^3) + (a + c*x^
2)^(3/2)/(3*f) - (Sqrt[c]*e*(3*a*f^2 + 2*c*(e^2 - 2*d*f))*ArcTanh[(Sqrt[c]*
x)/Sqrt[a + c*x^2]])/(2*f^4) - ((2*c*d*e*f*(2*a*f^2 + c*(e^2 - 2*d*f)) - (e
 - Sqrt[e^2 - 4*d*f])*(a^2*f^4 + 2*a*c*f^2*(e^2 - d*f) + c^2*(e^4 - 3*d*e^2
*f + d^2*f^2)))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt
[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[
2]*f^4*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f
])]) + ((2*c*d*e*f*(2*a*f^2 + c*(e^2 - 2*d*f)) - (e + Sqrt[e^2 - 4*d*f])*(a
^2*f^4 + 2*a*c*f^2*(e^2 - d*f) + c^2*(e^4 - 3*d*e^2*f + d^2*f^2)))*ArcTanh[
(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*
f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*f^4*Sqrt[e^2 - 4*d*f]
*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1020

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
eQ[p + q + 1, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1068

```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) +
(e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*p + 2*q + 3)
+ C*(-(c*e*(2*p + q + 2))) + 2*c*C*f*(p + q + 1)*x)*(a + c*x^2)^p*(d + e*x
+ f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f
^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^
q*Simp[p*(-(a*e))*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p +
q + 1)*(a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3
))) + (2*p*(c*d - a*f)*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) +
(p + q + 1)*(C*e*f*p*(-4*a*c)))*x + (p*(c*e)*(C*(c*e)*(q + 1) - c*(C*e - B*
f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(-4*a*c) - c^2*(C*(e^2 - 4*d*f)*
(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]
/; FreeQ[{a, c, d, e, f, A, B, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0
] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q
, 0]
```

Rule 1080

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
```

```
*c, 0]
```

Rubi steps

$$\int \frac{x\left(a+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx = \frac{\left(a+cx^2\right)^{3/2}}{3f} + \frac{\int \frac{\sqrt{a+cx^2}\left(-3(cd-af)x-3cex^2\right)}{d+ex+fx^2}\,dx}{3f}$$

$$= \frac{\left(2\left(af^2+c\left(e^2-df\right)\right)-cefx\right)\sqrt{a+cx^2}}{2f^3} + \frac{\left(a+cx^2\right)^{3/2}}{3f} - \frac{\int \frac{-3ac^2def+3c\left(ace^2f+2(cd-af)\left(ce^2-c\ldots\right.\right.}{\sqrt{a+cx^2}\left(\ldots\right.}}{6\ldots}$$

$$= \frac{\left(2\left(af^2+c\left(e^2-df\right)\right)-cefx\right)\sqrt{a+cx^2}}{2f^3} + \frac{\left(a+cx^2\right)^{3/2}}{3f} - \frac{\int \frac{-3ac^2def^2-3c^2de\left(3af^2+2c\left(e^2-2df\right)\right)\ldots}{\ldots}}{\ldots}$$

$$= \frac{\left(2\left(af^2+c\left(e^2-df\right)\right)-cefx\right)\sqrt{a+cx^2}}{2f^3} + \frac{\left(a+cx^2\right)^{3/2}}{3f} - \frac{\left(ce\left(3af^2+2c\left(e^2-2df\right)\right)\right)\mathrm{Su}\ldots}{2f^4}$$

$$= \frac{\left(2\left(af^2+c\left(e^2-df\right)\right)-cefx\right)\sqrt{a+cx^2}}{2f^3} + \frac{\left(a+cx^2\right)^{3/2}}{3f} - \frac{\sqrt{c}\,e\left(3af^2+2c\left(e^2-2df\right)\right)\tan\ldots}{2f^4}$$

$$= \frac{\left(2\left(af^2+c\left(e^2-df\right)\right)-cefx\right)\sqrt{a+cx^2}}{2f^3} + \frac{\left(a+cx^2\right)^{3/2}}{3f} - \frac{\sqrt{c}\,e\left(3af^2+2c\left(e^2-2df\right)\right)\tan\ldots}{2f^4}$$

**Mathematica [A]** time = 2.01, size = 755, normalized size = 1.37

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
```

```
[Out] (8*f^3*(-e + Sqrt[e^2 - 4*d*f])*(a + c*x^2)^(5/2)*Sqrt[1 + (c*x^2)/a] + 8*f
^3*(e + Sqrt[e^2 - 4*d*f])*(a + c*x^2)^(5/2)*Sqrt[1 + (c*x^2)/a] + 3*(e - S
qrt[e^2 - 4*d*f])*(2*Sqrt[c]*f^2*(e - Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2]*(a
*Sqrt[c]*x*(1 + (c*x^2)/a)^(3/2) + Sqrt[a]*(a + c*x^2)*ArcSinh[(Sqrt[c]*x)/
Sqrt[a]]) - a*(4*a*f^2 + c*(e - Sqrt[e^2 - 4*d*f])^2)*(1 + (c*x^2)/a)^(3/2)
*(2*f*Sqrt[a + c*x^2] + Sqrt[c]*(-e + Sqrt[e^2 - 4*d*f])*ArcTanh[(Sqrt[c]*x
)/Sqrt[a + c*x^2]] - Sqrt[2*c*e^2 - 4*c*d*f + 4*a*f^2 - 2*c*e*Sqrt[e^2 - 4*
```

```
d*f]]*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-
e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])) - 3*(e + Sqrt[e^2 -
 4*d*f])*(2*Sqrt[c]*f^2*(e + Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2]*(a*Sqrt[c]*
x*(1 + (c*x^2)/a)^(3/2) + Sqrt[a]*(a + c*x^2)*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])
 - a*(4*a*f^2 + c*(e + Sqrt[e^2 - 4*d*f])^2)*(1 + (c*x^2)/a)^(3/2)*(2*f*Sqr
t[a + c*x^2] - Sqrt[c]*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[a +
 c*x^2]] - Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[
(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*
Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])))/(48*a*f^4*Sqrt[e^2 - 4*d*f]*(1 + (
c*x^2)/a)^(3/2))
```

**IntegrateAlgebraic [C]** time = 0.89, size = 771, normalized size = 1.39

$$-\frac{\text{RootSum}\left[\#1^4 f-2\#1^3\sqrt{c}e-2\#1^2 af+4\#1^2 cd+2\#1 a\sqrt{c}e+a^2 f\&,\frac{-\#1^2a^2f^4\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)-\#1^2c^2d^2f^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+3\#1^2c^2de^2f\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+\#1^2\left(-c^2\right)e^4\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+2\#1^2acdf^3\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)-2\#1^2ace^2f^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+a^3f^4\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)-2a^2cdf^3\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+2a^2ce^2f^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)-4\#1c^{5/2}d^2ef\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+2\#1c^{5/2}de^3\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+4\#1ac^{3/2}def^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+ac^2d^2f^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)-3ac^2de^2f\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+ac^2e^4\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)}{2\#1^3f-3\#1^2\sqrt{c}e-2\#1af+4\#1cd+a\sqrt{c}e}\&\right]}{f^4}+\frac{\log\left(\sqrt{a+cx^2}-\sqrt{c}x\right)\left(3a\sqrt{c}ef^2-4c^{3/2}def+2c^{3/2}e^3\right)}{2f^4}+\frac{\sqrt{a+cx^2}\left(8af^2-6cdf+6ce^2-3cefx+2cf^2x^2\right)}{6f^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(a + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(6*c*e^2 - 6*c*d*f + 8*a*f^2 - 3*c*e*f*x + 2*c*f^2*x^2))/(
6*f^3) + ((2*c^(3/2)*e^3 - 4*c^(3/2)*d*e*f + 3*a*Sqrt[c]*e*f^2)*Log[-(Sqrt[
c]*x) + Sqrt[a + c*x^2]])/(2*f^4) - RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*
d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (a*c^2*e^4*Log[-(Sqrt[c
]*x) + Sqrt[a + c*x^2] - #1] - 3*a*c^2*d*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a +
c*x^2] - #1] + a*c^2*d^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + 2*a
^2*c*e^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - 2*a^2*c*d*f^3*Log[-
(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + a^3*f^4*Log[-(Sqrt[c]*x) + Sqrt[a + c
*x^2] - #1] + 2*c^(5/2)*d*e^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 -
 4*c^(5/2)*d^2*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + 4*a*c^(3/2
)*d*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - c^2*e^4*Log[-(Sqrt[
c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 + 3*c^2*d*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[
a + c*x^2] - #1]*#1^2 - c^2*d^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1
]*#1^2 - 2*a*c*e^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 + 2*a*
c*d*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 - a^2*f^4*Log[-(Sqrt[
c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*
Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]/f^4
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$sage_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage2
```

**maple [B]** time = 0.02, size = 14709, normalized size = 26.60

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,{\left(c\,x^2+a\right)}^{3/2}}{f\,x^2+e\,x+d} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + c*x^2)^(3/2))/(d + e*x + f*x^2),x)
```

```
[Out] int((x*(a + c*x^2)^(3/2))/(d + e*x + f*x^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \left(a + c x^{2}\right)^{\frac{3}{2}}}{d + e x + f x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x*(a + c*x**2)**(3/2)/(d + e*x + f*x**2), x)
```

## 3.60 $\int \frac{\left(a+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx$

**Optimal**. Leaf size=484

$$-\frac{\left(ce\left(e-\sqrt{e^2-4df}\,\right)\left(2af^2+c\left(e^2-2df\right)\right)-2f\left(-a^2f^3+2acdf^2+c^2d\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{\ldots}\right.}{\sqrt{2}\,\sqrt{a+cx^2}\,\sqrt{2af^2+c\left(-\ldots\right.}}\right.}{\sqrt{2}\,f^3\sqrt{e^2-4df}\,\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}\,-2df+e^2\right)}}$$

**Rubi** [A] time = 4.24, antiderivative size = 482, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {979, 1080, 217, 206, 1034, 725}

$$\frac{\left(-2a^2f^4-ce\left(e-\sqrt{e^2-4df}\,\right)\left(2af^2+c\left(e^2-2df\right)\right)+4acdf^3+2c^2df\left(e^2-df\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\,\right)}{\sqrt{2}\,\sqrt{a+cx^2}\,\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}\,-2df+e^2\right)}}\right)}{\sqrt{2}\,f^3\sqrt{e^2-4df}\,\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}\,-2df+e^2\right)}}-\frac{\left(-2a^2f^4-ce\left(\sqrt{e^2-4df}\,+e\right)\left(2af^2+c\left(e^2-2df\right)\right)+4acdf^3+2c^2df\left(e^2-df\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}\,+e\right)}{\sqrt{2}\,\sqrt{a+cx^2}\,\sqrt{2af^2+c\left(e\sqrt{e^2-4df}\,-2df+e^2\right)}}\right)}{\sqrt{2}\,f^3\sqrt{e^2-4df}\,\sqrt{2af^2+c\left(e\sqrt{e^2-4df}\,-2df+e^2\right)}}+\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)\left(3af^2+2c\left(e^2-df\right)\right)}{2f^3}-\frac{c\sqrt{a+cx^2}\,(2e-fx)}{2f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(3/2)/(d + e*x + f*x^2),x]
```

```
[Out] -(c*(2*e - f*x)*Sqrt[a + c*x^2])/(2*f^2) + (Sqrt[c]*(3*a*f^2 + 2*c*(e^2 - d
*f))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*f^3) + ((4*a*c*d*f^3 - 2*a^2*
f^4 + 2*c^2*d*f*(e^2 - d*f) - c*e*(e - Sqrt[e^2 - 4*d*f])*(2*a*f^2 + c*(e^2
 - 2*d*f)))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a
*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*f
^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])])
 - ((4*a*c*d*f^3 - 2*a^2*f^4 + 2*c^2*d*f*(e^2 - d*f) - c*e*(e + Sqrt[e^2 -
4*d*f])*(2*a*f^2 + c*(e^2 - 2*d*f)))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d
*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt
[a + c*x^2])])/(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f
 + e*Sqrt[e^2 - 4*d*f])])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 979

```
Int[((a_.) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> -Simp[(c*(e*(2*p + q) - 2*f*(p + q)*x)*(a + c*x^2)^(p - 1)*(d +
 e*x + f*x^2)^(q + 1))/(2*f^2*(p + q)*(2*p + 2*q + 1)), x] - Dist[1/(2*f^2*
(p + q)*(2*p + 2*q + 1)), Int[(a + c*x^2)^(p - 2)*(d + e*x + f*x^2)^q*Simp[
-(a*c*e^2*(1 - p)*(2*p + q)) + a*(p + q)*(-2*a*f^2*(2*p + 2*q + 1) + c*(2*d
*f - e^2*(2*p + q))) + (2*(c*d - a*f)*(c*e)*(1 - p)*(2*p + q) + 4*a*c*e*f*(
1 - p)*(p + q))*x + (p*c^2*e^2*(1 - p) - c*(p + q)*(2*a*f^2*(4*p + 2*q - 1)
 + c*(2*d*f*(1 - 2*p) + e^2*(3*p + q - 1))))*x^2, x], x], x] /; FreeQ[{a, c
, d, e, f, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 1] && NeQ[p + q, 0] && N
eQ[2*p + 2*q + 1, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1080

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx = -\frac{c(2e-fx)\sqrt{a+cx^2}}{2f^2} - \frac{\int \frac{af(cd-2af)-ce(2cd-af)x-c\left(3af^2+2c\left(e^2-df\right)\right)x^2}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{2f^2}$$

$$= -\frac{c(2e-fx)\sqrt{a+cx^2}}{2f^2} - \frac{\int \frac{af^2(cd-2af)+cd\left(3af^2+2c\left(e^2-df\right)\right)+\left(-cef(2cd-af)+ce\left(3af^2+2c\left(e^2-df\right)\right)\right)x}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{2f^3} +$$

$$= -\frac{c(2e-fx)\sqrt{a+cx^2}}{2f^2} + \frac{\left(c\left(3af^2+2c\left(e^2-df\right)\right)\right)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{2f^3} - \frac{\left(2f\left(af\right.\right.}{}$$

$$= -\frac{c(2e-fx)\sqrt{a+cx^2}}{2f^2} + \frac{\sqrt{c}\left(3af^2+2c\left(e^2-df\right)\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2f^3} + \frac{\left(2f\left(af^2(cd-2af)\right.\right.}{}$$

$$= -\frac{c(2e-fx)\sqrt{a+cx^2}}{2f^2} + \frac{\sqrt{c}\left(3af^2+2c\left(e^2-df\right)\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2f^3} - \frac{\left(ce\left(e-\sqrt{e^2-4df}\right.\right.}{}$$

**Mathematica** [A] time = 1.07, size = 603, normalized size = 1.25

$$\frac{\frac{2\left(2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)\left(-\sqrt{4af^2-2ce\sqrt{e^2-4df}-4cdf+2ce^2}\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)+\sqrt{c}\left(\sqrt{e^2-4df}-e\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)+2f\sqrt{a+cx^2}\right)}{f^2}+\frac{2\left(2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)\right)\left(\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{a+cx^2}\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)+\sqrt{c}\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)-2f\sqrt{a+cx^2}\right)}{f^2}+\frac{2\sqrt{c}\sqrt{a+cx^2}\left(\sqrt{e^2-4df}-e\right)\left(\sqrt{c}\,x\sqrt{\frac{cx^2}{a}+1}+\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\right)}{\sqrt{\frac{cx^2}{a}+1}}+\frac{2\sqrt{c}\sqrt{a+cx^2}\left(\sqrt{e^2-4df}+e\right)\left(\sqrt{c}\,x\sqrt{\frac{cx^2}{a}+1}+\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\right)}{\sqrt{\frac{cx^2}{a}+1}}}{8f\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(3/2)/(d + e*x + f*x^2),x]
```

```
[Out] ((2*Sqrt[c]*(-e + Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2]*(Sqrt[c]*x*Sqrt[1 + (c
*x^2)/a] + Sqrt[a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]]))/Sqrt[1 + (c*x^2)/a] + (2*
Sqrt[c]*(e + Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)
/a] + Sqrt[a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]]))/Sqrt[1 + (c*x^2)/a] + (2*(2*a*
f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))*(2*f*Sqrt[a + c*x^2] + Sqrt[c]
*(-e + Sqrt[e^2 - 4*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] - Sqrt[2*c*e
^2 - 4*c*d*f + 4*a*f^2 - 2*c*e*Sqrt[e^2 - 4*d*f]]*ArcTanh[(2*a*f + c*(-e +
Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*
f])]*Sqrt[a + c*x^2])]))/f^2 + (2*(2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 -
4*d*f]))*(-2*f*Sqrt[a + c*x^2] + Sqrt[c]*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(S
qrt[c]*x)/Sqrt[a + c*x^2]] + Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 -
 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c
*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]))/f^2)/(8*f*Sqrt[e^
2 - 4*d*f])
```

**IntegrateAlgebraic [C]** time = 0.70, size = 557, normalized size = 1.15

$$\frac{\text{RootSum}\left[\#1^4 f-2\#1^3\sqrt{c}\,e-2\#1^2 af+4\#1^2 cd+2\#1 a\sqrt{c}\,e+a^2 f\&,\frac{2\#1^2c^2def\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)+\#1^2(-c^2)e^3\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)-2\#1^2acef^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)+2a^2cef^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)-2\#1a^2\sqrt{c}f^3\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)-2\#1c^{5/2}d^2f\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)+2\#1c^{5/2}de^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)+4\#1ac^{3/2}df^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)-2ac^2def\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)+ac^2e^3\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)}{2\#1^3f-3\#1^2\sqrt{c}\,e-2\#1af+4\#1cd+a\sqrt{c}\,e}\&\right]}{f^3}+\frac{\log\left(\sqrt{a+cx^2}-\sqrt{c}\,x\right)\left(-3a\sqrt{c}\,f^2+2c^{3/2}df-2c^{3/2}e^2\right)}{2f^3}-\frac{c\sqrt{a+cx^2}\,(2e-fx)}{2f^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + c*x^2)^(3/2)/(d + e*x + f*x^2),x]
```

```
[Out] -1/2*(c*(2*e - f*x)*Sqrt[a + c*x^2])/f^2 + ((-2*c^(3/2)*e^2 + 2*c^(3/2)*d*f
 - 3*a*Sqrt[c]*f^2)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(2*f^3) + RootSum[
a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#
1^4 & , (a*c^2*e^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - 2*a*c^2*d*e*f
*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + 2*a^2*c*e*f^2*Log[-(Sqrt[c]*x)
+ Sqrt[a + c*x^2] - #1] + 2*c^(5/2)*d*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2
] - #1]*#1 - 2*c^(5/2)*d^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 +
4*a*c^(3/2)*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - 2*a^2*Sqrt[
c]*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - c^2*e^3*Log[-(Sqrt[c]*
x) + Sqrt[a + c*x^2] - #1]*#1^2 + 2*c^2*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + c
*x^2] - #1]*#1^2 - 2*a*c*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^
2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]/f^
3
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$sage_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage2
```

**maple [B]** time = 0.01, size = 8954, normalized size = 18.50

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+a\right)^{3/2}}{f\,x^2+e\,x+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(3/2)/(d + e*x + f*x^2),x)
```

```
[Out] int((a + c*x^2)^(3/2)/(d + e*x + f*x^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+cx^2\right)^{\frac{3}{2}}}{d+ex+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral((a + c*x**2)**(3/2)/(d + e*x + f*x**2), x)
```

## 3.61 $\int \frac{(a+cx^2)^{3/2}}{x(d+ex+fx^2)}\,dx$

**Optimal**. Leaf size=496

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}-\frac{\left(2ef\left(c^2d^2-a^2f^2\right)-\left(e-\sqrt{e^2-4df}\right)\left(c^2de^2-f(cd-af)^2\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{\right.}}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\right.}}\right.}{\sqrt{2}\,df^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

**Rubi [A]** time = 2.57, antiderivative size = 496, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {6728, 266, 50, 63, 208, 1020, 1080, 217, 206, 1034, 725}

$$-\frac{\left(2ef\left(c^2d^2-a^2f^2\right)-\left(e-\sqrt{e^2-4df}\right)\left(c^2de^2-f(cd-af)^2\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\,df^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(2ef\left(c^2d^2-a^2f^2\right)-\left(\sqrt{e^2-4df}+e\right)\left(c^2de^2-f(cd-af)^2\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\,df^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}-\frac{c^{3/2}e\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f^2}+\frac{\sqrt{a+cx^2}(cd-af)}{df}+\frac{a\sqrt{a+cx^2}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(3/2)/(x*(d + e*x + f*x^2)),x]
```

```
[Out] (a*Sqrt[a + c*x^2])/d + ((c*d - a*f)*Sqrt[a + c*x^2])/(d*f) - (c^(3/2)*e*Ar
cTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/f^2 - ((2*e*f*(c^2*d^2 - a^2*f^2) - (c^
2*d*e^2 - f*(c*d - a*f)^2)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e -
 Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2
- 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2
 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + ((2*e*f*(c^2*d^2 - a^2*f^2) -
(c^2*d*e^2 - f*(c*d - a*f)^2)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(
e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e
^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*
f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) - (a^(3/2)*ArcTanh[Sqrt[a + c
*x^2]/Sqrt[a]])/d
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1020

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
eQ[p + q + 1, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1080

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}}{x\left(d+ex+fx^2\right)}\,dx = \int \left(\frac{\left(a+cx^2\right)^{3/2}}{dx} + \frac{(-e-fx)\left(a+cx^2\right)^{3/2}}{d\left(d+ex+fx^2\right)}\right)dx$$

$$= \frac{\int \frac{\left(a+cx^2\right)^{3/2}}{x}\,dx}{d} + \frac{\int \frac{(-e-fx)\left(a+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx}{d}$$

$$= -\frac{\left(a+cx^2\right)^{3/2}}{3d} + \frac{\operatorname{Subst}\left(\int \frac{(a+cx)^{3/2}}{x}\,dx, x, x^2\right)}{2d} + \frac{\int \frac{\left(-3aef+3f(cd-af)x\right)\sqrt{a+cx^2}}{d+ex+fx^2}\,dx}{3df}$$

$$= \frac{(cd-af)\sqrt{a+cx^2}}{df} + \frac{a\operatorname{Subst}\left(\int \frac{\sqrt{a+cx}}{x}\,dx, x, x^2\right)}{2d} + \frac{\int \frac{-3a^2ef^2-3f(cd-af)^2x-3c^2defx^2}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{3df^2}$$

$$= \frac{a\sqrt{a+cx^2}}{d} + \frac{(cd-af)\sqrt{a+cx^2}}{df} + \frac{a^2\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2d} + \frac{\int \frac{3c^2d^2ef-3a^2ef^3+(3\cdots}{\sqrt{a+cx^2}\cdots}}{3\cdots}$$

$$= \frac{a\sqrt{a+cx^2}}{d} + \frac{(cd-af)\sqrt{a+cx^2}}{df} + \frac{a^2\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{cd} - \frac{\left(c^2e\right)\operatorname{Subst}\cdots}{\cdots}$$

$$= \frac{a\sqrt{a+cx^2}}{d} + \frac{(cd-af)\sqrt{a+cx^2}}{df} - \frac{c^{3/2}e\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{f^2} - \frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d} - \cdots$$

$$= \frac{a\sqrt{a+cx^2}}{d} + \frac{(cd-af)\sqrt{a+cx^2}}{df} - \frac{c^{3/2}e\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{f^2} - \frac{\left(2ef\left(c^2d^2-a^2f^2\right)-\left(c^2d\cdots\right.\right.}{\cdots}$$

**Mathematica [A]** time = 1.58, size = 746, normalized size = 1.50

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(3/2)/(x*(d + e*x + f*x^2)),x]
```

```
[Out] (c*Sqrt[a + c*x^2])/f - (c^(3/2)*e*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/f^
2 - ((c*d*(-e + Sqrt[e^2 - 4*d*f]) - a*f*(e + Sqrt[e^2 - 4*d*f]))*Sqrt[4*a*
f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f + c*(-e + Sq
rt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]
```

```
)]*Sqrt[a + c*x^2])])/(4*d*f^2*Sqrt[e^2 - 4*d*f]) - (c*Sqrt[a*f^2 + (c*(e^2
 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))/2]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*
f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*
x^2])])/(2*f^2) + (a*Sqrt[a*f^2 + (c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))/2
]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 -
2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(2*d*f) - (c*e*Sqrt[4*a*f^
2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e
^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sq
rt[a + c*x^2])])/(4*f^2*Sqrt[e^2 - 4*d*f]) - (a*e*Sqrt[4*a*f^2 + 2*c*(e^2 -
 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x
)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])
])/(4*d*f*Sqrt[e^2 - 4*d*f]) - (a^(3/2)*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d
```

**IntegrateAlgebraic [C]** time = 0.68, size = 559, normalized size = 1.13

$$\frac{\text{RootSum}\left[\#1^4 f - 2\#1^3\sqrt{c}\,e - 2\#1^2 af + 4\#1^2 cd + 2\#1 a\sqrt{c}\,e + a^2 f\&, \frac{\#1^2(-a^2)f^3\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right) - \#1^2c^2d^2f\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right) + \#1^2c^2de^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right) + 2\#1^2acdf^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right) + a^3f^3\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right) - 2a^2cdf^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right) + 2\#1a^2\sqrt{c}ef^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right) - 2\#1c^{5/2}d^2e\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right) + ac^2d^2f\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right) - ac^2de^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)}{2\#1^3f - 3\#1^2\sqrt{c}e - 2\#1af + 4\#1cd + a\sqrt{c}e}\&\right]}{df^2} + \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}} - \frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d} + \frac{c^{3/2}e\log\left(\sqrt{a+cx^2}-\sqrt{c}x\right)}{f^2} + \frac{c\sqrt{a+cx^2}}{f}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + c*x^2)^(3/2)/(x*(d + e*x + f*x^2)),x]
```

```
[Out] (c*Sqrt[a + c*x^2])/f + (2*a^(3/2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a] - Sqrt[a + c
*x^2]/Sqrt[a]])/d + (c^(3/2)*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/f^2 + R
ootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^
3 + f*#1^4 & , (-(a*c^2*d*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]) + a
*c^2*d^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - 2*a^2*c*d*f^2*Log[-(S
qrt[c]*x) + Sqrt[a + c*x^2] - #1] + a^3*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x
^2] - #1] - 2*c^(5/2)*d^2*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + 2
*a^2*Sqrt[c]*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + c^2*d*e^2*
Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 - c^2*d^2*f*Log[-(Sqrt[c]*x)
+ Sqrt[a + c*x^2] - #1]*#1^2 + 2*a*c*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^
2] - #1]*#1^2 - a^2*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*S
qrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]/(d*f^2)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/x/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/x/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

**maple [B]** time = 0.02, size = 9728, normalized size = 19.61

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)/x/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2 + a\right)^{\frac{3}{2}}}{\left(fx^2 + ex + d\right)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/x/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^(3/2)/((f*x^2 + e*x + d)*x), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2 + a\right)^{3/2}}{x\,\left(f\,x^2 + e\,x + d\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(3/2)/(x*(d + e*x + f*x^2)),x)
```

```
[Out] int((a + c*x^2)^(3/2)/(x*(d + e*x + f*x^2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + cx^2\right)^{\frac{3}{2}}}{x\left(d + ex + fx^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)/x/(f*x**2+e*x+d),x)
[Out] Integral((a + c*x**2)**(3/2)/(x*(d + e*x + f*x**2)), x)
```

## 3.62 $\int \frac{\left(a+cx^2\right)^{3/2}}{x^2\left(d+ex+fx^2\right)}\,dx$

**Optimal**. Leaf size=604

$$\frac{a^{3/2}e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}-\frac{\left(a^2f^2\left(e\sqrt{e^2-4df}-2df+e^2\right)+4acd^2f^2+c^2d^2\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)\tanh^{-1}\left(-\right.}{\sqrt{2}\,d^2f\sqrt{e^2-4df}\,\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right.}}$$

**Rubi [A]** time = 2.81, antiderivative size = 604, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.518, Rules used = {6728, 277, 195, 217, 206, 266, 50, 63, 208, 1020, 1068, 1080, 1034, 725}

$$-\frac{\left(a^2f^2\left(e\sqrt{e^2-4df}-2df+e^2\right)+4acd^2f^2+c^2d^2\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^2f\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(a^2f^2\left(-e\sqrt{e^2-4df}-2df+e^2\right)+4acd^2f^2+c^2d^2\left(e\sqrt{e^2-4df}-2df+e^2\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^2f\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{a^{3/2}e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}-\frac{ae\sqrt{a+cx^2}}{d^2}+\frac{\sqrt{a+cx^2}(2ae-cdx)}{2d^2}+\frac{\sqrt{c}(2cd-3af)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2df}-\frac{\left(a+cx^2\right)^{3/2}}{dx}+\frac{3cx\sqrt{a+cx^2}}{2d}+\frac{3a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(3/2)/(x^2*(d + e*x + f*x^2)),x]
```

```
[Out] -((a*e*Sqrt[a + c*x^2])/d^2) + (3*c*x*Sqrt[a + c*x^2])/(2*d) + ((2*a*e - c*
d*x)*Sqrt[a + c*x^2])/(2*d^2) - (a + c*x^2)^(3/2)/(d*x) + (3*a*Sqrt[c]*ArcT
anh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*d) + (Sqrt[c]*(2*c*d - 3*a*f)*ArcTanh[
(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*d*f) - ((4*a*c*d^2*f^2 + c^2*d^2*(e^2 - 2*
d*f - e*Sqrt[e^2 - 4*d*f]) + a^2*f^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*A
rcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2
 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^2*f*Sqrt[e^2
 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + ((4*a*c*
d^2*f^2 + a^2*f^2*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + c^2*d^2*(e^2 - 2*d*
f + e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sq
rt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]
)])/(Sqrt[2]*d^2*f*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt
[e^2 - 4*d*f])]) + (a^(3/2)*e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d^2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1020

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
eQ[p + q + 1, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1068

```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) +
(e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*p + 2*q + 3)
+ C*(-(c*e*(2*p + q + 2))) + 2*c*C*f*(p + q + 1)*x)*(a + c*x^2)^p*(d + e*x
+ f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f
^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^
q*Simp[p*(-(a*e))*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p +
q + 1)*(a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3
))) + (2*p*(c*d - a*f)*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) +
(p + q + 1)*(C*e*f*p*(-4*a*c)))*x + (p*(c*e)*(C*(c*e)*(q + 1) - c*(C*e - B*
f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(-4*a*c) - c^2*(C*(e^2 - 4*d*f)*
(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]
/; FreeQ[{a, c, d, e, f, A, B, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0
] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q
, 0]
```

Rule 1080

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
```

```
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}}{x^2\left(d+ex+fx^2\right)}\,dx = \int \left(\frac{\left(a+cx^2\right)^{3/2}}{dx^2} - \frac{e\left(a+cx^2\right)^{3/2}}{d^2x} + \frac{\left(e^2-df+efx\right)\left(a+cx^2\right)^{3/2}}{d^2\left(d+ex+fx^2\right)}\right)dx$$

$$= \frac{\int \frac{\left(e^2-df+efx\right)\left(a+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx}{d^2} + \frac{\int \frac{\left(a+cx^2\right)^{3/2}}{x^2}\,dx}{d} - \frac{e\int \frac{\left(a+cx^2\right)^{3/2}}{x}\,dx}{d^2}$$

$$= \frac{e\left(a+cx^2\right)^{3/2}}{3d^2} - \frac{\left(a+cx^2\right)^{3/2}}{dx} + \frac{(3c)\int \sqrt{a+cx^2}\,dx}{d} - \frac{e\,\text{Subst}\left(\int \frac{(a+cx)^{3/2}}{x}\,dx, x, x^2\right)}{2d^2} + \frac{\int}{}$$

$$= \frac{3cx\sqrt{a+cx^2}}{2d} + \frac{(2ae-cdx)\sqrt{a+cx^2}}{2d^2} - \frac{\left(a+cx^2\right)^{3/2}}{dx} + \frac{(3ac)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{2d} - \frac{(ae)\,\text{Subst}}{}$$

$$= -\frac{ae\sqrt{a+cx^2}}{d^2} + \frac{3cx\sqrt{a+cx^2}}{2d} + \frac{(2ae-cdx)\sqrt{a+cx^2}}{2d^2} - \frac{\left(a+cx^2\right)^{3/2}}{dx} + \frac{(3ac)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{ae\sqrt{a+cx^2}}{d^2} + \frac{3cx\sqrt{a+cx^2}}{2d} + \frac{(2ae-cdx)\sqrt{a+cx^2}}{2d^2} - \frac{\left(a+cx^2\right)^{3/2}}{dx} + \frac{3a\sqrt{c}\,\tanh^{-1}\left(\right.}{2d}$$

$$= -\frac{ae\sqrt{a+cx^2}}{d^2} + \frac{3cx\sqrt{a+cx^2}}{2d} + \frac{(2ae-cdx)\sqrt{a+cx^2}}{2d^2} - \frac{\left(a+cx^2\right)^{3/2}}{dx} + \frac{3a\sqrt{c}\,\tanh^{-1}\left(\right.}{2d}$$

$$= -\frac{ae\sqrt{a+cx^2}}{d^2} + \frac{3cx\sqrt{a+cx^2}}{2d} + \frac{(2ae-cdx)\sqrt{a+cx^2}}{2d^2} - \frac{\left(a+cx^2\right)^{3/2}}{dx} + \frac{3a\sqrt{c}\,\tanh^{-1}\left(\right.}{2d}$$

**Mathematica [C]** time = 4.16, size = 885, normalized size = 1.47

$$-\frac{x\left(2\sqrt{a}\sqrt{c}df\sqrt{e^2-4df}\sqrt{cx^2+a}\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)+\sqrt{\frac{cx^2}{a}+1}\left(-2c\sqrt{4af^2+2c\left(e^2+\sqrt{e^2-4df}e-2df\right)}\tanh^{-1}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{4af^2+2c\left(e^2+\sqrt{e^2-4df}e-2df\right)}\sqrt{cx^2+a}}\right)d^2-4\sqrt{c}(cd-af)\sqrt{e^2-4df}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{cx^2+a}}\right)d+2af\sqrt{4af^2+2c\left(e^2+\sqrt{e^2-4df}e-2df\right)}\tanh^{-1}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{4af^2+2c\left(e^2+\sqrt{e^2-4df}e-2df\right)}\sqrt{cx^2+a}}\right)d+2cf\sqrt{e^2-4df}x\sqrt{cx^2+a}d+\left(2cd^2+a\left(e^2+\sqrt{e^2-4df}e-2df\right)\right)\sqrt{4af^2-2c\left(-e^2+\sqrt{e^2-4df}e+2df\right)}\tanh^{-1}\left(\frac{2af+c\left(\sqrt{e^2-4df}-e\right)x}{\sqrt{4af^2-2c\left(-e^2+\sqrt{e^2-4df}e+2df\right)}\sqrt{cx^2+a}}\right)+\sqrt{2}ae\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2+\sqrt{e^2-4df}e-2df\right)}\tanh^{-1}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{4af^2+2c\left(e^2+\sqrt{e^2-4df}e-2df\right)}\sqrt{cx^2+a}}\right)-ae^2\sqrt{4af^2+2c\left(e^2+\sqrt{e^2-4df}e-2df\right)}\tanh^{-1}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{4af^2+2c\left(e^2+\sqrt{e^2-4df}e-2df\right)}\sqrt{cx^2+a}}\right)-4a^{3/2}ef\sqrt{e^2-4df}\tanh^{-1}\left(\frac{\sqrt{cx^2+a}}{\sqrt{a}}\right)\right)\right)-4adf\sqrt{e^2-4df}\sqrt{cx^2+a}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{cx^2}{a}\right)}{4d^2f\sqrt{e^2-4df}x\sqrt{\frac{cx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(3/2)/(x^2*(d + e*x + f*x^2)),x]
```

```
[Out] (-(x*(2*Sqrt[a]*Sqrt[c]*d*f*Sqrt[e^2 - 4*d*f]*Sqrt[a + c*x^2]*ArcSinh[(Sqrt
[c]*x)/Sqrt[a]] + Sqrt[1 + (c*x^2)/a]*(2*c*d*f*Sqrt[e^2 - 4*d*f]*x*Sqrt[a +
 c*x^2] - 4*Sqrt[c]*d*(c*d - a*f)*Sqrt[e^2 - 4*d*f]*ArcTanh[(Sqrt[c]*x)/Sqr
t[a + c*x^2]] + (2*c*d^2 + a*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*Sqrt[4*a*
f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f + c*(-e + Sq
rt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]
)]*Sqrt[a + c*x^2])] + Sqrt[2]*a*e*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2
- 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*
x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]
)] - 2*c*d^2*Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTan
h[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f +
e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])] - a*e^2*Sqrt[4*a*f^2 + 2*c*(e^2 - 2
*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/
(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]
+ 2*a*d*f*Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(
2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*S
qrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])] - 4*a^(3/2)*e*f*Sqrt[e^2 - 4*d*f]*ArcT
anh[Sqrt[a + c*x^2]/Sqrt[a]]))) - 4*a*d*f*Sqrt[e^2 - 4*d*f]*Sqrt[a + c*x^2]
*Hypergeometric2F1[-3/2, -1/2, 1/2, -((c*x^2)/a)])/(4*d^2*f*Sqrt[e^2 - 4*d*
f]*x*Sqrt[1 + (c*x^2)/a])
```

**IntegrateAlgebraic [C]** time = 0.74, size = 502, normalized size = 0.83

$$-\frac{\text{RootSum}\left[\#1^4f-2\#1^3\sqrt{c}e-2\#1^2af+4\#1^2cd+2\#1a\sqrt{c}e+a^2f\&,\frac{-\#1^2a^2ef^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+\#1^2c^2d^2e\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+a^3ef^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)-2\#1a^2\sqrt{c}df^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+2\#1a^2\sqrt{c}e^2f\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)-2\#1c^{5/2}d^3\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+4\#1ac^{3/2}d^2f\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)-ac^2d^2e\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)}{2\#1^3f-3\#1^2\sqrt{c}e-2\#1af+4\#1cd+a\sqrt{c}e}\&\right]}{d^2f}-\frac{2a^{3/2}e\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}-\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}-\frac{c^{3/2}\log\left(\sqrt{a+cx^2}-\sqrt{c}x\right)}{f}-\frac{a\sqrt{a+cx^2}}{dx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + c*x^2)^(3/2)/(x^2*(d + e*x + f*x^2)),x]
```

```
[Out] -((a*Sqrt[a + c*x^2])/(d*x)) - (2*a^(3/2)*e*ArcTanh[(Sqrt[c]*x)/Sqrt[a] - S
qrt[a + c*x^2]/Sqrt[a]])/d^2 - (c^(3/2)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]]
)/f - RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c
]*e*#1^3 + f*#1^4 & , (-(a*c^2*d^2*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #
1]) + a^3*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - 2*c^(5/2)*d^3*Lo
g[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + 4*a*c^(3/2)*d^2*f*Log[-(Sqrt[c]
*x) + Sqrt[a + c*x^2] - #1]*#1 + 2*a^2*Sqrt[c]*e^2*f*Log[-(Sqrt[c]*x) + Sqr
t[a + c*x^2] - #1]*#1 - 2*a^2*Sqrt[c]*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x
^2] - #1]*#1 + c^2*d^2*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 - a^
```

```
2*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d
*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]/(d^2*f)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/x^2/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$sage_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/x^2/(f*x^2+e*x+d),x, algorithm="giac")
[Out] sage2
```

**maple [B]** time = 0.02, size = 9912, normalized size = 16.41

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)/x^2/(f*x^2+e*x+d),x)
[Out] result too large to display
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2 + a\right)^{\frac{3}{2}}}{\left(fx^2 + ex + d\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/x^2/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate((c*x^2 + a)^(3/2)/((f*x^2 + e*x + d)*x^2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+a\right)^{3/2}}{x^2\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(3/2)/(x^2*(d + e*x + f*x^2)),x)
```

```
[Out] int((a + c*x^2)^(3/2)/(x^2*(d + e*x + f*x^2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + cx^2\right)^{\frac{3}{2}}}{x^2 \left(d + ex + fx^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)/x**2/(f*x**2+e*x+d),x)
```

```
[Out] Integral((a + c*x**2)**(3/2)/(x**2*(d + e*x + f*x**2)), x)
```

## 3.63 $\int \frac{\left(a+cx^2\right)^{3/2}}{x^3\left(d+ex+fx^2\right)} \, dx$

**Optimal**. Leaf size=668

$$-\frac{a^{3/2}\left(e^2-df\right)\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3}+\frac{\left(a^2f\left(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3\right)+2acd^2f\left(\sqrt{e^2-4df}+e\right)+c\cdots}{\sqrt{2}\,d^3\sqrt{e^2-4df}\,\sqrt{2af^2+c\left(-e\sqrt{e\cdots}\right.}}$$

**Rubi [A]** time = 3.46, antiderivative size = 668, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 15, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {6728, 266, 47, 50, 63, 208, 277, 195, 217, 206, 1020, 1068, 1080, 1034, 725}

$$\frac{\left(a^2f\left(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3\right)+2acd^2f\left(\sqrt{e^2-4df}+e\right)+c^2d^3\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\,d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(a^2f\left(-e^2\sqrt{e^2-4df}+df\sqrt{e^2-4df}-3def+e^3\right)+2acd^2f\left(e-\sqrt{e^2-4df}\right)+c^2d^3\left(\sqrt{e^2-4df}+e\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\,d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{a^{3/2}\left(e^2-df\right)\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3}+\frac{a\sqrt{a+cx^2}\left(e^2-df\right)}{d^3}-\frac{\sqrt{a+cx^2}\left(2\left(a\left(e^2-df\right)+cd^2\right)-cdex\right)}{2d^3}+\frac{e\left(a+cx^2\right)^{3/2}}{d^2x}-\frac{3cex\sqrt{a+cx^2}}{2d^2}-\frac{\left(a+cx^2\right)^{3/2}}{2dx^2}+\frac{3c\sqrt{a+cx^2}}{2d}-\frac{3\sqrt{a}\,c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(3/2)/(x^3*(d + e*x + f*x^2)),x]
```

```
[Out] (3*c*Sqrt[a + c*x^2])/(2*d) + (a*(e^2 - d*f)*Sqrt[a + c*x^2])/d^3 - (3*c*e*
x*Sqrt[a + c*x^2])/(2*d^2) - ((2*(c*d^2 + a*(e^2 - d*f)) - c*d*e*x)*Sqrt[a
+ c*x^2])/(2*d^3) - (a + c*x^2)^(3/2)/(2*d*x^2) + (e*(a + c*x^2)^(3/2))/(d^
2*x) + ((c^2*d^3*(e - Sqrt[e^2 - 4*d*f]) + 2*a*c*d^2*f*(e + Sqrt[e^2 - 4*d*
f]) + a^2*f*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f])
)*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(
e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^3*Sqrt[e^
2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - ((2*a*c
*d^2*f*(e - Sqrt[e^2 - 4*d*f]) + c^2*d^3*(e + Sqrt[e^2 - 4*d*f]) + a^2*f*(e
^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*d*f] + d*f*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a
*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f +
e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqr
t[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) - (3*Sqrt[a]*c*ArcTanh[
Sqrt[a + c*x^2]/Sqrt[a]])/(2*d) - (a^(3/2)*(e^2 - d*f)*ArcTanh[Sqrt[a + c*x
^2]/Sqrt[a]])/d^3
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
```

```
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1020

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
eQ[p + q + 1, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1068

```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) +
(e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*p + 2*q + 3)
+ C*(-(c*e*(2*p + q + 2))) + 2*c*C*f*(p + q + 1)*x)*(a + c*x^2)^p*(d + e*x
+ f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f
^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^
q*Simp[p*(-(a*e))*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p +
q + 1)*(a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3
```

```
))) + (2*p*(c*d - a*f)*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) +
(p + q + 1)*(C*e*f*p*(-4*a*c)))*x + (p*(c*e)*(C*(c*e)*(q + 1) - c*(C*e - B*
f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(-4*a*c) - c^2*(C*(e^2 - 4*d*f)*
(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]
/; FreeQ[{a, c, d, e, f, A, B, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0
] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q
, 0]
```

Rule 1080

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}}{x^3\left(d+ex+fx^2\right)}\,dx = \int\left(\frac{\left(a+cx^2\right)^{3/2}}{dx^3} - \frac{e\left(a+cx^2\right)^{3/2}}{d^2x^2} + \frac{\left(e^2-df\right)\left(a+cx^2\right)^{3/2}}{d^3x} + \frac{\left(-e\left(e^2-2df\right)-f\left(e^2-d\right.\right.}{d^3\left(d+ex+f\right.}\right.$$

$$= \frac{\int \frac{\left(-e\left(e^2-2df\right)-f\left(e^2-df\right)x\right)\left(a+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx}{d^3} + \frac{\int \frac{\left(a+cx^2\right)^{3/2}}{x^3}\,dx}{d} - \frac{e\int \frac{\left(a+cx^2\right)^{3/2}}{x^2}\,dx}{d^2} + \frac{\left(e^2-df\right)\int \frac{(a+}{}}{d^3}$$

$$= -\frac{\left(e^2-df\right)\left(a+cx^2\right)^{3/2}}{3d^3} + \frac{e\left(a+cx^2\right)^{3/2}}{d^2x} + \frac{\text{Subst}\left(\int \frac{(a+cx)^{3/2}}{x^2}\,dx, x, x^2\right)}{2d} - \frac{(3ce)\int\sqrt{a+}}{d^2}$$

$$= -\frac{3cex\sqrt{a+cx^2}}{2d^2} - \frac{\left(2\left(cd^2+a\left(e^2-df\right)\right)-cdex\right)\sqrt{a+cx^2}}{2d^3} - \frac{\left(a+cx^2\right)^{3/2}}{2dx^2} + \frac{e\left(a+cx^2\right.}{d^2x}$$

$$= \frac{3c\sqrt{a+cx^2}}{2d} + \frac{a\left(e^2-df\right)\sqrt{a+cx^2}}{d^3} - \frac{3cex\sqrt{a+cx^2}}{2d^2} - \frac{\left(2\left(cd^2+a\left(e^2-df\right)\right)-cdex\right)}{2d^3}$$

$$= \frac{3c\sqrt{a+cx^2}}{2d} + \frac{a\left(e^2-df\right)\sqrt{a+cx^2}}{d^3} - \frac{3cex\sqrt{a+cx^2}}{2d^2} - \frac{\left(2\left(cd^2+a\left(e^2-df\right)\right)-cdex\right)}{2d^3}$$

$$= \frac{3c\sqrt{a+cx^2}}{2d} + \frac{a\left(e^2-df\right)\sqrt{a+cx^2}}{d^3} - \frac{3cex\sqrt{a+cx^2}}{2d^2} - \frac{\left(2\left(cd^2+a\left(e^2-df\right)\right)-cdex\right)}{2d^3}$$

$$= \frac{3c\sqrt{a+cx^2}}{2d} + \frac{a\left(e^2-df\right)\sqrt{a+cx^2}}{d^3} - \frac{3cex\sqrt{a+cx^2}}{2d^2} - \frac{\left(2\left(cd^2+a\left(e^2-df\right)\right)-cdex\right)}{2d^3}$$

**<u>Mathematica</u> [C]** time = 3.22, size = 904, normalized size = 1.35

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(3/2)/(x^3*(d + e*x + f*x^2)),x]
```

```
[Out] (-5*(e^2 - d*f - (e*(e^2 - 3*d*f))/Sqrt[e^2 - 4*d*f])*(a + c*x^2)^(3/2) - 5
*(e^2 - d*f + (e*(e^2 - 3*d*f))/Sqrt[e^2 - 4*d*f])*(a + c*x^2)^(3/2) + (15*
(-e^2 + d*f - (e*(e^2 - 3*d*f))/Sqrt[e^2 - 4*d*f])*((2*Sqrt[c]*(-e + Sqrt[e
```

```
^2 - 4*d*f])*Sqrt[a + c*x^2]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/a] + Sqrt[a]*ArcSi
nh[(Sqrt[c]*x)/Sqrt[a]]))/Sqrt[1 + (c*x^2)/a] + (2*(2*a*f^2 + c*(e^2 - 2*d*
f - e*Sqrt[e^2 - 4*d*f]))*(2*f*Sqrt[a + c*x^2] + Sqrt[c]*(-e + Sqrt[e^2 - 4
*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] - Sqrt[2*c*e^2 - 4*c*d*f + 4*a*
f^2 - 2*c*e*Sqrt[e^2 - 4*d*f]]*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*
x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2
])]))/f^2))/(8*f) - (15*(-e^2 + d*f + (e*(e^2 - 3*d*f))/Sqrt[e^2 - 4*d*f])*
((2*Sqrt[c]*(e + Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2]*(Sqrt[c]*x*Sqrt[1 + (c*
x^2)/a] + Sqrt[a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]]))/Sqrt[1 + (c*x^2)/a] + (2*(
2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*(-2*f*Sqrt[a + c*x^2] + Sq
rt[c]*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] + Sqrt[4
*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + S
qrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]
)]*Sqrt[a + c*x^2])]))/f^2))/(8*f) + 10*(e^2 - d*f)*(Sqrt[a + c*x^2]*(4*a +
 c*x^2) - 3*a^(3/2)*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]) + (30*a*d*e*Sqrt[a +
c*x^2]*Hypergeometric2F1[-3/2, -1/2, 1/2, -((c*x^2)/a)])/(x*Sqrt[1 + (c*x^2
)/a]) + (6*c*d^2*(a + c*x^2)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 + (c*x^
2)/a])/a^2)/(30*d^3)
```

**IntegrateAlgebraic [C]** time = 0.95, size = 624, normalized size = 0.93

$$-\frac{\operatorname{RootSum}\left[\#1^4 f-2\#1^3\sqrt{c}\,e-2\#1^2 af+4\#1^2 cd+2\#1 a\sqrt{c}\,e+a^2 f\&,\frac{-\#1^2a^2df^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)+\#1^2a^2e^2f\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)-\#1^2c^2d^3\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)+2\#1^2acd^2f\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)+a^3df^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)-a^3e^2f\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)-2a^2cd^2f\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)+4\#1a^2\sqrt{c}\,def\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)-2\#1a^2\sqrt{c}\,e^3\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)-4\#1ac^{3/2}d^2e\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)+ac^2d^3\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)}{2\#1^3f-3\#1^2\sqrt{c}\,e-2\#1af+4\#1cd+a\sqrt{c}\,e}\&\right]}{d^3}+\frac{2a^{3/2}\left(df-e^2\right)\tanh^{-1}\left(\frac{\sqrt{a+cx^2}-\sqrt{c}\,x}{\sqrt{a}}\right)}{d^3}-\frac{a\sqrt{a+cx^2}\,(d-2ex)}{2d^2x^2}+\frac{3\sqrt{a}\,c\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}-\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + c*x^2)^(3/2)/(x^3*(d + e*x + f*x^2)),x]
```

```
[Out] -1/2*(a*(d - 2*e*x)*Sqrt[a + c*x^2])/(d^2*x^2) + (2*a^(3/2)*(-e^2 + d*f)*Ar
cTanh[(-(Sqrt[c]*x) + Sqrt[a + c*x^2])/Sqrt[a]])/d^3 + (3*Sqrt[a]*c*ArcTanh
[(Sqrt[c]*x)/Sqrt[a] - Sqrt[a + c*x^2]/Sqrt[a]])/d - RootSum[a^2*f + 2*a*Sq
rt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (a*c^2
*d^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - 2*a^2*c*d^2*f*Log[-(Sqrt[c]
*x) + Sqrt[a + c*x^2] - #1] - a^3*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]
 - #1] + a^3*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - 4*a*c^(3/2)*d^
2*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - 2*a^2*Sqrt[c]*e^3*Log[-(S
qrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + 4*a^2*Sqrt[c]*d*e*f*Log[-(Sqrt[c]*x)
 + Sqrt[a + c*x^2] - #1]*#1 - c^2*d^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] -
#1]*#1^2 + 2*a*c*d^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 + a^2*
e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 - a^2*d*f^2*Log[-(Sqrt[
c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*
Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]/d^3
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/x^3/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$sage_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/x^3/(f*x^2+e*x+d),x, algorithm="giac")
[Out] sage2
```

**maple [B]** time = 0.02, size = 10298, normalized size = 15.42

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)/x^3/(f*x^2+e*x+d),x)
[Out] result too large to display
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2+a\right)^{\frac{3}{2}}}{\left(fx^2+ex+d\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/x^3/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate((c*x^2 + a)^(3/2)/((f*x^2 + e*x + d)*x^3), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+a\right)^{3/2}}{x^3\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(3/2)/(x^3*(d + e*x + f*x^2)),x)
[Out] int((a + c*x^2)^(3/2)/(x^3*(d + e*x + f*x^2)), x)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)/x**3/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

## 3.64 $\int \frac{x^3}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx$

**Optimal**. Leaf size=380

$$-\frac{\left(2def-\left(e^2-df\right)\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(2def-\left(e^2-df\right)\left(\sqrt{e^2-4df}\right.\right.}{\sqrt{2}f^2\sqrt{e^2-4df}}$$

**Rubi [A]** time = 1.17, antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6728, 217, 206, 261, 1034, 725}

$$-\frac{\left(2def-\left(e^2-df\right)\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(2def-\left(e^2-df\right)\left(\sqrt{e^2-4df}+e\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{e\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}f^2}+\frac{\sqrt{a+cx^2}}{cf}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] Sqrt[a + c*x^2]/(c*f) - (e*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(Sqrt[c]*f
^2) - ((2*d*e*f - (e^2 - d*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(
e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e
^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^
2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + ((2*d*e*f - (e^2 - d*f)*(e +
Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*
Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(S
qrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4
*d*f])])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx = \int \left(-\frac{e}{f^2\sqrt{a+cx^2}}+\frac{x}{f\sqrt{a+cx^2}}+\frac{de+\left(e^2-df\right)x}{f^2\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\right)dx$$

$$= \frac{\int \frac{de+\left(e^2-df\right)x}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{f^2} - \frac{e\int \frac{1}{\sqrt{a+cx^2}}\,dx}{f^2} + \frac{\int \frac{x}{\sqrt{a+cx^2}}\,dx}{f}$$

$$= \frac{\sqrt{a+cx^2}}{cf} - \frac{e\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{f^2} + \frac{\left(2def-\left(e^2-df\right)\left(e-\sqrt{e^2-4\right.\right.}{f^2\sqrt{e^2}$$

$$= \frac{\sqrt{a+cx^2}}{cf} - \frac{e\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,f^2} - \frac{\left(2def-\left(e^2-df\right)\left(e-\sqrt{e^2-4df}\right)\right)\text{Subst}\left(\right.}{f^2\sqrt{e}$$

$$= \frac{\sqrt{a+cx^2}}{cf} - \frac{e\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,f^2} - \frac{\left(2def-\left(e^2-df\right)\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}}{\sqrt{2}\,f^2\sqrt{e^2-4df}\,\sqrt{2af^2+c\left(e^2\right.}}$$

**Mathematica [A]** time = 1.33, size = 378, normalized size = 0.99

$$-\frac{\frac{\sqrt{2}\left(\left(e^2-df\right)\left(\sqrt{e^2-4df}-e\right)+2def\right)\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\sqrt{2}\left(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{a+cx^2}\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{2e\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}}-\frac{2f\sqrt{a+cx^2}}{c}}{2f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -1/2*((-2*f*Sqrt[a + c*x^2])/c + (2*e*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])
/Sqrt[c] + (Sqrt[2]*(2*d*e*f + (e^2 - d*f)*(-e + Sqrt[e^2 - 4*d*f]))*ArcTan
h[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f
+ e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2
+ c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (Sqrt[2]*(e^3 - 3*d*e*f + e^2*S
qrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2
- 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[
a + c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2
 - 4*d*f])]))/f^2
```

**IntegrateAlgebraic [C]** time = 0.49, size = 315, normalized size = 0.83

$$-\frac{\text{RootSum}\left[\#1^4 f-2\#1^3\sqrt{c}e-2\#1^2 af+4\#1^2 cd+2\#1 a\sqrt{c}e+a^2 f\&,\frac{\#1^2 df\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+\#1^2 e^2\left(-\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)\right)+2\#1\sqrt{c}de\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)-adf\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+ae^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)}{2\#1^3 f-3\#1^2\sqrt{c}e-2\#1 af+4\#1 cd+a\sqrt{c}e}\&\right]}{f^2}+\frac{e\log\left(\sqrt{a+cx^2}-\sqrt{c}x\right)}{\sqrt{c}f^2}+\frac{\sqrt{a+cx^2}}{cf}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] Sqrt[a + c*x^2]/(c*f) + (e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(Sqrt[c]*f^
2) - RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]
*e*#1^3 + f*#1^4 & , (a*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - a*d*
f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + 2*Sqrt[c]*d*e*Log[-(Sqrt[c]*x)
 + Sqrt[a + c*x^2] - #1]*#1 - e^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*
#1^2 + d*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*
c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]/f^2
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 11.34sym2poly/r2sym(const g
en & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

**maple [B]** time = 0.03, size = 2397, normalized size = 6.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] (c*x^2+a)^(1/2)/c/f-1/f^2*e*ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)+1/2/f^2*2
^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*
d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x
+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2
-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(
-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)
*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*d
-1/2/f^3*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)
*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(
1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*
c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1
/2))/f))*e^2+3/2/f^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f
+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2
*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1
/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4
*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(
1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*d*e-1/2/f^3/(-4*d*f+e^2)^(1/2)*2^(1
/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f
+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*
c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*
d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*e^3+
1/2/f^2*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)
```

```
*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^
(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f
)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-
4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+
e^2)^(1/2))/f))*d-1/2/f^3*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f
+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-
c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((
-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4
*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2
)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(
x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*e^2-3/2/f^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((
-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2
)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-
e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*
d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d
*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*d*
e+1/2/f^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*
d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f
^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)
*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+
(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+
e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)
)/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*e^3
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(x^3/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a + cx^2}\left(d + ex + fx^2\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

## 3.65 $\int \frac{x^2}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx$

**Optimal**. Leaf size=344

$$-\frac{\left(-e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(2df-e\left(\sqrt{e^2-4df}+e\right)\right)\tanh^{-1}\left(\frac{}{\sqrt{2}\cdots}\right.}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\cdots\right.}}$$

**Rubi** [A] time = 0.54, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1081, 217, 206, 1034, 725}

$$-\frac{\left(-e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(2df-e\left(\sqrt{e^2-4df}+e\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}f}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]/(Sqrt[c]*f) - ((e^2 - 2*d*f - e*Sqrt[e
^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*
a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*
f*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])])
- ((2*d*f - e*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4
*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sq
rt[a + c*x^2])])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f
 + e*Sqrt[e^2 - 4*d*f])])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1081

```
Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2], x], x] + Dis
t[1/c, Int[(A*c - a*C - b*C*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]), x], x]
/; FreeQ[{a, b, c, d, f, A, C}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx = \frac{\int \frac{1}{\sqrt{a+cx^2}}\,dx}{f} + \frac{\int \frac{-d-ex}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{f} + \frac{\left(e^2-2df-e\sqrt{e^2-4df}\right)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a}}}{f\sqrt{e^2-4df}}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,f} - \frac{\left(e^2-2df-e\sqrt{e^2-4df}\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e-\sqrt{e^2-4df}\right)^2-x^2}\,dx\right.}{f\sqrt{e^2-4df}}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,f} - \frac{\left(e^2-2df-e\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4}\right.}}\right.}{\sqrt{2}\,f\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4d}\right.}}$$

**Mathematica [A]** time = 0.70, size = 334, normalized size = 0.97

$$\frac{\frac{\sqrt{2}\left(e\sqrt{e^2-4df}+2df-e^2\right)\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\sqrt{2}\left(e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{a+cx^2}\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ((2*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/Sqrt[c] + (Sqrt[2]*(-e^2 + 2*d*f
+ e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt
[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sq
rt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (S
qrt[2]*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2
 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt
[a + c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^
2 - 4*d*f])]))/(2*f)
```

**IntegrateAlgebraic [C]** time = 0.43, size = 229, normalized size = 0.67

$$\frac{\text{RootSum}\left[\#1^4 f - 2\#1^3\sqrt{c}\,e - 2\#1^2 a f + 4\#1^2 c d + 2\#1 a\sqrt{c}\,e + a^2 f\&, \frac{\#1^2(-e)\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)+2\#1\sqrt{c}\,d\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)+ae\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)}{2\#1^3 f-3\#1^2\sqrt{c}\,e-2\#1 a f+4\#1 c d+a\sqrt{c}\,e}\&\right]}{f} - \frac{\log\left(\sqrt{a+cx^2}-\sqrt{c}\,x\right)}{\sqrt{c}\,f}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -(Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]]/(Sqrt[c]*f)) + RootSum[a^2*f + 2*a*Sq
rt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (a*e*L
og[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + 2*Sqrt[c]*d*Log[-(Sqrt[c]*x) + Sq
rt[a + c*x^2] - #1]*#1 - e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(
a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]/f
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 6.03Error index.cc index_gc
d Error: Bad Argument Value
```

**maple [B]** time = 0.02, size = 1796, normalized size = 5.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/f*ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)+1/2/f^2*2^(1/2)/((2*a*f^2-2*c*d*f
+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e+(-4*d*f+e^2)^(1/2))*(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c
*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1
/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(x+1/2
*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*
c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*e-1/(-4*d*f+e^2)^(1/2)/f
*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e
+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+
c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d
*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+
(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f
+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)
)*d+1/2/(-4*d*f+e^2)^(1/2)/f^2*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)
^(1/2)*c*e)/f^2)^(1/2)*ln((-(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(
1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)
*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e+(-
4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1
/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x
+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*e^2+1/2/f^2*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2
-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+
(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f
^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(
4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e
+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e
)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*e+1/(-4*d*f+e^2)^(1/2)/f*2
^(1/2)/((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e-(
-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c
*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*
f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e-
(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*
f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/
f))*d-1/2/(-4*d*f+e^2)^(1/2)/f^2*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^
2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2
)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1
/2)*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-
e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^
2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2
```

```
))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*e^2
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(x^2/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a+cx^2}\,\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

## 3.66 $\int \frac{x}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx$

__Optimal__. Leaf size=294

$$\frac{\left(e-\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}\right.}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-\right.}}}\right.}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2d\right.}}$$

__Rubi__ [A] time = 0.24, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1034, 725, 206}

$$\frac{\left(e-\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ((e - Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqr
t[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])
])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 -
4*d*f])]) - ((e + Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d
*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt
[a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e
*Sqrt[e^2 - 4*d*f])])
```

__Rule 206__

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

__Rule 725__

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

__Rule 1034__

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
```

```
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx = -\left(\left(-1-\frac{e}{\sqrt{e^2-4df}}\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx\right)+\left(1-\frac{e}{\sqrt{e^2-4d}}\right.$$

$$= \left(-1+\frac{e}{\sqrt{e^2-4df}}\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e-\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{2af-c\left(e\right.}{\sqrt{a}}\right.$$

$$= -\frac{\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\tanh^{-1}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} - \frac{\left(1+\frac{e}{\sqrt{e^2-4df}}\right)\tan}{\sqrt{2}\sqrt{2a}}$$

**Mathematica [A]** time = 0.32, size = 275, normalized size = 0.94

$$\frac{\sqrt{2}\left(-\frac{\left(\sqrt{e^2-4df}-e\right)\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)}{2\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{a+cx^2}\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{2\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] (Sqrt[2]*(-1/2*((-e + Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2
- 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt
[a + c*x^2])])/Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])] - ((e
+ Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*
a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(2*Sqrt
[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])))/Sqrt[e^2 - 4*d*f]
```

**IntegrateAlgebraic [C]** time = 0.38, size = 156, normalized size = 0.53

$$\text{RootSum}\left[\#1^4 f-2\#1^3\sqrt{c}\,e-2\#1^2 af+4\#1^2 cd+2\#1 a\sqrt{c}\,e+a^2 f\&, \frac{\#1^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)-a\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)}{2\#1^3 f-3\#1^2\sqrt{c}\,e-2\#1 af+4\#1 cd+a\sqrt{c}\,e}\&\right]$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1
^3 + f*#1^4 & , (-(a*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]) + Log[-(Sqrt
[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3
*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]
```

**<u>fricas</u> [B]** time = 1.65, size = 5085, normalized size = 17.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f + (c^2*d^2*e^2 + a*c*e^4 - 4*a
^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(
a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^
3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*
d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 +
2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a
^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f))*log((4*a*c*d^2*e*x - 2*a^2*d*
e^2 + sqrt(2)*(a^2*e^4 - 4*a^2*d*e^2*f - (2*c^3*d^4*e^2 + 3*a*c^2*d^2*e^4 +
 a^2*c*e^6 + 8*a^3*d^2*f^4 - 6*(4*a^2*c*d^3 + a^3*d*e^2)*f^3 + (24*a*c^2*d^
4 + 22*a^2*c*d^2*e^2 + a^3*e^4)*f^2 - 2*(4*c^3*d^5 + 9*a*c^2*d^3*e^2 + 4*a^
2*c*d*e^4)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4
*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2
)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 +
 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))*sqrt(c*x^2 + a)*sqrt((2*c*d^2 + a*
e^2 - 2*a*d*f + (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2
)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*
d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*
a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*
c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*
e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*
a*c*d*e^2)*f)) + 2*(a*c^2*d^3*e^2 + a^2*c*d*e^4 - 4*a^3*d^2*f^3 + (8*a^2*c*
d^3 + a^3*d*e^2)*f^2 - 2*(2*a*c^2*d^4 + 3*a^2*c*d^2*e^2)*f)*sqrt(a^2*e^2/(c
^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 +
a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a
^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*
d*e^4)*f)))/x) + 1/4*sqrt(2)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f + (c^2*d^2*e^2
 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c
*d*e^2)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^
4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f
^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*
```

a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f))*log((4*a*c*d^2*e*x - 2*a^2*d*e^2 - sqrt(2)*(a^2*e^4 - 4*a^2*d*e^2*f - (2*c^3*d^4*e^2 + 3*a*c^2*d^2*e^4 + a^2*c*e^6 + 8*a^3*d^2*f^4 - 6*(4*a^2*c*d^3 + a^3*d*e^2)*f^3 + (24*a*c^2*d^4 + 22*a^2*c*d^2*e^2 + a^3*e^4)*f^2 - 2*(4*c^3*d^5 + 9*a*c^2*d^3*e^2 + 4*a^2*c*d*e^4)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))*sqrt(c*x^2 + a)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f + (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)) + 2*(a*c^2*d^3*e^2 + a^2*c*d*e^4 - 4*a^3*d^2*f^3 + (8*a^2*c*d^3 + a^3*d*e^2)*f^2 - 2*(2*a*c^2*d^4 + 3*a^2*c*d^2*e^2)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/x) - 1/4*sqrt(2)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f - (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f))*log((4*a*c*d^2*e*x - 2*a^2*d*e^2 + sqrt(2)*(a^2*e^4 - 4*a^2*d*e^2*f + (2*c^3*d^4*e^2 + 3*a*c^2*d^2*e^4 + a^2*c*e^6 + 8*a^3*d^2*f^4 - 6*(4*a^2*c*d^3 + a^3*d*e^2)*f^3 + (24*a*c^2*d^4 + 22*a^2*c*d^2*e^2 + a^3*e^4)*f^2 - 2*(4*c^3*d^5 + 9*a*c^2*d^3*e^2 + 4*a^2*c*d*e^4)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))*sqrt(c*x^2 + a)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f - (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)) - 2*(a*c^2*d^3*e^2 + a^2*c*d*e^4 - 4*a^3*d^2*f^3 + (8*a^2*c*d^3 + a^3*d*e^2)*f^2 - 2*(2*a*c^2*d^4 + 3*a^2*c*d^2*e^2)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^

```
5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/x) + 1/4*sqrt(2)*sqrt((2*c*d^2
+ a*e^2 - 2*a*d*f - (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2
*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*
c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12
*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 +
a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*
d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3
+ 3*a*c*d*e^2)*f))*log((4*a*c*d^2*e*x - 2*a^2*d*e^2 - sqrt(2)*(a^2*e^4 - 4*
a^2*d*e^2*f + (2*c^3*d^4*e^2 + 3*a*c^2*d^2*e^4 + a^2*c*e^6 + 8*a^3*d^2*f^4
- 6*(4*a^2*c*d^3 + a^3*d*e^2)*f^3 + (24*a*c^2*d^4 + 22*a^2*c*d^2*e^2 + a^3*
e^4)*f^2 - 2*(4*c^3*d^5 + 9*a*c^2*d^3*e^2 + 4*a^2*c*d*e^4)*f)*sqrt(a^2*e^2/
(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2
+ a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11
*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^
2*d*e^4)*f)))*sqrt(c*x^2 + a)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f - (c^2*d^2*e^
2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*
c*d*e^2)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a
^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*
f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3
*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3
 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)) - 2*(a*c^2*d
^3*e^2 + a^2*c*d*e^4 - 4*a^3*d^2*f^3 + (8*a^2*c*d^3 + a^3*d*e^2)*f^2 - 2*(2
*a*c^2*d^4 + 3*a^2*c*d^2*e^2)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^
4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^
2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)
*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/x)
```

**giac** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 1.77index.cc index_m operat
or + Error: Bad Argument Value
```

**maple** [**B**] time = 0.01, size = 1172, normalized size = 3.99

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] -1/2/(-4*d*f+e^2)^(1/2)/f*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2
)*c*e)/f^2)^(1/2)*ln((-(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))
/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*
a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f
+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/
f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+1/2*
(e+(-4*d*f+e^2)^(1/2))/f))*e-1/2/f*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+
e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^
2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(
1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(
e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2
)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)
)/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))-1/2/f*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2-(
-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-
4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2
+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*
(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(
-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/
f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))+1/2/(-4*d*f+e^2)^(1/2)/f*2^(
1/2)/((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e-(-4
*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e
^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+
e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e-(-
4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+
c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)
)*e
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(x/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a + cx^2}\left(d + ex + fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

## 3.67 $\int \frac{1}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx$

**Optimal**. Leaf size=266

$$\frac{\sqrt{2}\, f \tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\,\sqrt{a+cx^2}\,\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\,\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\sqrt{2}\, f \tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\,\sqrt{a+cx^2}\,\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\,\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

**Rubi [A]** time = 0.15, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {985, 725, 206}

$$\frac{\sqrt{2}\, f \tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\,\sqrt{a+cx^2}\,\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\,\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\sqrt{2}\, f \tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\,\sqrt{a+cx^2}\,\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\,\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -((Sqrt[2]*f*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*
a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[e^2
 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])])) + (Sqrt[2
]*f*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c
*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[e^2 - 4*d*f]
*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 985

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f_.)*(x_)^2]), x_Sym
bol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[1/((b - q + 2*c*x)
```

```
*Sqrt[d + f*x^2]), x], x] - Dist[(2*c)/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, f}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ
[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx = \frac{(2f)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{\sqrt{e^2-4df}} - \frac{(2f)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{\sqrt{e^2-4df}}$$

$$= -\frac{(2f)\,\mathrm{Subst}\left(\int \frac{1}{4af^2+c\left(e-\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{a+cx^2}}\right)}{\sqrt{e^2-4df}} + \frac{(2f)\,\mathrm{Subst}\Big(\int}{}$$

$$= -\frac{\sqrt{2}\,f\tanh^{-1}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} + \frac{\sqrt{2}\,f\tanh^{-1}\Big(\frac{2a}{\sqrt{2}\sqrt{2af^2+c}}}{\sqrt{e^2-4df}\sqrt{2af^2+c\,(e}}$$

**Mathematica [A]** time = 0.29, size = 247, normalized size = 0.93

$$\frac{2\sqrt{2}f\left(\frac{\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{a+cx^2}\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{2\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}} - \frac{\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)}{2\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] (2*Sqrt[2]*f*(-1/2*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a
*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]/Sqrt[2*a
*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])] + ArcTanh[(2*a*f - c*(e + Sqr
t[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]
*Sqrt[a + c*x^2])]/(2*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]
)))/Sqrt[e^2 - 4*d*f]
```

**IntegrateAlgebraic [C]** time = 0.36, size = 131, normalized size = 0.49

$$-2\sqrt{c}\,\text{RootSum}\left[\#1^4f-2\#1^3\sqrt{c}\,e-2\#1^2af+4\#1^2cd+2\#1a\sqrt{c}\,e+a^2f\&,\frac{\#1\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)}{2\#1^3f-3\#1^2\sqrt{c}\,e-2\#1af+4\#1cd+a\sqrt{c}\,e}\&\right]$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -2*Sqrt[c]*RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*S
qrt[c]*e*#1^3 + f*#1^4 & , (Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1)/(a
*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]
```

**fricas [B]** time = 1.62, size = 5073, normalized size = 19.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*sqrt((c*e^2 - 2*c*d*f + 2*a*f^2 + (c^2*d^2*e^2 + a*c*e^4 - 4*a
^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(
c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^
3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*
d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 +
2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a
^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f))*log((4*c^2*d*e*f*x - 2*a*c*e^
2*f + sqrt(2)*(c^2*d*e^3 + 4*a*c*d*e*f^2 - (4*c^2*d^2*e + a*c*e^3)*f - (c^3
*d^3*e^3 + a*c^2*d*e^5 - 4*a^3*d*e*f^4 + (4*a^2*c*d^2*e + a^3*e^3)*f^3 + (4
*a*c^2*d^3*e - 5*a^2*c*d*e^3)*f^2 - (4*c^3*d^4*e + 5*a*c^2*d^2*e^3 - a^2*c*
e^5)*f)*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d
*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3
+ 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c
^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))*sqrt(c*x^2 + a)*sqrt((c*e^2 - 2*c*d*f +
2*a*f^2 + (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2
- 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^
4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^
2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)
*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 +
a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*
e^2)*f)) + 2*(4*a^3*d*f^4 - (8*a^2*c*d^2 + a^3*e^2)*f^3 + 2*(2*a*c^2*d^3 +
3*a^2*c*d*e^2)*f^2 - (a*c^2*d^2*e^2 + a^2*c*e^4)*f)*sqrt(c^2*e^2/(c^4*d^4*e
^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)
*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d
^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f
```

```
)))/x) + 1/4*sqrt(2)*sqrt((c*e^2 - 2*c*d*f + 2*a*f^2 + (c^2*d^2*e^2 + a*c*e
^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*
f)*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5
+ (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(
8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^
3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c
*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f))*log((4*c^2*d*e*f*x -
2*a*c*e^2*f - sqrt(2)*(c^2*d*e^3 + 4*a*c*d*e*f^2 - (4*c^2*d^2*e + a*c*e^3)*
f - (c^3*d^3*e^3 + a*c^2*d*e^5 - 4*a^3*d*e*f^4 + (4*a^2*c*d^2*e + a^3*e^3)*
f^3 + (4*a*c^2*d^3*e - 5*a^2*c*d*e^3)*f^2 - (4*c^3*d^4*e + 5*a*c^2*d^2*e^3
- a^2*c*e^5)*f)*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 -
 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e
^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5
 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))*sqrt(c*x^2 + a)*sqrt((c*e^2 - 2*
c*d*f + 2*a*f^2 + (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e
^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^
3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(
2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^
3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^
2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 +
3*a*c*d*e^2)*f)) + 2*(4*a^3*d*f^4 - (8*a^2*c*d^2 + a^3*e^2)*f^3 + 2*(2*a*c^
2*d^3 + 3*a^2*c*d*e^2)*f^2 - (a*c^2*d^2*e^2 + a^2*c*e^4)*f)*sqrt(c^2*e^2/(c
^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 +
a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a
^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*
d*e^4)*f)))/x) - 1/4*sqrt(2)*sqrt((c*e^2 - 2*c*d*f + 2*a*f^2 - (c^2*d^2*e^2
 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c
*d*e^2)*f)*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^
4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f
^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*
a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3
+ (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f))*log((4*c^2*d*
e*f*x - 2*a*c*e^2*f + sqrt(2)*(c^2*d*e^3 + 4*a*c*d*e*f^2 - (4*c^2*d^2*e + a
*c*e^3)*f + (c^3*d^3*e^3 + a*c^2*d*e^5 - 4*a^3*d*e*f^4 + (4*a^2*c*d^2*e + a
^3*e^3)*f^3 + (4*a*c^2*d^3*e - 5*a^2*c*d*e^3)*f^2 - (4*c^3*d^4*e + 5*a*c^2*
d^2*e^3 - a^2*c*e^5)*f)*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c
^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a
^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*
(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))*sqrt(c*x^2 + a)*sqrt((c*
e^2 - 2*c*d*f + 2*a*f^2 - (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2
 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(c^2*e^2/(c^4*d^4*e^2
+ 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^
4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*
e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))
/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^
```

```
2*d^3 + 3*a*c*d*e^2)*f)) - 2*(4*a^3*d*f^4 - (8*a^2*c*d^2 + a^3*e^2)*f^3 + 2
*(2*a*c^2*d^3 + 3*a^2*c*d*e^2)*f^2 - (a*c^2*d^2*e^2 + a^2*c*e^4)*f)*sqrt(c^
2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*
c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^
4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*
a^2*c^2*d*e^4)*f)))/x) + 1/4*sqrt(2)*sqrt((c*e^2 - 2*c*d*f + 2*a*f^2 - (c^2
*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3
 + 3*a*c*d*e^2)*f)*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^
6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*
d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*
d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^
2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f))*log((
4*c^2*d*e*f*x - 2*a*c*e^2*f - sqrt(2)*(c^2*d*e^3 + 4*a*c*d*e*f^2 - (4*c^2*d
^2*e + a*c*e^3)*f + (c^3*d^3*e^3 + a*c^2*d*e^5 - 4*a^3*d*e*f^4 + (4*a^2*c*d
^2*e + a^3*e^3)*f^3 + (4*a*c^2*d^3*e - 5*a^2*c*d*e^3)*f^2 - (4*c^3*d^4*e +
5*a*c^2*d^2*e^3 - a^2*c*e^5)*f)*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4
 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2
*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*
f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))*sqrt(c*x^2 + a)*
sqrt((c*e^2 - 2*c*d*f + 2*a*f^2 - (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8
*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(c^2*e^2/(c^4*
d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4
*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*
c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e
^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 -
 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)) - 2*(4*a^3*d*f^4 - (8*a^2*c*d^2 + a^3*e^2)
*f^3 + 2*(2*a*c^2*d^3 + 3*a^2*c*d*e^2)*f^2 - (a*c^2*d^2*e^2 + a^2*c*e^4)*f)
*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 +
(16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*
a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*
e^2 + 2*a^2*c^2*d*e^4)*f)))/x)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 1.45Error index.cc index_gc
d Error: Bad Argument Value
```

**maple** [B] time = 0.01, size = 589, normalized size = 2.21

$$-\frac{\sqrt{2}\,\ln\left(\frac{-\frac{\left(e-\sqrt{-4df+e^2}\right)\left(x-\frac{-e+\sqrt{-4df+e^2}}{2f}\right)c}{f}+\frac{2af^2-2cdf+ce^2-\sqrt{-4df+e^2}\,ce}{f^2}+\frac{\sqrt{2}\,\sqrt{\frac{2af^2-2cdf+ce^2-\sqrt{-4df+e^2}\,ce}{f^2}}\,\sqrt{4\left(x-\frac{-e+\sqrt{-4df+e^2}}{2f}\right)^2 c-\frac{4\left(e-\sqrt{-4df+e^2}\right)\left(x-\frac{-e+\sqrt{-4df+e^2}}{2f}\right)c}{f}+\frac{4af^2-4cdf+2ce^2-2\sqrt{-4df+e^2}\,ce}{f^2}}}{2}}{x-\frac{-e+\sqrt{-4df+e^2}}{2f}}\right)}{\sqrt{-4df+e^2}\,\sqrt{\frac{2af^2-2cdf+ce^2-\sqrt{-4df+e^2}\,ce}{f^2}}}+\frac{\sqrt{2}\,\ln\left(\frac{-\frac{\left(e+\sqrt{-4df+e^2}\right)\left(x+\frac{e+\sqrt{-4df+e^2}}{2f}\right)c}{f}+\frac{2af^2-2cdf+ce^2+\sqrt{-4df+e^2}\,ce}{f^2}+\frac{\sqrt{2}\,\sqrt{\frac{2af^2-2cdf+ce^2+\sqrt{-4df+e^2}\,ce}{f^2}}\,\sqrt{4\left(x+\frac{e+\sqrt{-4df+e^2}}{2f}\right)^2 c-\frac{4\left(e+\sqrt{-4df+e^2}\right)\left(x+\frac{e+\sqrt{-4df+e^2}}{2f}\right)c}{f}+\frac{4af^2-4cdf+2ce^2+2\sqrt{-4df+e^2}\,ce}{f^2}}}{2}}{x+\frac{e+\sqrt{-4df+e^2}}{2f}}\right)}{\sqrt{-4df+e^2}\,\sqrt{\frac{2af^2-2cdf+ce^2+\sqrt{-4df+e^2}\,ce}{f^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e
)/f^2)^(1/2)*ln((-(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c
/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2
-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)
^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/
f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+1/2*(e+(-
4*d*f+e^2)^(1/2))/f))-1/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2-
(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(
-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^
2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4
*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+
(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)
/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(1/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + cx^2}\,\left(d + ex + fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
[Out] Integral(1/(sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

## 3.68 $\int \frac{1}{x\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx$

**Optimal**. Leaf size=330

$$\frac{f\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{f\left(e-\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-\right.}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(e\sqrt{}}\right.}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-\right.}}$$

**Rubi** [A] time = 0.82, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6728, 266, 63, 208, 1034, 725, 206}

$$\frac{f\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{f\left(e-\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] (f*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(S
qrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2
])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^
2 - 4*d*f])]) - (f*(e - Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2
 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])
]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2
*d*f + e*Sqrt[e^2 - 4*d*f])]) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(Sqrt[a]*d
)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx &= \int \left(\frac{1}{dx\sqrt{a+cx^2}}+\frac{-e-fx}{d\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\right)dx \\
&= \frac{\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d}+\frac{\int \frac{-e-fx}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{d} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx,x,x^2\right)}{2d}-\frac{\left(f\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{d}-
\\
&= \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx,x,\sqrt{a+cx^2}\right)}{cd}+\frac{\left(f\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e+\sqrt{e^2-\right.}\right.}\right.}{d}
\\
&= \frac{f\left(1+\frac{e}{\sqrt{e^2-4df}}\right)\tanh^{-1}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}\,d\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}+\frac{f\left(1-\frac{e}{\sqrt{e^2-4df}}\right.}{\sqrt{2}\,d}
\end{aligned}
$$

**Mathematica [A]** time = 0.74, size = 319, normalized size = 0.97

$$
\frac{\frac{\sqrt{2}f\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\sqrt{2}f\left(\sqrt{e^2-4df}-e\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{a+cx^2}\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}}}{2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ((Sqrt[2]*f*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d
*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a +
c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4
*d*f])]) + (Sqrt[2]*f*(-e + Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt
[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*
Sqrt[a + c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqr
t[e^2 - 4*d*f])]) - (2*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/Sqrt[a])/(2*d)
```

**IntegrateAlgebraic [C]** time = 0.45, size = 242, normalized size = 0.73

$$
\frac{2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}-\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d}-\frac{\text{RootSum}\left[\#1^4f-2\#1^3\sqrt{c}e-2\#1^2af+4\#1^2cd+2\#1a\sqrt{c}e+a^2f\&,\frac{\#1^2(-f)\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+2\#1\sqrt{c}e\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+af\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)}{-2\#1^3f+3\#1^2\sqrt{c}e+2\#1af-4\#1cd-a\sqrt{c}e}\&\right]}{d}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] (2*ArcTanh[(Sqrt[c]*x)/Sqrt[a] - Sqrt[a + c*x^2]/Sqrt[a]])/(Sqrt[a]*d) - Ro
otSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3
 + f*#1^4 & , (a*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + 2*Sqrt[c]*e*L
og[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - f*Log[-(Sqrt[c]*x) + Sqrt[a +
c*x^2] - #1]*#1^2)/(-(a*Sqrt[c]*e) - 4*c*d*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*#1^2
 - 2*f*#1^3) & ]/d
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.02, size = 681, normalized size = 2.06

$$-\frac{2\sqrt{2}\, f \ln\left(\frac{-\frac{\left(e-\sqrt{-4df+e^2}\right)\left(x-\frac{-e+\sqrt{-4df+e^2}}{2f}\right)c}{f}+\frac{2af^2-2cdf+ce^2-\sqrt{-4df+e^2}\, ce}{f^2}+\frac{\sqrt{2}\sqrt{\frac{2af^2-2cdf+ce^2-\sqrt{-4df+e^2}\, ce}{f^2}}\sqrt{4\left(x-\frac{-e+\sqrt{-4df+e^2}}{2f}\right)^2 c-\frac{4\left(e-\sqrt{-4df+e^2}\right)\left(x-\frac{-e+\sqrt{-4df+e^2}}{2f}\right)c}{f}+\frac{4af^2-4cdf+2ce^2-2\sqrt{-4df+e^2}\, ce}{f^2}}}{2}}{x-\frac{-e+\sqrt{-4df+e^2}}{2f}}\right)}{\left(-e+\sqrt{-4df+e^2}\right)\sqrt{-4df+e^2}\,\sqrt{\frac{2af^2-2cdf+ce^2-\sqrt{-4df+e^2}\, ce}{f^2}}}-\frac{2\sqrt{2}\, f \ln\left(\frac{-\frac{\left(e+\sqrt{-4df+e^2}\right)\left(x+\frac{e+\sqrt{-4df+e^2}}{2f}\right)c}{f}+\frac{2af^2-2cdf+ce^2+\sqrt{-4df+e^2}\, ce}{f^2}+\frac{\sqrt{2}\sqrt{\frac{2af^2-2cdf+ce^2+\sqrt{-4df+e^2}\, ce}{f^2}}\sqrt{4\left(x+\frac{e+\sqrt{-4df+e^2}}{2f}\right)^2 c-\frac{4\left(e+\sqrt{-4df+e^2}\right)\left(x+\frac{e+\sqrt{-4df+e^2}}{2f}\right)c}{f}+\frac{4af^2-4cdf+2ce^2+2\sqrt{-4df+e^2}\, ce}{f^2}}}{2}}{x+\frac{e+\sqrt{-4df+e^2}}{2f}}\right)}{\left(e+\sqrt{-4df+e^2}\right)\sqrt{-4df+e^2}\,\sqrt{\frac{2af^2-2cdf+ce^2+\sqrt{-4df+e^2}\, ce}{f^2}}}+\frac{4f\ln\left(\frac{2a+2\sqrt{cx^2+a}\,\sqrt{a}}{x}\right)}{\left(-e+\sqrt{-4df+e^2}\right)\left(e+\sqrt{-4df+e^2}\right)\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] -2*f/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-2*c*d*f+c*
e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(
e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)
/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)
*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e
+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e
)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))-2*f/(-e+(-4*d*f+e^2)^(1/2))
```

```
/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)
/f^2)^(1/2)*ln((-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c
/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2
-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2
)^(1/2))/f)^2*c-4*(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*
c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x-1/2*(-e
+(-4*d*f+e^2)^(1/2))/f))+4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))
/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+a}\left(fx^2+ex+d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*x), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\,\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
[Out] int(1/(x*(a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+cx^2}\,\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
[Out] Integral(1/(x*sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

## 3.69 $\int \frac{1}{x^2\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx$

**Optimal**. Leaf size=367

$$-\frac{f\left(e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(-e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\cdots\right.}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(\cdots\right.}}$$

**Rubi [A]** time = 1.20, antiderivative size = 367, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6728, 264, 266, 63, 208, 1034, 725, 206}

$$-\frac{f\left(e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(-e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^2}-\frac{\sqrt{a+cx^2}}{adx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -(Sqrt[a + c*x^2]/(a*d*x)) - (f*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*ArcTanh
[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d
*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f
]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (f*(e^2 - 2*d*f
- e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[
2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])
/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2
- 4*d*f])]) + (e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx = \int \left(\frac{1}{dx^2\sqrt{a+cx^2}} - \frac{e}{d^2x\sqrt{a+cx^2}} + \frac{e^2-df+efx}{d^2\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\right)dx$$

$$= \frac{\int \frac{e^2-df+efx}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{d^2} + \frac{\int \frac{1}{x^2\sqrt{a+cx^2}}\,dx}{d} - \frac{e\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d^2}$$

$$= -\frac{\sqrt{a+cx^2}}{adx} - \frac{e\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2d^2} - \frac{\left(f\left(e^2-2df-e\sqrt{e^2-4df}\right)\right)\int \cdots}{d^2\sqrt{e^2-4 \cdots}}$$

$$= -\frac{\sqrt{a+cx^2}}{adx} - \frac{e\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{cd^2} + \frac{\left(f\left(e^2-2df-e\sqrt{e^2-4d \cdots}\right.\right.}{\cdots}$$

$$= -\frac{\sqrt{a+cx^2}}{adx} - \frac{f\left(e^2-2df+e\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\cdots}\right.}{\sqrt{2}\,d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}\cdots$$

**Mathematica [A]** time = 0.84, size = 356, normalized size = 0.97

$$-\frac{\frac{\sqrt{2}f\left(e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}} + \frac{\sqrt{2}f\left(e\sqrt{e^2-4df}+2df-e^2\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{a+cx^2}\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}} + \frac{2d\sqrt{a+cx^2}}{ax} - \frac{2e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -1/2*((2*d*Sqrt[a + c*x^2])/(a*x) + (Sqrt[2]*f*(e^2 - 2*d*f + e*Sqrt[e^2 -
4*d*f])*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*
(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[e^2 - 4*d*f]
*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (Sqrt[2]*f*(-e^2
+ 2*d*f + e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x
)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])
])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]
) - (2*e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/Sqrt[a])/d^2
```

**IntegrateAlgebraic [C]** time = 0.55, size = 304, normalized size = 0.83

$$-\frac{\mathrm{RootSum}\left[\#1^4f-2\#1^3\sqrt{c}e-2\#1^2af+4\#1^2cd+2\#1a\sqrt{c}e+a^2f\&, \frac{-\#1^2ef\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)-2\#1\sqrt{c}df\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+2\#1\sqrt{c}e^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+aef\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)}{2\#1^3f-3\#1^2\sqrt{c}e-2\#1af+4\#1cd+a\sqrt{c}e}\&\right]}{d^2} - \frac{2e\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}-\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^2} - \frac{\sqrt{a+cx^2}}{adx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -(Sqrt[a + c*x^2]/(a*d*x)) - (2*e*ArcTanh[(Sqrt[c]*x)/Sqrt[a] - Sqrt[a + c*
x^2]/Sqrt[a]])/(Sqrt[a]*d^2) - RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^
2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (a*e*f*Log[-(Sqrt[c]*x) + Sq
rt[a + c*x^2] - #1] + 2*Sqrt[c]*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1
]*#1 - 2*Sqrt[c]*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - e*f*Log[
-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*
#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]/d^2
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 9.45Error index.cc index_gc
d Error: Bad Argument Value
```

**maple [B]** time = 0.02, size = 736, normalized size = 2.01

$$\frac{16 e f^{2} \ln\left(\frac{2a+2\sqrt{c x^{2}+a}\,\sqrt{a}}{x}\right)}{\left(-e+\sqrt{-4df+e^{2}}\right)^{2}\left(e+\sqrt{-4df+e^{2}}\right)^{2}\sqrt{a}}-\frac{4\sqrt{2}\, f^{2}\ln\left(\frac{-\frac{\left(e-\sqrt{-4df+e^{2}}\right)\left(x-\frac{-e+\sqrt{-4df+e^{2}}}{2f}\right)c}{f}+\frac{2af^{2}-2cdf+ce^{2}-\sqrt{-4df+e^{2}}\,ce}{f^{2}}+\frac{\sqrt{2}\,\sqrt{\frac{2af^{2}-2cdf+ce^{2}-\sqrt{-4df+e^{2}}\,ce}{f^{2}}}\,\sqrt{4\left(x-\frac{-e+\sqrt{-4df+e^{2}}}{2f}\right)^{2}c-\frac{4\left(e-\sqrt{-4df+e^{2}}\right)\left(x-\frac{-e+\sqrt{-4df+e^{2}}}{2f}\right)c}{f}+\frac{4af^{2}-4cdf+2ce^{2}-2\sqrt{-4df+e^{2}}\,ce}{f^{2}}}}{2}}{x-\frac{-e+\sqrt{-4df+e^{2}}}{2f}}\right)}{\left(-e+\sqrt{-4df+e^{2}}\right)^{2}\sqrt{-4df+e^{2}}\,\sqrt{\frac{2af^{2}-2cdf+ce^{2}-\sqrt{-4df+e^{2}}\,ce}{f^{2}}}}+\frac{4\sqrt{2}\, f^{2}\ln\left(\frac{-\frac{\left(e+\sqrt{-4df+e^{2}}\right)\left(x+\frac{e+\sqrt{-4df+e^{2}}}{2f}\right)c}{f}+\frac{2af^{2}-2cdf+ce^{2}+\sqrt{-4df+e^{2}}\,ce}{f^{2}}+\frac{\sqrt{2}\,\sqrt{\frac{2af^{2}-2cdf+ce^{2}+\sqrt{-4df+e^{2}}\,ce}{f^{2}}}\,\sqrt{4\left(x+\frac{e+\sqrt{-4df+e^{2}}}{2f}\right)^{2}c-\frac{4\left(e+\sqrt{-4df+e^{2}}\right)\left(x+\frac{e+\sqrt{-4df+e^{2}}}{2f}\right)c}{f}+\frac{4af^{2}-4cdf+2ce^{2}+2\sqrt{-4df+e^{2}}\,ce}{f^{2}}}}{2}}{x+\frac{e+\sqrt{-4df+e^{2}}}{2f}}\right)}{\left(e+\sqrt{-4df+e^{2}}\right)^{2}\sqrt{-4df+e^{2}}\,\sqrt{\frac{2af^{2}-2cdf+ce^{2}+\sqrt{-4df+e^{2}}\,ce}{f^{2}}}}+\frac{4\sqrt{c x^{2}+a}\, f}{\left(-e+\sqrt{-4df+e^{2}}\right)\left(e+\sqrt{-4df+e^{2}}\right) a x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] 4*f^2/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-2*c*d*f
+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e+(-4*d*f+e^2)^(1/2))*(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c
*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1
```

```
/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(x+1/2
*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*
c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))+4*f/(-e+(-4*d*f+e^2)^(1/
2))/(e+(-4*d*f+e^2)^(1/2))/a/x*(c*x^2+a)^(1/2)-4*f^2/(-e+(-4*d*f+e^2)^(1/2)
)^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c
*e)/f^2)^(1/2)*ln((-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f
)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*
f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+
e^2)^(1/2))/f)^2*c-4*(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/
f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x-1/2*
(-e+(-4*d*f+e^2)^(1/2))/f))+16*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+(-4*d*f+e
^2)^(1/2))^2/a^(1/2)*ln((2*a+2*(c*x^2+a)^(1/2)*a^(1/2))/x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+a}\left(fx^2+ex+d\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*x^2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
[Out] int(1/(x^2*(a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
[Out] Integral(1/(x**2*sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

## 3.70 $\int \frac{1}{x^3 \sqrt{a+cx^2} \left(d+ex+fx^2\right)} \, dx$

**Optimal**. Leaf size=457

$$\frac{c \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d} - \frac{\left(e^2-df\right)\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\, d^3} + \frac{f\left(-\left(e^2-df\right)\left(e-\sqrt{e^2-4df}\right)-4def+2e^3\right)\tanh^{-1}\left(\frac{\quad}{\sqrt{2}\sqrt{a+cx^2}\ \cdots}\right.}{\sqrt{2}\, d^3 \sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}\ -2\cdots\right.}}$$

**Rubi [A]** time = 1.86, antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6728, 266, 51, 63, 208, 264, 1034, 725, 206}

$$\frac{c \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d} + \frac{f\left(-\left(e^2-df\right)\left(e-\sqrt{e^2-4df}\right)-4def+2e^3\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\, d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}} - \frac{f\left(-\left(e^2-df\right)\left(\sqrt{e^2-4df}+e\right)-4def+2e^3\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\, d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}} - \frac{\left(e^2-df\right)\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\, d^3} + \frac{e\sqrt{a+cx^2}}{ad^2x} - \frac{\sqrt{a+cx^2}}{2adx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -Sqrt[a + c*x^2]/(2*a*d*x^2) + (e*Sqrt[a + c*x^2])/(a*d^2*x) + (f*(2*e^3 -
4*d*e*f - (e^2 - d*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt
[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d
*f])]*Sqrt[a + c*x^2])]])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e
^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (f*(2*e^3 - 4*d*e*f - (e^2 - d*f)*(e
+ Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2
]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/
(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 -
 4*d*f])]) + (c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*a^(3/2)*d) - ((e^2 - d
*f)*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^3)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
```

```
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx = \int \left(\frac{1}{dx^3\sqrt{a+cx^2}} - \frac{e}{d^2x^2\sqrt{a+cx^2}} + \frac{e^2-df}{d^3x\sqrt{a+cx^2}} + \frac{-e\left(e^2-2df\right)-f\left(e^2\right.}{d^3\sqrt{a+cx^2}\left(d+ex\right.}\right.$$

$$= \frac{\int \frac{-e\left(e^2-2df\right)-f\left(e^2-df\right)x}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{d^3} + \frac{\int \frac{1}{x^3\sqrt{a+cx^2}}\,dx}{d} - \frac{e\int \frac{1}{x^2\sqrt{a+cx^2}}\,dx}{d^2} + \frac{\left(e^2-df\right)\int -}{d^3}$$

$$= \frac{e\sqrt{a+cx^2}}{ad^2x} + \frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+cx}}\,dx, x, x^2\right)}{2d} + \frac{\left(e^2-df\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x\right.}{2d^3}$$

$$= -\frac{\sqrt{a+cx^2}}{2adx^2} + \frac{e\sqrt{a+cx^2}}{ad^2x} - \frac{c\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{4ad} + \frac{\left(e^2-df\right)\text{Subst}}{}$$

$$= -\frac{\sqrt{a+cx^2}}{2adx^2} + \frac{e\sqrt{a+cx^2}}{ad^2x} + \frac{f\left(2e^3-4def-\left(e^2-df\right)\left(e-\sqrt{e^2-4df}\right)\right)\tan}{\sqrt{2}\,d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\right.}}$$

$$= -\frac{\sqrt{a+cx^2}}{2adx^2} + \frac{e\sqrt{a+cx^2}}{ad^2x} + \frac{f\left(2e^3-4def-\left(e^2-df\right)\left(e-\sqrt{e^2-4df}\right)\right)\tan}{\sqrt{2}\,d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\right.}}$$

**Mathematica [A]** time = 1.57, size = 460, normalized size = 1.01

$$-\frac{\frac{cd^2\sqrt{a+cx^2}\left(\frac{a}{cx^2}-\frac{\tanh^{-1}\left(\sqrt{\frac{cx^2}{a}+1}\right)}{\sqrt{\frac{cx^2}{a}+1}}\right)}{a^2}+\frac{2\left(e^2-df\right)\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{2}f\left(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3\right)\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\sqrt{2}f\left(-e^2\sqrt{e^2-4df}+df\sqrt{e^2-4df}-3def+e^3\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{a+cx^2}\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{2de\sqrt{a+cx^2}}{ax}}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -1/2*((-2*d*e*Sqrt[a + c*x^2])/(a*x) - (Sqrt[2]*f*(e^3 - 3*d*e*f + e^2*Sqrt
[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 -
4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a
 + c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2
```

```
- 4*d*f])]) + (Sqrt[2]*f*(e^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*d*f] + d*f*Sqrt[
e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 +
 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[e^2 - 4*
d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) + (2*(e^2 - d*f
)*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/Sqrt[a] + (c*d^2*Sqrt[a + c*x^2]*(a/(c*
x^2) - ArcTanh[Sqrt[1 + (c*x^2)/a]]/Sqrt[1 + (c*x^2)/a]))/a^2)/d^3
```

**IntegrateAlgebraic [C]** time = 0.84, size = 430, normalized size = 0.94

$$\frac{\text{RootSum}\left[\#1^4 f-2\#1^3\sqrt{c}e-2\#1^2 af+4\#1^2 cd+2\#1a\sqrt{c}e+a^2 f\&,\frac{\#1^2 df^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)-\#1^2 e^2 f\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)-4\#1\sqrt{c}def\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)-adf^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+2\#1\sqrt{c}e^3\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+ae^2 f\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)}{2\#1^3 f-3\#1^2\sqrt{c}e-2\#1af+4\#1cd+a\sqrt{c}e}\&\right]}{d^3}-\frac{c\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}-\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2\left(df-e^2\right)\tanh^{-1}\left(\frac{\sqrt{a+cx^2}-\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}d^3}+\frac{\sqrt{a+cx^2}(2ex-d)}{2ad^2x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ((-d + 2*e*x)*Sqrt[a + c*x^2])/(2*a*d^2*x^2) + (2*(-e^2 + d*f)*ArcTanh[(-(S
qrt[c]*x) + Sqrt[a + c*x^2])/Sqrt[a]])/(Sqrt[a]*d^3) - (c*ArcTanh[(Sqrt[c]*
x)/Sqrt[a] - Sqrt[a + c*x^2]/Sqrt[a]])/(a^(3/2)*d) + RootSum[a^2*f + 2*a*Sq
rt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (a*e^2
*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - a*d*f^2*Log[-(Sqrt[c]*x) + Sq
rt[a + c*x^2] - #1] + 2*Sqrt[c]*e^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1
]*#1 - 4*Sqrt[c]*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - e^2*f*
Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 + d*f^2*Log[-(Sqrt[c]*x) + Sq
rt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*
#1^2 + 2*f*#1^3) & ]/d^3
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.02, size = 911, normalized size = 1.99

$$-\frac{64 d f^{4} \ln\left(\frac{2a+2\sqrt{c x^{2}+a}\,\sqrt{a}}{x}\right)}{\left(-e+\sqrt{-4df+e^{2}}\right)^{3}\left(e+\sqrt{-4df+e^{2}}\right)^{3}\sqrt{a}}+\frac{64 e^{2} f^{3} \ln\left(\frac{2a+2\sqrt{c x^{2}+a}\,\sqrt{a}}{x}\right)}{\left(-e+\sqrt{-4df+e^{2}}\right)^{3}\left(e+\sqrt{-4df+e^{2}}\right)^{3}\sqrt{a}}-\frac{8\sqrt{2}\, f^{3}\ln\left(\frac{-\frac{\left(e-\sqrt{-4df+e^{2}}\right)\left(x-\frac{-e+\sqrt{-4df+e^{2}}}{2f}\right)c}{f}+\frac{2af^{2}-2cdf+ce^{2}-\sqrt{-4df+e^{2}}\,ce}{f^{2}}+\frac{\sqrt{2}\sqrt{\frac{2af^{2}-2cdf+ce^{2}-\sqrt{-4df+e^{2}}\,ce}{f^{2}}}\sqrt{4\left(x-\frac{-e+\sqrt{-4df+e^{2}}}{2f}\right)^{2}c-\frac{4\left(e-\sqrt{-4df+e^{2}}\right)\left(x-\frac{-e+\sqrt{-4df+e^{2}}}{2f}\right)c}{f}+\frac{4af^{2}-4cdf+2ce^{2}-2\sqrt{-4df+e^{2}}\,ce}{f^{2}}}}{2}}{x-\frac{-e+\sqrt{-4df+e^{2}}}{2f}}\right)}{\left(-e+\sqrt{-4df+e^{2}}\right)^{3}\sqrt{-4df+e^{2}}\,\sqrt{\frac{2af^{2}-2cdf+ce^{2}-\sqrt{-4df+e^{2}}\,ce}{f^{2}}}}-\frac{8\sqrt{2}\, f^{3}\ln\left(\frac{-\frac{\left(e+\sqrt{-4df+e^{2}}\right)\left(x+\frac{e+\sqrt{-4df+e^{2}}}{2f}\right)c}{f}+\frac{2af^{2}-2cdf+ce^{2}+\sqrt{-4df+e^{2}}\,ce}{f^{2}}+\frac{\sqrt{2}\sqrt{\frac{2af^{2}-2cdf+ce^{2}+\sqrt{-4df+e^{2}}\,ce}{f^{2}}}\sqrt{4\left(x+\frac{e+\sqrt{-4df+e^{2}}}{2f}\right)^{2}c-\frac{4\left(e+\sqrt{-4df+e^{2}}\right)\left(x+\frac{e+\sqrt{-4df+e^{2}}}{2f}\right)c}{f}+\frac{4af^{2}-4cdf+2ce^{2}+2\sqrt{-4df+e^{2}}\,ce}{f^{2}}}}{2}}{x+\frac{e+\sqrt{-4df+e^{2}}}{2f}}\right)}{\left(e+\sqrt{-4df+e^{2}}\right)^{3}\sqrt{-4df+e^{2}}\,\sqrt{\frac{2af^{2}-2cdf+ce^{2}+\sqrt{-4df+e^{2}}\,ce}{f^{2}}}}-\frac{2cf\ln\left(\frac{2a+2\sqrt{c x^{2}+a}\,\sqrt{a}}{x}\right)}{\left(-e+\sqrt{-4df+e^{2}}\right)\left(e+\sqrt{-4df+e^{2}}\right)a^{\frac{3}{2}}}+\frac{16\sqrt{c x^{2}+a}\, e f^{2}}{\left(-e+\sqrt{-4df+e^{2}}\right)^{2}\left(e+\sqrt{-4df+e^{2}}\right)^{2} a x}+\frac{2\sqrt{c x^{2}+a}\, f}{\left(-e+\sqrt{-4df+e^{2}}\right)\left(e+\sqrt{-4df+e^{2}}\right) a x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] -8*f^3/(e+(-4*d*f+e^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-2*c*d*
f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e+(-4*d*f+e^2)^(1/2))*(x+1
/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*
c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(
1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)
*c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))+16*f^2*e/(-e+(-4*d*f+e^
2)^(1/2))^2/(e+(-4*d*f+e^2)^(1/2))^2/a/x*(c*x^2+a)^(1/2)+2*f/(-e+(-4*d*f+e^
2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a/x^2*(c*x^2+a)^(1/2)-2*f/(-e+(-4*d*f+e^2)
^(1/2))/(e+(-4*d*f+e^2)^(1/2))*c/a^(3/2)*ln((2*a+2*(c*x^2+a)^(1/2)*a^(1/2))
/x)-8*f^3/(-e+(-4*d*f+e^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-2*
c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e-(-4*d*f+e^2)^(1/2))*
(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(
1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f
^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e-(-4*d*f+e^2)^(1/2))
*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2
)^(1/2)*c*e)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))-64*f^4/(-e+(-4*
d*f+e^2)^(1/2))^3/(e+(-4*d*f+e^2)^(1/2))^3/a^(1/2)*ln((2*a+2*(c*x^2+a)^(1/2
)*a^(1/2))/x)*d+64*f^3/(-e+(-4*d*f+e^2)^(1/2))^3/(e+(-4*d*f+e^2)^(1/2))^3/a
^(1/2)*ln((2*a+2*(c*x^2+a)^(1/2)*a^(1/2))/x)*e^2
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^{2}+a}\left(fx^{2}+ex+d\right)x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*x^3), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{3}\,\sqrt{c\,x^{2}+a}\,\left(f\,x^{2}+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(1/(x^3*(a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{a+cx^2}\,\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

## 3.71 $\int \frac{x^3}{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx$

**Optimal**. Leaf size=499

$$\frac{cex\left(a\left(e^2-2df\right)+cd^2\right)+af\left(a\left(e^2-df\right)+cd^2\right)}{af^2\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}-\frac{\left(2adef-\left(e-\sqrt{e^2-4df}\right)\left(a\left(e^2-df\right)+cd^2\right)\right)\tanh^{-1}\left(\frac{\cdots}{\sqrt{2}\sqrt{a\cdots}}\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e\cdots}\right.}}$$

**Rubi [A]** time = 2.11, antiderivative size = 499, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6728, 191, 261, 1017, 1034, 725, 206}

$$\frac{cex\left(a\left(e^2-2df\right)+cd^2\right)+af\left(a\left(e^2-df\right)+cd^2\right)}{af^2\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}-\frac{\left(2adef-\left(e-\sqrt{e^2-4df}\right)\left(a\left(e^2-df\right)+cd^2\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(2adef-\left(\sqrt{e^2-4df}+e\right)\left(a\left(e^2-df\right)+cd^2\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{ex}{af^2\sqrt{a+cx^2}}-\frac{1}{cf\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] -(1/(c*f*Sqrt[a + c*x^2])) - (e*x)/(a*f^2*Sqrt[a + c*x^2]) + (a*f*(c*d^2 +
a*(e^2 - d*f)) + c*e*(c*d^2 + a*(e^2 - 2*d*f))*x)/(a*f^2*(a*c*e^2 + (c*d -
a*f)^2)*Sqrt[a + c*x^2]) - ((2*a*d*e*f - (e - Sqrt[e^2 - 4*d*f])*(c*d^2 + a
*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[
2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[2
]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f
 - e*Sqrt[e^2 - 4*d*f])]) + ((2*a*d*e*f - (e + Sqrt[e^2 - 4*d*f])*(c*d^2 +
a*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt
[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[
2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*
f + e*Sqrt[e^2 - 4*d*f])]])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1017

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + c*x^2)^(p + 1)*(d + e*x + f*x^2)^
(q + 1)*(g*c*(2*a*c*e) + (-(a*h))*(2*c^2*d - c*(2*a*f)) + c*(g*(2*c^2*d - c
*(2*a*f)) - h*(-2*a*c*e))*x))/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)),
 x] + Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^
(p + 1)*(d + e*x + f*x^2)^q*Simp[(-2*g*c)*((c*d - a*f)^2 - (-(a*e))*(c*e))*
(p + 1) + (2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(a*f*(p + 1) - c*d*(p + 2))
- e*((g*c)*(2*a*c*e) + (-(a*h))*(2*c^2*d - c*((Plus[2])*a*f)))*(p + q + 2)
- (2*f*((g*c)*(2*a*c*e) + (-(a*h))*(2*c^2*d - c*((Plus[2])*a*f)))*(p + q +
2) - (2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(-(c*e*(2*p + q + 4))))*x - c*f*(
2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(2*p + 2*q + 5)*x^2, x], x], x] /; Free
Q[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[
a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^3}{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx = \int \left(-\frac{e}{f^2\left(a+cx^2\right)^{3/2}} + \frac{x}{f\left(a+cx^2\right)^{3/2}} + \frac{de+\left(e^2-df\right)x}{f^2\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\right) dx$$

$$= \frac{\int \frac{de+\left(e^2-df\right)x}{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx}{f^2} - \frac{e\int \frac{1}{\left(a+cx^2\right)^{3/2}}\,dx}{f^2} + \frac{\int \frac{x}{\left(a+cx^2\right)^{3/2}}\,dx}{f}$$

$$= -\frac{1}{cf\sqrt{a+cx^2}} - \frac{ex}{af^2\sqrt{a+cx^2}} + \frac{af\left(cd^2+a\left(e^2-df\right)\right)+ce\left(cd^2+a\left(e^2-2\right.\right.}{af^2\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}}$$

$$= -\frac{1}{cf\sqrt{a+cx^2}} - \frac{ex}{af^2\sqrt{a+cx^2}} + \frac{af\left(cd^2+a\left(e^2-df\right)\right)+ce\left(cd^2+a\left(e^2-2\right.\right.}{af^2\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}}$$

$$= -\frac{1}{cf\sqrt{a+cx^2}} - \frac{ex}{af^2\sqrt{a+cx^2}} + \frac{af\left(cd^2+a\left(e^2-df\right)\right)+ce\left(cd^2+a\left(e^2-2\right.\right.}{af^2\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}}$$

$$= -\frac{1}{cf\sqrt{a+cx^2}} - \frac{ex}{af^2\sqrt{a+cx^2}} + \frac{af\left(cd^2+a\left(e^2-df\right)\right)+ce\left(cd^2+a\left(e^2-2\right.\right.}{af^2\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}}$$

**Mathematica [A]** time = 2.70, size = 577, normalized size = 1.16

$$\frac{\left(-\frac{e(e^2-3df)}{\sqrt{e^2-4df}}-df+e^2\right)\left(2af+cx\left(e-\sqrt{e^2-4df}\right)\right)}{af^2\sqrt{a+cx^2}\left(4af^2+c\left(e-\sqrt{e^2-4df}\right)^2\right)}+\frac{\left(\frac{e(e^2-3df)}{\sqrt{e^2-4df}}-df+e^2\right)\left(2af+cx\left(\sqrt{e^2-4df}+e\right)\right)}{af^2\sqrt{a+cx^2}\left(4af^2+c\left(\sqrt{e^2-4df}+e\right)^2\right)}+\frac{\sqrt{2}\left(-e^2\sqrt{e^2-4df}+df\sqrt{e^2-4df}-3def+e^3\right)\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)}{\sqrt{e^2-4df}\left(2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)^{3/2}}-\frac{\sqrt{2}\left(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{a+cx^2}\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\left(2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)\right)^{3/2}}-\frac{ex}{af^2\sqrt{a+cx^2}}-\frac{1}{cf\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] -(1/(c*f*Sqrt[a + c*x^2])) - (e*x)/(a*f^2*Sqrt[a + c*x^2]) + ((e^2 - d*f -
(e*(e^2 - 3*d*f))/Sqrt[e^2 - 4*d*f])*(2*a*f + c*(e - Sqrt[e^2 - 4*d*f])*x))
/(a*f^2*(4*a*f^2 + c*(e - Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + c*x^2]) + ((e^2 -
d*f + (e*(e^2 - 3*d*f))/Sqrt[e^2 - 4*d*f])*(2*a*f + c*(e + Sqrt[e^2 - 4*d*f
])*x))/(a*f^2*(4*a*f^2 + c*(e + Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + c*x^2]) + (S
qrt[2]*(e^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*d*f] + d*f*Sqrt[e^2 - 4*d*f])*ArcT
anh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*
f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[e^2 - 4*d*f]*(2*a*f^2 +
c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))^(3/2)) - (Sqrt[2]*(e^3 - 3*d*e*f + e
^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[
```

```
e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*S
qrt[a + c*x^2])])/(Sqrt[e^2 - 4*d*f]*(2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2
 - 4*d*f]))^(3/2))
```

**IntegrateAlgebraic [C]** time = 0.76, size = 425, normalized size = 0.85

$$\frac{\text{RootSum}\left[\#1^4 f-2\#1^3\sqrt{c}\,e-2\#1^2 af+4\#1^2 cd+2\#1 a\sqrt{c}\,e+a^2 f\&,\frac{\#1^2cd^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)-\#1^2adf\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)+\#1^2ae^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)+a^2df\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)+a^2\left(-e^2\right)\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)-acd^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)-2\#1a\sqrt{c}\,de\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)}{2\#1^3f-3\#1^2\sqrt{c}\,e-2\#1af+4\#1cd+a\sqrt{c}\,e}\&\right]}{a^2f^2-2acdf+ace^2+c^2d^2}+\frac{a^2(-f)+acd-acex}{c\sqrt{a+cx^2}\left(a^2f^2-2acdf+ace^2+c^2d^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (a*c*d - a^2*f - a*c*e*x)/(c*(c^2*d^2 + a*c*e^2 - 2*a*c*d*f + a^2*f^2)*Sqrt
[a + c*x^2]) + RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 -
 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (-(a*c*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2
] - #1]) - a^2*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + a^2*d*f*Log[-
(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - 2*a*Sqrt[c]*d*e*Log[-(Sqrt[c]*x) + Sq
rt[a + c*x^2] - #1]*#1 + c*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^
2 + a*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 - a*d*f*Log[-(Sqrt[
c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*
Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]/(c^2*d^2 + a*c*e^2 - 2*a*c*d*f + a^2*f^2)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$sage_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage2
```

**maple [B]** time = 0.03, size = 6124, normalized size = 12.27

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

**maxima** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

**mupad** [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\left(c\,x^2+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(x^3/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

**sympy** [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x**3/((a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

## 3.72 $\int \frac{x^2}{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx$

**Optimal**. Leaf size=410

$$-\frac{f\left(2d(cd-af)+ae\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(2d(cd-af)+ae\left(\sqrt{e^2-\cdots\right.\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-a\cdots\right.}$$

**Rubi** [A] time = 0.71, antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1063, 1034, 725, 206}

$$-\frac{f\left(2d(cd-af)+ae\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(2d(cd-af)+ae\left(\sqrt{e^2-4df}+e\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{x(cd-af)+ae}{\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] -((a*e + (c*d - a*f)*x)/((a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2])) - (f*(
2*d*(c*d - a*f) + a*e*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt
[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d
*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^
2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (f*(2*d*(c*d -
a*f) + a*e*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*
f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[
a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a
*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1063

```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) + (e_.)*(x_) +
(f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + c*x^2)^(p + 1)*(d + e*x + f*x^2
)^(q + 1)*((A*c - a*C)*(2*a*c*e) + c*(A*(2*c^2*d - c*(2*a*f)) + C*(-2*a*(c*
d - a*f)))*x))/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((
-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^(p + 1)*(d + e*
x + f*x^2)^q*Simp[(-2*A*c - 2*a*C)*((c*d - a*f)^2 - (-(a*e))*(c*e))*(p + 1)
 + (2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) -
e*((A*c - a*C)*(2*a*c*e))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e))*(p + q
 + 2) - (2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(-(c*e*(2*p + q + 4))))*x
 - c*f*(2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x
], x] /; FreeQ[{a, c, d, e, f, A, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p
, -1] && NeQ[a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]
) &&  !IGtQ[q, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx = -\frac{ae+(cd-af)x}{\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} + \frac{\int \frac{2acd(cd-af)-2a^2cefx}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{2ac\left(ace^2+(cd-af)^2\right)}$$

$$= -\frac{ae+(cd-af)x}{\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} + \frac{\left(f\left(2d(cd-af)+ae\left(e-\sqrt{e^2-4df}\right)\right)\right)\int}{\sqrt{e^2-4df}\left(ace^2+(cd\right.}$$

$$= -\frac{ae+(cd-af)x}{\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} - \frac{\left(f\left(2d(cd-af)+ae\left(e-\sqrt{e^2-4df}\right)\right)\right)\mathrm{S}}{\sqrt{e^2-4d}}$$

$$= -\frac{ae+(cd-af)x}{\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} - \frac{f\left(2d(cd-af)+ae\left(e-\sqrt{e^2-4df}\right)\right)\tan}{\sqrt{2}\sqrt{e^2-4df}\left(ace^2+(cd-af)^2\right)\sqrt{}}$$

**Mathematica [A]** time = 2.07, size = 509, normalized size = 1.24

$$\frac{-\frac{\left(\frac{2df-e^2}{\sqrt{e^2-4df}}+e\right)\left(2af+cx\left(e-\sqrt{e^2-4df}\right)\right)}{a\sqrt{a+cx^2}\left(4af^2+c\left(e-\sqrt{e^2-4df}\right)^2\right)}-\frac{\left(\frac{e^2-2df}{\sqrt{e^2-4df}}+e\right)\left(2af+cx\left(\sqrt{e^2-4df}+e\right)\right)}{a\sqrt{a+cx^2}\left(4af^2+c\left(\sqrt{e^2-4df}+e\right)^2\right)}+\frac{\sqrt{2}f^2\left(e\sqrt{e^2-4df}+2df-e^2\right)\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)}{\sqrt{e^2-4df}\left(2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)^{3/2}}+\frac{\sqrt{2}f^2\left(e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{a+cx^2}\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\left(2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)\right)^{3/2}}+\frac{x}{a\sqrt{a+cx^2}}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (x/(a*Sqrt[a + c*x^2]) - ((e + (-e^2 + 2*d*f)/Sqrt[e^2 - 4*d*f])*(2*a*f + c
*(e - Sqrt[e^2 - 4*d*f])*x))/(a*(4*a*f^2 + c*(e - Sqrt[e^2 - 4*d*f])^2)*Sqr
t[a + c*x^2]) - ((e + (e^2 - 2*d*f)/Sqrt[e^2 - 4*d*f])*(2*a*f + c*(e + Sqrt
[e^2 - 4*d*f])*x))/(a*(4*a*f^2 + c*(e + Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + c*x^
2]) + (Sqrt[2]*f^2*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f + c*
(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2
- 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[e^2 - 4*d*f]*(2*a*f^2 + c*(e^2 - 2*d*f
- e*Sqrt[e^2 - 4*d*f]))^(3/2)) + (Sqrt[2]*f^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4
*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e
^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[e^2 - 4*d*f]*(2
*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))^(3/2)))/f
```

**IntegrateAlgebraic [C]** time = 0.63, size = 323, normalized size = 0.79

$$\frac{\mathrm{RootSum}\left[\#1^4 f-2\#1^3\sqrt{c}e-2\#1^2af+4\#1^2cd+2\#1a\sqrt{c}e+a^2f\&,\frac{-\#1^2aef\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+a^2ef\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)-2\#1c^{3/2}d^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+2\#1a\sqrt{c}df\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)}{2\#1^3f-3\#1^2\sqrt{c}e-2\#1af+4\#1cd+a\sqrt{c}e}\&\right]}{a^2f^2-2acdf+ace^2+c^2d^2}+\frac{-ae+afx-cdx}{\sqrt{a+cx^2}\left(a^2f^2-2acdf+ace^2+c^2d^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (-(a*e) - c*d*x + a*f*x)/((c^2*d^2 + a*c*e^2 - 2*a*c*d*f + a^2*f^2)*Sqrt[a
+ c*x^2]) + RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*
Sqrt[c]*e*#1^3 + f*#1^4 & , (a^2*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #
1] - 2*c^(3/2)*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + 2*a*Sqrt[c
]*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - a*e*f*Log[-(Sqrt[c]*x)
+ Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c
]*e*#1^2 + 2*f*#1^3) & ]/(c^2*d^2 + a*c*e^2 - 2*a*c*d*f + a^2*f^2)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

*sage*$_2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage2
```

**maple [B]** time = 0.02, size = 4752, normalized size = 11.59

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] 1/f*x/a/(c*x^2+a)^(1/2)-1/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/((
x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*
f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2
)^(1/2)*e+2/(-4*d*f+e^2)^(1/2)*f/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*
c*e)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e
+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c
*e)/f^2)^(1/2)*d-1/(-4*d*f+e^2)^(1/2)/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(
1/2)*c*e)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2)^(1/2))*(x+1
/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1
/2)*c*e)/f^2)^(1/2)*e^2-2*(-4*d*f+e^2)^(1/2)/f*c^2/(2*a*f^2-2*c*d*f+c*e^2+(
-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/(
(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d
*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^
2)^(1/2)*x*e+4*c^2/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*
c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)
^2*c-(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f
^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*x*d-4/f*c^2/(2*a*f^2-2*
c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-
4*d*f+e^2))/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2)^(1/2))*(x
+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^
(1/2)*c*e)/f^2)^(1/2)*x*e^2+4/(-4*d*f+e^2)^(1/2)*c^2/(2*a*f^2-2*c*d*f+c*e^2
+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))
/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4
*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/
f^2)^(1/2)*e*x*d-2/(-4*d*f+e^2)^(1/2)/f*c^2/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+
e^2)^(1/2)*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/((x+1/2*
```

(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)
^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2
)*e^3*x+1/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/((2*a*f^2-
2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e+(-4*d*f+e^2)^(1/2)
)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^
(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/
f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))
*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)
^(1/2)*c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*e-2/(-4*d*f+e^2)^
(1/2)*f/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/((2*a*f^2-2*
c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e+(-4*d*f+e^2)^(1/2))*
(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1
/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^
2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(
x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(
1/2)*c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*d+1/(-4*d*f+e^2)^(1
/2)/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/((2*a*f^2-2*c*d*
f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e+(-4*d*f+e^2)^(1/2))*(x+1
/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*
c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(
1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)
*c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*e^2-1/(2*a*f^2-2*c*d*f+
c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e-(-4
*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f
+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*e-2/(-4*d*f+e^2)^(1/2)*f/(2*a*f^2
-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2
*c-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^
2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*d+1/(-4*d*f+e^2)^(1/2)/(
2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2
))/f)^2*c-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*
(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*e^2+2*(-4*d*f+e^2
)^(1/2)/f*c^2/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2/f
*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c
-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-
2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*x*e+4*c^2/(2*a*f^2-2*c*d*f
+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f
+e^2))/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e-(-4*d*f+e^2)^(1/2))*(x-1/2
*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/
2)*c*e)/f^2)^(1/2)*x*d-4/f*c^2/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*
e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/((x-1/2*(-e+(-4*d*f+e
^2)^(1/2))/f)^2*c-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*
c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*x*e^2-4/(
-4*d*f+e^2)^(1/2)*c^2/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c
-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/((x-1/2*(-e+(-4*d*f+e^2)^(1/2)

```
)/f)^2*c-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(
2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*e*x*d+2/(-4*d*f+e^
2)^(1/2)/f*c^2/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2/
f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*
c-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2
-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*e^3*x+1/(2*a*f^2-2*c*d*f+
c*e^2-(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^
(1/2)*c*e)/f^2)^(1/2)*ln((-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(
1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)
*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(
-4*d*f+e^2)^(1/2))/f)^2*c-4*(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^
(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/
(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*e+2/(-4*d*f+e^2)^(1/2)*f/(2*a*f^2-2*c*d*
f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2
)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)
^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/
2)*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e
+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2
)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)
)/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*d-1/(-4*d*f+e^2)^(1/2)/(2*a*f^2-2*c*d*
f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2
)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)
^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/
2)*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e
+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2
)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)
)/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*e^2
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(c\,x^2+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(x^2/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

**sympy [F]**   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x**2/((a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

## 3.73 $\int \frac{x}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

**Optimal**. Leaf size=411

$$\frac{f\left(2cde-\left(e-\sqrt{e^2-4df}\right)(cd-af)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{f\left(2cde-\left(\sqrt{e^2-4df}+e\right)(\cdots\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-a\cdots\right.}$$

**Rubi** [A] time = 0.83, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1017, 1034, 725, 206}

$$\frac{f\left(2cde-\left(e-\sqrt{e^2-4df}\right)(cd-af)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{f\left(2cde-\left(\sqrt{e^2-4df}+e\right)(cd-af)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{-af+cd-cex}{\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] -((c*d - a*f - c*e*x)/((a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2])) + (f*(2*
c*d*e - (c*d - a*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e
^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f
])]*Sqrt[a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)
*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (f*(2*c*d*e - (c*
d - a*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f]
)*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a
+ c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f
^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1017

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + c*x^2)^(p + 1)*(d + e*x + f*x^2)^
(q + 1)*(g*c*(2*a*c*e) + (-(a*h))*(2*c^2*d - c*(2*a*f)) + c*(g*(2*c^2*d - c
*(2*a*f)) - h*(-2*a*c*e))*x))/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)),
 x] + Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^
(p + 1)*(d + e*x + f*x^2)^q*Simp[(-2*g*c)*((c*d - a*f)^2 - (-(a*e))*(c*e))*
(p + 1) + (2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(a*f*(p + 1) - c*d*(p + 2))
- e*((g*c)*(2*a*c*e) + (-(a*h))*(2*c^2*d - c*((Plus[2])*a*f)))*(p + q + 2)
- (2*f*((g*c)*(2*a*c*e) + (-(a*h))*(2*c^2*d - c*((Plus[2])*a*f)))*(p + q +
2) - (2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(-(c*e*(2*p + q + 4))))*x - c*f*(
2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(2*p + 2*q + 5)*x^2, x], x], x] /; Free
Q[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[
a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x}{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx = -\frac{cd-af-cex}{\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} + \frac{\int \frac{-2ac^2de-2acf(cd-af)x}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{2ac\left(ace^2+(cd-af)^2\right)}$$

$$= -\frac{cd-af-cex}{\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} - \frac{\left(f\left(2cde-(cd-af)\left(e-\sqrt{e^2-4df}\right)\right)\right)\int \frac{}{(}}{\sqrt{e^2-4df}\left(ace^2+(cd-\right.}$$

$$= -\frac{cd-af-cex}{\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} + \frac{\left(f\left(2cde-(cd-af)\left(e-\sqrt{e^2-4df}\right)\right)\right)\mathrm{Sul}}{\sqrt{e^2-4df}}$$

$$= -\frac{cd-af-cex}{\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} + \frac{f\left(2cde-(cd-af)\left(e-\sqrt{e^2-4df}\right)\right)\tanh}{\sqrt{2}\sqrt{e^2-4df}\left(ace^2+(cd-af)^2\right)\sqrt{2}}$$

**Mathematica [A]** time = 0.81, size = 457, normalized size = 1.11

$$\frac{\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\left(2af+cx\left(e-\sqrt{e^2-4df}\right)\right)}{a\sqrt{a+cx^2}\left(4af^2+c\left(e-\sqrt{e^2-4df}\right)^2\right)}+\frac{\left(\frac{e}{\sqrt{e^2-4df}}+1\right)\left(2af+cx\left(\sqrt{e^2-4df}+e\right)\right)}{a\sqrt{a+cx^2}\left(4af^2+c\left(\sqrt{e^2-4df}+e\right)^2\right)}+\frac{\sqrt{2}f^2\left(e-\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)}{\sqrt{e^2-4df}\left(2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)^{3/2}}-\frac{\sqrt{2}f^2\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{a+cx^2}\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\left(2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] ((1 - e/Sqrt[e^2 - 4*d*f])*(2*a*f + c*(e - Sqrt[e^2 - 4*d*f])*x))/(a*(4*a*f
^2 + c*(e - Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + c*x^2]) + ((1 + e/Sqrt[e^2 - 4*d
*f])*(2*a*f + c*(e + Sqrt[e^2 - 4*d*f])*x))/(a*(4*a*f^2 + c*(e + Sqrt[e^2 -
 4*d*f])^2)*Sqrt[a + c*x^2]) + (Sqrt[2]*f^2*(e - Sqrt[e^2 - 4*d*f])*ArcTanh
[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f +
 e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[e^2 - 4*d*f]*(2*a*f^2 + c*(
e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))^(3/2)) - (Sqrt[2]*f^2*(e + Sqrt[e^2 - 4
*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e
^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[e^2 - 4*d*f]*(2
*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))^(3/2))
```

**IntegrateAlgebraic [C]** time = 0.64, size = 352, normalized size = 0.86

$$\frac{\text{RootSum}\left[\#1^4 f-2\#1^3\sqrt{c}e-2\#1^2af+4\#1^2cd+2\#1a\sqrt{c}e+a^2f\&,\frac{\#1^2cdf\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+\#1^2(-a)f^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+a^2f^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)-2\#1c^{3/2}de\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)-acdf\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)}{2\#1^3f-3\#1^2\sqrt{c}e-2\#1af+4\#1cd+a\sqrt{c}e}\&\right]}{-a^2f^2+2acdf-ace^2-c^2d^2}+\frac{af-cd+cex}{\sqrt{a+cx^2}\left(a^2f^2-2acdf+ace^2+c^2d^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (-(c*d) + a*f + c*e*x)/((c^2*d^2 + a*c*e^2 - 2*a*c*d*f + a^2*f^2)*Sqrt[a +
c*x^2]) + RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sq
rt[c]*e*#1^3 + f*#1^4 & , (-(a*c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #
1]) + a^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - 2*c^(3/2)*d*e*Log[
-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a +
 c*x^2] - #1]*#1^2 - a*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(
a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]/(-(c^2
*d^2) - a*c*e^2 + 2*a*c*d*f - a^2*f^2)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$sage_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage2
```

**maple [B]** time = 0.02, size = 3000, normalized size = 7.30

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] 1/(-4*d*f+e^2)^(1/2)*f*e/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/((x
+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f
+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)
^(1/2)+4*e*c^2/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2/
f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c
-(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2
*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*x+2/(-4*d*f+e^2)^(1/2)*c^2/
(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2
-c^2/f^2*(-4*d*f+e^2))/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2
)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-
4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*e^2*x-1/(-4*d*f+e^2)^(1/2)*f*e/(2*a*f^2-2*
c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f
+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e
^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^
(1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*
(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^
2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2
))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))+f/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^
(1/2)*c*e)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2)^(1/2))*(x+
1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(
1/2)*c*e)/f^2)^(1/2)+2*(-4*d*f+e^2)^(1/2)*c^2/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*
f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/((x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^
2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1
/2)*x-f/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/((2*a*f^2-2*
c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e+(-4*d*f+e^2)^(1/2))*
(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1
/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^
```

```
2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(
x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(
1/2)*c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))+f/(2*a*f^2-2*c*d*f+
c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e-(-4
*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f
+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)-2*(-4*d*f+e^2)^(1/2)*c^2/(2*a*f^2
-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2
*(-4*d*f+e^2))/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e-(-4*d*f+e^2)^(1/2)
)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+
e^2)^(1/2)*c*e)/f^2)^(1/2)*x+4*c^2/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2
)*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/((x-1/2*(-e+(-4*d
*f+e^2)^(1/2))/f)^2*c-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))
/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*e*x-f
/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/((2*a*f^2-2*c*d*f+c
*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*
(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*
e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/
2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e-(-4*d*f+e^2)^(1/2))*(x-1/2
*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)
*c*e)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))-1/(-4*d*f+e^2)^(1/2)*f
*e/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/((x-1/2*(-e+(-4*d*f+e^2)^
(1/2))/f)^2*c-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+
1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)-2/(-4*d*f+e^2
)^(1/2)*c^2/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2/f*d
+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-(
e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*
c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*e^2*x+1/(-4*d*f+e^2)^(1/2)*f
*e/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/((2*a*f^2-2*c*d*f
+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e-(-4*d*f+e^2)^(1/2))*(x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*
c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(
1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e-(-4*d*f+e^2)^(1/2))*(x-1
/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/
2)*c*e)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(c\,x^2+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
[Out] int(x/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
[Out] Integral(x/((a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

## 3.74 $\int \frac{1}{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx$

**Optimal**. Leaf size=416

$$-\frac{f\left(2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(2af^2+c\left(-e\sqrt{e^2-4dj\right.\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd\right.}$$

**Rubi** [A] time = 0.62, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {976, 1034, 725, 206}

$$-\frac{f\left(2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{c(x(cd-af)+ae)}{a\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (c*(a*e + (c*d - a*f)*x))/(a*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2]) - (
f*(2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e -
 Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2
- 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d -
a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (f*(2*a*f^
2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2
 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])
]*Sqrt[a + c*x^2])]])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*S
qrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 976

```
Int[((a_.) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((2*a*c^2*e + c*(2*c^2*d - c*(2*a*f))*x)*(a + c*x^2)^(p +
1)*(d + e*x + f*x^2)^(q + 1))/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)),
 x] - Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^
(p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - (-(a*e))*(c*e))*(p +
1) - (2*c^2*d - c*(2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(-2*a*c^2*e)*(p
+ q + 2) + (2*f*(2*a*c^2*e)*(p + q + 2) - (2*c^2*d - c*(2*a*f))*(-(c*e*(2*p
 + q + 4))))*x + c*f*(2*c^2*d - c*(2*a*f))*(2*p + 2*q + 5)*x^2, x], x], x]
/; FreeQ[{a, c, d, e, f, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ
[a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[
q, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx = \frac{c(ae+(cd-af)x)}{a\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} - \frac{\int \frac{-2ac\left(af^2+c\left(e^2-df\right)\right)-2ac^2efx}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{2ac\left(ace^2+(cd-af)^2\right)}$$

$$= \frac{c(ae+(cd-af)x)}{a\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} - \frac{\left(f\left(2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)\right)\right)\int}{\sqrt{e^2-4df}\,\left(ace^2+(cd-\right.}$$

$$= \frac{c(ae+(cd-af)x)}{a\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} + \frac{\left(f\left(2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)\right)\right)\mathrm{Su}}{\sqrt{e^2-4df}}$$

$$= \frac{c(ae+(cd-af)x)}{a\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} - \frac{f\left(2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)\right)\tanh}{\sqrt{2}\,\sqrt{e^2-4df}\,\left(ace^2+(cd-af)^2\right)\sqrt{2}}$$

**Mathematica** [A] time = 2.06, size = 320, normalized size = 0.77

$$\frac{c(a(e-fx)+cdx)}{a\sqrt{a+cx^2}\left(a^2f^2+ac\left(e^2-2df\right)+c^2d^2\right)}-\frac{2\sqrt{2}f^3\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)}{\sqrt{e^2-4df}\left(2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)^{3/2}}+\frac{2\sqrt{2}f^3\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{a+cx^2}\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\left(2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (c*(c*d*x + a*(e - f*x)))/(a*(c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f))*Sqrt[a
 + c*x^2]) - (2*Sqrt[2]*f^3*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/
(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]
)/(Sqrt[e^2 - 4*d*f]*(2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))^(3/2
)) + (2*Sqrt[2]*f^3*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a
*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[e^
2 - 4*d*f]*(2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))^(3/2))
```

**IntegrateAlgebraic** [C] time = 0.64, size = 368, normalized size = 0.88

$$\frac{\text{RootSum}\left[\#1^4f-2\#1^3\sqrt{c}e-2\#1^2af+4\#1^2cd+2\#1a\sqrt{c}e+a^2f\&,\frac{-\#1^2cef\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)-2\#1c^{3/2}df\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+2\#1c^{3/2}e^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+acef\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+2\#1a\sqrt{c}f^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)}{2\#1^3f-3\#1^2\sqrt{c}e-2\#1af+4\#1cd+a\sqrt{c}e}\&\right]}{-a^2f^2+2acdf-ace^2-c^2d^2}+\frac{ace-acfx+c^2dx}{a\sqrt{a+cx^2}\left(a^2f^2-2acdf+ace^2+c^2d^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (a*c*e + c^2*d*x - a*c*f*x)/(a*(c^2*d^2 + a*c*e^2 - 2*a*c*d*f + a^2*f^2)*Sq
rt[a + c*x^2]) + RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2
 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (a*c*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2
] - #1] + 2*c^(3/2)*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - 2*c^(
3/2)*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + 2*a*Sqrt[c]*f^2*Log[
-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - c*e*f*Log[-(Sqrt[c]*x) + Sqrt[a +
 c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 +
 2*f*#1^3) & ]/(-(c^2*d^2) - a*c*e^2 + 2*a*c*d*f - a^2*f^2)
```

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage2
```

**maple [B]** time = 0.02, size = 1713, normalized size = 4.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] -2/(-4*d*f+e^2)^(1/2)/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)*f^2/((
x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*
f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2
)^(1/2)-4*c^2*f/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2
/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*
c-(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-
2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*x-4/(-4*d*f+e^2)^(1/2)*c^2
*f/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*
e^2-c^2/f^2*(-4*d*f+e^2))/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+
e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2
+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*e*x+2/(-4*d*f+e^2)^(1/2)/(2*a*f^2-2*c*d
*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)*f^2*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2+(-4*d*
f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+
e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2
^(1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2
*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e
^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/
2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))+2/(-4*d*f+e^2)^(1/2)/(2*a*f^2-2*c*d*f
+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)*f^2/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-(
e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*
c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)-4*c^2*f/(2*a*f^2-2*c*d*f+c*e
^2-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2
))/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e
+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c
*e)/f^2)^(1/2)*x+4/(-4*d*f+e^2)^(1/2)*c^2*f/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+
e^2)^(1/2)*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/((x-1/2*
(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^
2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1
/2)*e*x-2/(-4*d*f+e^2)^(1/2)/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)
*f^2*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln(
(-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c
*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2-
(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c
```

```
-4*(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-
2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(
1/2))/f))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(c\,x^2+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(1/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+cx^{2}\right)^{\frac{3}{2}}\left(d+ex+fx^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(1/((a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

## 3.75 $\int \frac{1}{x\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx$

**Optimal**. Leaf size=526

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{a\left(af^2+c\left(e^2-df\right)\right)+c^2dex}{ad\sqrt{a+cx^2}\,\left((cd-af)^2+ace^2\right)}+\frac{f\left(2e\left(af^2+c\left(e^2-2df\right)\right)-\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-\right.\right.\right.}{\sqrt{2}\,d\sqrt{e^2-4df}\,\left((cd-af)^2+ace^2\right)\sqrt{2af}}$$

**Rubi [A]** time = 2.18, antiderivative size = 526, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6728, 266, 51, 63, 208, 1017, 1034, 725, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{a\left(af^2+c\left(e^2-df\right)\right)+c^2dex}{ad\sqrt{a+cx^2}\,\left((cd-af)^2+ace^2\right)}+\frac{f\left(2e\left(af^2+c\left(e^2-2df\right)\right)-\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\,d\sqrt{e^2-4df}\,\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{f\left(2e\left(af^2+c\left(e^2-2df\right)\right)-\left(\sqrt{e^2-4df}+e\right)\left(af^2+c\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\,d\sqrt{e^2-4df}\,\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{1}{ad\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] 1/(a*d*Sqrt[a + c*x^2]) - (a*(a*f^2 + c*(e^2 - d*f)) + c^2*d*e*x)/(a*d*(a*c
*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2]) + (f*(2*e*(a*f^2 + c*(e^2 - 2*d*f))
- (e - Sqrt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e -
Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 -
 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d -
 a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (f*(2*e*(
a*f^2 + c*(e^2 - 2*d*f)) - (e + Sqrt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - d*f)))
*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e
^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 -
 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^
2 - 4*d*f])]) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(a^(3/2)*d)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1017

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + c*x^2)^(p + 1)*(d + e*x + f*x^2)^
(q + 1)*(g*c*(2*a*c*e) + (-(a*h))*(2*c^2*d - c*(2*a*f)) + c*(g*(2*c^2*d - c
*(2*a*f)) - h*(-2*a*c*e))*x))/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)),
 x] + Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^
(p + 1)*(d + e*x + f*x^2)^q*Simp[(-2*g*c)*((c*d - a*f)^2 - (-(a*e))*(c*e))*
(p + 1) + (2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(a*f*(p + 1) - c*d*(p + 2))
- e*((g*c)*(2*a*c*e) + (-(a*h))*(2*c^2*d - c*((Plus[2])*a*f)))*(p + q + 2)
- (2*f*((g*c)*(2*a*c*e) + (-(a*h))*(2*c^2*d - c*((Plus[2])*a*f)))*(p + q +
2) - (2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(-(c*e*(2*p + q + 4))))*x - c*f*(
2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(2*p + 2*q + 5)*x^2, x], x], x] /; Free
Q[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[
a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx = \int \left(\frac{1}{dx\left(a+cx^2\right)^{3/2}}+\frac{-e-fx}{d\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\right)dx$$

$$= \frac{\int \frac{1}{x\left(a+cx^2\right)^{3/2}}\,dx}{d}+\frac{\int \frac{-e-fx}{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx}{d}$$

$$= -\frac{a\left(af^2+c\left(e^2-df\right)\right)+c^2dex}{ad\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}}+\frac{\text{Subst}\left(\int \frac{1}{x(a+cx)^{3/2}}\,dx,x,x^2\right)}{2d}+\frac{\int \frac{-2ace(\cdots}{\cdots}}{\cdots}$$

$$= \frac{1}{ad\sqrt{a+cx^2}}-\frac{a\left(af^2+c\left(e^2-df\right)\right)+c^2dex}{ad\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx,x,x^{\cdots}\right.}{2ad}$$

$$= \frac{1}{ad\sqrt{a+cx^2}}-\frac{a\left(af^2+c\left(e^2-df\right)\right)+c^2dex}{ad\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}}+\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx,x,\sqrt{\cdots}\right.}{acd}$$

$$= \frac{1}{ad\sqrt{a+cx^2}}-\frac{a\left(af^2+c\left(e^2-df\right)\right)+c^2dex}{ad\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}}+\frac{f\left(2e\left(af^2+c\left(e^2-2df\right)\cdots\right.\right.}{\sqrt{2}\,d\sqrt{\cdots}}$$

**Mathematica [C]** time = 3.50, size = 497, normalized size = 0.94

$$\frac{-\frac{f\left(\frac{e}{\sqrt{e^2-4df}}+1\right)\left(2af+cx\left(e-\sqrt{e^2-4df}\right)\right)}{a\sqrt{a+cx^2}\left(4af^2+c\left(e-\sqrt{e^2-4df}\right)^2\right)}-\frac{f\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\left(2af+cx\left(\sqrt{e^2-4df}+e\right)\right)}{a\sqrt{a+cx^2}\left(4af^2+c\left(\sqrt{e^2-4df}+e\right)^2\right)}+\frac{\sqrt{2}f^3\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)}{\sqrt{e^2-4df}\left(2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)^{3/2}}+\frac{\sqrt{2}f^3\left(\sqrt{e^2-4df}-e\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{a+cx^2}\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\left(2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)\right)^{3/2}}+\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{cx^2}{a}+1\right)}{a\sqrt{a+cx^2}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (-((f*(1 + e/Sqrt[e^2 - 4*d*f])*(2*a*f + c*(e - Sqrt[e^2 - 4*d*f])*x))/(a*(
4*a*f^2 + c*(e - Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + c*x^2])) - (f*(1 - e/Sqrt[e
^2 - 4*d*f])*(2*a*f + c*(e + Sqrt[e^2 - 4*d*f])*x))/(a*(4*a*f^2 + c*(e + Sq
rt[e^2 - 4*d*f])^2)*Sqrt[a + c*x^2]) + (Sqrt[2]*f^3*(e + Sqrt[e^2 - 4*d*f])
*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 +
 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[e^2 - 4*d*f]*(2*a*f
^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))^(3/2)) + (Sqrt[2]*f^3*(-e + Sqr
t[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2
 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[e^2 -
4*d*f]*(2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))^(3/2)) + Hypergeom
etric2F1[-1/2, 1, 1/2, 1 + (c*x^2)/a]/(a*Sqrt[a + c*x^2]))/d
```

**IntegrateAlgebraic [C]** time = 0.95, size = 552, normalized size = 1.05

$$\frac{\text{RootSum}\left[\#1^4 f-2\#1^3\sqrt{c}\,e-2\#1^2 af+4\#1^2 cd+2\#1 a\sqrt{c}\,e+a^2 f\&,\frac{\#1^2 cdf^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)-\#1^2 ce^2 f\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)-\#1^2 af^3\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)+a^2f^3\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)-4\#1c^{3/2}def\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)+2\#1c^{3/2}e^3\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)-acdf^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)+ace^2f\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)+2\#1a\sqrt{c}\,ef^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}\,x\right)}{2\#1^3 f-3\#1^2\sqrt{c}\,e-2\#1af+4\#1cd+a\sqrt{c}\,e}\&\right]}{d\left(a^2f^2-2acdf+ace^2+c^2d^2\right)}+\frac{2\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}-\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{-acf+c^2d-c^2ex}{a\sqrt{a+cx^2}\left(a^2f^2-2acdf+ace^2+c^2d^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (c^2*d - a*c*f - c^2*e*x)/(a*(c^2*d^2 + a*c*e^2 - 2*a*c*d*f + a^2*f^2)*Sqrt
[a + c*x^2]) + (2*ArcTanh[(Sqrt[c]*x)/Sqrt[a] - Sqrt[a + c*x^2]/Sqrt[a]])/(
a^(3/2)*d) + RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2
*Sqrt[c]*e*#1^3 + f*#1^4 & , (a*c*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]
- #1] - a*c*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + a^2*f^3*Log[-(
Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + 2*c^(3/2)*e^3*Log[-(Sqrt[c]*x) + Sqrt[
a + c*x^2] - #1]*#1 - 4*c^(3/2)*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] -
#1]*#1 + 2*a*Sqrt[c]*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - c*
e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 + c*d*f^2*Log[-(Sqrt[c]
*x) + Sqrt[a + c*x^2] - #1]*#1^2 - a*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]
 - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1
^3) & ]/(d*(c^2*d^2 + a*c*e^2 - 2*a*c*d*f + a^2*f^2))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.02, size = 1945, normalized size = 3.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] 4*f^3/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/(2*a*f^2-2*c*d*f+c*e^2+(-4*
d*f+e^2)^(1/2)*c*e)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2)^(
1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d
*f+e^2)^(1/2)*c*e)/f^2)^(1/2)+8*f^2/(e+(-4*d*f+e^2)^(1/2))*c^2/(2*a*f^2-2*c
*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2
)*c^2/f^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2)^(1/2))*(x+
1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(
1/2)*c*e)/f^2)^(1/2)*x+8*f^2/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*c^2/
(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2
-(-4*d*f+e^2)*c^2/f^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2
)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-
4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*e*x-4*f^3/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e
^2)^(1/2)/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/((2*a*f^2-
2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e+(-4*d*f+e^2)^(1/2)
)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^
(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/
f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))
*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)
^(1/2)*c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))+4*f^3/(-e+(-4*d*f
+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c
*e)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-
e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*
c*e)/f^2)^(1/2)-8*f^2/(-e+(-4*d*f+e^2)^(1/2))*c^2/(2*a*f^2-2*c*d*f+c*e^2-(-
4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2)*c^2/f^2)/((
x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*
d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f
^2)^(1/2)*x+8*f^2/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*c^2/(2*a*f^2-2
*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e
^2)*c^2/f^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e-(-4*d*f+e^2)^(1/2))*
```

```
(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^
2)^(1/2)*c*e)/f^2)^(1/2)*e*x-4*f^3/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/
2)/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/((2*a*f^2-2*c*d*f
+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e-(-4*d*f+e^2)^(1/2))*(x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*
c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(
1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e-(-4*d*f+e^2)^(1/2))*(x-1
/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/
2)*c*e)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))-4*f/(-e+(-4*d*f+e^2)
^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a/(c*x^2+a)^(1/2)+4*f/(-e+(-4*d*f+e^2)^(1/2)
)/(e+(-4*d*f+e^2)^(1/2))/a^(3/2)*ln((2*a+2*(c*x^2+a)^(1/2)*a^(1/2))/x)
```

**maxima** **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{{\left(cx^{2} + a\right)}^{\frac{3}{2}} {\left(fx^{2} + ex + d\right)} x}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate(1/((c*x^2 + a)^(3/2)*(f*x^2 + e*x + d)*x), x)
```

**mupad** **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\,{\left(c\,x^2+a\right)}^{3/2}\,\left(f\,x^2+e\,x+d\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
[Out] int(1/(x*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

**sympy** **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \left(a + cx^{2}\right)^{\frac{3}{2}} \left(d + ex + fx^{2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
[Out] Integral(1/(x*(a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

## 3.76 $\int \frac{1}{x^2(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

**Optimal**. Leaf size=618

$$\frac{e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^2}-\frac{2cx}{a^2d\sqrt{a+cx^2}}+\frac{cdx\left(af^2+c\left(e^2-df\right)\right)+ae\left(af^2+c\left(e^2-2df\right)\right)}{ad^2\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}+\frac{f\left(e\left(e-\sqrt{e^2-4df}\right)\left(af^2 \ldots\right.\right.}{\ldots}$$

**Rubi [A]** time = 2.28, antiderivative size = 618, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {6728, 271, 191, 266, 51, 63, 208, 1017, 1034, 725, 206}

$$\frac{e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^2}-\frac{2cx}{a^2d\sqrt{a+cx^2}}+\frac{cdx\left(af^2+c\left(e^2-df\right)\right)+ae\left(af^2+c\left(e^2-2df\right)\right)}{ad^2\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}+\frac{f\left(e\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-2df\right)\right)-2\left(af^2\left(e^2-df\right)+c\left(d^2f^2-3de^2f+e^4\right)\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{f\left(e\left(\sqrt{e^2-4df}+e\right)\left(af^2+c\left(e^2-2df\right)\right)-2\left(af^2\left(e^2-df\right)+c\left(d^2f^2-3de^2f+e^4\right)\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{e}{ad^2\sqrt{a+cx^2}}-\frac{1}{adx\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] -(e/(a*d^2*Sqrt[a + c*x^2])) - 1/(a*d*x*Sqrt[a + c*x^2]) - (2*c*x)/(a^2*d*S
qrt[a + c*x^2]) + (a*e*(a*f^2 + c*(e^2 - 2*d*f)) + c*d*(a*f^2 + c*(e^2 - d*
f))*x)/(a*d^2*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2]) + (f*(e*(e - Sqrt[
e^2 - 4*d*f])*(a*f^2 + c*(e^2 - 2*d*f)) - 2*(a*f^2*(e^2 - d*f) + c*(e^4 - 3
*d*e^2*f + d^2*f^2)))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2
]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/
(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(
e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (f*(e*(e + Sqrt[e^2 - 4*d*f])*(a*f^2
 + c*(e^2 - 2*d*f)) - 2*(a*f^2*(e^2 - d*f) + c*(e^4 - 3*d*e^2*f + d^2*f^2))
)*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(
e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[2]*d^2*Sqrt[e^
2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt
[e^2 - 4*d*f])]) + (e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(a^(3/2)*d^2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1017

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + c*x^2)^(p + 1)*(d + e*x + f*x^2)^
```

```
(q + 1)*(g*c*(2*a*c*e) + (-(a*h))*(2*c^2*d - c*(2*a*f)) + c*(g*(2*c^2*d - c
*(2*a*f)) - h*(-2*a*c*e))*x))/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)),
 x] + Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^
(p + 1)*(d + e*x + f*x^2)^q*Simp[(-2*g*c)*((c*d - a*f)^2 - (-(a*e))*(c*e))*
(p + 1) + (2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(a*f*(p + 1) - c*d*(p + 2))
- e*((g*c)*(2*a*c*e) + (-(a*h))*(2*c^2*d - c*((Plus[2])*a*f)))*(p + q + 2)
- (2*f*((g*c)*(2*a*c*e) + (-(a*h))*(2*c^2*d - c*((Plus[2])*a*f)))*(p + q +
2) - (2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(-(c*e*(2*p + q + 4))))*x - c*f*(
2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(2*p + 2*q + 5)*x^2, x], x], x] /; Free
Q[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[
a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^2 \left(a + cx^2\right)^{3/2} \left(d + ex + fx^2\right)} dx = \int \left( \frac{1}{dx^2 \left(a + cx^2\right)^{3/2}} - \frac{e}{d^2 x \left(a + cx^2\right)^{3/2}} + \frac{e^2 - df + efx}{d^2 \left(a + cx^2\right)^{3/2} \left(d + ex + fx^2\right)} \right)$$

$$= \frac{\int \frac{e^2-df+efx}{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)} dx}{d^2} + \frac{\int \frac{1}{x^2\left(a+cx^2\right)^{3/2}} dx}{d} - \frac{e \int \frac{1}{x\left(a+cx^2\right)^{3/2}} dx}{d^2}$$

$$= -\frac{1}{adx\sqrt{a + cx^2}} + \frac{ae\left(af^2 + c\left(e^2 - 2df\right)\right) + cd\left(af^2 + c\left(e^2 - df\right)\right)x}{ad^2\left(ace^2 + (cd - af)^2\right)\sqrt{a + cx^2}} - \text{—} \quad (2$$

$$= -\frac{e}{ad^2\sqrt{a + cx^2}} - \frac{1}{adx\sqrt{a + cx^2}} - \frac{2cx}{a^2 d\sqrt{a + cx^2}} + \frac{ae\left(af^2 + c\left(e^2 - 2df\right)\right.}{ad^2\left(ace^2 + (c\right.}$$

$$= -\frac{e}{ad^2\sqrt{a + cx^2}} - \frac{1}{adx\sqrt{a + cx^2}} - \frac{2cx}{a^2 d\sqrt{a + cx^2}} + \frac{ae\left(af^2 + c\left(e^2 - 2df\right)\right.}{ad^2\left(ace^2 + (c\right.}$$

$$= -\frac{e}{ad^2\sqrt{a + cx^2}} - \frac{1}{adx\sqrt{a + cx^2}} - \frac{2cx}{a^2 d\sqrt{a + cx^2}} + \frac{ae\left(af^2 + c\left(e^2 - 2df\right)\right.}{ad^2\left(ace^2 + (c\right.}$$

**Mathematica [C]** time = 3.60, size = 557, normalized size = 0.90

$$-\frac{\frac{d\left(a+2cx^2\right)}{a^2 x\sqrt{a+cx^2}} - \frac{f\left(\frac{e^2-2df}{\sqrt{e^2-4df}}+e\right)\left(2af+cx\left(e-\sqrt{e^2-4df}\right)\right)}{a\sqrt{a+cx^2}\left(4af^2+c\left(e-\sqrt{e^2-4df}\right)^2\right)} - \frac{f\left(\frac{2df-e^2}{\sqrt{e^2-4df}}+e\right)\left(2af+cx\left(\sqrt{e^2-4df}+e\right)\right)}{a\sqrt{a+cx^2}\left(4af^2+c\left(\sqrt{e^2-4df}+e\right)^2\right)} + \frac{\sqrt{2}f^3\left(e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)}{\sqrt{e^2-4df}\left(2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)^{3/2}} + \frac{\sqrt{2}f^3\left(e\sqrt{e^2-4df}+2df-e^2\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{a+cx^2}\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\left(2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)\right)^{3/2}} + \frac{e\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{cx^2}{a}+1\right)}{a\sqrt{a+cx^2}}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] -((-((f*(e + (e^2 - 2*d*f)/Sqrt[e^2 - 4*d*f])*(2*a*f + c*(e - Sqrt[e^2 - 4*
d*f])*x))/(a*(4*a*f^2 + c*(e - Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + c*x^2])) - (f
*(e + (-e^2 + 2*d*f)/Sqrt[e^2 - 4*d*f])*(2*a*f + c*(e + Sqrt[e^2 - 4*d*f])*
x))/(a*(4*a*f^2 + c*(e + Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + c*x^2]) + (d*(a + 2
*c*x^2))/(a^2*x*Sqrt[a + c*x^2]) + (Sqrt[2]*f^3*(e^2 - 2*d*f + e*Sqrt[e^2 -
 4*d*f])*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c
*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[e^2 - 4*d*f
]*(2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))^(3/2)) + (Sqrt[2]*f^3*(
-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*
```

```
f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*
x^2])])/(Sqrt[e^2 - 4*d*f]*(2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])
)^(3/2)) + (e*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (c*x^2)/a])/(a*Sqrt[a + c
*x^2]))/d^2)
```

**IntegrateAlgebraic [C]** time = 1.29, size = 720, normalized size = 1.17

$$-\frac{\text{RootSum}\left[\#1^4 f-2\#1^3\sqrt{c}e-2\#1^2af+4\#1^2cd+2\#1a\sqrt{c}e+a^2f\&,\frac{2\#1^2cdef^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)-\#1^2ce^3f\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)-\#1^2aef^3\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+a^2ef^3\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+2\#1c^{3/2}d^2f^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)-6\#1c^{3/2}de^2f\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+2\#1c^{3/2}e^4\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)-2acdef^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)-2\#1a\sqrt{c}df^3\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+ace^3f\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)+2\#1a\sqrt{c}e^2f^2\log\left(-\#1+\sqrt{a+cx^2}-\sqrt{c}x\right)}{2\#1^3f-3\#1^2\sqrt{c}e-2\#1af+4\#1cd+a\sqrt{c}e}\&\right]}{d^2\left(a^2f^2-2acdf+ace^2+c^2d^2\right)}-\frac{2e\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}-\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^2}+\frac{a^3\left(-f^2\right)+2a^2cdf-a^2ce^2-a^2cf^2x^2-ac^2d^2-ac^2dex+3ac^2dfx^2-ac^2e^2x^2-2c^3d^2x^2}{a^2dx\sqrt{a+cx^2}\left(a^2f^2-2acdf+ace^2+c^2d^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (-(a*c^2*d^2) - a^2*c*e^2 + 2*a^2*c*d*f - a^3*f^2 - a*c^2*d*e*x - 2*c^3*d^2
*x^2 - a*c^2*e^2*x^2 + 3*a*c^2*d*f*x^2 - a^2*c*f^2*x^2)/(a^2*d*(c^2*d^2 + a
*c*e^2 - 2*a*c*d*f + a^2*f^2)*x*Sqrt[a + c*x^2]) - (2*e*ArcTanh[(Sqrt[c]*x)
/Sqrt[a] - Sqrt[a + c*x^2]/Sqrt[a]])/(a^(3/2)*d^2) - RootSum[a^2*f + 2*a*Sq
rt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (a*c*e
^3*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - 2*a*c*d*e*f^2*Log[-(Sqrt[c]
*x) + Sqrt[a + c*x^2] - #1] + a^2*e*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]
- #1] + 2*c^(3/2)*e^4*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - 6*c^(3/
2)*d*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + 2*c^(3/2)*d^2*f^2*
Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + 2*a*Sqrt[c]*e^2*f^2*Log[-(Sqr
t[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - 2*a*Sqrt[c]*d*f^3*Log[-(Sqrt[c]*x) + S
qrt[a + c*x^2] - #1]*#1 - c*e^3*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*
#1^2 + 2*c*d*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 - a*e*f^3*
Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*
a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]/(d^2*(c^2*d^2 + a*c*e^2 - 2*a*c*d
*f + a^2*f^2))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$sage_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

[Out] sage2

**maple [B]** time = 0.02, size = 2046, normalized size = 3.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] -8*f^4/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)/(2*a*f^2-2*c*d*f+c*e^2+(
-4*d*f+e^2)^(1/2)*c*e)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2
)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-
4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)-16*f^3/(e+(-4*d*f+e^2)^(1/2))^2*c^2/(2*a*f
^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d
*f+e^2)*c^2/f^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2)^(1/2
))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+
e^2)^(1/2)*c*e)/f^2)^(1/2)*x-16*f^3/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(
1/2)*c^2/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^
2*e^2/f^2-(-4*d*f+e^2)*c^2/f^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-
4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f
+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*e*x+8*f^4/(e+(-4*d*f+e^2)^(1/2))^
2/(-4*d*f+e^2)^(1/2)/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)
/((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e+(-4*d*f
+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-
4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^
(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+
e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(
-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))+4*f/(-
e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a/x/(c*x^2+a)^(1/2)+8*f/(-e+(-
4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*c/a^2*x/(c*x^2+a)^(1/2)+8*f^4/(-e+
(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2
)^(1/2)*c*e)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e-(-4*d*f+e^2)^(1/2))*
(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^
2)^(1/2)*c*e)/f^2)^(1/2)-16*f^3/(-e+(-4*d*f+e^2)^(1/2))^2*c^2/(2*a*f^2-2*c*
d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2)
*c^2/f^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e-(-4*d*f+e^2)^(1/2))*(x-
1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^
(1/2)*c*e)/f^2)^(1/2)*x+16*f^3/(-e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)
*c^2/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^
2/f^2-(-4*d*f+e^2)*c^2/f^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e-(-4*d
*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c
*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*e*x-8*f^4/(-e+(-4*d*f+e^2)^(1/2))^2
/(-4*d*f+e^2)^(1/2)/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/
((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e-(-4*d*f+
e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-
```

```
4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^
(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e-(-4*d*f
+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2
-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))-16*
f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+(-4*d*f+e^2)^(1/2))^2/a/(c*x^2+a)^(1/2)+
16*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+(-4*d*f+e^2)^(1/2))^2/a^(3/2)*ln((2*a
+2*(c*x^2+a)^(1/2)*a^(1/2))/x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^2+a\right)^{\frac{3}{2}}\left(fx^2+ex+d\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)^(3/2)*(f*x^2 + e*x + d)*x^2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\left(c\,x^2+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(1/(x^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(1/(x**2*(a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

## 3.77 $\int \frac{x^3\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$

**Optimal**. Leaf size=392

$$-\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}f}+\frac{b(b+2cx)\sqrt{a+bx+cx^2}}{8c^2f}-\frac{d\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{-2a\sqrt{f}+ \cdots}{2\sqrt{a+bx+cx^2} \cdots}\right.}{2f^{5/2}}$$

**Rubi** [A] time = 0.95, antiderivative size = 392, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {6725, 640, 612, 621, 206, 1021, 1078, 1033, 724}

$$-\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}f}+\frac{b(b+2cx)\sqrt{a+bx+cx^2}}{8c^2f}-\frac{d\sqrt{a+bx+cx^2}}{f^2}-\frac{bd\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2}-\frac{d\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^{5/2}}+\frac{d\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^{5/2}}-\frac{\left(a+bx+cx^2\right)^{3/2}}{3cf}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] -((d*Sqrt[a + b*x + c*x^2])/f^2) + (b*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(8
*c^2*f) - (a + b*x + c*x^2)^(3/2)/(3*c*f) - (b*d*ArcTanh[(b + 2*c*x)/(2*Sqr
t[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*f^2) - (b*(b^2 - 4*a*c)*ArcTanh[(b
 + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(5/2)*f) - (d*Sqrt[c*d
- b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d]
- b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2
])])/(2*f^(5/2)) + (d*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d
] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt
[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1021

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3\sqrt{a+bx+cx^2}}{d-fx^2}\,dx &= \int \left(-\frac{x\sqrt{a+bx+cx^2}}{f}+\frac{dx\sqrt{a+bx+cx^2}}{f\left(d-fx^2\right)}\right)dx \\
&= -\frac{\int x\sqrt{a+bx+cx^2}\,dx}{f}+\frac{d\int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2}\,dx}{f} \\
&= -\frac{d\sqrt{a+bx+cx^2}}{f^2}-\frac{\left(a+bx+cx^2\right)^{3/2}}{3cf}+\frac{d\int \frac{\frac{bd}{2}+(cd+af)x+\frac{1}{2}bfx^2}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{f^2}+\frac{b\int \sqrt{a+bx+cx^2 \cdots}}{2cf} \\
&= -\frac{d\sqrt{a+bx+cx^2}}{f^2}+\frac{b(b+2cx)\sqrt{a+bx+cx^2}}{8c^2f}-\frac{\left(a+bx+cx^2\right)^{3/2}}{3cf}-\frac{d\int \frac{-bdf-f(cd+a \cdots}{\sqrt{a+bx+cx^2}\left(d- \cdots\right.}}{f^3} \\
&= -\frac{d\sqrt{a+bx+cx^2}}{f^2}+\frac{b(b+2cx)\sqrt{a+bx+cx^2}}{8c^2f}-\frac{\left(a+bx+cx^2\right)^{3/2}}{3cf}-\frac{(bd)\,\mathrm{Subst}\left(\int \frac{\cdots}{4c \cdots}\right.}{\cdots} \\
&= -\frac{d\sqrt{a+bx+cx^2}}{f^2}+\frac{b(b+2cx)\sqrt{a+bx+cx^2}}{8c^2f}-\frac{\left(a+bx+cx^2\right)^{3/2}}{3cf}-\frac{bd\tanh^{-1}\left(\frac{\cdots}{2\sqrt{c}\cdots}\right.}{2\sqrt{c}\,f^2 \cdots} \\
&= -\frac{d\sqrt{a+bx+cx^2}}{f^2}+\frac{b(b+2cx)\sqrt{a+bx+cx^2}}{8c^2f}-\frac{\left(a+bx+cx^2\right)^{3/2}}{3cf}-\frac{bd\tanh^{-1}\left(\frac{\cdots}{2\sqrt{c}\cdots}\right.}{2\sqrt{c}\,f^2 \cdots}
\end{aligned}
$$

**Mathematica [A]** time = 0.89, size = 327, normalized size = 0.83

$$
\frac{-\frac{2\sqrt{f}\sqrt{a+x(b+cx)}\left(2cf(4a+bx)-3b^2f+8c^2\left(3d+fx^2\right)\right)}{c^2}-\frac{3b\sqrt{f}\left(-4acf+b^2f+8c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{c^{5/2}}+24d\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)-24d\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{48f^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] ((-2*Sqrt[f]*Sqrt[a + x*(b + c*x)]*(-3*b^2*f + 2*c*f*(4*a + b*x) + 8*c^2*(3
*d + f*x^2)))/c^2 - (3*b*Sqrt[f]*(8*c^2*d + b^2*f - 4*a*c*f)*ArcTanh[(b + 2
```

```
*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/c^(5/2) + 24*d*Sqrt[c*d + b*Sqrt[
d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt
[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] - 24*
d*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x
 + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a +
 x*(b + c*x)])])/(48*f^(5/2))
```

**IntegrateAlgebraic [C]** time = 1.05, size = 414, normalized size = 1.06

$$-\frac{d\text{RootSum}\left[\#1^4(-f)+2\#1^2af+4\#1^2cd-4\#1b\sqrt{c}\,d-a^2f+b^2d\&,\frac{\#1^2cd\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)+\#1^2af\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)+a^2(-f)\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)+b^2d\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)-acd\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)-2\#1b\sqrt{c}\,d\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)}{\#1^3f-\#1af-2\#1cd+b\sqrt{c}\,d}\&\right]}{2f^2}+\frac{\left(-4abcf+b^3f+8bc^2d\right)\log\left(-2\sqrt{c}\sqrt{a+bx+cx^2}+b+2cx\right)}{16c^{5/2}f^2}+\frac{\sqrt{a+bx+cx^2}\left(-8acf+3b^2f-2bcfx-24c^2d-8c^2fx^2\right)}{24c^2f^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] (Sqrt[a + b*x + c*x^2]*(-24*c^2*d + 3*b^2*f - 8*a*c*f - 2*b*c*f*x - 8*c^2*f
*x^2))/(24*c^2*f^2) + ((8*b*c^2*d + b^3*f - 4*a*b*c*f)*Log[b + 2*c*x - 2*Sq
rt[c]*Sqrt[a + b*x + c*x^2]])/(16*c^(5/2)*f^2) - (d*RootSum[b^2*d - a^2*f -
 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (b^2*d*Log[-(Sqrt[
c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x
 + c*x^2] - #1] - a^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*
b*Sqrt[c]*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + c*d*Log[-(S
qrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + a*f*Log[-(Sqrt[c]*x) + Sqrt[
a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ]
)/(2*f^2)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

**maple [B]** time = 0.02, size = 1817, normalized size = 4.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -1/3*(c*x^2+b*x+a)^(3/2)/c/f+1/4/f*b/c*x*(c*x^2+b*x+a)^(1/2)+1/8/f*b^2/c^2*
(c*x^2+b*x+a)^(1/2)+1/4/f*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1
/2))*a-1/16/f*b^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/2/f
^2*d*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+
c*d-(d*f)^(1/2)*b)/f)^(1/2)+1/2/f^3*d*ln((1/2/f*(b*f-2*(d*f)^(1/2)*c)+c*(x+
(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*
f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*(d*f)^(1/2)-1/4/f^2
*d*ln((1/2/f*(b*f-2*(d*f)^(1/2)*c)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(
1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*
b)/f)^(1/2))/c^(1/2)*b-1/2/f^3*d/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a
*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c
*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+
(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*(d*f)
^(1/2)*b+1/2/f^2*d/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(
1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)
*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)
/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*a+1/2/f^3*d^2/((a*f
+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(
1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(
1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*
b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*c-1/2/f^2*d*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(
d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)-1/2/f^3*
d*ln((1/2*(b*f+2*(d*f)^(1/2)*c)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1
/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b
)/f)^(1/2))*c^(1/2)*(d*f)^(1/2)-1/4/f^2*d*ln((1/2*(b*f+2*(d*f)^(1/2)*c)/f+c
*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x
-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b+1/2/f^3*d/((a
*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)
^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)
^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2
)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*(d*f)^(1/2)*b+1/2/f^2*d/((a*f+c*d+(d*f)^(
1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(
d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+
(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))
/(x-(d*f)^(1/2)/f))*a+1/2/f^3*d^2/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(
a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+
c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x
-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                    /(2*f)                  +(c*d)/f+a))
     /f^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\sqrt{c\,x^2+b\,x+a}}{d-f\,x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x + c*x^2)^(1/2))/(d - f*x^2),x)
```

```
[Out] int((x^3*(a + b*x + c*x^2)^(1/2))/(d - f*x^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^{3} \sqrt{a + b x + c x^{2}}}{- d + f x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x**3*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)
```

## 3.78 $\int \frac{x^2 \sqrt{a+bx+cx^2}}{d-fx^2} \, dx$

**Optimal**. Leaf size=316

$$-\frac{\left(4acf+b^2(-f)+8c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}f^2}+\frac{\sqrt{d}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)}}\right.}{2f^2}$$

**Rubi** [A] time = 0.49, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1071, 1078, 621, 206, 1033, 724}

$$-\frac{\left(4acf+b^2(-f)+8c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}f^2}+\frac{\sqrt{d}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^2}+\frac{\sqrt{d}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^2}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] -((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c*f) - ((8*c^2*d - b^2*f + 4*a*c*f)
*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2)*f^2) +
(Sqrt[d]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[
f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sq
rt[a + b*x + c*x^2])]])/(2*f^2) + (Sqrt[d]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*
f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[
c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]])/(2*f^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1071

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) +
(f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*
(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x^
2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*(C*(-(b*f))*(q + 1)) + (p + q + 1)*(b
^2*C*d*f*p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*
f)*(C*(-(b*f))*(q + 1)) + (p + q + 1)*(-(b*c*(C*(-4*d*f)*(2*p + q + 2) + f*
(2*C*d + 2*A*f)*(2*p + 2*q + 3)))))*x + (p*(-(b*f))*(C*(-(b*f))*(q + 1)) +
(p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C
*d + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, A,
C, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2
*p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rubi steps

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf} - \frac{\int \frac{-\frac{1}{4}\left(b^2+4ac\right)df-2bcdfx-\frac{1}{4}f\left(8c^2d-b^2f+4acf\right)x^2}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{2cf^2}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf} + \frac{\int \frac{\frac{1}{4}\left(b^2+4ac\right)df^2+\frac{1}{4}df\left(8c^2d-b^2f+4acf\right)+2bcdf^2x}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{2cf^3} - \frac{\left(8c^2d-b^2f\right.}{}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf} - \frac{\left(\sqrt{d}\left(cd-b\sqrt{d}\sqrt{f}+af\right)\right)\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2f^{3/2}} +$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf} - \frac{\left(8c^2d-b^2f+4acf\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}f^2} + \frac{\left(\sqrt{d}\left(cd\right.\right.}{}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf} - \frac{\left(8c^2d-b^2f+4acf\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}f^2} + \frac{\sqrt{d}\sqrt{cd\ldots}}{}$$

**Mathematica** [A] time = 0.49, size = 302, normalized size = 0.96

$$\frac{\left(-4acf+b^2f-8c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-2\sqrt{c}\left(-2c\sqrt{d}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)-2c\sqrt{d}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)+f(b+2cx)\sqrt{a+x(b+cx)}\right)}{8c^{3/2}f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] ((-8*c^2*d + b^2*f - 4*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b
+ c*x)])] - 2*Sqrt[c]*(f*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] - 2*c*Sqrt[d]*Sq
rt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sq
rt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b
 + c*x)])] - 2*c*Sqrt[d]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(-2*a*
Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*
Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]))/(8*c^(3/2)*f^2)
```

**IntegrateAlgebraic** [C] time = 0.83, size = 327, normalized size = 1.03

$$-\frac{d\,\text{RootSum}\left[\#1^4(-f)+2\#1^2af+4\#1^2cd-4\#1b\sqrt{c}d-a^2f+b^2d\&,\frac{\#1^2bf\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2\#1c^{3/2}d\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+bcd\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2\#1a\sqrt{c}f\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)}{\#1^3f-\#1af-2\#1cd+b\sqrt{c}d}\&\right]}{2f^2}+\frac{\left(4acf+b^2(-f)+8c^2d\right)\log\left(-2c^{3/2}\sqrt{a+bx+cx^2}+bc+2c^2x\right)}{8c^{3/2}f^2}+\frac{(-b-2cx)\sqrt{a+bx+cx^2}}{4cf}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] ((-b - 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c*f) + ((8*c^2*d - b^2*f + 4*a*c*f)
*Log[b*c + 2*c^2*x - 2*c^(3/2)*Sqrt[a + b*x + c*x^2]])/(8*c^(3/2)*f^2) - (d
*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^
4 & , (b*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*c^(3/2)*d*L
og[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*a*Sqrt[c]*f*Log[-(Sqrt
[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + b*f*Log[-(Sqrt[c]*x) + Sqrt[a + b
*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ])/(2*
f^2)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

**maple [B]** time = 0.02, size = 1810, normalized size = 5.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -1/2/f*x*(c*x^2+b*x+a)^(1/2)-1/4/f/c*(c*x^2+b*x+a)^(1/2)*b-1/2/f/c^(1/2)*ln
((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+1/8/f/c^(3/2)*ln((c*x+1/2*b)/c^
(1/2)+(c*x^2+b*x+a)^(1/2))*b^2+1/2*d/(d*f)^(1/2)/f*((x+(d*f)^(1/2)/f)^2*c+(
b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)-1
/2*d/f^2*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(
d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^
(1/2)*b)/f)^(1/2))*c^(1/2)+1/4*d/(d*f)^(1/2)/f*ln(((x+(d*f)^(1/2)/f)*c+1/2*
(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*
c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b+1/2*d/f^
2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*
```

```
(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+
(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)
^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*b-1/2*d/(d*f)^(1/2)/f/((a*f+c*d-(d*f
)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(
x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2
*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/
2))/(x+(d*f)^(1/2)/f))*a-1/2*d^2/(d*f)^(1/2)/f^2/((a*f+c*d-(d*f)^(1/2)*b)/f
)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2
)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d
*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)
^(1/2)/f))*c-1/2*d/(d*f)^(1/2)/f*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*
c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)-1/2*d/f^2*ln(((x-(d
*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+
(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))
*c^(1/2)-1/4*d/(d*f)^(1/2)/f*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)
*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/
f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b+1/2*d/f^2/((a*f+c*d+(d*f)^
(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-
(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c
+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)
)/(x-(d*f)^(1/2)/f))*b+1/2*d/(d*f)^(1/2)/f/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2
)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f
+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1
/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)
/f))*a+1/2*d^2/(d*f)^(1/2)/f^2/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f
+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d
+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d
*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                    /(2*f)                   +(c*d)/f+a))
     /f^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\sqrt{c\,x^2+b\,x+a}}{d-f\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x + c*x^2)^(1/2))/(d - f*x^2),x)
[Out] int((x^2*(a + b*x + c*x^2)^(1/2))/(d - f*x^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
[Out] -Integral(x**2*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)
```

## 3.79 $\int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$

**Optimal**. Leaf size=282

$$-\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^{3/2}}+\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+\right.}{2\sqrt{a+bx+cx^2}\sqrt{a}}\right.}{2f^{3/2}}$$

**Rubi** [A] time = 0.29, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1021, 1078, 621, 206, 1033, 724}

$$-\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^{3/2}}+\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^{3/2}}-\frac{\sqrt{a+bx+cx^2}}{f}-\frac{b\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] -(Sqrt[a + b*x + c*x^2]/f) - (b*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x
 + c*x^2])])/(2*Sqrt[c]*f) - (Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(
b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt
[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(3/2)) + (Sqrt[c*d + b*Sqr
t[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqr
t[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2
*f^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1021

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rubi steps

$$\int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = -\frac{\sqrt{a+bx+cx^2}}{f} + \frac{\int \frac{\frac{bd}{2}+(cd+af)x+\frac{1}{2}bfx^2}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx}{f}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{f} - \frac{\int \frac{-bdf-f(cd+af)x}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx}{f^2} - \frac{b\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2f}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{f} - \frac{b\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f} + \frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)\int \frac{\quad}{(-\sqrt{d}\sqrt{\cdots}}}{2f}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{f} - \frac{b\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f} - \frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)\mathrm{Subst}\left(\int \frac{\quad}{4cdf-4b\sqrt{\cdots}}\right.}{\quad}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{f} - \frac{b\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f} - \frac{\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\,\tanh^{-1}\left(\frac{b\sqrt{d}-2a\cdots}{2\sqrt{cd-b\sqrt{d}\cdots}}\right.}{2f^{3/2}}$$

**Mathematica** [A] time = 0.29, size = 272, normalized size = 0.96

$$\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)-\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)-2\sqrt{f}\sqrt{a+x(b+cx)}-\frac{b\sqrt{f}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}}}{2f^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] (-2*Sqrt[f]*Sqrt[a + x*(b + c*x)] - (b*Sqrt[f]*ArcTanh[(b + 2*c*x)/(2*Sqrt[
c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*A
rcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d
+ b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] - Sqrt[c*d - b*Sqrt[d]*S
qrt[f] + a*f]*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*
x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(2*f^(3
/2))
```

**IntegrateAlgebraic** [C] time = 0.58, size = 361, normalized size = 1.28

$$-\frac{\mathrm{RootSum}\left[\#1^4(-f)+2\#1^2af+4\#1^2cd-4\#1b\sqrt{c}d-a^2f+b^2d\&,\frac{\#1^2cd\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+\#1^2af\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+a^2(-f)\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+b^2d\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-acd\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2\#1b\sqrt{c}d\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)}{\#1^3f-\#1af-2\#1cd+b\sqrt{c}d}\&\right]}{2f}-\frac{\sqrt{a+bx+cx^2}}{f}+\frac{b\log\left(-2\sqrt{c}f\sqrt{a+bx+cx^2}+bf+2cfx\right)}{2\sqrt{c}f}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] -(Sqrt[a + b*x + c*x^2]/f) + (b*Log[b*f + 2*c*f*x - 2*Sqrt[c]*f*Sqrt[a + b*
x + c*x^2]])/(2*Sqrt[c]*f) - RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c
*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (b^2*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x +
c*x^2] - #1] - a*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a^2*f
*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*b*Sqrt[c]*d*Log[-(Sqrt[
c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*
x + c*x^2] - #1]*#1^2 + a*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*
#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ]/(2*f)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

**maple [B]** time = 0.01, size = 1667, normalized size = 5.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -1/2/f*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*
f+c*d-(d*f)^(1/2)*b)/f)^(1/2)+1/2/f^2*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d
*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*
f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*(d*f)^(1/2)-1/4/f*l
n(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)
/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f
)^(1/2))/c^(1/2)*b-1/2/f^2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d
-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*
f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^
(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*(d*f)^(1/2)
```

```
*b+1/2/f/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+
(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/
2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*
d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*a+1/2/f^2/((a*f+c*d-(d*f)^(1/
2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*
f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b
*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(
x+(d*f)^(1/2)/f))*c*d-1/2/f*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x
-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)-1/2/f^2*ln(((x-(d*f)^(1/
2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*
(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2
)*(d*f)^(1/2)-1/4/f*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^
(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f
+c*d+(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b+1/2/f^2/((a*f+c*d+(d*f)^(1/2)*b)/f)
^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)
/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*
f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^
(1/2)/f))*(d*f)^(1/2)*b+1/2/f/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+
c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+
(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*
f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a+1/2/f^
2/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*
(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-
(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)
^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c*d
```

**maxima** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                     /(2*f)                   +(c*d)/f+a))
     /f^2 positive, negative or zero?
```

**mupad** [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\sqrt{c\,x^2+b\,x+a}}{d-f\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x + c*x^2)^(1/2))/(d - f*x^2),x)
```

```
[Out] int((x*(a + b*x + c*x^2)^(1/2))/(d - f*x^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)
```

## 3.80 $\int \frac{\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$

**Optimal**. Leaf size=266

$$\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\,\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}\,f}+\frac{\sqrt{af+b\sqrt{d}\,\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\right.}{2\sqrt{a+bx+cx^2}\,\sqrt{af+}}\right.}{2\sqrt{d}\,f}$$

**Rubi** [A] time = 0.23, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {990, 621, 206, 1033, 724}

$$\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\,\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}\,f}+\frac{\sqrt{af+b\sqrt{d}\,\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\,\sqrt{af+b\sqrt{d}\,\sqrt{f}+cd}}\right)}{2\sqrt{d}\,f}-\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(d - f*x^2),x]
```

```
[Out] -((Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f) + (Sq
rt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*S
qrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x
+ c*x^2])])/(2*Sqrt[d]*f) + (Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b
*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[
d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 990

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (f_.)*(x_)^2), x_Symbol]
 :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*
f - b*f*x)/(Sqrt[a + b*x + c*x^2]*(d + f*x^2)), x], x] /; FreeQ[{a, b, c, d
, f}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx+cx^2}}{d-fx^2}\,dx &= \frac{\int \frac{cd+af+bfx}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{f} - \frac{c\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{f}\\
&= -\frac{(2c)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f} + \frac{1}{2}\left(b-\frac{cd+af}{\sqrt{d}\sqrt{f}}\right)\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+c}}\\
&= -\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f} + \left(-b-\frac{cd+af}{\sqrt{d}\sqrt{f}}\right)\text{Subst}\left(\int \frac{1}{4cdf+4b\sqrt{d}f^{3/2}+4af^2-x^2}\right.\\
&= -\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f} + \frac{\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}f}
\end{aligned}$$

**Mathematica [A]** time = 0.17, size = 253, normalized size = 0.95

$$\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\sqrt{d}-b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)+\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)-2\sqrt{c}\sqrt{d}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{2\sqrt{d}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(d - f*x^2),x]
```

```
[Out] (-2*Sqrt[c]*Sqrt[d]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]
+ Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + 2*
c*Sqrt[d]*x - b*Sqrt[f]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a +
x*(b + c*x)])] + Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2
*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] +
 a*f]*Sqrt[a + x*(b + c*x)])])/(2*Sqrt[d]*f)
```

**IntegrateAlgebraic [C]** time = 0.42, size = 273, normalized size = 1.03

$$\frac{\sqrt{c}\log\left(-2\sqrt{c}f\sqrt{a+bx+cx^2}+bf+2cfx\right)}{f}-\frac{\text{RootSum}\left[\#1^4(-f)+2\#1^2af+4\#1^2cd-4\#1b\sqrt{c}d-a^2f+b^2d\&,\frac{\#1^2bf\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2\#1c^{3/2}d\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+bcd\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2\#1a\sqrt{c}f\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)}{\#1^3f-\#1af-2\#1cd+b\sqrt{c}d}\&\right]}{2f}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x + c*x^2]/(d - f*x^2),x]
```

```
[Out] (Sqrt[c]*Log[b*f + 2*c*f*x - 2*Sqrt[c]*f*Sqrt[a + b*x + c*x^2]])/f - RootSu
m[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (
b*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*c^(3/2)*d*Log[-(Sq
rt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*a*Sqrt[c]*f*Log[-(Sqrt[c]*x)
+ Sqrt[a + b*x + c*x^2] - #1]*#1 + b*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*
x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ]/(2*f)
```

**fricas [B]** time = 98.76, size = 1139, normalized size = 4.28

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/4*(f*sqrt((d*f^2*sqrt(b^2/(d*f^3)) + c*d + a*f)/(d*f^2))*log((2*b*c*x +
2*sqrt(c*x^2 + b*x + a)*b*f*sqrt((d*f^2*sqrt(b^2/(d*f^3)) + c*d + a*f)/(d*f
^2)) + b^2 + (b*f^2*x + 2*a*f^2)*sqrt(b^2/(d*f^3)))/x) - f*sqrt((d*f^2*sqrt
(b^2/(d*f^3)) + c*d + a*f)/(d*f^2))*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*
b*f*sqrt((d*f^2*sqrt(b^2/(d*f^3)) + c*d + a*f)/(d*f^2)) + b^2 + (b*f^2*x +
2*a*f^2)*sqrt(b^2/(d*f^3)))/x) + f*sqrt(-(d*f^2*sqrt(b^2/(d*f^3)) - c*d - a
*f)/(d*f^2))*log((2*b*c*x + 2*sqrt(c*x^2 + b*x + a)*b*f*sqrt(-(d*f^2*sqrt(b
^2/(d*f^3)) - c*d - a*f)/(d*f^2)) + b^2 - (b*f^2*x + 2*a*f^2)*sqrt(b^2/(d*f
^3)))/x) - f*sqrt(-(d*f^2*sqrt(b^2/(d*f^3)) - c*d - a*f)/(d*f^2))*log((2*b*
c*x - 2*sqrt(c*x^2 + b*x + a)*b*f*sqrt(-(d*f^2*sqrt(b^2/(d*f^3)) - c*d - a*
f)/(d*f^2)) + b^2 - (b*f^2*x + 2*a*f^2)*sqrt(b^2/(d*f^3)))/x) + 2*sqrt(c)*l
og(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c)
 - 4*a*c))/f, 1/4*(f*sqrt((d*f^2*sqrt(b^2/(d*f^3)) + c*d + a*f)/(d*f^2))*lo
g((2*b*c*x + 2*sqrt(c*x^2 + b*x + a)*b*f*sqrt((d*f^2*sqrt(b^2/(d*f^3)) + c*
```

```
d + a*f)/(d*f^2)) + b^2 + (b*f^2*x + 2*a*f^2)*sqrt(b^2/(d*f^3)))/x) - f*sqr
t((d*f^2*sqrt(b^2/(d*f^3)) + c*d + a*f)/(d*f^2))*log((2*b*c*x - 2*sqrt(c*x^
2 + b*x + a)*b*f*sqrt((d*f^2*sqrt(b^2/(d*f^3)) + c*d + a*f)/(d*f^2)) + b^2
+ (b*f^2*x + 2*a*f^2)*sqrt(b^2/(d*f^3)))/x) + f*sqrt(-(d*f^2*sqrt(b^2/(d*f^
3)) - c*d - a*f)/(d*f^2))*log((2*b*c*x + 2*sqrt(c*x^2 + b*x + a)*b*f*sqrt(-
(d*f^2*sqrt(b^2/(d*f^3)) - c*d - a*f)/(d*f^2)) + b^2 - (b*f^2*x + 2*a*f^2)*
sqrt(b^2/(d*f^3)))/x) - f*sqrt(-(d*f^2*sqrt(b^2/(d*f^3)) - c*d - a*f)/(d*f^
2))*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*b*f*sqrt(-(d*f^2*sqrt(b^2/(d*f^3
)) - c*d - a*f)/(d*f^2)) + b^2 - (b*f^2*x + 2*a*f^2)*sqrt(b^2/(d*f^3)))/x)
+ 4*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2
 + b*c*x + a*c)))/f]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

**maple [B]** time = 0.01, size = 1669, normalized size = 6.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] 1/2/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)
/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)-1/2/f*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b
*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)
*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)+1/4/(d*f)^(1
/2)*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^
(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)
*b)/f)^(1/2))/c^(1/2)*b+1/2/f/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+
c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-
(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*
f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*b-1/2/(d
*f)^(1/2)/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f
+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1
/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c
*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*a-1/2/(d*f)^(1/2)/f/((a*f+c*
d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2
)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2
```

```
)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/
f)^(1/2))/(x+(d*f)^(1/2)/f))*c*d-1/2/(d*f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*
f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)-1/2
/f*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(
1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*
b)/f)^(1/2))*c^(1/2)-1/4/(d*f)^(1/2)*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*
f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f
)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b+1/2/f/((a*f+c*d+(d
*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)
*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)
^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(
1/2))/(x-(d*f)^(1/2)/f))*b+1/2/(d*f)^(1/2)/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2
)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f
+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1
/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)
/f))*a+1/2/(d*f)^(1/2)/f/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(
d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)
^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1
/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c*d
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                  /(2*f)                +(c*d)/f+a))
     /f^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}}{d-f\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/(d - f*x^2),x)
```

```
[Out] int((a + b*x + c*x^2)^(1/2)/(d - f*x^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
[Out] -Integral(sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)
```

## 3.81 $\int \frac{\sqrt{a+bx+cx^2}}{x\left(d-fx^2\right)}\,dx$

**Optimal**. Leaf size=267

$$-\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\,\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d\sqrt{f}}+\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+\cdots\right.}{2\sqrt{a+bx+cx^2}\,\sqrt{a\cdots}}\right.}{2d\sqrt{f}}$$

**Rubi [A]** time = 0.78, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {6725, 734, 843, 621, 206, 724, 1021, 1078, 1033}

$$-\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\,\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d\sqrt{f}}+\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\,\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d\sqrt{f}}-\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(x*(d - f*x^2)),x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/d) - (Sq
rt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*S
qrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x
+ c*x^2])])/(2*d*Sqrt[f]) + (Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b
*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[
d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d*Sqrt[f])
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1021

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
```

```
, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{x\left(d-fx^2\right)}\,dx = \int \left(\frac{\sqrt{a+bx+cx^2}}{dx} - \frac{fx\sqrt{a+bx+cx^2}}{d\left(-d+fx^2\right)}\right)dx$$

$$= \frac{\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx}{d} - \frac{f\int \frac{x\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx}{d}$$

$$= -\frac{\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx}{2d} + \frac{\int \frac{-\frac{bd}{2}-(cd+af)x-\frac{1}{2}bfx^2}{\sqrt{a+bx+cx^2}\left(-d+fx^2\right)}\,dx}{d}$$

$$= \frac{a\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{d} + \frac{\int \frac{-bdf+f(-cd-af)x}{\sqrt{a+bx+cx^2}\left(-d+fx^2\right)}\,dx}{df}$$

$$= -\frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d} - \frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)\int \frac{1}{\left(\sqrt{d}\sqrt{f}+fx\right)\sqrt{a+bx+cx^2}}\,dx}{2d}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d} + \frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)\mathrm{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx,\right.}{d}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d} - \frac{\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right.}{2d\sqrt{f}}$$

**Mathematica [A]** time = 0.30, size = 255, normalized size = 0.96

$$-\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\sqrt{d}-b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)-\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)+2\sqrt{a}\sqrt{f}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{2d\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(x*(d - f*x^2)),x]
```

```
[Out] -1/2*(2*Sqrt[a]*Sqrt[f]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)
])] + Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f]
+ 2*c*Sqrt[d]*x - b*Sqrt[f]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[
a + x*(b + c*x)])] - Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d]
 + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[
f] + a*f]*Sqrt[a + x*(b + c*x)])]))/(d*Sqrt[f])
```

**IntegrateAlgebraic [C]** time = 0.39, size = 342, normalized size = 1.28

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}-\frac{\sqrt{a+bx+cx^2}}{\sqrt{a}}\right)}{d}-\frac{\text{RootSum}\left[\#1^4(-f)+2\#1^2af+4\#1^2cd-4\#1b\sqrt{c}\,d-a^2f+b^2d\&,\frac{\#1^2cd\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)+\#1^2af\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)+a^2(-f)\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)+b^2d\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)-acd\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)-2\#1b\sqrt{c}\,d\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)}{\#1^3f-\#1af-2\#1cd+b\sqrt{c}\,d}\&\right]}{2d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x + c*x^2]/(x*(d - f*x^2)),x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[(Sqrt[c]*x)/Sqrt[a] - Sqrt[a + b*x + c*x^2]/Sqrt[a]])/d
- RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1
^4 & , (b^2*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*c*d*Log[-(
Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a^2*f*Log[-(Sqrt[c]*x) + Sqrt[a
+ b*x + c*x^2] - #1] - 2*b*Sqrt[c]*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^
2] - #1]*#1 + c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + a*f
*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*
#1 - a*f*#1 + f*#1^3) & ]/(2*d)
```

**fricas [B]** time = 17.02, size = 1253, normalized size = 4.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/4*(d*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d + a*f)/(d^2*f))*log((2*sqrt(c*x
^2 + b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d +
 a*f)/(d^2*f)) + 2*b*c*x + b^2 + (b*d*f*x + 2*a*d*f)*sqrt(b^2/(d^3*f)))/x)
- d*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d + a*f)/(d^2*f))*log(-(2*sqrt(c*x^2
+ b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d + a*
f)/(d^2*f)) - 2*b*c*x - b^2 - (b*d*f*x + 2*a*d*f)*sqrt(b^2/(d^3*f)))/x) - d
*sqrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d - a*f)/(d^2*f))*log((2*sqrt(c*x^2 + b
*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d - a*f)
/(d^2*f)) + 2*b*c*x + b^2 - (b*d*f*x + 2*a*d*f)*sqrt(b^2/(d^3*f)))/x) + d*s
qrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d - a*f)/(d^2*f))*log(-(2*sqrt(c*x^2 + b*
x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d - a*f)/
(d^2*f)) - 2*b*c*x - b^2 + (b*d*f*x + 2*a*d*f)*sqrt(b^2/(d^3*f)))/x) + 2*sq
rt(a)*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*
a)*sqrt(a) + 8*a^2)/x^2))/d, 1/4*(d*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d + a
*f)/(d^2*f))*log((2*sqrt(c*x^2 + b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt((d^2
```

```
*f*sqrt(b^2/(d^3*f)) + c*d + a*f)/(d^2*f)) + 2*b*c*x + b^2 + (b*d*f*x + 2*a
*d*f)*sqrt(b^2/(d^3*f)))/x) - d*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d + a*f)/
(d^2*f))*log(-(2*sqrt(c*x^2 + b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt((d^2*f*
sqrt(b^2/(d^3*f)) + c*d + a*f)/(d^2*f)) - 2*b*c*x - b^2 - (b*d*f*x + 2*a*d*
f)*sqrt(b^2/(d^3*f)))/x) - d*sqrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d - a*f)/(d
^2*f))*log((2*sqrt(c*x^2 + b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt(-(d^2*f*sq
rt(b^2/(d^3*f)) - c*d - a*f)/(d^2*f)) + 2*b*c*x + b^2 - (b*d*f*x + 2*a*d*f)
*sqrt(b^2/(d^3*f)))/x) + d*sqrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d - a*f)/(d^2
*f))*log(-(2*sqrt(c*x^2 + b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt(-(d^2*f*sqr
t(b^2/(d^3*f)) - c*d - a*f)/(d^2*f)) - 2*b*c*x - b^2 + (b*d*f*x + 2*a*d*f)*
sqrt(b^2/(d^3*f)))/x) + 4*sqrt(-a)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x +
2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)))/d]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$sage_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] sage2
```

**maple [B]** time = 0.02, size = 1764, normalized size = 6.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/x/(-f*x^2+d),x)
```

```
[Out] -1/2/d*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*
f+c*d-(d*f)^(1/2)*b)/f)^(1/2)+1/2/d/f*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d
*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*
f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*(d*f)^(1/2)-1/4/d*l
n(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)
/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f
)^(1/2))/c^(1/2)*b-1/2/d/f/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d
-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*
f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^
(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*(d*f)^(1/2)
*b+1/2/d/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+
(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/
2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*
d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*a+1/2/f/((a*f+c*d-(d*f)^(1/2)
*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)
^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f
```

```
-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+
(d*f)^(1/2)/f))*c-1/2/d*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*
f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)-1/2/d/f*ln(((x-(d*f)^(1/2)/f
)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f
)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*(d
*f)^(1/2)-1/4/d*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2
)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d
+(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b+1/2/d/f/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/
2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/
f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(
1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2
)/f))*(d*f)^(1/2)*b+1/2/d/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+
(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f
)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(
1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a+1/2/f/((a*
f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^
(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^
(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)
*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c+1/d*(c*x^2+b*x+a)^(1/2)+1/2/d*b*ln((c*x+
1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-1/d*a^(1/2)*ln((2*a+b*x+2*a^(1/
2)*(c*x^2+b*x+a)^(1/2))/x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{cx^2 + bx + a}}{(fx^2 - d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 - d)*x), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2 + b\,x + a}}{x\,\left(d - f\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/(x*(d - f*x^2)),x)
```

```
[Out] int((a + b*x + c*x^2)^(1/2)/(x*(d - f*x^2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{a+bx+cx^2}}{-dx+fx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/x/(-f*x**2+d),x)
```

```
[Out] -Integral(sqrt(a + b*x + c*x**2)/(-d*x + f*x**3), x)
```

## 3.82 $\int \frac{\sqrt{a+bx+cx^2}}{x^2\left(d-fx^2\right)}\,dx$

**Optimal**. Leaf size=286

$$\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\,\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^{3/2}}+\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{\right.}}{2\sqrt{a+bx+cx^2}\,\sqrt{af+b}}\right.}{2d^{3/2}}$$

**Rubi [A]** time = 0.71, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6725, 732, 843, 621, 206, 724, 990, 1033}

$$\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\,\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^{3/2}}+\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\,\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^{3/2}}-\frac{\sqrt{a+bx+cx^2}}{dx}-\frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(x^2*(d - f*x^2)),x]
```

```
[Out] -(Sqrt[a + b*x + c*x^2]/(d*x)) - (b*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a +
 b*x + c*x^2])])/(2*Sqrt[a]*d) + (Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTa
nh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*
Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^(3/2)) + (Sqrt[c*d + b
*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b
*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]
)/(2*d^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 990

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (f_.)*(x_)^2), x_Symbol]
 :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*
f - b*f*x)/(Sqrt[a + b*x + c*x^2]*(d + f*x^2)), x], x] /; FreeQ[{a, b, c, d
, f}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2\left(d-fx^2\right)}\,dx = \int \left(\frac{\sqrt{a+bx+cx^2}}{dx^2}+\frac{f\sqrt{a+bx+cx^2}}{d\left(d-fx^2\right)}\right)dx$$

$$= \frac{\int \frac{\sqrt{a+bx+cx^2}}{x^2}\,dx}{d}+\frac{f\int \frac{\sqrt{a+bx+cx^2}}{d-fx^2}\,dx}{d}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx}+\frac{\int \frac{b+2cx}{x\sqrt{a+bx+cx^2}}\,dx}{2d}+\frac{\int \frac{cd+af+bfx}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{d}-\frac{c\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{d}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx}+\frac{b\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{2d}+\frac{c\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{d}-\frac{(2c)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b}{\sqrt{a+}}\right.}{d}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx}-\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{d}-\frac{b\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d}+(2c$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx}-\frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}\,d}+\frac{\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\,\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}}{2\sqrt{cd-b\sqrt{d}\sqrt{}}}\right.}{2d^{3/2}}$$

**Mathematica [A]** time = 0.43, size = 275, normalized size = 0.96

$$\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)+\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)-\frac{2\sqrt{d}\sqrt{a+x(b+cx)}}{x}-\frac{b\sqrt{d}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{\sqrt{a}}}{2d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(x^2*(d - f*x^2)),x]
```

```
[Out] ((-2*Sqrt[d]*Sqrt[a + x*(b + c*x)])/x - (b*Sqrt[d]*ArcTanh[(2*a + b*x)/(2*S
qrt[a]*Sqrt[a + x*(b + c*x)])])/Sqrt[a] + Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*
f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[
c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] + Sqrt[c*d - b*Sqrt[
d]*Sqrt[f] + a*f]*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt
[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(2*
d^(3/2))
```

**IntegrateAlgebraic [C]** time = 0.53, size = 299, normalized size = 1.05

$$-\frac{\text{RootSum}\left[\#1^4(-f)+2\#1^2af+4\#1^2cd-4\#1b\sqrt{c}d-a^2f+b^2d\&,\frac{\#1^2bf\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2\#1c^{3/2}d\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+bcd\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2\#1a\sqrt{c}f\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)}{\#1^3f-\#1af-2\#1cd+b\sqrt{c}d}\&\right]}{2d}-\frac{\sqrt{a+bx+cx^2}}{dx}+\frac{b\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}-\frac{\sqrt{a+bx+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x + c*x^2]/(x^2*(d - f*x^2)),x]
```

```
[Out] -(Sqrt[a + b*x + c*x^2]/(d*x)) + (b*ArcTanh[(Sqrt[c]*x)/Sqrt[a] - Sqrt[a +
b*x + c*x^2]/Sqrt[a]])/(Sqrt[a]*d) - RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*
#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (b*c*d*Log[-(Sqrt[c]*x) + Sqrt[a
+ b*x + c*x^2] - #1] - 2*c^(3/2)*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2]
 - #1]*#1 - 2*a*Sqrt[c]*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1
 + b*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d -
2*c*d*#1 - a*f*#1 + f*#1^3) & ]/(2*d)
```

**fricas [B]** time = 24.59, size = 1094, normalized size = 3.83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^2/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/4*(a*d*x*sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3)*log((2*b*c*x + 2*sq
rt(c*x^2 + b*x + a)*b*d*sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3) + b^2 +
 (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x) - a*d*x*sqrt((d^3*sqrt(b^2*f/d^5)
+ c*d + a*f)/d^3)*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt((d^3*sqrt
(b^2*f/d^5) + c*d + a*f)/d^3) + b^2 + (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/
x) + a*d*x*sqrt(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3)*log((2*b*c*x + 2*sq
rt(c*x^2 + b*x + a)*b*d*sqrt(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3) + b^2
- (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x) - a*d*x*sqrt(-(d^3*sqrt(b^2*f/d^5
) - c*d - a*f)/d^3)*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt(-(d^3*s
qrt(b^2*f/d^5) - c*d - a*f)/d^3) + b^2 - (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5
))/x) + sqrt(a)*b*x*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x
+ a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*sqrt(c*x^2 + b*x + a)*a)/(a*d*x)
, 1/4*(a*d*x*sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3)*log((2*b*c*x + 2*s
qrt(c*x^2 + b*x + a)*b*d*sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3) + b^2
+ (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x) - a*d*x*sqrt((d^3*sqrt(b^2*f/d^5)
 + c*d + a*f)/d^3)*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt((d^3*sqr
t(b^2*f/d^5) + c*d + a*f)/d^3) + b^2 + (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))
/x) + a*d*x*sqrt(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3)*log((2*b*c*x + 2*s
qrt(c*x^2 + b*x + a)*b*d*sqrt(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3) + b^2
 - (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x) - a*d*x*sqrt(-(d^3*sqrt(b^2*f/d^
5) - c*d - a*f)/d^3)*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt(-(d^3*
sqrt(b^2*f/d^5) - c*d - a*f)/d^3) + b^2 - (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^
5))/x) + 2*sqrt(-a)*b*x*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-
a)/(a*c*x^2 + a*b*x + a^2)) - 4*sqrt(c*x^2 + b*x + a)*a)/(a*d*x)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^2/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.87sym2poly/r2sym(const ge
n & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

**maple [B]** time = 0.02, size = 1819, normalized size = 6.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/x^2/(-f*x^2+d),x)
```

```
[Out] 1/2*f/d/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(
1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)-1/2/d*ln(((x+(d*f)^(1/2)/f)*c+1/
2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2
)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)+1/4*f/d/
(d*f)^(1/2)*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((
x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*
f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b+1/2/d/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln(
(2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((
a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c
)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*
b-1/2*f/d/(d*f)^(1/2)/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f
)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1
/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)
/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*a-1/2/(d*f)^(1/2
)/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*
(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+
(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)
^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*c-1/d/a/x*(c*x^2+b*x+a)^(3/2)+1/d*b/
a*(c*x^2+b*x+a)^(1/2)-1/2/d*b/a^(1/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(
1/2))/x)+1/d*c/a*(c*x^2+b*x+a)^(1/2)*x+1/d*c^(1/2)*ln((c*x+1/2*b)/c^(1/2)+(
c*x^2+b*x+a)^(1/2))-1/2*f/d/(d*f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)
^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)-1/2/d*ln(((x
-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2
*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/
2))*c^(1/2)-1/4*f/d/(d*f)^(1/2)*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1
/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/
2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b+1/2/d/((a*f+c*d+(d*f)^(
1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(
d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+
(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))
/(x-(d*f)^(1/2)/f))*b+1/2*f/d/(d*f)^(1/2)/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)
```

```
*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+
2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/
2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/
f))*a+1/2/(d*f)^(1/2)/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f
)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1
/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)
/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{cx^2+bx+a}}{\left(fx^2-d\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^2/(-f*x^2+d),x, algorithm="maxima")
[Out] -integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 - d)*x^2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}}{x^2\,\left(d-f\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/(x^2*(d - f*x^2)),x)
[Out] int((a + b*x + c*x^2)^(1/2)/(x^2*(d - f*x^2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{a+bx+cx^2}}{-dx^2+fx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/x**2/(-f*x**2+d),x)
[Out] -Integral(sqrt(a + b*x + c*x**2)/(-d*x**2 + f*x**4), x)
```

## 3.83 $\int \frac{\sqrt{a+bx+cx^2}}{x^3\left(d-fx^2\right)}\,dx$

**Optimal**. Leaf size=353

$$\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}d}-\frac{\sqrt{a}\,f\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2}-\frac{\sqrt{f}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{-2a\sqrt{f}}{2\sqrt{a+bx+}}\right.}{2d^2}$$

**Rubi [A]** time = 0.88, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6725, 720, 724, 206, 734, 843, 621, 1021, 1078, 1033}

$$\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}d}-\frac{\sqrt{a}\,f\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2}-\frac{\sqrt{f}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^2}+\frac{\sqrt{f}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^2}-\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4adx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(x^3*(d - f*x^2)),x]
```

```
[Out] -((2*a + b*x)*Sqrt[a + b*x + c*x^2])/(4*a*d*x^2) + ((b^2 - 4*a*c)*ArcTanh[(
2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*a^(3/2)*d) - (Sqrt[a]*f*A
rcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/d^2 - (Sqrt[f]*Sqrt[
c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt
[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c
*x^2])])/(2*d^2) + (Sqrt[f]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*
Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d
]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
```

```
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1021

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
```

```
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{x^3\left(d-fx^2\right)}\,dx = \int \left(\frac{\sqrt{a+bx+cx^2}}{dx^3}+\frac{f\sqrt{a+bx+cx^2}}{d^2x}+\frac{f^2x\sqrt{a+bx+cx^2}}{d^2\left(d-fx^2\right)}\right)dx$$

$$= \frac{\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx}{d}+\frac{f\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx}{d^2}+\frac{f^2\int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2}\,dx}{d^2}$$

$$= -\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4adx^2}-\frac{\left(b^2-4ac\right)\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{8ad}-\frac{f\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx}{2d^2}+\frac{f\int \frac{\frac{bd}{2}+(c\cdots}{\sqrt{a+b\cdots}}\cdots}{\cdots}$$

$$= -\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4adx^2}-\frac{\int \frac{-bdf-f(cd+af)x}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{d^2}+\frac{\left(b^2-4ac\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a\cdots}{\sqrt{a+\cdots}}\right)}{4ad}\cdots$$

$$= -\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4adx^2}+\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}d}-\frac{(2af)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,d\cdots\right)}{d^2}\cdots$$

$$= -\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4adx^2}+\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}d}-\frac{\sqrt{a}\,f\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx\cdots}}\right)}{d^2}\cdots$$

$$= -\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4adx^2}+\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}d}-\frac{\sqrt{a}\,f\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx\cdots}}\right)}{d^2}\cdots$$

**Mathematica [A]** time = 0.52, size = 316, normalized size = 0.90

$$\frac{x^2\left(b^2d-4a(2af+cd)\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)-2\sqrt{a}\left(-2a\sqrt{f}x^2\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)+2a\sqrt{f}x^2\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)+d(2a+bx)\sqrt{a+x(b+cx)}\right)}{8a^{3/2}d^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(x^3*(d - f*x^2)),x]
```

```
[Out] ((b^2*d - 4*a*(c*d + 2*a*f))*x^2*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*
(b + c*x)])] - 2*Sqrt[a]*(d*(2*a + b*x)*Sqrt[a + x*(b + c*x)] - 2*a*Sqrt[f]
*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*x^2*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] +
 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a
 + x*(b + c*x)])] + 2*a*Sqrt[f]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*x^2*Arc
Tanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d -
 b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]))/(8*a^(3/2)*d^2*x^2)
```

**IntegrateAlgebraic [C]** time = 0.86, size = 392, normalized size = 1.11

$$-\frac{f\text{RootSum}\left[\#1^4(-f)+2\#1^2af+4\#1^2cd-4\#1b\sqrt{c}d-a^2f+b^2d\&,\frac{\#1^2cd\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+\#1^2af\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+a^2(-f)\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+b^2d\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-acd\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2\#1b\sqrt{c}d\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)}{\#1^3f-\#1af-2\#1cd+b\sqrt{c}d}\&\right]}{2d^2}+\frac{\left(-8a^2f-4acd+b^2d\right)\tanh^{-1}\left(\frac{\sqrt{a+bx+cx^2}-\sqrt{c}x}{\sqrt{a}}\right)}{4a^{3/2}d^2}+\frac{(-2a-bx)\sqrt{a+bx+cx^2}}{4adx^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x + c*x^2]/(x^3*(d - f*x^2)),x]
```

```
[Out] ((-2*a - b*x)*Sqrt[a + b*x + c*x^2])/(4*a*d*x^2) + ((b^2*d - 4*a*c*d - 8*a^
2*f)*ArcTanh[(-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2])/Sqrt[a]])/(4*a^(3/2)*d^
2) - (f*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2
- f*#1^4 & , (b^2*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*c*d*
Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a^2*f*Log[-(Sqrt[c]*x) + S
qrt[a + b*x + c*x^2] - #1] - 2*b*Sqrt[c]*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x
+ c*x^2] - #1]*#1 + c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2
 + a*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d -
2*c*d*#1 - a*f*#1 + f*#1^3) & ])/(2*d^2)
```

**fricas [B]** time = 133.83, size = 1485, normalized size = 4.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^3/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/16*(4*a^2*d^2*x^2*sqrt((d^4*sqrt(b^2*f^3/d^7) + c*d*f + a*f^2)/d^4)*log(
(2*sqrt(c*x^2 + b*x + a)*d^5*sqrt(b^2*f^3/d^7)*sqrt((d^4*sqrt(b^2*f^3/d^7)
+ c*d*f + a*f^2)/d^4) + 2*b*c*f^2*x + b^2*f^2 + (b*d^3*f*x + 2*a*d^3*f)*sqr
t(b^2*f^3/d^7))/x) - 4*a^2*d^2*x^2*sqrt((d^4*sqrt(b^2*f^3/d^7) + c*d*f + a*
f^2)/d^4)*log(-(2*sqrt(c*x^2 + b*x + a)*d^5*sqrt(b^2*f^3/d^7)*sqrt((d^4*sqr
```

```
t(b^2*f^3/d^7) + c*d*f + a*f^2)/d^4) - 2*b*c*f^2*x - b^2*f^2 - (b*d^3*f*x +
 2*a*d^3*f)*sqrt(b^2*f^3/d^7))/x) - 4*a^2*d^2*x^2*sqrt(-(d^4*sqrt(b^2*f^3/d
^7) - c*d*f - a*f^2)/d^4)*log((2*sqrt(c*x^2 + b*x + a)*d^5*sqrt(b^2*f^3/d^7
)*sqrt(-(d^4*sqrt(b^2*f^3/d^7) - c*d*f - a*f^2)/d^4) + 2*b*c*f^2*x + b^2*f^
2 - (b*d^3*f*x + 2*a*d^3*f)*sqrt(b^2*f^3/d^7))/x) + 4*a^2*d^2*x^2*sqrt(-(d^
4*sqrt(b^2*f^3/d^7) - c*d*f - a*f^2)/d^4)*log(-(2*sqrt(c*x^2 + b*x + a)*d^5
*sqrt(b^2*f^3/d^7)*sqrt(-(d^4*sqrt(b^2*f^3/d^7) - c*d*f - a*f^2)/d^4) - 2*b
*c*f^2*x - b^2*f^2 + (b*d^3*f*x + 2*a*d^3*f)*sqrt(b^2*f^3/d^7))/x) + (8*a^2
*f - (b^2 - 4*a*c)*d)*sqrt(a)*x^2*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqr
t(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(a*b*d*x + 2*a^2*d
)*sqrt(c*x^2 + b*x + a))/(a^2*d^2*x^2), 1/8*(2*a^2*d^2*x^2*sqrt((d^4*sqrt(b
^2*f^3/d^7) + c*d*f + a*f^2)/d^4)*log((2*sqrt(c*x^2 + b*x + a)*d^5*sqrt(b^2
*f^3/d^7)*sqrt((d^4*sqrt(b^2*f^3/d^7) + c*d*f + a*f^2)/d^4) + 2*b*c*f^2*x +
 b^2*f^2 + (b*d^3*f*x + 2*a*d^3*f)*sqrt(b^2*f^3/d^7))/x) - 2*a^2*d^2*x^2*sq
rt((d^4*sqrt(b^2*f^3/d^7) + c*d*f + a*f^2)/d^4)*log(-(2*sqrt(c*x^2 + b*x +
a)*d^5*sqrt(b^2*f^3/d^7)*sqrt((d^4*sqrt(b^2*f^3/d^7) + c*d*f + a*f^2)/d^4)
- 2*b*c*f^2*x - b^2*f^2 - (b*d^3*f*x + 2*a*d^3*f)*sqrt(b^2*f^3/d^7))/x) - 2
*a^2*d^2*x^2*sqrt(-(d^4*sqrt(b^2*f^3/d^7) - c*d*f - a*f^2)/d^4)*log((2*sqrt
(c*x^2 + b*x + a)*d^5*sqrt(b^2*f^3/d^7)*sqrt(-(d^4*sqrt(b^2*f^3/d^7) - c*d*
f - a*f^2)/d^4) + 2*b*c*f^2*x + b^2*f^2 - (b*d^3*f*x + 2*a*d^3*f)*sqrt(b^2*
f^3/d^7))/x) + 2*a^2*d^2*x^2*sqrt(-(d^4*sqrt(b^2*f^3/d^7) - c*d*f - a*f^2)/
d^4)*log(-(2*sqrt(c*x^2 + b*x + a)*d^5*sqrt(b^2*f^3/d^7)*sqrt(-(d^4*sqrt(b^
2*f^3/d^7) - c*d*f - a*f^2)/d^4) - 2*b*c*f^2*x - b^2*f^2 + (b*d^3*f*x + 2*a
*d^3*f)*sqrt(b^2*f^3/d^7))/x) + (8*a^2*f - (b^2 - 4*a*c)*d)*sqrt(-a)*x^2*ar
ctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)
) - 2*(a*b*d*x + 2*a^2*d)*sqrt(c*x^2 + b*x + a))/(a^2*d^2*x^2)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^3/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 1.36sym2poly/r2sym(const ge
n & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

**maple [B]** time = 0.02, size = 1953, normalized size = 5.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/x^3/(-f*x^2+d),x)
```

```
[Out] -1/2*f/d^2*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f
+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)+1/2/d^2*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-
2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x
+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*(d*f)^(1/2)-1/4
*f/d^2*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*
f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1
/2)*b)/f)^(1/2))/c^(1/2)*b-1/2/d^2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*
(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f
+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(
x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*(d*
f)^(1/2)*b+1/2*f/d^2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)
^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/
2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/
f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*a+1/2/d/((a*f+c*d
-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)
*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)
/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f
)^(1/2))/(x+(d*f)^(1/2)/f))*c-1/2*f/d^2*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)
^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)-1/2/d^2*ln((
(x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)
^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(
1/2))*c^(1/2)*(d*f)^(1/2)-1/4*f/d^2*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f
)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)
^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b+1/2/d^2/((a*f+c*d+(
d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c
)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f
)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^
(1/2))/(x-(d*f)^(1/2)/f))*(d*f)^(1/2)*b+1/2*f/d^2/((a*f+c*d+(d*f)^(1/2)*b)/
f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/
2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(
d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f
)^(1/2)/f))*a+1/2/d/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^
(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2
)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f
)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c-1/2/d/a/x^2*(c*x
^2+b*x+a)^(3/2)+1/4/d*b/a^2/x*(c*x^2+b*x+a)^(3/2)-1/4/d*b^2/a^2*(c*x^2+b*x+
a)^(1/2)+1/8/d*b^2/a^(3/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-1/
4/d*b/a^2*c*(c*x^2+b*x+a)^(1/2)*x+1/2/d*c/a*(c*x^2+b*x+a)^(1/2)-1/2/d*c/a^(
1/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)+f/d^2*(c*x^2+b*x+a)^(1/2
)+1/2*f/d^2*b*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-f/d^2*a^(
1/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{cx^2 + bx + a}}{\left(fx^2 - d\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^3/(-f*x^2+d),x, algorithm="maxima")
[Out] -integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 - d)*x^3), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2 + b\,x + a}}{x^3\,\left(d - f\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/(x^3*(d - f*x^2)),x)
[Out] int((a + b*x + c*x^2)^(1/2)/(x^3*(d - f*x^2)), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{a + bx + cx^2}}{-dx^3 + fx^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/x**3/(-f*x**2+d),x)
[Out] -Integral(sqrt(a + b*x + c*x**2)/(-d*x**3 + f*x**5), x)
```

## 3.84 $\int \frac{x^3\left(a+bx+cx^2\right)^{3/2}}{d-fx^2}\,dx$

**Optimal**. Leaf size=501

$$\frac{3b\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{7/2}f}-\frac{3b\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}}{128c^3f}-\frac{d\sqrt{a+bx+cx^2}\left(8acf+b^2f+2\right.}{8cf^3}$$

**Rubi** [A] time = 1.41, antiderivative size = 501, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6725, 640, 612, 621, 206, 1021, 1070, 1078, 1033, 724}

$$-\frac{d\sqrt{a+bx+cx^2}\left(8acf+b^2f+2bcfx+8c^2d\right)}{8cf^3}-\frac{bd\left(12acf+b^2(-f)+24c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}f^3}-\frac{3b\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}}{128c^3f}+\frac{3b\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{7/2}f}+\frac{b(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{16c^2f}-\frac{d\left(a+bx+cx^2\right)^{3/2}}{3f^2}-\frac{d\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^{7/2}}+\frac{d\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^{7/2}}-\frac{\left(a+bx+cx^2\right)^{5/2}}{5cf}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]
```

```
[Out] (-3*b*(b^2 - 4*a*c)*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(128*c^3*f) - (d*(8*
c^2*d + b^2*f + 8*a*c*f + 2*b*c*f*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^3) - (d*
(a + b*x + c*x^2)^(3/2))/(3*f^2) + (b*(b + 2*c*x)*(a + b*x + c*x^2)^(3/2))/
(16*c^2*f) - (a + b*x + c*x^2)^(5/2)/(5*c*f) + (3*b*(b^2 - 4*a*c)^2*ArcTanh
[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*c^(7/2)*f) - (b*d*(24
*c^2*d - b^2*f + 12*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*
x^2])])/(16*c^(3/2)*f^3) - (d*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh
[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sq
rt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(7/2)) + (d*(c*d + b*Sqr
t[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] +
 b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2]
)])/(2*f^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1021

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1070

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*p + 2*q + 3)
+ C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1
))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*
(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*
```

```
(C*(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*
p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(-(
b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(-(b*c*(C*(-4*d*f
)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))))*x + (p*(-(b*f))*(C*
(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2
- 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3
))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, A, B, C, q}, x] && NeQ[b^2 -
4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !I
GtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x^3 \left(a+bx+cx^2\right)^{3/2}}{d-fx^2}\,dx = \int \left(-\frac{x\left(a+bx+cx^2\right)^{3/2}}{f} + \frac{dx\left(a+bx+cx^2\right)^{3/2}}{f\left(d-fx^2\right)}\right) dx$$

$$= -\frac{\int x\left(a+bx+cx^2\right)^{3/2}\,dx}{f} + \frac{d\int \frac{x\left(a+bx+cx^2\right)^{3/2}}{d-fx^2}\,dx}{f}$$

$$= -\frac{d\left(a+bx+cx^2\right)^{3/2}}{3f^2} - \frac{\left(a+bx+cx^2\right)^{5/2}}{5cf} + \frac{d\int \frac{\sqrt{a+bx+cx^2}\left(\frac{3bd}{2}+3(cd+af)x+\frac{3}{2}bfx^2\right)}{d-fx^2}\,dx}{3f^2} + \ldots b$$

$$= -\frac{d\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cf^3} - \frac{d\left(a+bx+cx^2\right)^{3/2}}{3f^2} + \frac{b(b+2cx)\,(\ldots}{\ldots}$$

$$= -\frac{3b\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}}{128c^3f} - \frac{d\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+\ldots}}{8cf^3}$$

$$= -\frac{3b\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}}{128c^3f} - \frac{d\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+\ldots}}{8cf^3}$$

$$= -\frac{3b\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}}{128c^3f} - \frac{d\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+\ldots}}{8cf^3}$$

$$= -\frac{3b\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}}{128c^3f} - \frac{d\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+\ldots}}{8cf^3}$$

**Mathematica** [A] time = 1.49, size = 447, normalized size = 0.89

$$\frac{-\frac{\sqrt{f}\sqrt{a+x(b+cx)}\left(24c^2f\left(16a^2f+7abfx+b^2\left(10d+fx^2\right)\right)-30b^2cf^2(10a+bx)+16c^3f\left(160ad+48afx^2+70bdx+33bfx^3\right)+45b^4f^2+128c^4\left(15d^2+5dfx^2+3f^2x^4\right)\right)}{c^3}+960d\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}-b\sqrt{d}+b\sqrt{f}x-2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)-960d\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{-2\left(a\sqrt{f}+c\sqrt{d}x\right)-b\left(\sqrt{d}+\sqrt{f}x\right)}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{1920f^{7/2}}+\frac{b\left(16c^2f\left(3a^2f+b^2d\right)-24ab^2cf^2-192ac^3df+3b^4f^2-384c^4d^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{256c^{7/2}f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]
```

```
[Out] (b*(-384*c^4*d^2 - 192*a*c^3*d*f + 3*b^4*f^2 - 24*a*b^2*c*f^2 + 16*c^2*f*(b
^2*d + 3*a^2*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(2
56*c^(7/2)*f^3) + (-((Sqrt[f]*Sqrt[a + x*(b + c*x)]*(45*b^4*f^2 - 30*b^2*c*
f^2*(10*a + b*x) + 16*c^3*f*(160*a*d + 70*b*d*x + 48*a*f*x^2 + 33*b*f*x^3)
+ 128*c^4*(15*d^2 + 5*d*f*x^2 + 3*f^2*x^4) + 24*c^2*f*(16*a^2*f + 7*a*b*f*x
```

```
 + b^2*(10*d + f*x^2))))/c^3) + 960*d*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)
*ArcTanh[(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt
[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] - 960*d*(c*d + b*Sq
rt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[
d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x
)])])/(1920*f^(7/2))
```

**IntegrateAlgebraic [C]** time = 3.07, size = 774, normalized size = 1.54

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]
```

```
[Out] (Sqrt[a + b*x + c*x^2]*(-1920*c^4*d^2 - 240*b^2*c^2*d*f - 2560*a*c^3*d*f -
45*b^4*f^2 + 300*a*b^2*c*f^2 - 384*a^2*c^2*f^2 - 1120*b*c^3*d*f*x + 30*b^3*
c*f^2*x - 168*a*b*c^2*f^2*x - 640*c^4*d*f*x^2 - 24*b^2*c^2*f^2*x^2 - 768*a*
c^3*f^2*x^2 - 528*b*c^3*f^2*x^3 - 384*c^4*f^2*x^4))/(1920*c^3*f^3) + ((384*
b*c^4*d^2 - 16*b^3*c^2*d*f + 192*a*b*c^3*d*f - 3*b^5*f^2 + 24*a*b^3*c*f^2 -
 48*a^2*b*c^2*f^2)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(256*c
^(7/2)*f^3) - (d*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*
a*f*#1^2 - f*#1^4 & , (2*b^2*c*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2]
 - #1] - a*c^2*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a*b^2*d
*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*a^2*c*d*f*Log[-(Sqrt[
c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a^3*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b
*x + c*x^2] - #1] - 4*b*c^(3/2)*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2
] - #1]*#1 - 4*a*b*Sqrt[c]*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #
1]*#1 + c^2*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + b^2*d
*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + 2*a*c*d*f*Log[-(Sq
rt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + a^2*f^2*Log[-(Sqrt[c]*x) + Sq
rt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3)
& ])/(2*f^3)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

**maple [B]** time = 0.03, size = 4884, normalized size = 9.75

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] -1/5*(c*x^2+b*x+a)^(5/2)/c/f+1/16/f*b^2/c^2*(c*x^2+b*x+a)^(3/2)-3/128/f*b^4
/c^3*(c*x^2+b*x+a)^(1/2)+3/256/f*b^5/c^(7/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))-1/2/f^2*d*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*
f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*a-1/2/f^3*d^2*((x-(d*f)^(1/2
)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/
f)^(1/2)*c-1/2/f^2*d*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^
(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*a-1/2/f^3*d^2*((x+(d*f)^(1/2)/f
)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^
(1/2)*c+1/2/f^4*d^3/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^
(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2
)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f
)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c^2-1/8/f^2*d*((x+
(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)
^(1/2)*b)/f)^(1/2)*x*b+5/8/f^3*d*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*
c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b*(d*f)^(1/2)-1/16/
f^2*d/c*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a
*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b^2+1/32/f^2*d/c^(3/2)*ln(((x+(d*f)^(1/2)/f)
*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)
^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*b^3-3/4/f^3
*d^2*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)
^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2
)*b)/f)^(1/2))*c^(1/2)*b+1/2/f^4*d^2*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*
f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f
)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(3/2)*(d*f)^(1/2)+1/2/f^3*
d^2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-
2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((
x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*
f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*b^2+1/2/f^2*d/((a*f+c*d-(d*f)^(1/2
)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f
)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*
f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x
```

```
+(d*f)^(1/2)/f))*a^2+1/2/f^4*d^3/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a
*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c
*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+
(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*c^2+1
/8/f*b/c*x*(c*x^2+b*x+a)^(3/2)-3/64/f*b^3/c^2*(c*x^2+b*x+a)^(1/2)*x+3/32/f*
b^2/c^2*(c*x^2+b*x+a)^(1/2)*a+3/16/f*b/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^
2+b*x+a)^(1/2))*a^2-3/32/f*b^3/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)
^(1/2))*a-1/4/f^3*d*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(
1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*x*c*(d*f)^(1/2)-3/4/f^3*d*ln(((x
-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2
*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/
2))*c^(1/2)*(d*f)^(1/2)*a-3/8/f^2*d/c^(1/2)*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*
f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*
(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*a*b-3/16/f^3*d*ln(((x
-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2
*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/
2))/c^(1/2)*b^2*(d*f)^(1/2)+1/f^3*d^2/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln(
(2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((
a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c
)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*
a*c+3/16/f^3*d*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)
+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-
(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b^2*(d*f)^(1/2)+1/f^3*d^2/((a*f+c*d-(d*f)^
(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+
(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c
+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)
)/(x+(d*f)^(1/2)/f))*a*c+3/16/f*b/c*(c*x^2+b*x+a)^(1/2)*x*a+1/4/f^3*d*((x+(
d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^
(1/2)*b)/f)^(1/2)*x*c*(d*f)^(1/2)+3/4/f^3*d*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*
f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*
(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*(d*f)^(1/2)*a
-3/8/f^2*d/c^(1/2)*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(
1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+
c*d-(d*f)^(1/2)*b)/f)^(1/2))*a*b+1/2/f^3*d^2/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1
/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)
/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^
(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/
2)/f))*b^2+1/2/f^2*d/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)
^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/
2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/
f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a^2+1/32/f^2*d/c^
(3/2)*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f
)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/
2)*b)/f)^(1/2))*b^3-3/4/f^3*d^2*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1
/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/
```

```
2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*b-1/2/f^4*d^2*ln(((x-(d*f
)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b
*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*c
^(3/2)*(d*f)^(1/2)-1/16/f^2*d/c*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c
)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*b^2-1/6/f^2*d*((x-(d
*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(
1/2)*b)/f)^(3/2)-1/6/f^2*d*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+
(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(3/2)+1/f^3*d/((a*f+c*d+(d*f)^(
1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(
d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+
(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))
/(x-(d*f)^(1/2)/f))*b*(d*f)^(1/2)*a+1/f^4*d^2/((a*f+c*d+(d*f)^(1/2)*b)/f)^(
1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f
)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)
^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1
/2)/f))*b*(d*f)^(1/2)*c-1/f^3*d/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*
f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*
d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(
d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*b*(d*f
)^(1/2)*a-1/f^4*d^2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^
(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2
)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f
)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*b*(d*f)^(1/2)*c-1/
8/f^2*d*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a
*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*x*b-5/8/f^3*d*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*
(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*b*(d*f)
^(1/2)
```

**maxima** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                   /(2*f)                   +(c*d)/f+a))
     /f^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\left(c\,x^2+b\,x+a\right)^{3/2}}{d-f\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x)
[Out] int((x^3*(a + b*x + c*x^2)^(3/2))/(d - f*x^2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
[Out] Timed out
```

## 3.85 $\int \frac{x^2(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx$

**Optimal**. Leaf size=417

$$-\frac{\left(48c^2f\left(a^2f+b^2d\right)-24ab^2cf^2+192ac^3df+3b^4f^2+128c^4d^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{5/2}f^3}-\frac{\sqrt{a+bx+cx^2}\left(2cx\left(1\right.\right.}{}$$

**Rubi [A]** time = 1.02, antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1071, 1070, 1078, 621, 206, 1033, 724}

$$-\frac{\left(48c^2f\left(a^2f+b^2d\right)-24ab^2cf^2+192ac^3df+3b^4f^2+128c^4d^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{5/2}f^3}-\frac{\sqrt{a+bx+cx^2}\left(2cx\left(12acf-3b^2f+16c^2d\right)+b\left(12acf-3b^2f+80c^2d\right)\right)}{64c^2f^2}+\frac{\sqrt{d}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^3}+\frac{\sqrt{d}\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^3}-\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8cf}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]
```

```
[Out] -((b*(80*c^2*d - 3*b^2*f + 12*a*c*f) + 2*c*(16*c^2*d - 3*b^2*f + 12*a*c*f)*
x)*Sqrt[a + b*x + c*x^2])/(64*c^2*f^2) - ((b + 2*c*x)*(a + b*x + c*x^2)^(3/
2))/(8*c*f) - ((128*c^4*d^2 + 192*a*c^3*d*f + 3*b^4*f^2 - 24*a*b^2*c*f^2 +
48*c^2*f*(b^2*d + a^2*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x
^2])])/(128*c^(5/2)*f^3) + (Sqrt[d]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*A
rcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d
- b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]])/(2*f^3) + (Sqrt[d]*(c*d
 + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*S
qrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x
+ c*x^2])]])/(2*f^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1070

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*p + 2*q + 3)
+ C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1
))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*
(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*
(C*(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*
p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(-(
b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(-(b*c*(C*(-4*d*f
)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))))*x + (p*(-(b*f))*(C*
(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2
- 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3
))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, A, B, C, q}, x] && NeQ[b^2 -
4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !I
GtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1071

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) +
(f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*
(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x^
2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*(C*(-(b*f))*(q + 1)) + (p + q + 1)*(b
^2*C*d*f*p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*
f)*(C*(-(b*f))*(q + 1)) + (p + q + 1)*(-(b*c*(C*(-4*d*f)*(2*p + q + 2) + f*
(2*C*d + 2*A*f)*(2*p + 2*q + 3)))))*x + (p*(-(b*f))*(C*(-(b*f))*(q + 1)) +
(p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C
*d + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, A,
C, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2
*p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rubi steps

$$\int \frac{x^2\left(a+bx+cx^2\right)^{3/2}}{d-fx^2}\,dx = -\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8cf} - \frac{\int \frac{\sqrt{a+bx+cx^2}\left(-\frac{3}{4}\left(3b^2+4ac\right)df-12bcdfx-\frac{3}{4}f\left(16c^2d-3\left(b^2-4ac\right)f\right)x^2 \cdots}{d-fx^2}}{12cf^2} \cdots$$

$$= -\frac{\left(b\left(80c^2d-3b^2f+12acf\right)+2c\left(16c^2d-3b^2f+12acf\right)x\right)\sqrt{a+bx+cx^2}}{64c^2f^2} - \frac{(b+2 \cdots}{\cdots}$$

$$= -\frac{\left(b\left(80c^2d-3b^2f+12acf\right)+2c\left(16c^2d-3b^2f+12acf\right)x\right)\sqrt{a+bx+cx^2}}{64c^2f^2} - \frac{(b+2 \cdots}{\cdots}$$

$$= -\frac{\left(b\left(80c^2d-3b^2f+12acf\right)+2c\left(16c^2d-3b^2f+12acf\right)x\right)\sqrt{a+bx+cx^2}}{64c^2f^2} - \frac{(b+2 \cdots}{\cdots}$$

$$= -\frac{\left(b\left(80c^2d-3b^2f+12acf\right)+2c\left(16c^2d-3b^2f+12acf\right)x\right)\sqrt{a+bx+cx^2}}{64c^2f^2} - \frac{(b+2 \cdots}{\cdots}$$

$$= -\frac{\left(b\left(80c^2d-3b^2f+12acf\right)+2c\left(16c^2d-3b^2f+12acf\right)x\right)\sqrt{a+bx+cx^2}}{64c^2f^2} - \frac{(b+2 \cdots}{\cdots}$$

**Mathematica [A]** time = 1.02, size = 395, normalized size = 0.95

$$\frac{-\left(\left(48c^2f\left(a^2f+b^2d\right)-24ab^2cf^2+192ac^3df+3b^4f^2+128c^4d^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)-2\sqrt{c}\left(f\sqrt{a+x(b+cx)}\left(4bc\left(5af+20cd+6cfx^2\right)+8c^2x\left(5af+4cd+2cfx^2\right)-3b^3f+2b^2cfx\right)+32c^2\sqrt{d}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}-b\sqrt{d}+b\sqrt{f}x-2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)+32c^2\sqrt{d}\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{-2\left(a\sqrt{f}+c\sqrt{d}x\right)-b\left(\sqrt{d}+\sqrt{f}x\right)}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)\right)}{128c^{5/2}f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]
```

```
[Out] (-((128*c^4*d^2 + 192*a*c^3*d*f + 3*b^4*f^2 - 24*a*b^2*c*f^2 + 48*c^2*f*(b^
2*d + a^2*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]) - 2*S
```

```
qrt[c]*(f*Sqrt[a + x*(b + c*x)]*(-3*b^3*f + 2*b^2*c*f*x + 8*c^2*x*(4*c*d +
5*a*f + 2*c*f*x^2) + 4*b*c*(20*c*d + 5*a*f + 6*c*f*x^2)) + 32*c^2*Sqrt[d]*(
c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(-(b*Sqrt[d]) + 2*a*Sqrt[f] -
2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a
+ x*(b + c*x)])] + 32*c^2*Sqrt[d]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*Arc
Tanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d +
 b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]]))/(128*c^(5/2)*f^3)
```

**IntegrateAlgebraic [C]**   time = 2.56, size = 620, normalized size = 1.49

$$-\frac{d\,\mathrm{RootSum}\left[\#1^4(-f)+2\#1^2af+4\#1^2cd-4\#1b\sqrt{c}\,d-a^2f+b^2d\&,\frac{2\#1^2bcdf\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)+2\#1^2abf^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)-a^2bf^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)-2\#1a^2\sqrt{c}\,f^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)+b^3df\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)-2\#1b^2\sqrt{c}\,df\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)-2\#1c^{5/2}d^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)-4\#1ac^{3/2}df\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)+bc^2d^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)}{\#1^3f-\#1af-2\#1cd+b\sqrt{c}\,d}\&\right]}{2f^3}+\frac{\left(48a^2c^2f^2-24ab^2cf^2+192ac^3df+3b^4f^2+48b^2c^2df+128c^4d^2\right)\log\left(-2\sqrt{c}\sqrt{a+bx+cx^2}+b+2cx\right)}{128c^{5/2}f^3}+\frac{\sqrt{a+bx+cx^2}\left(-20abcf-40ac^2fx+3b^3f-2b^2cfx-80bc^2d-24bc^2fx^2-32c^3dx-16c^3fx^3\right)}{64c^2f^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]
```

```
[Out] (Sqrt[a + b*x + c*x^2]*(-80*b*c^2*d + 3*b^3*f - 20*a*b*c*f - 32*c^3*d*x - 2
*b^2*c*f*x - 40*a*c^2*f*x - 24*b*c^2*f*x^2 - 16*c^3*f*x^3))/(64*c^2*f^2) +
((128*c^4*d^2 + 48*b^2*c^2*d*f + 192*a*c^3*d*f + 3*b^4*f^2 - 24*a*b^2*c*f^2
 + 48*a^2*c^2*f^2)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(128*c
^(5/2)*f^3) - (d*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*
a*f*#1^2 - f*#1^4 & , (b*c^2*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] -
 #1] + b^3*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a^2*b*f^2*L
og[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*c^(5/2)*d^2*Log[-(Sqrt[c]
*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*b^2*Sqrt[c]*d*f*Log[-(Sqrt[c]*x) +
 Sqrt[a + b*x + c*x^2] - #1]*#1 - 4*a*c^(3/2)*d*f*Log[-(Sqrt[c]*x) + Sqrt[a
 + b*x + c*x^2] - #1]*#1 - 2*a^2*Sqrt[c]*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*
x + c*x^2] - #1]*#1 + 2*b*c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] -
#1]*#1^2 + 2*a*b*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(
b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ])/(2*f^3)
```

**fricas [F(-1)]**   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]**   time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

**maple [B]** time = 0.02, size = 4900, normalized size = 11.75

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] -3/8/f/c^(1/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2-3/128/f/c^(5
/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^4-1/6*d/(d*f)^(1/2)/f*((x
-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f
)^(1/2)*b)/f)^(3/2)-5/8*d/f^2*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*
(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*b-1/2*d^2/f^3*ln(((x-(
d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c
+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)
)*c^(3/2)+1/6*d/(d*f)^(1/2)/f*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*
(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(3/2)-5/8*d/f^2*((x+(d*f)^(1
/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b
)/f)^(1/2)*b-1/2*d^2/f^3*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/
f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f
+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(3/2)-1/8/f/c*(c*x^2+b*x+a)^(3/2)*b-3/
8/f*(c*x^2+b*x+a)^(1/2)*x*a+3/64/f/c^2*(c*x^2+b*x+a)^(1/2)*b^3-d^2/(d*f)^(1
/2)/f^2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(
b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2
)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d
-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*a*c+3/8*d/(d*f)^(1/2)/f/c^(1/2
)*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1
/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b
)/f)^(1/2))*a*b-3/8*d/(d*f)^(1/2)/f/c^(1/2)*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*
f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*
(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*a*b+1/2*d/(d*f)^(1/2)
/f/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2
*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x
-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f
)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a^2+1/2*d^3/(d*f)^(1/2)/f^3/((a*f+c
*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/
2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/
2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)
/f)^(1/2))/(x-(d*f)^(1/2)/f))*c^2-1/8*d/(d*f)^(1/2)/f*((x-(d*f)^(1/2)/f)^2*
c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2
)*x*b-1/16*d/(d*f)^(1/2)/f/c*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(
```

x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*b^2+1/32*d/(d*f)^(1/2)/
f/c^(3/2)*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-
(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)
^(1/2)*b)/f)^(1/2))*b^3-3/4*d^2/(d*f)^(1/2)/f^2*ln(((x-(d*f)^(1/2)/f)*c+1/2
*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)
*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*b+1/2*d^2
/(d*f)^(1/2)/f^2/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/
2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b
)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f
+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*b^2+d/f^2/((a*f+c*d+(
d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c
)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f
)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^
(1/2))/(x-(d*f)^(1/2)/f))*b*a+d^2/f^3/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln(
(2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((
a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c
)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*
b*c+d^2/f^3/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)
/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^
(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f
+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*b*c-1/2*d/(d*f)^(1/2)/f/((
a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f
)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f
)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/
2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*a^2-1/2*d^3/(d*f)^(1/2)/f^3/((a*f+c*d-(d
*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)
*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)
^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(
1/2))/(x+(d*f)^(1/2)/f))*c^2+1/8*d/(d*f)^(1/2)/f*((x+(d*f)^(1/2)/f)^2*c+(b*
f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*b
+1/16*d/(d*f)^(1/2)/f/c*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*
f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b^2-1/32*d/(d*f)^(1/2)/f/c^(
3/2)*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)
^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2
)*b)/f)^(1/2))*b^3+3/4*d^2/(d*f)^(1/2)/f^2*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f
-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(
x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*b-1/2*d^2/(d*f
)^(1/2)/f^2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)
/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^
(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f
+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*b^2+d/f^2/((a*f+c*d-(d*f)^
(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+
(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c
+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)
)/(x+(d*f)^(1/2)/f))*b*a+d^2/(d*f)^(1/2)/f^2/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1

```
/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)
/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^
(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/
2)/f))*a*c-1/4/f*x*(c*x^2+b*x+a)^(3/2)-3/16*d/f^2*ln(((x+(d*f)^(1/2)/f)*c+1
/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/
2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b^2+1/2
*d/(d*f)^(1/2)/f*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2
)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*a+1/2*d^2/(d*f)^(1/2)/f^2*((x+(d*f)
^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2
)*b)/f)^(1/2)*c-1/4*d/f^2*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(
d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*x*c-3/4*d/f^2*ln(((x-(d*f)
^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*
f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*c^
(1/2)*a-3/16*d/f^2*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(
1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+
c*d+(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b^2-1/2*d/(d*f)^(1/2)/f*((x-(d*f)^(1/2
)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/
f)^(1/2)*a-1/2*d^2/(d*f)^(1/2)/f^2*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2
)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*c+3/32/f/c*(c*x^2
+b*x+a)^(1/2)*x*b^2-3/16/f/c*(c*x^2+b*x+a)^(1/2)*b*a+3/16/f/c^(3/2)*ln((c*x
+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2*a-1/4*d/f^2*((x+(d*f)^(1/2)/f)^2*c
+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)
*x*c-3/4*d/f^2*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)
+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-
(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*a
```

**maxima** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                   /(2*f)                  +(c*d)/f+a))
     /f^2 positive, negative or zero?
```

**mupad** [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}{d-f\,x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x)
```

```
[Out] int((x^2*(a + b*x + c*x^2)^(3/2))/(d - f*x^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax^2\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx-\int \frac{bx^3\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx-\int \frac{cx^4\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(a*x**2*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(b*x**3
*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(c*x**4*sqrt(a + b*x +
c*x**2)/(-d + f*x**2), x)
```

## 3.86 $\int \frac{x\left(a+bx+cx^2\right)^{3/2}}{d-fx^2}\,dx$

**Optimal**. Leaf size=349

$$-\frac{\sqrt{a+bx+cx^2}\left(8acf+b^2f+2bcfx+8c^2d\right)}{8cf^2}-\frac{b\left(12acf+b^2(-f)+24c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}f^2}-\frac{\left(af+b\left(-\sqrt{\right.}\right.}{}$$

**Rubi [A]** time = 0.52, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1021, 1070, 1078, 621, 206, 1033, 724}

$$-\frac{\sqrt{a+bx+cx^2}\left(8acf+b^2f+2bcfx+8c^2d\right)}{8cf^2}-\frac{b\left(12acf+b^2(-f)+24c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}f^2}-\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^{5/2}}+\frac{\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^{5/2}}-\frac{\left(a+bx+cx^2\right)^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]
```

```
[Out] -((8*c^2*d + b^2*f + 8*a*c*f + 2*b*c*f*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^2)
- (a + b*x + c*x^2)^(3/2)/(3*f) - (b*(24*c^2*d - b^2*f + 12*a*c*f)*ArcTanh[
(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*f^2) - ((c*d -
b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt
[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c
*x^2])])/(2*f^(5/2)) + ((c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sq
rt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*
Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1021

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1070

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*p + 2*q + 3)
+ C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1
))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*
(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*
(C*(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*
p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(-(
b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(-(b*c*(C*(-4*d*f
)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))))*x + (p*(-(b*f))*(C*
(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2
- 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3
))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, A, B, C, q}, x] && NeQ[b^2 -
4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !I
GtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
```

, 0]

Rubi steps

$$\int \frac{x\left(a+bx+cx^2\right)^{3/2}}{d-fx^2}\,dx = -\frac{\left(a+bx+cx^2\right)^{3/2}}{3f} + \frac{\int \frac{\sqrt{a+bx+cx^2}\left(\frac{3bd}{2}+3(cd+af)x+\frac{3}{2}bfx^2\right)}{d-fx^2}\,dx}{3f}$$

$$= -\frac{\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cf^2} - \frac{\left(a+bx+cx^2\right)^{3/2}}{3f} - \frac{\int \frac{-\frac{3}{8}bdf\left(8c^2d+b^2f\right.}{}}{}$$

$$= -\frac{\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cf^2} - \frac{\left(a+bx+cx^2\right)^{3/2}}{3f} + \frac{\int \frac{\frac{3}{8}bdf^2\left(24c^2d-b^2\right.}{}}{}$$

$$= -\frac{\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cf^2} - \frac{\left(a+bx+cx^2\right)^{3/2}}{3f} + \frac{\left(cd-b\sqrt{d}\sqrt{f}\right.}{}$$

$$= -\frac{\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cf^2} - \frac{\left(a+bx+cx^2\right)^{3/2}}{3f} - \frac{b\left(24c^2d-b^2f\right.}{}$$

$$= -\frac{\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cf^2} - \frac{\left(a+bx+cx^2\right)^{3/2}}{3f} - \frac{b\left(24c^2d-b^2f\right.}{}$$

**Mathematica [A]** time = 0.80, size = 330, normalized size = 0.95

$$\frac{b\left(-12acf+b^2f-24c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{16c^{3/2}f^2} - \frac{\sqrt{f}\sqrt{a+x(b+cx)}\left(2cf(16a+7bx)+3b^2f+8c^2\left(3d+fx^2\right)\right)-12c\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}-b\sqrt{d}+b\sqrt{f}x-2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)+12c\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{-2\left(a\sqrt{f}+c\sqrt{d}x\right)-b\left(\sqrt{d}+\sqrt{f}x\right)}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{24cf^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]
```

```
[Out] (b*(-24*c^2*d + b^2*f - 12*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x
*(b + c*x)])])/(16*c^(3/2)*f^2) - (Sqrt[f]*Sqrt[a + x*(b + c*x)]*(3*b^2*f +
 2*c*f*(16*a + 7*b*x) + 8*c^2*(3*d + f*x^2)) - 12*c*(c*d - b*Sqrt[d]*Sqrt[f
] + a*f)^(3/2)*ArcTanh[(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt
[f]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] + 12*
c*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*
x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a
 + x*(b + c*x)])])/(24*c*f^(5/2))
```

**IntegrateAlgebraic [C]** time = 1.29, size = 626, normalized size = 1.79

$$-\frac{\text{RootSum}\left[\#1^4(-f)+2\#1^2af+4\#1^2cd-4\#1b\sqrt{c}d-a^2f+b^2d\&,\frac{\#1^2a^2f^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+\#1^2b^2df\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+\#1^2c^2d^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+2\#1^2acdf\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+a^3(-f^2)\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2a^2cdf\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+2b^2cd^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+ab^2df\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-4\#1bc^{3/2}d^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-ac^2d^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-4\#1ab\sqrt{c}df\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)}{\#1^3f-\#1af-2\#1cd+b\sqrt{c}d}\&\right]}{2f^2}+\frac{\left(12abcf+b^3(-f)+24bc^2d\right)\log\left(-2c^{3/2}\sqrt{a+bx+cx^2}+bc+2c^2x\right)}{16c^{3/2}f^2}+\frac{\sqrt{a+bx+cx^2}\left(-32acf-3b^2f-14bcfx-24c^2d-8c^2fx^2\right)}{24cf^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]
```

```
[Out] (Sqrt[a + b*x + c*x^2]*(-24*c^2*d - 3*b^2*f - 32*a*c*f - 14*b*c*f*x - 8*c^2
*f*x^2))/(24*c*f^2) + ((24*b*c^2*d - b^3*f + 12*a*b*c*f)*Log[b*c + 2*c^2*x
- 2*c^(3/2)*Sqrt[a + b*x + c*x^2]])/(16*c^(3/2)*f^2) - RootSum[b^2*d - a^2*
f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (2*b^2*c*d^2*Lo
g[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*c^2*d^2*Log[-(Sqrt[c]*x) +
 Sqrt[a + b*x + c*x^2] - #1] + a*b^2*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x +
c*x^2] - #1] - 2*a^2*c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] -
 a^3*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 4*b*c^(3/2)*d^2*L
og[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 4*a*b*Sqrt[c]*d*f*Log[-(
Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + c^2*d^2*Log[-(Sqrt[c]*x) + Sq
rt[a + b*x + c*x^2] - #1]*#1^2 + b^2*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x +
c*x^2] - #1]*#1^2 + 2*a*c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1
]*#1^2 + a^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sq
rt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ]/(2*f^2)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

**maple [B]** time = 0.02, size = 4567, normalized size = 13.09

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] -1/6/f*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*
f+c*d+(d*f)^(1/2)*b)/f)^(3/2)-1/6/f*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/
2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(3/2)-1/2/f^2*((x-(d*f
)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/
2)*b)/f)^(1/2)*c*d+1/2/f/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(
d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)
^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1
/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a^2-1/8/f*((x
-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f
)^(1/2)*b)/f)^(1/2)*x*b-5/8/f^2*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c
)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*b*(d*f)^(1/2)-1/16/f
/c*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*
d+(d*f)^(1/2)*b)/f)^(1/2)*b^2+1/32/f/c^(3/2)*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b
*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)
*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*b^3-1/8/f*((x+(d*f)^
(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)
*b)/f)^(1/2)*x*b+5/8/f^2*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d
*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b*(d*f)^(1/2)-1/16/f/c*((x+
(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)
^(1/2)*b)/f)^(1/2)*b^2+1/32/f/c^(3/2)*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d
*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*
f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*b^3-1/2/f^2*((x+(d*f)^(1/2)
/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f
)^(1/2)*c*d+1/2/f/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1
/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*
b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/
f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*a^2+1/f^2/((a*f+c*d+
(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*
c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/
f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)
^(1/2))/(x-(d*f)^(1/2)/f))*(d*f)^(1/2)*b*a+1/f^2/((a*f+c*d+(d*f)^(1/2)*b)/f
)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2
)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d
*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)
^(1/2)/f))*a*c*d-1/f^2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*
f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(
1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2
)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*(d*f)^(1/2)*b*a
+1/f^2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b
*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)
*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-
```

```
(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*a*c*d-1/2/f*((x-(d*f)^(1/2)/f)^
2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1
/2)*a-1/2/f*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/
f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*a-3/8/f/c^(1/2)*ln(((x-(d*f)^(1/2)/f)*c+
1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1
/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*a*b-3/4/f^2*ln
(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/
f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)
^(1/2))*c^(1/2)*(d*f)^(1/2)*a-3/16/f^2*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(
d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d
*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b^2*(d*f)^(1/2)-3/
4/f^2*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f
)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/
2)*b)/f)^(1/2))*c^(1/2)*d*b-1/2/f^3*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f
)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)
^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*c^(3/2)*(d*f)^(1/2)*d+1/2/f^2
/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(
d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(
d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^
(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*d*b^2+1/2/f^3/((a*f+c*d+(d*f)^(1/2)*b
)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(
1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2
*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d
*f)^(1/2)/f))*c^2*d^2+1/4/f^2*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*
(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*(d*f)^(1/2)*c+3/4/f^
2*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1
/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b
)/f)^(1/2))*c^(1/2)*(d*f)^(1/2)*a-3/8/f/c^(1/2)*ln(((x+(d*f)^(1/2)/f)*c+1/2
*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)
*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*a*b+3/16/f^2*ln((
(x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)
^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(
1/2))/c^(1/2)*b^2*(d*f)^(1/2)-3/4/f^2*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d
*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*
f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*d*b+1/2/f^3*ln(((x+
(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*
c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2
))*c^(3/2)*(d*f)^(1/2)*d+1/2/f^2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a
*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c
*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+
(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*d*b^2
+1/2/f^3/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+
(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/
2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*
d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*c^2*d^2-1/4/f^2*((x-(d*f)^(1/
```

```
2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)
/f)^(1/2)*x*(d*f)^(1/2)*c+1/f^3/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*
f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*
d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(
d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*(d*f)^
(1/2)*b*c*d-1/f^3/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1
/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*
b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/
f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*(d*f)^(1/2)*b*c*d
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                   /(2*f)                   +(c*d)/f+a))
     /f^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}{d-f\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x)
```

```
[Out] int((x*(a + b*x + c*x^2)^(3/2))/(d - f*x^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx-\int \frac{bx^2\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx-\int \frac{cx^3\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(a*x*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(b*x**2*sq
rt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(c*x**3*sqrt(a + b*x + c*x
**2)/(-d + f*x**2), x)
```

## 3.87 $\int \frac{(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx$

**Optimal**. Leaf size=315

$$-\frac{\left(12acf+3b^2f+8c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}f^2}+\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}f^2}$$

**Rubi** [A] time = 0.52, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {978, 1078, 621, 206, 1033, 724}

$$-\frac{\left(12acf+3b^2f+8c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}f^2}+\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}f^2}+\frac{\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}f^2}-\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(d - f*x^2),x]
```

```
[Out] -((5*b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*f) - ((8*c^2*d + 3*b^2*f + 12*a*c
*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]*f^2)
 + ((c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f]
+ (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[
a + b*x + c*x^2]])])/(2*Sqrt[d]*f^2) + ((c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2
)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c
*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2]])])/(2*Sqrt[d]*f^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 978

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((b*(3*p + 2*q) + 2*c*(p + q)*x)*(a + b*x + c*x^2)^(p - 1)
*(d + f*x^2)^(q + 1))/(2*f*(p + q)*(2*p + 2*q + 1)), x] - Dist[1/(2*f*(p +
q)*(2*p + 2*q + 1)), Int[(a + b*x + c*x^2)^(p - 2)*(d + f*x^2)^q*Simp[b^2*d
*(p - 1)*(2*p + q) - (p + q)*(b^2*d*(1 - p) - 2*a*(c*d - a*f*(2*p + 2*q + 1
))) - (2*b*(c*d - a*f)*(1 - p)*(2*p + q) - 2*(p + q)*b*(2*c*d*(2*p + q) - (
c*d + a*f)*(2*p + 2*q + 1)))*x + (b^2*f*p*(1 - p) + 2*c*(p + q)*(c*d*(2*p -
 1) - a*f*(4*p + 2*q - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x]
 && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*q + 1,
 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{d-fx^2}\,dx = -\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4f} + \frac{\int \frac{\frac{1}{4}\left(5b^2d+4a(cd+2af)\right)+4b(cd+af)x+\frac{1}{4}\left(8c^2d+3b^2f+12acf\right)x^2}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{2f}$$

$$= -\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4f} - \frac{\int \frac{-\frac{1}{4}d\left(8c^2d+3b^2f+12acf\right)-\frac{1}{4}f\left(5b^2d+4a(cd+2af)\right)-4bf(cd+af)x}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{2f^2} -$$

$$= -\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4f} - \frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)^2\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2\sqrt{d}\,f^{3/2}} + \frac{(cd}{}$$

$$= -\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4f} - \frac{\left(8c^2d+3b^2f+12acf\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}\,f^2} + \frac{(cd-}{}$$

$$= -\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4f} - \frac{\left(8c^2d+3b^2f+12acf\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}\,f^2} + \frac{(cd-}{}$$

**Mathematica** [A] time = 0.81, size = 298, normalized size = 0.95

$$-\frac{\frac{\left(12acf+3b^2f+8c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}}+\frac{4\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}-b\sqrt{d}+b\sqrt{f}x-2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{\sqrt{d}}+\frac{4\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{-2\left(a\sqrt{f}+c\sqrt{d}x\right)-b\left(\sqrt{d}+\sqrt{f}x\right)}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{d}}+2f(5b+2cx)\sqrt{a+x(b+cx)}}{8f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(d - f*x^2),x]
```

```
[Out] -1/8*(2*f*(5*b + 2*c*x)*Sqrt[a + x*(b + c*x)] + ((8*c^2*d + 3*b^2*f + 12*a*
c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + (4*(
c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(-(b*Sqrt[d]) + 2*a*Sqrt[f] -
2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a
+ x*(b + c*x)])])/Sqrt[d] + (4*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTan
h[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*
Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/Sqrt[d])/f^2
```

**IntegrateAlgebraic** [C] time = 1.00, size = 524, normalized size = 1.66

$$-\frac{\text{RootSum}\left[\#1^4(-f)+2\#1^2af+4\#1^2cd-4\#1b\sqrt{c}d-a^2f+b^2d\&,\frac{2\#1^2bcdf\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+2\#1^2abf^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-a^2bf^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2\#1a^2\sqrt{c}f^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+b^3df\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2\#1b^2\sqrt{c}df\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2\#1c^{5/2}d^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-4\#1ac^{3/2}df\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+bc^2d^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)}{\#1^3f-\#1af-2\#1cd+b\sqrt{c}d}\&\right]}{2f^2}+\frac{\left(12acf+3b^2f+8c^2d\right)\log\left(-2\sqrt{c}\sqrt{a+bx+cx^2}+b+2cx\right)}{8\sqrt{c}\,f^2}+\frac{(-5b-2cx)\sqrt{a+bx+cx^2}}{4f}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)^(3/2)/(d - f*x^2),x]
```

```
[Out] ((-5*b - 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*f) + ((8*c^2*d + 3*b^2*f + 12*a*c
*f)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(8*Sqrt[c]*f^2) - Roo
tSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 &
, (b*c^2*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + b^3*d*f*Log[-
(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a^2*b*f^2*Log[-(Sqrt[c]*x) + Sq
rt[a + b*x + c*x^2] - #1] - 2*c^(5/2)*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x +
 c*x^2] - #1]*#1 - 2*b^2*Sqrt[c]*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^
2] - #1]*#1 - 4*a*c^(3/2)*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1
]*#1 - 2*a^2*Sqrt[c]*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1
+ 2*b*c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + 2*a*b*f^2
*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*
#1 - a*f*#1 + f*#1^3) & ]/(2*f^2)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

**maple [B]** time = 0.02, size = 4574, normalized size = 14.52

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] -1/6/(d*f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2
)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(3/2)+1/6/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^
2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(3
/2)-1/2/f^2*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((
```

```
x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*
f)^(1/2)*b)/f)^(1/2))*c^(3/2)*d-1/2/(d*f)^(1/2)/((a*f+c*d-(d*f)^(1/2)*b)/f)
^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)
/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*
f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^
(1/2)/f))*a^2+1/32/(d*f)^(1/2)/c^(3/2)*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(
d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d
*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*b^3-1/2/f^2*ln(((x-(d*f)^(
1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+
2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*c^(3
/2)*d+1/2/(d*f)^(1/2)/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f
)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1
/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)
/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a^2-1/4/f*((x+(d
*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(
1/2)*b)/f)^(1/2)*x*c+1/8/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1
/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*b+1/16/(d*f)^
(1/2)/c*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a
*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b^2-3/4/f*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2
*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+
(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*a-3/16/f*ln(((x+
(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*
c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2
))/c^(1/2)*b^2-1/32/(d*f)^(1/2)/c^(3/2)*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*
(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(
d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*b^3-1/4/f*((x-(d*f)^(1/2)
/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f
)^(1/2)*x*c-1/8/(d*f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x
-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*x*b-1/16/(d*f)^(1/2)/c*(
(x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d
*f)^(1/2)*b)/f)^(1/2)*b^2-3/4/f*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1
/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/
2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*a-3/16/f*ln(((x-(d*f)^(1/
2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*
(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2
)*b^2+1/(d*f)^(1/2)/f/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f
)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1
/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)
/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a*c*d+1/f^2/((a*
f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^
(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^
(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)
*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*b*c*d+1/2/(d*f)^(1/2)/f^2/((a*f+c*d+(d*f)^
(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-
(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c
```

```
+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)
)/(x-(d*f)^(1/2)/f))*c^2*d^2-3/4/(d*f)^(1/2)/f*ln(((x-(d*f)^(1/2)/f)*c+1/2*
(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*
c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*d*b+1/2/(d
*f)^(1/2)/f/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)
/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^
(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f
+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*d*b^2+3/4/(d*f)^(1/2)/f*ln
(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/
f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)
^(1/2))*c^(1/2)*d*b-1/2/(d*f)^(1/2)/f/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln(
(2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((
a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c
)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*
d*b^2+1/f^2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)
/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^
(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f
+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*b*c*d-1/2/(d*f)^(1/2)/f^2/
((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d
*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d
*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(
1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*c^2*d^2-5/8/f*((x-(d*f)^(1/2)/f)^2*c+(
b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*b
-1/2/(d*f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2
)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*a-5/8/f*((x+(d*f)^(1/2)/f)^2*c+(b*f
-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b+1/
2/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f
)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*a+3/8/(d*f)^(1/2)/c^(1/2)*ln(((x+(d*f)
^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*
f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*a*
b+1/2/(d*f)^(1/2)/f*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(
1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*c*d+1/f/((a*f+c*d-(d*f)^(1/2)*b)
/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1
/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*
(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*
f)^(1/2)/f))*b*a-3/8/(d*f)^(1/2)/c^(1/2)*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2
*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-
(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*a*b-1/2/(d*f)^(1/2)/f*((
x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*
f)^(1/2)*b)/f)^(1/2)*c*d+1/f/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c
*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(
d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f
)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*b*a-1/(d*
f)^(1/2)/f/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/
f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(
```

```
1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+
c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*a*c*d
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                   /(2*f)                  +(c*d)/f+a))
     /f^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^{3/2}}{d-f\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)/(d - f*x^2),x)
```

```
[Out] int((a + b*x + c*x^2)^(3/2)/(d - f*x^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx - \int \frac{bx\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx - \int \frac{cx^2\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(a*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(b*x*sqrt(a
+ b*x + c*x**2)/(-d + f*x**2), x) - Integral(c*x**2*sqrt(a + b*x + c*x**2)/
(-d + f*x**2), x)
```

## 3.88 $\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x\left(d-fx^2\right)}\,dx$

**Optimal**. Leaf size=469

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d}-\frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}d}-\frac{\sqrt{a+bx+cx^2}\left(8acf+b^2f+2bcfx+8c^2d\right.}{8cdf}$$

**Rubi [A]** time = 1.27, antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {6725, 734, 814, 843, 621, 206, 724, 1021, 1070, 1078, 1033}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d}-\frac{\sqrt{a+bx+cx^2}\left(8acf+b^2f+2bcfx+8c^2d\right)}{8cdf}-\frac{b\left(12acf+b^2(-f)+24c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}df}-\frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}d}+\frac{\left(8ac+b^2+2bcx\right)\sqrt{a+bx+cx^2}}{8cd}-\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2df^{3/2}}+\frac{\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2df^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(x*(d - f*x^2)),x]
```

```
[Out] ((b^2 + 8*a*c + 2*b*c*x)*Sqrt[a + b*x + c*x^2])/(8*c*d) - ((8*c^2*d + b^2*f
 + 8*a*c*f + 2*b*c*f*x)*Sqrt[a + b*x + c*x^2])/(8*c*d*f) - (a^(3/2)*ArcTanh
[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/d - (b*(b^2 - 12*a*c)*ArcT
anh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*d) - (b*(24
*c^2*d - b^2*f + 12*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*
x^2])])/(16*c^(3/2)*d*f) - ((c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(
b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt
[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d*f^(3/2)) + ((c*d + b*Sqrt[
d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b
*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]
)/(2*d*f^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1021

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
```

```
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1070

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*p + 2*q + 3)
+ C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1
))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*
(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*
(C*(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*
p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(-(
b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(-(b*c*(C*(-4*d*f
)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))))*x + (p*(-(b*f))*(C*
(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2
- 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3
))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, A, B, C, q}, x] && NeQ[b^2 -
4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !I
GtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x\left(d-fx^2\right)}\,dx &= \int \left(\frac{\left(a+bx+cx^2\right)^{3/2}}{dx}-\frac{fx\left(a+bx+cx^2\right)^{3/2}}{d\left(-d+fx^2\right)}\right)dx\\
&= \frac{\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x}\,dx}{d}-\frac{f\int \frac{x\left(a+bx+cx^2\right)^{3/2}}{-d+fx^2}\,dx}{d}\\
&= \frac{\int \frac{\sqrt{a+bx+cx^2}\left(-\frac{3bd}{2}-3(cd+af)x-\frac{3}{2}bfx^2\right)}{-d+fx^2}\,dx}{3d}-\frac{\int \frac{(-2a-bx)\sqrt{a+bx+cx^2}}{x}\,dx}{2d}\\
&= \frac{\left(b^2+8ac+2bcx\right)\sqrt{a+bx+cx^2}}{8cd}-\frac{\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cdf}+\cdots\\
&= \frac{\left(b^2+8ac+2bcx\right)\sqrt{a+bx+cx^2}}{8cd}-\frac{\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cdf}+\cdots\\
&= \frac{\left(b^2+8ac+2bcx\right)\sqrt{a+bx+cx^2}}{8cd}-\frac{\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cdf}-\cdots\\
&= \frac{\left(b^2+8ac+2bcx\right)\sqrt{a+bx+cx^2}}{8cd}-\frac{\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cdf}-\cdots\\
&= \frac{\left(b^2+8ac+2bcx\right)\sqrt{a+bx+cx^2}}{8cd}-\frac{\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cdf}-\cdots
\end{aligned}
$$

**Mathematica [A]** time = 0.50, size = 755, normalized size = 1.61

$$
-\frac{2a^{3/2}f^{3/2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)+2cd\sqrt{f}\sqrt{a+x(b+cx)}-af\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{2a\sqrt{f}-b\sqrt{d}+b\sqrt{f}x-2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)+af\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{-2\left(a\sqrt{f}+c\sqrt{d}x\right)-b\left(\sqrt{d}+\sqrt{f}x\right)}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)+3b\sqrt{c}d\sqrt{f}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)+b\sqrt{d}\sqrt{f}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{2a\sqrt{f}-b\sqrt{d}+b\sqrt{f}x-2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)+b\sqrt{d}\sqrt{f}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{-2\left(a\sqrt{f}+c\sqrt{d}x\right)-b\left(\sqrt{d}+\sqrt{f}x\right)}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)-cd\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{2a\sqrt{f}-b\sqrt{d}+b\sqrt{f}x-2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)+cd\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{-2\left(a\sqrt{f}+c\sqrt{d}x\right)-b\left(\sqrt{d}+\sqrt{f}x\right)}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2df^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(x*(d - f*x^2)),x]
```

```
[Out] -1/2*(2*c*d*Sqrt[f]*Sqrt[a + x*(b + c*x)] + 2*a^(3/2)*f^(3/2)*ArcTanh[(2*a
+ b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])] + 3*b*Sqrt[c]*d*Sqrt[f]*ArcTanh[(
b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] - c*d*Sqrt[c*d - b*Sqrt[d]*Sq
rt[f] + a*f]*ArcTanh[(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f
```

```
]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] + b*Sqr
t[d]*Sqrt[f]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(-(b*Sqrt[d]) + 2*
a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] +
a*f]*Sqrt[a + x*(b + c*x)])] - a*f*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcT
anh[(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d
- b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] + c*d*Sqrt[c*d + b*Sqrt[
d]*Sqrt[f] + a*f]*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt
[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] + b*
Sqrt[d]*Sqrt[f]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(-2*(a*Sqrt[f]
+ c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] +
 a*f]*Sqrt[a + x*(b + c*x)])] + a*f*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Arc
Tanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d +
 b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(d*f^(3/2))
```

**<u>IntegrateAlgebraic</u> [C]** time = 1.02, size = 618, normalized size = 1.32

$$-\frac{\text{RootSum}\left[\#1^4(-f)+2\#1^2af+4\#1^2cd-4\#1b\sqrt{c}d-a^2f+b^2d\&,\frac{\#1^2a^2f^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+\#1^2b^2df\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+\#1^2c^2d^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+2\#1^2acdf\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+a^3(-f^2)\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2a^2cdf\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+2b^2cd^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+ab^2df\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-4\#1bc^{3/2}d^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-ac^2d^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-4\#1ab\sqrt{c}df\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)}{\#1^3f-\#1af-2\#1cd+b\sqrt{c}d}\&\right]}{2df}+\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}-\frac{\sqrt{a+bx+cx^2}}{\sqrt{a}}\right)}{d}-\frac{c\sqrt{a+bx+cx^2}}{f}+\frac{3b\sqrt{c}\log\left(-2\sqrt{c}f\sqrt{a+bx+cx^2}+bf+2cfx\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)^(3/2)/(x*(d - f*x^2)),x]
```

```
[Out] -((c*Sqrt[a + b*x + c*x^2])/f) + (2*a^(3/2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a] - S
qrt[a + b*x + c*x^2]/Sqrt[a]])/d + (3*b*Sqrt[c]*Log[b*f + 2*c*f*x - 2*Sqrt[
c]*f*Sqrt[a + b*x + c*x^2]])/(2*f) - RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*
#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (2*b^2*c*d^2*Log[-(Sqrt[c]*x) + S
qrt[a + b*x + c*x^2] - #1] - a*c^2*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*
x^2] - #1] + a*b^2*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*a
^2*c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a^3*f^2*Log[-(Sqr
t[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 4*b*c^(3/2)*d^2*Log[-(Sqrt[c]*x) +
Sqrt[a + b*x + c*x^2] - #1]*#1 - 4*a*b*Sqrt[c]*d*f*Log[-(Sqrt[c]*x) + Sqrt[
a + b*x + c*x^2] - #1]*#1 + c^2*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2
] - #1]*#1^2 + b^2*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2
+ 2*a*c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + a^2*f^2*L
og[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1
 - a*f*#1 + f*#1^3) & ]/(2*d*f)
```

**<u>fricas</u> [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/x/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/x/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m operator + Error: Bad Argum
ent Value
```

**maple [B]** time = 0.02, size = 4765, normalized size = 10.16

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/x/(-f*x^2+d),x)
```

```
[Out] 1/3/d*(c*x^2+b*x+a)^(3/2)-1/6/d*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c
)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(3/2)-1/6/d*((x+(d*f)^(1/2
)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/
f)^(3/2)+1/4/d*b*(c*x^2+b*x+a)^(1/2)*x+1/8/d/c*(c*x^2+b*x+a)^(1/2)*b^2-1/16
/d/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^3-1/8/d*((x-(d*f)^
(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)
*b)/f)^(1/2)*x*b-1/16/d/c*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(
d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*b^2-3/4/f*ln(((x-(d*f)^(1/
2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*
(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2
)*b-1/2/f^2*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((
x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*
f)^(1/2)*b)/f)^(1/2))*c^(3/2)*(d*f)^(1/2)+1/2/f/((a*f+c*d+(d*f)^(1/2)*b)/f)
^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)
/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*
f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^
(1/2)/f))*b^2+1/2/d/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^
(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2
)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f
)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a^2+1/2/d/((a*f+c*
d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2
)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2
)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/
f)^(1/2))/(x+(d*f)^(1/2)/f))*a^2-1/8/d*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^
(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*b-1/16/d/c*
((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(
```

```
d*f)^(1/2)*b)/f)^(1/2)*b^2+1/32/d/c^(3/2)*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-
2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x
+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*b^3-3/4/f*ln(((x+(d*f)^
(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f
-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(
1/2)*b+1/2/f^2*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)
+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-
(d*f)^(1/2)*b)/f)^(1/2))*c^(3/2)*(d*f)^(1/2)+1/2/f/((a*f+c*d-(d*f)^(1/2)*b)
/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1
/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*
(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*
f)^(1/2)/f))*b^2+1/32/d/c^(3/2)*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1
/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/
2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*b^3-1/4/d/f*((x-(d*f)^(1/2)/f)^2*
c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2
)*x*(d*f)^(1/2)*c-3/4/d/f*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)
/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/
f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*(d*f)^(1/2)*a-3/16/d/f*ln(((x-(
d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c
+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)
)/c^(1/2)*b^2*(d*f)^(1/2)+1/f^2/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*
f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*
d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(
d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*(d*f)^
(1/2)*b*c+3/16/d/f*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(
1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+
c*d-(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b^2*(d*f)^(1/2)-1/f^2/((a*f+c*d-(d*f)^
(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+
(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c
+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)
)/(x+(d*f)^(1/2)/f))*(d*f)^(1/2)*b*c+1/4/d/f*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*
(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*(d*f)
^(1/2)*c+3/4/d/f*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/
2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*
d-(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*(d*f)^(1/2)*a+1/f/((a*f+c*d+(d*f)^(1/2)*
b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^
(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+
2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(
d*f)^(1/2)/f))*a*c+1/2*d/f^2/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c
*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(
d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f
)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c^2+3/4/d
*b/c^(1/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+1/f/((a*f+c*d-(d*f
)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(
x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2
```

```
*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/
2))/(x+(d*f)^(1/2)/f))*a*c+1/2*d/f^2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((
2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a
*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)
*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*c
^2+5/8/d/f*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f
+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b*(d*f)^(1/2)-3/8/d/c^(1/2)*ln(((x+(d*f)^
(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f
-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*a*b
+1/d/f/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b
*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)
*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+
(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*(d*f)^(1/2)*b*a-1/d/f/((a*f+c*d
-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)
*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)
/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f
)^(1/2))/(x+(d*f)^(1/2)/f))*(d*f)^(1/2)*b*a-1/2/d*((x+(d*f)^(1/2)/f)^2*c+(b
*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*a-
1/2/f*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f
+c*d-(d*f)^(1/2)*b)/f)^(1/2)*c+1/d*a*(c*x^2+b*x+a)^(1/2)-1/d*a^(3/2)*ln((b*
x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-1/2/d*((x-(d*f)^(1/2)/f)^2*c+(b*f+2
*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*a-1/2/
f*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d
+(d*f)^(1/2)*b)/f)^(1/2)*c-5/8/d/f*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2
)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*b*(d*f)^(1/2)-3/8
/d/c^(1/2)*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x
-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f
)^(1/2)*b)/f)^(1/2))*a*b
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(cx^2+bx+a\right)^{\frac{3}{2}}}{\left(fx^2-d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/x/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^2 + b*x + a)^(3/2)/((f*x^2 - d)*x), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^{3/2}}{x\,\left(d-f\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)/(x*(d - f*x^2)),x)
```

```
[Out] int((a + b*x + c*x^2)^(3/2)/(x*(d - f*x^2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a\sqrt{a+bx+cx^2}}{-dx+fx^3}\,dx-\int \frac{bx\sqrt{a+bx+cx^2}}{-dx+fx^3}\,dx-\int \frac{cx^2\sqrt{a+bx+cx^2}}{-dx+fx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/x/(-f*x**2+d),x)
```

```
[Out] -Integral(a*sqrt(a + b*x + c*x**2)/(-d*x + f*x**3), x) - Integral(b*x*sqrt(
a + b*x + c*x**2)/(-d*x + f*x**3), x) - Integral(c*x**2*sqrt(a + b*x + c*x*
*2)/(-d*x + f*x**3), x)
```

## 3.89 $\int \frac{(a+bx+cx^2)^{3/2}}{x^2(d-fx^2)}\,dx$

**Optimal**. Leaf size=463

$$-\frac{\left(12acf+3b^2f+8c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}\,df}+\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}\,d}+\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right.}{}$$

**Rubi [A]** time = 1.20, antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6725, 732, 814, 843, 621, 206, 724, 978, 1078, 1033}

$$-\frac{\left(12acf+3b^2f+8c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}\,df}+\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}\,d}+\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^{3/2}f}+\frac{\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^{3/2}f}-\frac{\left(a+bx+cx^2\right)^{3/2}}{dx}+\frac{3(3b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{3\sqrt{a}\,b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(x^2*(d - f*x^2)),x]
```

```
[Out] (3*(3*b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*d) - ((5*b + 2*c*x)*Sqrt[a + b*x
 + c*x^2])/(4*d) - (a + b*x + c*x^2)^(3/2)/(d*x) - (3*Sqrt[a]*b*ArcTanh[(2*
a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*d) + (3*(b^2 + 4*a*c)*ArcTa
nh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]*d) - ((8*c^2*
d + 3*b^2*f + 12*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2
])])/(8*Sqrt[c]*d*f) + ((c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sq
rt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*
Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^(3/2)*f) + ((c*d + b*Sqrt[d]*S
qrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqr
t[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2
*d^(3/2)*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 978

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((b*(3*p + 2*q) + 2*c*(p + q)*x)*(a + b*x + c*x^2)^(p - 1)
*(d + f*x^2)^(q + 1))/(2*f*(p + q)*(2*p + 2*q + 1)), x] - Dist[1/(2*f*(p +
q)*(2*p + 2*q + 1)), Int[(a + b*x + c*x^2)^(p - 2)*(d + f*x^2)^q*Simp[b^2*d
*(p - 1)*(2*p + q) - (p + q)*(b^2*d*(1 - p) - 2*a*(c*d - a*f*(2*p + 2*q + 1
```

```
))) - (2*b*(c*d - a*f)*(1 - p)*(2*p + q) - 2*(p + q)*b*(2*c*d*(2*p + q) - (
c*d + a*f)*(2*p + 2*q + 1)))*x + (b^2*f*p*(1 - p) + 2*c*(p + q)*(c*d*(2*p -
 1) - a*f*(4*p + 2*q - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x]
 && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*q + 1,
 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^2\left(d-fx^2\right)}\,dx=\int\left(\frac{\left(a+bx+cx^2\right)^{3/2}}{dx^2}+\frac{f\left(a+bx+cx^2\right)^{3/2}}{d\left(d-fx^2\right)}\right)dx$$

$$=\frac{\int\frac{\left(a+bx+cx^2\right)^{3/2}}{x^2}\,dx}{d}+\frac{f\int\frac{\left(a+bx+cx^2\right)^{3/2}}{d-fx^2}\,dx}{d}$$

$$=-\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{\left(a+bx+cx^2\right)^{3/2}}{dx}+\frac{\int\frac{\frac{1}{4}\left(5b^2d+4a(cd+2af)\right)+4b(cd+af)x+\frac{1}{4}\left(8c^2d+3\right.}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}}{2d}$$

$$=\frac{3(3b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{\left(a+bx+cx^2\right)^{3/2}}{dx}-\frac{3\int\frac{-4abc}{x\sqrt{a}}}{}$$

$$=\frac{3(3b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{\left(a+bx+cx^2\right)^{3/2}}{dx}+\frac{(3ab)\int}{}$$

$$=\frac{3(3b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{\left(a+bx+cx^2\right)^{3/2}}{dx}-\frac{\left(8c^2d+3\right.}{}$$

$$=\frac{3(3b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{\left(a+bx+cx^2\right)^{3/2}}{dx}-\frac{3\sqrt{a}\,b\tan}{}$$

**Mathematica** [A] time = 0.64, size = 765, normalized size = 1.65

$$-\frac{2c^{3/2}d^{3/2}x\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)+2a\sqrt{d}f\sqrt{a+x(b+cx)}+cdx\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}-b\sqrt{d}+b\sqrt{f}x-2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)+cdx\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2\left(a\sqrt{f}+c\sqrt{d}x\right)-b\left(\sqrt{d}+\sqrt{f}x\right)}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)+3\sqrt{a}b\sqrt{d}fx\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)-b\sqrt{d}\sqrt{f}x\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}-b\sqrt{d}+b\sqrt{f}x-2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)+b\sqrt{d}\sqrt{f}x\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2\left(a\sqrt{f}+c\sqrt{d}x\right)-b\left(\sqrt{d}+\sqrt{f}x\right)}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)+afx\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}-b\sqrt{d}+b\sqrt{f}x-2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)+afx\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2\left(a\sqrt{f}+c\sqrt{d}x\right)-b\left(\sqrt{d}+\sqrt{f}x\right)}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^{3/2}fx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(x^2*(d - f*x^2)),x]
```

```
[Out] -1/2*(2*a*Sqrt[d]*f*Sqrt[a + x*(b + c*x)] + 3*Sqrt[a]*b*Sqrt[d]*f*x*ArcTanh
[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])] + 2*c^(3/2)*d^(3/2)*x*ArcTa
nh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + c*d*Sqrt[c*d - b*Sqrt[d
]*Sqrt[f] + a*f]*x*ArcTanh[(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*
Sqrt[f]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] -
 b*Sqrt[d]*Sqrt[f]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*x*ArcTanh[(-(b*Sqrt[
d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sq
rt[f] + a*f]*Sqrt[a + x*(b + c*x)])] + a*f*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a
```

```
*f]*x*ArcTanh[(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2
*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] + c*d*Sqrt[c*d
 + b*Sqrt[d]*Sqrt[f] + a*f]*x*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sq
rt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b +
c*x)])] + b*Sqrt[d]*Sqrt[f]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*x*ArcTanh[(
-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqr
t[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] + a*f*Sqrt[c*d + b*Sqrt[d]*Sqrt
[f] + a*f]*x*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x
))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(d^(3/2)
*f*x)
```

**IntegrateAlgebraic [C]** time = 0.89, size = 550, normalized size = 1.19

$$-\frac{\text{RootSum}\left[\#1^4(-f)+2\#1^2af+4\#1^2cd-4\#1b\sqrt{c}d-a^2f+b^2d\&,\frac{2\#1^2bcdf\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+2\#1^2abf^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-a^2bf^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2\#1a^2\sqrt{c}f^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+b^3df\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2\#1b^2\sqrt{c}df\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2\#1c^{5/2}d^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-4\#1ac^{3/2}df\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+bc^2d^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)}{\#1^3f-\#1af-2\#1cd+b\sqrt{c}d}\&\right]}{2df}+\frac{c^{3/2}\log\left(-2\sqrt{c}f\sqrt{a+bx+cx^2}+bf+2cfx\right)}{f}-\frac{a\sqrt{a+bx+cx^2}}{dx}+\frac{3\sqrt{a}b\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}-\frac{\sqrt{a+bx+cx^2}}{\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)^(3/2)/(x^2*(d - f*x^2)),x]
```

```
[Out] -((a*Sqrt[a + b*x + c*x^2])/(d*x)) + (3*Sqrt[a]*b*ArcTanh[(Sqrt[c]*x)/Sqrt[
a] - Sqrt[a + b*x + c*x^2]/Sqrt[a]])/d + (c^(3/2)*Log[b*f + 2*c*f*x - 2*Sqr
t[c]*f*Sqrt[a + b*x + c*x^2]])/f - RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1
 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (b*c^2*d^2*Log[-(Sqrt[c]*x) + Sqrt[
a + b*x + c*x^2] - #1] + b^3*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] -
 #1] - a^2*b*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*c^(5/2)
*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*b^2*Sqrt[c]*d*f*
Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 4*a*c^(3/2)*d*f*Log[-(S
qrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*a^2*Sqrt[c]*f^2*Log[-(Sqrt[c
]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*b*c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a
 + b*x + c*x^2] - #1]*#1^2 + 2*a*b*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*
x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ]/(2*d*f)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/x^2/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/x^2/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

**maple [B]** time = 0.02, size = 4799, normalized size = 10.37

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/x^2/(-f*x^2+d),x)
```

```
[Out] -1/6*f/d/(d*f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^
(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(3/2)-1/4/d*((x-(d*f)^(1/2)/f)^2*c+(b
*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*x*
c-3/4/d*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d
*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(
1/2)*b)/f)^(1/2))*c^(1/2)*a-3/16/d*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)
^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^
(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b^2-3/4/(d*f)^(1/2)*ln
(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/
f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)
^(1/2))*c^(1/2)*b+1/2/(d*f)^(1/2)/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(
a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+
c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x
-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*b^2+
3/8/d*b^2/c^(1/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/d*b/a*(c*x^
2+b*x+a)^(3/2)+1/6*f/d/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2
)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(3/2)-1/4/d*((x+(d*f)^(
1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*
b)/f)^(1/2)*x*c-3/4/d*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/
c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a
*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*a-3/16/d*ln(((x+(d*f)^(1/2)/f)*c+1/
2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2
)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b^2+3/4/
(d*f)^(1/2)*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((
x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*
f)^(1/2)*b)/f)^(1/2))*c^(1/2)*b-1/2/(d*f)^(1/2)/((a*f+c*d-(d*f)^(1/2)*b)/f)
^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)
/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*
f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^
(1/2)/f))*b^2-1/8*f/d/(d*f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)
*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*x*b-1/16*f/d/(d*f)
^(1/2)/c*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(
a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*b^2+1/32*f/d/(d*f)^(1/2)/c^(3/2)*ln(((x-(d*
```

```
f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(
b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*
b^3+1/2*f/d/(d*f)^(1/2)/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d
*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^
(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/
2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a^2+1/2/f*d/(d
*f)^(1/2)/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f
+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1
/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c
*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c^2+1/8*f/d/(d*f)^(1/2)*((x+
(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)
^(1/2)*b)/f)^(1/2)*x*b+1/16*f/d/(d*f)^(1/2)/c*((x+(d*f)^(1/2)/f)^2*c+(b*f-2
*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b^2-1/
32*f/d/(d*f)^(1/2)/c^(3/2)*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c
)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)
/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*b^3-1/2*f/d/(d*f)^(1/2)/((a*f+c*d-(d*f
)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(
x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2
*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/
2))/(x+(d*f)^(1/2)/f))*a^2-1/2/f*d/(d*f)^(1/2)/((a*f+c*d-(d*f)^(1/2)*b)/f)^
(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/
f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f
)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(
1/2)/f))*c^2+1/d/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/
2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b
)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f
+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*b*a+1/f/((a*f+c*d+(d*
f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*
(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^
2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1
/2))/(x-(d*f)^(1/2)/f))*b*c+1/2*f/d/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f
-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*a+1/
d/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*
(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+
(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)
^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*b*a+1/f/((a*f+c*d-(d*f)^(1/2)*b)/f)^
(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/
f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f
)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(
1/2)/f))*b*c-1/(d*f)^(1/2)/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d
-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*
f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^
(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*a*c+1/d*c/a
*(c*x^2+b*x+a)^(3/2)*x-5/8/d*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(
x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*b-1/2/(d*f)^(1/2)*((x-(
```

```
d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^
(1/2)*b)/f)^(1/2)*c-1/2/f*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)
/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/
f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*c^(3/2)+9/4/d*b*(c*x^2+b*x+a)^(1/2)-5/8
/d*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*
d-(d*f)^(1/2)*b)/f)^(1/2)*b+1/2/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(
d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*c-1/2/f*
ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2
)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/
f)^(1/2))*c^(3/2)+3/2/d*c*(c*x^2+b*x+a)^(1/2)*x+3/2/d*c^(1/2)*a*ln((c*x+1/2
*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/d/a/x*(c*x^2+b*x+a)^(5/2)+3/8*f/d/(d*f)^
(1/2)/c^(1/2)*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+
((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(
d*f)^(1/2)*b)/f)^(1/2))*a*b-3/8*f/d/(d*f)^(1/2)/c^(1/2)*ln(((x-(d*f)^(1/2)/
f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*
f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*a*b+1/(d*
f)^(1/2)/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+
(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/
2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*
d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a*c-1/2*f/d/(d*f)^(1/2)*((x-(
d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^
(1/2)*b)/f)^(1/2)*a-3/2/d*b*a^(1/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/
2))/x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(cx^2+bx+a\right)^{\frac{3}{2}}}{\left(fx^2-d\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/x^2/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^2 + b*x + a)^(3/2)/((f*x^2 - d)*x^2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^{3/2}}{x^2\,\left(d-f\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)/(x^2*(d - f*x^2)),x)
```

```
[Out] int((a + b*x + c*x^2)^(3/2)/(x^2*(d - f*x^2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a\sqrt{a+bx+cx^2}}{-dx^2+fx^4}\,dx-\int \frac{bx\sqrt{a+bx+cx^2}}{-dx^2+fx^4}\,dx-\int \frac{cx^2\sqrt{a+bx+cx^2}}{-dx^2+fx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/x**2/(-f*x**2+d),x)
```

```
[Out] -Integral(a*sqrt(a + b*x + c*x**2)/(-d*x**2 + f*x**4), x) - Integral(b*x*sq
rt(a + b*x + c*x**2)/(-d*x**2 + f*x**4), x) - Integral(c*x**2*sqrt(a + b*x
+ c*x**2)/(-d*x**2 + f*x**4), x)
```

## 3.90 $\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^3\left(d-fx^2\right)}\,dx$

**Optimal**. Leaf size=614

$$-\frac{a^{3/2}f\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2}-\frac{bf\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}d^2}-\frac{\sqrt{a+bx+cx^2}\left(8acf+b^2f+2bcfx+8c\right.}{8cd^2}$$

**Rubi [A]** time = 1.44, antiderivative size = 614, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {6725, 732, 812, 843, 621, 206, 724, 734, 814, 1021, 1070, 1078, 1033}

$$-\frac{a^{3/2}f\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2}-\frac{\sqrt{a+bx+cx^2}\left(8acf+b^2f+2bcfx+8c^2d\right)}{8cd^2}-\frac{bf\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}d^2}-\frac{b\left(12acf+b^2(-f)+24c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}d^2}+\frac{f\left(8ac+b^2+2bcx\right)\sqrt{a+bx+cx^2}}{8cd^2}-\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{a}d}-\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^2\sqrt{f}}+\frac{\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^2\sqrt{f}}-\frac{\left(a+bx+cx^2\right)^{3/2}}{2dx^2}-\frac{3(b-2cx)\sqrt{a+bx+cx^2}}{4dx}+\frac{3b\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(x^3*(d - f*x^2)),x]
```

```
[Out] (-3*(b - 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*d*x) + (f*(b^2 + 8*a*c + 2*b*c*x)
*Sqrt[a + b*x + c*x^2])/(8*c*d^2) - ((8*c^2*d + b^2*f + 8*a*c*f + 2*b*c*f*x
)*Sqrt[a + b*x + c*x^2])/(8*c*d^2) - (a + b*x + c*x^2)^(3/2)/(2*d*x^2) - (3
*(b^2 + 4*a*c)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*S
qrt[a]*d) - (a^(3/2)*f*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2]
)])/d^2 + (3*b*Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2]
)])/(2*d) - (b*(b^2 - 12*a*c)*f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x
 + c*x^2])])/(16*c^(3/2)*d^2) - (b*(24*c^2*d - b^2*f + 12*a*c*f)*ArcTanh[(b
 + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*d^2) - ((c*d - b*
Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d
] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x
^2])])/(2*d^2*Sqrt[f]) + ((c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*
Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d
]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^2*Sqrt[f])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
```

```
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1021

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1070

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*p + 2*q + 3)
+ C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1
))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*
(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*
```

```
(C*(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*
p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(-(
b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(-(b*c*(C*(-4*d*f
)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))))*x + (p*(-(b*f))*(C*
(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2
- 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3
))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, A, B, C, q}, x] && NeQ[b^2 -
4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !I
GtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^3\left(d-fx^2\right)}\,dx = \int \left(\frac{\left(a+bx+cx^2\right)^{3/2}}{dx^3} + \frac{f\left(a+bx+cx^2\right)^{3/2}}{d^2x} + \frac{f^2x\left(a+bx+cx^2\right)^{3/2}}{d^2\left(d-fx^2\right)}\right)dx$$

$$= \frac{\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^3}\,dx}{d} + \frac{f\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x}\,dx}{d^2} + \frac{f^2\int \frac{x\left(a+bx+cx^2\right)^{3/2}}{d-fx^2}\,dx}{d^2}$$

$$= -\frac{\left(a+bx+cx^2\right)^{3/2}}{2dx^2} + \frac{3\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x^2}\,dx}{4d} + \frac{f\int \frac{\sqrt{a+bx+cx^2}\left(\frac{3bd}{2}+3(cd+af)x+\frac{3}{2}bfx^2\right)}{d-fx^2}\,dx}{3d^2} - \cdots$$

$$= -\frac{3(b-2cx)\sqrt{a+bx+cx^2}}{4dx} + \frac{f\left(b^2+8ac+2bcx\right)\sqrt{a+bx+cx^2}}{8cd^2} - \frac{\left(8c^2d+b^2f+8ac\cdots\right.}{\cdots}$$

$$= -\frac{3(b-2cx)\sqrt{a+bx+cx^2}}{4dx} + \frac{f\left(b^2+8ac+2bcx\right)\sqrt{a+bx+cx^2}}{8cd^2} - \frac{\left(8c^2d+b^2f+8ac\cdots\right.}{\cdots}$$

$$= -\frac{3(b-2cx)\sqrt{a+bx+cx^2}}{4dx} + \frac{f\left(b^2+8ac+2bcx\right)\sqrt{a+bx+cx^2}}{8cd^2} - \frac{\left(8c^2d+b^2f+8ac\cdots\right.}{\cdots}$$

$$= -\frac{3(b-2cx)\sqrt{a+bx+cx^2}}{4dx} + \frac{f\left(b^2+8ac+2bcx\right)\sqrt{a+bx+cx^2}}{8cd^2} - \frac{\left(8c^2d+b^2f+8ac\cdots\right.}{\cdots}$$

$$= -\frac{3(b-2cx)\sqrt{a+bx+cx^2}}{4dx} + \frac{f\left(b^2+8ac+2bcx\right)\sqrt{a+bx+cx^2}}{8cd^2} - \frac{\left(8c^2d+b^2f+8ac\cdots\right.}{\cdots}$$

**Mathematica [A]** time = 0.97, size = 303, normalized size = 0.49

$$-\frac{\frac{\left(4a(2af+3cd)+3b^2d\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{\sqrt{a}} - \frac{4\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}-b\sqrt{d}+b\sqrt{f}x-2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{\sqrt{f}} + \frac{4\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{-2\left(a\sqrt{f}+c\sqrt{d}x\right)-b\left(\sqrt{d}+\sqrt{f}x\right)}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{f}} + \frac{2d(2a+5bx)\sqrt{a+x(b+cx)}}{x^2}}{8d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(x^3*(d - f*x^2)),x]
```

```
[Out] -1/8*((2*d*(2*a + 5*b*x)*Sqrt[a + x*(b + c*x)])/x^2 + ((3*b^2*d + 4*a*(3*c*
d + 2*a*f))*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/Sqrt[a]
 - (4*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(-(b*Sqrt[d]) + 2*a*Sqr
t[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*
```

```
Sqrt[a + x*(b + c*x)])])/Sqrt[f] + (4*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)
*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c
*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/Sqrt[f])/d^2
```

**IntegrateAlgebraic [C]** time = 1.17, size = 597, normalized size = 0.97

$$-\frac{\text{RootSum}\left[\#1^4(-f)+2\#1^2af+4\#1^2cd-4\#1b\sqrt{c}d-a^2f+b^2d\&,\frac{\#1^2a^2f^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+\#1^2b^2df\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+\#1^2c^2d^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+2\#1^2acdf\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+a^3(-f^2)\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2a^2cdf\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+2b^2cd^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+ab^2df\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-4\#1bc^{3/2}d^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-ac^2d^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-4\#1ab\sqrt{c}df\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)}{\#1^3f-\#1af-2\#1cd+b\sqrt{c}d}\&\right]}{2d^2}+\frac{\left(-8a^2f-12acd-3b^2d\right)\tanh^{-1}\left(\frac{\sqrt{a+bx+cx^2}-\sqrt{c}x}{\sqrt{a}}\right)}{4\sqrt{a}d^2}+\frac{(-2a-5bx)\sqrt{a+bx+cx^2}}{4dx^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)^(3/2)/(x^3*(d - f*x^2)),x]
```

```
[Out] ((-2*a - 5*b*x)*Sqrt[a + b*x + c*x^2])/(4*d*x^2) + ((-3*b^2*d - 12*a*c*d -
8*a^2*f)*ArcTanh[(-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2])/Sqrt[a]])/(4*Sqrt[a
]*d^2) - RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2
 - f*#1^4 & , (2*b^2*c*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] -
 a*c^2*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a*b^2*d*f*Log[-
(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*a^2*c*d*f*Log[-(Sqrt[c]*x) +
Sqrt[a + b*x + c*x^2] - #1] - a^3*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x
^2] - #1] - 4*b*c^(3/2)*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*
#1 - 4*a*b*Sqrt[c]*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 +
c^2*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + b^2*d*f*Log[-
(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + 2*a*c*d*f*Log[-(Sqrt[c]*x)
 + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + a^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b
*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ]/(2*d
^2)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/x^3/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$sage_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/x^3/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] sage2
```

**maple [B]** time = 0.02, size = 5056, normalized size = 8.23

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/x^3/(-f*x^2+d),x)
```

```
[Out] result too large to display
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(cx^2+bx+a\right)^{\frac{3}{2}}}{\left(fx^2-d\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/x^3/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^2 + b*x + a)^(3/2)/((f*x^2 - d)*x^3), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^{3/2}}{x^3\,\left(d-f\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)/(x^3*(d - f*x^2)),x)
```

```
[Out] int((a + b*x + c*x^2)^(3/2)/(x^3*(d - f*x^2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a\sqrt{a+bx+cx^2}}{-dx^3+fx^5}\,dx-\int \frac{bx\sqrt{a+bx+cx^2}}{-dx^3+fx^5}\,dx-\int \frac{cx^2\sqrt{a+bx+cx^2}}{-dx^3+fx^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/x**3/(-f*x**2+d),x)
```

```
[Out] -Integral(a*sqrt(a + b*x + c*x**2)/(-d*x**3 + f*x**5), x) - Integral(b*x*sq
rt(a + b*x + c*x**2)/(-d*x**3 + f*x**5), x) - Integral(c*x**2*sqrt(a + b*x
+ c*x**2)/(-d*x**3 + f*x**5), x)
```

## 3.91 $\int \frac{\left(a+bx+cx^2\right)^{3/2}}{1-x^2} \, dx$

**Optimal**. Leaf size=189

$$-\frac{\left(12ac+3b^2+8c^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}}-\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2}-\frac{1}{2}(a-b+c)^{3/2}\tanh^{-1}\left(\frac{2a+x(b-2}{2\sqrt{a-b+c}\sqrt{a-}\right.$$

**Rubi [A]** time = 0.28, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {978, 1078, 621, 206, 1033, 724}

$$-\frac{\left(12ac+3b^2+8c^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}}-\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2}-\frac{1}{2}(a-b+c)^{3/2}\tanh^{-1}\left(\frac{2a+x(b-2c)-b}{2\sqrt{a-b+c}\sqrt{a+bx+cx^2}}\right)+\frac{1}{2}(a+b+c)^{3/2}\tanh^{-1}\left(\frac{2a+x(b+2c)+b}{2\sqrt{a+b+c}\sqrt{a+bx+cx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(1 - x^2),x]
```

```
[Out] -((5*b + 2*c*x)*Sqrt[a + b*x + c*x^2])/4 - ((a - b + c)^(3/2)*ArcTanh[(2*a
- b + (b - 2*c)*x)/(2*Sqrt[a - b + c]*Sqrt[a + b*x + c*x^2])])/2 - ((3*b^2
+ 12*a*c + 8*c^2)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(
8*Sqrt[c]) + ((a + b + c)^(3/2)*ArcTanh[(2*a + b + (b + 2*c)*x)/(2*Sqrt[a +
 b + c]*Sqrt[a + b*x + c*x^2])])/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 978

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((b*(3*p + 2*q) + 2*c*(p + q)*x)*(a + b*x + c*x^2)^(p - 1)
*(d + f*x^2)^(q + 1))/(2*f*(p + q)*(2*p + 2*q + 1)), x] - Dist[1/(2*f*(p +
q)*(2*p + 2*q + 1)), Int[(a + b*x + c*x^2)^(p - 2)*(d + f*x^2)^q*Simp[b^2*d
*(p - 1)*(2*p + q) - (p + q)*(b^2*d*(1 - p) - 2*a*(c*d - a*f*(2*p + 2*q + 1
))) - (2*b*(c*d - a*f)*(1 - p)*(2*p + q) - 2*(p + q)*b*(2*c*d*(2*p + q) - (
c*d + a*f)*(2*p + 2*q + 1)))*x + (b^2*f*p*(1 - p) + 2*c*(p + q)*(c*d*(2*p -
 1) - a*f*(4*p + 2*q - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x]
 && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*q + 1,
 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{1-x^2}\,dx = -\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2} + \frac{1}{2}\int \frac{\frac{1}{4}\left(8a^2+5b^2+4ac\right)+4b(a+c)x+\frac{1}{4}\left(3b^2+12ac\right.}{\left(1-x^2\right)\sqrt{a+bx+cx^2}}$$

$$= -\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2} - \frac{1}{2}\int \frac{\frac{1}{4}\left(-8a^2-5b^2-4ac\right)+\frac{1}{4}\left(-3b^2-12ac-8c^2\right)-4}{\left(1-x^2\right)\sqrt{a+bx+cx^2}}$$

$$= -\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2} - \frac{1}{2}(a-b+c)^2\int \frac{1}{(-1-x)\sqrt{a+bx+cx^2}}\,dx + \frac{1}{2}(a+b$$

$$= -\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2} - \frac{\left(3b^2+12ac+8c^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}} + (a-b+$$

$$= -\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2} - \frac{1}{2}(a-b+c)^{3/2}\tanh^{-1}\left(\frac{2a-b+(b-2c)x}{2\sqrt{a-b+c}\sqrt{a+bx+cx^2}}\right) -$$

**Mathematica** [A]   time = 0.58, size = 181, normalized size = 0.96

$$\frac{1}{8}\left(-\frac{\left(4c(3a+2c)+3b^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}} - 2(5b+2cx)\sqrt{a+x(b+cx)} - 4(a-b+c)^{3/2}\tanh^{-1}\left(\frac{2a+b(x-1)-2cx}{2\sqrt{a-b+c}\sqrt{a+x(b+cx)}}\right) + 4(a+b+c)^{3/2}\tanh^{-1}\left(\frac{2a+bx+b+2cx}{2\sqrt{a+b+c}\sqrt{a+x(b+cx)}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(1 - x^2),x]
```

```
[Out] (-2*(5*b + 2*c*x)*Sqrt[a + x*(b + c*x)] - 4*(a - b + c)^(3/2)*ArcTanh[(2*a
+ b*(-1 + x) - 2*c*x)/(2*Sqrt[a - b + c]*Sqrt[a + x*(b + c*x)])] - ((3*b^2
+ 4*c*(3*a + 2*c))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/
Sqrt[c] + 4*(a + b + c)^(3/2)*ArcTanh[(2*a + b + b*x + 2*c*x)/(2*Sqrt[a + b
 + c]*Sqrt[a + x*(b + c*x)])])/8
```

**IntegrateAlgebraic** [A]   time = 0.82, size = 314, normalized size = 1.66

$$\frac{\left(12ac+3b^2+8c^2\right)\log\left(-2\sqrt{c}\sqrt{a+bx+cx^2}+b+2cx\right)}{8\sqrt{c}} + \frac{1}{4}(-5b-2cx)\sqrt{a+bx+cx^2} + \left(a\sqrt{-a-b-c}+c\sqrt{-a-b-c}+b\sqrt{-a-b-c}\right)\tan^{-1}\left(\frac{\sqrt{a+bx+cx^2}}{\sqrt{-a-b-c}} - \frac{\sqrt{c}\,x}{\sqrt{-a-b-c}} + \frac{\sqrt{c}}{\sqrt{-a-b-c}}\right) + \left(a\sqrt{-a+b-c}+c\sqrt{-a+b-c}-b\sqrt{-a+b-c}\right)\tan^{-1}\left(-\frac{\sqrt{a+bx+cx^2}}{\sqrt{-a+b-c}} + \frac{\sqrt{c}\,x}{\sqrt{-a+b-c}} + \frac{\sqrt{c}}{\sqrt{-a+b-c}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)^(3/2)/(1 - x^2),x]
```

```
[Out] ((-5*b - 2*c*x)*Sqrt[a + b*x + c*x^2])/4 + (a*Sqrt[-a - b - c] + b*Sqrt[-a
- b - c] + Sqrt[-a - b - c]*c)*ArcTan[Sqrt[c]/Sqrt[-a - b - c] - (Sqrt[c]*x
)/Sqrt[-a - b - c] + Sqrt[a + b*x + c*x^2]/Sqrt[-a - b - c]] + (a*Sqrt[-a +
```

```
 b - c] - b*Sqrt[-a + b - c] + Sqrt[-a + b - c]*c)*ArcTan[Sqrt[c]/Sqrt[-a +
 b - c] + (Sqrt[c]*x)/Sqrt[-a + b - c] - Sqrt[a + b*x + c*x^2]/Sqrt[-a + b
- c]] + ((3*b^2 + 12*a*c + 8*c^2)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x +
c*x^2]])/(8*Sqrt[c])
```

**fricas [A]** time = 164.30, size = 2579, normalized size = 13.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(-x^2+1),x, algorithm="fricas")
```

```
[Out] [1/16*((3*b^2 + 12*a*c + 8*c^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*
sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*((a - b)*c + c^2)*sq
rt(a - b + c)*log(-((b^2 + 4*(a - 2*b)*c + 8*c^2)*x^2 - 4*sqrt(c*x^2 + b*x
+ a)*((b - 2*c)*x + 2*a - b)*sqrt(a - b + c) + 8*a^2 - 8*a*b + b^2 + 4*a*c
+ 2*(4*a*b - 3*b^2 - 4*(a - b)*c)*x)/(x^2 + 2*x + 1)) + 4*((a + b)*c + c^2)
*sqrt(a + b + c)*log(-((b^2 + 4*(a + 2*b)*c + 8*c^2)*x^2 + 4*sqrt(c*x^2 + b
*x + a)*((b + 2*c)*x + 2*a + b)*sqrt(a + b + c) + 8*a^2 + 8*a*b + b^2 + 4*a
*c + 2*(4*a*b + 3*b^2 + 4*(a + b)*c)*x)/(x^2 - 2*x + 1)) - 4*(2*c^2*x + 5*b
*c)*sqrt(c*x^2 + b*x + a))/c, -1/16*(8*((a - b)*c + c^2)*sqrt(-a + b - c)*a
rctan(-1/2*sqrt(c*x^2 + b*x + a)*((b - 2*c)*x + 2*a - b)*sqrt(-a + b - c)/(
((a - b)*c + c^2)*x^2 + a^2 - a*b + a*c + (a*b - b^2 + b*c)*x)) - (3*b^2 +
12*a*c + 8*c^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x
 + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*((a + b)*c + c^2)*sqrt(a + b + c)*lo
g(-((b^2 + 4*(a + 2*b)*c + 8*c^2)*x^2 + 4*sqrt(c*x^2 + b*x + a)*((b + 2*c)*
x + 2*a + b)*sqrt(a + b + c) + 8*a^2 + 8*a*b + b^2 + 4*a*c + 2*(4*a*b + 3*b
^2 + 4*(a + b)*c)*x)/(x^2 - 2*x + 1)) + 4*(2*c^2*x + 5*b*c)*sqrt(c*x^2 + b*
x + a))/c, -1/16*(8*((a + b)*c + c^2)*sqrt(-a - b - c)*arctan(1/2*sqrt(c*x^
2 + b*x + a)*((b + 2*c)*x + 2*a + b)*sqrt(-a - b - c)/(((a + b)*c + c^2)*x^
2 + a^2 + a*b + a*c + (a*b + b^2 + b*c)*x)) - (3*b^2 + 12*a*c + 8*c^2)*sqrt
(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sq
rt(c) - 4*a*c) - 4*((a - b)*c + c^2)*sqrt(a - b + c)*log(-((b^2 + 4*(a - 2*
b)*c + 8*c^2)*x^2 - 4*sqrt(c*x^2 + b*x + a)*((b - 2*c)*x + 2*a - b)*sqrt(a
- b + c) + 8*a^2 - 8*a*b + b^2 + 4*a*c + 2*(4*a*b - 3*b^2 - 4*(a - b)*c)*x)
/(x^2 + 2*x + 1)) + 4*(2*c^2*x + 5*b*c)*sqrt(c*x^2 + b*x + a))/c, -1/16*(8*
((a - b)*c + c^2)*sqrt(-a + b - c)*arctan(-1/2*sqrt(c*x^2 + b*x + a)*((b -
2*c)*x + 2*a - b)*sqrt(-a + b - c)/(((a - b)*c + c^2)*x^2 + a^2 - a*b + a*c
 + (a*b - b^2 + b*c)*x)) + 8*((a + b)*c + c^2)*sqrt(-a - b - c)*arctan(1/2*
sqrt(c*x^2 + b*x + a)*((b + 2*c)*x + 2*a + b)*sqrt(-a - b - c)/(((a + b)*c
+ c^2)*x^2 + a^2 + a*b + a*c + (a*b + b^2 + b*c)*x)) - (3*b^2 + 12*a*c + 8*
c^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*
x + b)*sqrt(c) - 4*a*c) + 4*(2*c^2*x + 5*b*c)*sqrt(c*x^2 + b*x + a))/c, 1/8
*((3*b^2 + 12*a*c + 8*c^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x
 + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*((a - b)*c + c^2)*sqrt(a - b +
c)*log(-((b^2 + 4*(a - 2*b)*c + 8*c^2)*x^2 - 4*sqrt(c*x^2 + b*x + a)*((b -
```

```
2*c)*x + 2*a - b)*sqrt(a - b + c) + 8*a^2 - 8*a*b + b^2 + 4*a*c + 2*(4*a*b
- 3*b^2 - 4*(a - b)*c)*x)/(x^2 + 2*x + 1)) + 2*((a + b)*c + c^2)*sqrt(a + b
 + c)*log(-((b^2 + 4*(a + 2*b)*c + 8*c^2)*x^2 + 4*sqrt(c*x^2 + b*x + a)*((b
 + 2*c)*x + 2*a + b)*sqrt(a + b + c) + 8*a^2 + 8*a*b + b^2 + 4*a*c + 2*(4*a
*b + 3*b^2 + 4*(a + b)*c)*x)/(x^2 - 2*x + 1)) - 2*(2*c^2*x + 5*b*c)*sqrt(c*
x^2 + b*x + a))/c, -1/8*(4*((a - b)*c + c^2)*sqrt(-a + b - c)*arctan(-1/2*s
qrt(c*x^2 + b*x + a)*((b - 2*c)*x + 2*a - b)*sqrt(-a + b - c)/(((a - b)*c +
 c^2)*x^2 + a^2 - a*b + a*c + (a*b - b^2 + b*c)*x)) - (3*b^2 + 12*a*c + 8*c
^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2
 + b*c*x + a*c)) - 2*((a + b)*c + c^2)*sqrt(a + b + c)*log(-((b^2 + 4*(a +
2*b)*c + 8*c^2)*x^2 + 4*sqrt(c*x^2 + b*x + a)*((b + 2*c)*x + 2*a + b)*sqrt(
a + b + c) + 8*a^2 + 8*a*b + b^2 + 4*a*c + 2*(4*a*b + 3*b^2 + 4*(a + b)*c)*
x)/(x^2 - 2*x + 1)) + 2*(2*c^2*x + 5*b*c)*sqrt(c*x^2 + b*x + a))/c, -1/8*(4
*((a + b)*c + c^2)*sqrt(-a - b - c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*((b +
2*c)*x + 2*a + b)*sqrt(-a - b - c)/(((a + b)*c + c^2)*x^2 + a^2 + a*b + a*c
 + (a*b + b^2 + b*c)*x)) - (3*b^2 + 12*a*c + 8*c^2)*sqrt(-c)*arctan(1/2*sqr
t(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*((a -
b)*c + c^2)*sqrt(a - b + c)*log(-((b^2 + 4*(a - 2*b)*c + 8*c^2)*x^2 - 4*sqr
t(c*x^2 + b*x + a)*((b - 2*c)*x + 2*a - b)*sqrt(a - b + c) + 8*a^2 - 8*a*b
+ b^2 + 4*a*c + 2*(4*a*b - 3*b^2 - 4*(a - b)*c)*x)/(x^2 + 2*x + 1)) + 2*(2*
c^2*x + 5*b*c)*sqrt(c*x^2 + b*x + a))/c, -1/8*(4*((a - b)*c + c^2)*sqrt(-a
+ b - c)*arctan(-1/2*sqrt(c*x^2 + b*x + a)*((b - 2*c)*x + 2*a - b)*sqrt(-a
+ b - c)/(((a - b)*c + c^2)*x^2 + a^2 - a*b + a*c + (a*b - b^2 + b*c)*x)) +
 4*((a + b)*c + c^2)*sqrt(-a - b - c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*((b
+ 2*c)*x + 2*a + b)*sqrt(-a - b - c)/(((a + b)*c + c^2)*x^2 + a^2 + a*b + a
*c + (a*b + b^2 + b*c)*x)) - (3*b^2 + 12*a*c + 8*c^2)*sqrt(-c)*arctan(1/2*s
qrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(2*c
^2*x + 5*b*c)*sqrt(c*x^2 + b*x + a))/c]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(-x^2+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

**maple [B]** time = 0.02, size = 1346, normalized size = 7.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(-x^2+1),x)
```

```
[Out] -1/2*c^(3/2)*ln((1/2*b-c+c*(x+1))/c^(1/2)+((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^(
1/2))-1/2*c^(3/2)*ln((1/2*b+c+c*(x-1))/c^(1/2)+((x-1)^2*c+(b+2*c)*(x-1)+a+b
+c)^(1/2))+1/2*a*((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^(1/2)+1/2*c*((x+1)^2*c+(b-
2*c)*(x+1)+a-b+c)^(1/2)-5/8*((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^(1/2)*b-1/2*a*(
(x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^(1/2)-1/2*c*((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^
(1/2)-5/8*((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^(1/2)*b+3/8*b/c^(1/2)*ln((1/2*b-c
+c*(x+1))/c^(1/2)+((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^(1/2))*a-3/8*b/c^(1/2)*ln
((1/2*b+c+c*(x-1))/c^(1/2)+((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^(1/2))*a-1/2*a*(
a-b+c)^(1/2)*ln((2*a-2*b+2*c+(b-2*c)*(x+1)+2*(a-b+c)^(1/2)*((x+1)^2*c+(b-2*
c)*(x+1)+a-b+c)^(1/2))/(x+1))+3/4*b*ln((1/2*b-c+c*(x+1))/c^(1/2)+((x+1)^2*c
+(b-2*c)*(x+1)+a-b+c)^(1/2))*c^(1/2)+1/2*b*(a-b+c)^(1/2)*ln((2*a-2*b+2*c+(b
-2*c)*(x+1)+2*(a-b+c)^(1/2)*((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^(1/2))/(x+1))-1
/2*c*(a-b+c)^(1/2)*ln((2*a-2*b+2*c+(b-2*c)*(x+1)+2*(a-b+c)^(1/2)*((x+1)^2*c
+(b-2*c)*(x+1)+a-b+c)^(1/2))/(x+1))+1/8*b*((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^(
1/2)*x+1/16/c*((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^(1/2)*b^2-1/32/c^(3/2)*ln((1/
2*b-c+c*(x+1))/c^(1/2)+((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^(1/2))*b^3-1/4*c*((x
+1)^2*c+(b-2*c)*(x+1)+a-b+c)^(1/2)*x+1/2*a*(a+b+c)^(1/2)*ln((2*a+2*b+2*c+(b
+2*c)*(x-1)+2*(a+b+c)^(1/2)*((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^(1/2))/(x-1))-3
/4*b*ln((1/2*b+c+c*(x-1))/c^(1/2)+((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^(1/2))*c^
(1/2)+1/2*b*(a+b+c)^(1/2)*ln((2*a+2*b+2*c+(b+2*c)*(x-1)+2*(a+b+c)^(1/2)*((x
-1)^2*c+(b+2*c)*(x-1)+a+b+c)^(1/2))/(x-1))+1/2*c*(a+b+c)^(1/2)*ln((2*a+2*b+
2*c+(b+2*c)*(x-1)+2*(a+b+c)^(1/2)*((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^(1/2))/(x
-1))-1/8*b*((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^(1/2)*x-1/16/c*((x-1)^2*c+(b+2*c
)*(x-1)+a+b+c)^(1/2)*b^2+1/32/c^(3/2)*ln((1/2*b+c+c*(x-1))/c^(1/2)+((x-1)^2
*c+(b+2*c)*(x-1)+a+b+c)^(1/2))*b^3-1/4*c*((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^(1
/2)*x-3/16/c^(1/2)*ln((1/2*b-c+c*(x+1))/c^(1/2)+((x+1)^2*c+(b-2*c)*(x+1)+a-
b+c)^(1/2))*b^2-3/4*c^(1/2)*ln((1/2*b-c+c*(x+1))/c^(1/2)+((x+1)^2*c+(b-2*c)
*(x+1)+a-b+c)^(1/2))*a-3/4*c^(1/2)*ln((1/2*b+c+c*(x-1))/c^(1/2)+((x-1)^2*c+
(b+2*c)*(x-1)+a+b+c)^(1/2))*a-3/16/c^(1/2)*ln((1/2*b+c+c*(x-1))/c^(1/2)+((x
-1)^2*c+(b+2*c)*(x-1)+a+b+c)^(1/2))*b^2+1/6*((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)
^(3/2)-1/6*((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^(3/2)
```

**maxima** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(-x^2+1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(c\,x^2+b\,x+a\right)^{3/2}}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*x + c*x^2)^(3/2)/(x^2 - 1),x)
```

```
[Out] -int((a + b*x + c*x^2)^(3/2)/(x^2 - 1), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a\sqrt{a+bx+cx^2}}{x^2-1}\,dx-\int \frac{bx\sqrt{a+bx+cx^2}}{x^2-1}\,dx-\int \frac{cx^2\sqrt{a+bx+cx^2}}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(-x**2+1),x)
```

```
[Out] -Integral(a*sqrt(a + b*x + c*x**2)/(x**2 - 1), x) - Integral(b*x*sqrt(a + b
*x + c*x**2)/(x**2 - 1), x) - Integral(c*x**2*sqrt(a + b*x + c*x**2)/(x**2
- 1), x)
```

## 3.92 $\int \frac{\sqrt{-1-x+x^2}}{1-x^2}\,dx$

**Optimal**. Leaf size=75

$$-\frac{1}{2}\tan^{-1}\left(\frac{3-x}{2\sqrt{x^2-x-1}}\right)+\tanh^{-1}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{3x+1}{2\sqrt{x^2-x-1}}\right)$$

**Rubi** [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {990, 621, 206, 1033, 724, 204}

$$-\frac{1}{2}\tan^{-1}\left(\frac{3-x}{2\sqrt{x^2-x-1}}\right)+\tanh^{-1}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{3x+1}{2\sqrt{x^2-x-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 - x + x^2]/(1 - x^2),x]
```

```
[Out] -ArcTan[(3 - x)/(2*Sqrt[-1 - x + x^2])]/2 + ArcTanh[(1 - 2*x)/(2*Sqrt[-1 -
x + x^2])] + ArcTanh[(1 + 3*x)/(2*Sqrt[-1 - x + x^2])]/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 990

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (f_.)*(x_)^2), x_Symbol]
 :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*
f - b*f*x)/(Sqrt[a + b*x + c*x^2]*(d + f*x^2)), x], x] /; FreeQ[{a, b, c, d
, f}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1-x+x^2}}{1-x^2}\,dx &= -\int \frac{1}{\sqrt{-1-x+x^2}}\,dx - \int \frac{x}{\left(1-x^2\right)\sqrt{-1-x+x^2}}\,dx \\
&= -\left(\frac{1}{2}\int \frac{1}{(-1-x)\sqrt{-1-x+x^2}}\,dx\right) - \frac{1}{2}\int \frac{1}{(1-x)\sqrt{-1-x+x^2}}\,dx - 2\,\text{Subst}\left(\int \frac{1}{4-x^2}\right. \\
&= \tanh^{-1}\left(\frac{1-2x}{2\sqrt{-1-x+x^2}}\right) + \text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, \frac{3-x}{\sqrt{-1-x+x^2}}\right) + \text{Subst}\left(\int \frac{1}{4-x^2}\right. \\
&= -\frac{1}{2}\tan^{-1}\left(\frac{3-x}{2\sqrt{-1-x+x^2}}\right) + \tanh^{-1}\left(\frac{1-2x}{2\sqrt{-1-x+x^2}}\right) + \frac{1}{2}\tanh^{-1}\left(\frac{1+3x}{2\sqrt{-1-x+x^2}}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 75, normalized size = 1.00

$$-\frac{1}{2}\tan^{-1}\left(\frac{3-x}{2\sqrt{x^2-x-1}}\right) + \tanh^{-1}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right) + \frac{1}{2}\tanh^{-1}\left(\frac{3x+1}{2\sqrt{x^2-x-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 - x + x^2]/(1 - x^2),x]

[Out] -1/2*ArcTan[(3 - x)/(2*Sqrt[-1 - x + x^2])] + ArcTanh[(1 - 2*x)/(2*Sqrt[-1
- x + x^2])] + ArcTanh[(1 + 3*x)/(2*Sqrt[-1 - x + x^2])]/2
```

**IntegrateAlgebraic [A]** time = 0.14, size = 57, normalized size = 0.76

$$\log\left(2\sqrt{x^2-x-1}-2x+1\right)+\tan^{-1}\left(\sqrt{x^2-x-1}-x+1\right)+\tanh^{-1}\left(-\sqrt{x^2-x-1}+x+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-1 - x + x^2]/(1 - x^2),x]
```

```
[Out] ArcTan[1 - x + Sqrt[-1 - x + x^2]] + ArcTanh[1 + x - Sqrt[-1 - x + x^2]] +
Log[1 - 2*x + 2*Sqrt[-1 - x + x^2]]
```

**fricas [A]** time = 0.42, size = 70, normalized size = 0.93

$$\arctan\left(-x+\sqrt{x^2-x-1}+1\right)-\frac{1}{2}\log\left(-x+\sqrt{x^2-x-1}\right)+\frac{1}{2}\log\left(-x+\sqrt{x^2-x-1}-2\right)+\log\left(-2x+2\sqrt{x^2-x-1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x-1)^(1/2)/(-x^2+1),x, algorithm="fricas")
```

```
[Out] arctan(-x + sqrt(x^2 - x - 1) + 1) - 1/2*log(-x + sqrt(x^2 - x - 1)) + 1/2*
log(-x + sqrt(x^2 - x - 1) - 2) + log(-2*x + 2*sqrt(x^2 - x - 1) + 1)
```

**giac [A]** time = 0.20, size = 73, normalized size = 0.97

$$\arctan\left(-x+\sqrt{x^2-x-1}+1\right)-\frac{1}{2}\log\left(\left|-x+\sqrt{x^2-x-1}\right|\right)+\frac{1}{2}\log\left(\left|-x+\sqrt{x^2-x-1}-2\right|\right)+\log\left(\left|-2x+2\sqrt{x^2-x-1}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x-1)^(1/2)/(-x^2+1),x, algorithm="giac")
```

```
[Out] arctan(-x + sqrt(x^2 - x - 1) + 1) - 1/2*log(abs(-x + sqrt(x^2 - x - 1))) +
 1/2*log(abs(-x + sqrt(x^2 - x - 1) - 2)) + log(abs(-2*x + 2*sqrt(x^2 - x -
 1) + 1))
```

**maple [A]** time = 0.02, size = 102, normalized size = 1.36

$$-\frac{\operatorname{arctanh}\left(\frac{-3x-1}{2\sqrt{-3x+(x+1)^2-2}}\right)}{2}+\frac{\arctan\left(\frac{x-3}{2\sqrt{x+(x-1)^2-2}}\right)}{2}-\frac{3\ln\left(x-\frac{1}{2}+\sqrt{-3x+(x+1)^2-2}\right)}{4}-\frac{\ln\left(x-\frac{1}{2}+\sqrt{x+(x-1)^2-2}\right)}{4}-\frac{\sqrt{x+(x-1)^2-2}}{2}+\frac{\sqrt{-3x+(x+1)^2-2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-x-1)^(1/2)/(-x^2+1),x)
```

```
[Out] -1/2*((x-1)^2+x-2)^(1/2)-1/4*ln(-1/2+x+((x-1)^2+x-2)^(1/2))+1/2*arctan(1/2*
(-3+x)/((x-1)^2+x-2)^(1/2))+1/2*((x+1)^2-3*x-2)^(1/2)-3/4*ln(-1/2+x+((x+1)^
2-3*x-2)^(1/2))-1/2*arctanh(1/2*(-1-3*x)/((x+1)^2-3*x-2)^(1/2))
```

**maxima [A]** time = 0.97, size = 83, normalized size = 1.11

$$\frac{1}{2}\arcsin\left(\frac{2\sqrt{5}x}{5|2x-2|}-\frac{6\sqrt{5}}{5|2x-2|}\right)-\log\left(x+\sqrt{x^2-x-1}-\frac{1}{2}\right)-\frac{1}{2}\log\left(\frac{2\sqrt{x^2-x-1}}{|2x+2|}+\frac{2}{|2x+2|}-\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x-1)^(1/2)/(-x^2+1),x, algorithm="maxima")
[Out] 1/2*arcsin(2/5*sqrt(5)*x/abs(2*x - 2) - 6/5*sqrt(5)/abs(2*x - 2)) - log(x +
 sqrt(x^2 - x - 1) - 1/2) - 1/2*log(2*sqrt(x^2 - x - 1)/abs(2*x + 2) + 2/ab
s(2*x + 2) - 3/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\sqrt{x^2-x-1}}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - x - 1)^(1/2)/(x^2 - 1),x)
[Out] -int((x^2 - x - 1)^(1/2)/(x^2 - 1), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{x^2-x-1}}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-x-1)**(1/2)/(-x**2+1),x)
[Out] -Integral(sqrt(x**2 - x - 1)/(x**2 - 1), x)
```

## 3.93 $\int \frac{(x+x^2)^{3/2}}{1+x^2} dx$

**Optimal**. Leaf size=130

$$\frac{1}{4}\sqrt{x^2+x}\,(2x+5)+\sqrt{1+\sqrt{2}}\,\tan^{-1}\left(\frac{-x+\sqrt{2}+1}{\sqrt{2\left(1+\sqrt{2}\right)}\,\sqrt{x^2+x}}\right)-\sqrt{\sqrt{2}-1}\,\tanh^{-1}\left(\frac{-x-\sqrt{2}+1}{\sqrt{2\left(\sqrt{2}-1\right)}\,\sqrt{x^2+x}}\right)-\frac{5}{4}\tanh$$

**Rubi [A]** time = 0.16, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {978, 1078, 620, 206, 12, 1036, 1030, 207, 203}

$$\frac{1}{4}\sqrt{x^2+x}\,(2x+5)+\sqrt{1+\sqrt{2}}\,\tan^{-1}\left(\frac{-x+\sqrt{2}+1}{\sqrt{2\left(1+\sqrt{2}\right)}\,\sqrt{x^2+x}}\right)-\sqrt{\sqrt{2}-1}\,\tanh^{-1}\left(\frac{-x-\sqrt{2}+1}{\sqrt{2\left(\sqrt{2}-1\right)}\,\sqrt{x^2+x}}\right)-\frac{5}{4}\tanh^{-1}\left(\frac{x}{\sqrt{x^2+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x + x^2)^(3/2)/(1 + x^2),x]
```

```
[Out] ((5 + 2*x)*Sqrt[x + x^2])/4 + Sqrt[1 + Sqrt[2]]*ArcTan[(1 + Sqrt[2] - x)/(S
qrt[2*(1 + Sqrt[2])]*Sqrt[x + x^2])] - Sqrt[-1 + Sqrt[2]]*ArcTanh[(1 - Sqrt
[2] - x)/(Sqrt[2*(-1 + Sqrt[2])]*Sqrt[x + x^2])] - (5*ArcTanh[x/Sqrt[x + x^
2]])/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 978

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((b*(3*p + 2*q) + 2*c*(p + q)*x)*(a + b*x + c*x^2)^(p - 1)
*(d + f*x^2)^(q + 1))/(2*f*(p + q)*(2*p + 2*q + 1)), x] - Dist[1/(2*f*(p +
q)*(2*p + 2*q + 1)), Int[(a + b*x + c*x^2)^(p - 2)*(d + f*x^2)^q*Simp[b^2*d
*(p - 1)*(2*p + q) - (p + q)*(b^2*d*(1 - p) - 2*a*(c*d - a*f*(2*p + 2*q + 1
))) - (2*b*(c*d - a*f)*(1 - p)*(2*p + q) - 2*(p + q)*b*(2*c*d*(2*p + q) - (
c*d + a*f)*(2*p + 2*q + 1)))*x + (b^2*f*p*(1 - p) + 2*c*(p + q)*(c*d*(2*p -
 1) - a*f*(4*p + 2*q - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x]
 && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*q + 1,
 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1030

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1036

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[-(a*h*e) - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[-(a*h*e) - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[-(a*c)]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
```

, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\left(x+x^2\right)^{3/2}}{1+x^2}\,dx &= \frac{1}{4}(5+2x)\sqrt{x+x^2}-\frac{1}{2}\int \frac{\frac{5}{4}+4x+\frac{5x^2}{4}}{\left(1+x^2\right)\sqrt{x+x^2}}\,dx\\
&= \frac{1}{4}(5+2x)\sqrt{x+x^2}-\frac{1}{2}\int \frac{4x}{\left(1+x^2\right)\sqrt{x+x^2}}\,dx-\frac{5}{8}\int \frac{1}{\sqrt{x+x^2}}\,dx\\
&= \frac{1}{4}(5+2x)\sqrt{x+x^2}-\frac{5}{4}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{x+x^2}}\right)-2\int \frac{x}{\left(1+x^2\right)\sqrt{x+x^2}}\,dx\\
&= \frac{1}{4}(5+2x)\sqrt{x+x^2}-\frac{5}{4}\tanh^{-1}\left(\frac{x}{\sqrt{x+x^2}}\right)+\frac{\int \frac{-1+\left(-1-\sqrt{2}\right)x}{\left(1+x^2\right)\sqrt{x+x^2}}\,dx}{\sqrt{2}}-\frac{\int \frac{-1+\left(-1+\sqrt{2}\right)x}{\left(1+x^2\right)\sqrt{x+x^2}}\,dx}{\sqrt{2}}\\
&= \frac{1}{4}(5+2x)\sqrt{x+x^2}-\frac{5}{4}\tanh^{-1}\left(\frac{x}{\sqrt{x+x^2}}\right)+\left(-2+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{2\left(1-\sqrt{2}\right)+x^2}\,dx,x,\frac{-1}{\cdots}\right.\\
&= \frac{1}{4}(5+2x)\sqrt{x+x^2}+\sqrt{1+\sqrt{2}}\tan^{-1}\left(\frac{1+\sqrt{2}-x}{\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+x^2}}\right)-\sqrt{-1+\sqrt{2}}\tanh^{-1}\left(\frac{1\cdots}{\sqrt{2\left(-\cdots\right.}}\right.
\end{aligned}$$

**Mathematica [C]** time = 0.16, size = 120, normalized size = 0.92

$$\frac{\sqrt{x}\sqrt{x+1}\left(2\sqrt{x+1}x^{3/2}+5\sqrt{x+1}\sqrt{x}+4(-1+i)^{3/2}\tan^{-1}\left(\sqrt{-1+i}\sqrt{\frac{x}{x+1}}\right)-5\sinh^{-1}\left(\sqrt{x}\right)+4(1+i)^{3/2}\tanh^{-1}\left(\sqrt{1+i}\sqrt{\frac{x}{x+1}}\right)\right)}{4\sqrt{x(x+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^2)^(3/2)/(1 + x^2),x]
```

```
[Out] (Sqrt[x]*Sqrt[1 + x]*(5*Sqrt[x]*Sqrt[1 + x] + 2*x^(3/2)*Sqrt[1 + x] - 5*Arc
Sinh[Sqrt[x]] + 4*(-1 + I)^(3/2)*ArcTan[Sqrt[-1 + I]*Sqrt[x/(1 + x)]] + 4*(
1 + I)^(3/2)*ArcTanh[Sqrt[1 + I]*Sqrt[x/(1 + x)]]))/(4*Sqrt[x*(1 + x)])
```

**IntegrateAlgebraic [C]** time = 0.43, size = 103, normalized size = 0.79

$$\frac{1}{4}\sqrt{x^2+x}\,(2x+5)+\sqrt{2+2i}\tan^{-1}\left(\frac{\sqrt{-\frac{1}{2}-\frac{i}{2}}\sqrt{x^2+x}}{x}\right)+\sqrt{2-2i}\tan^{-1}\left(\frac{\sqrt{-\frac{1}{2}+\frac{i}{2}}\sqrt{x^2+x}}{x}\right)-\frac{5}{4}\tanh^{-1}\left(\frac{\sqrt{x^2+x}}{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x + x^2)^(3/2)/(1 + x^2),x]
```

```
[Out] ((5 + 2*x)*Sqrt[x + x^2])/4 + Sqrt[2 + 2*I]*ArcTan[(Sqrt[-1/2 - I/2]*Sqrt[x
 + x^2])/x] + Sqrt[2 - 2*I]*ArcTan[(Sqrt[-1/2 + I/2]*Sqrt[x + x^2])/x] - (5
*ArcTanh[Sqrt[x + x^2]/x])/4
```

**fricas [B]** time = 0.46, size = 777, normalized size = 5.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)^(3/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/8*8^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2)*log(8*x^2 - 8*sqrt(x^2 + x)*
x + 2*(8^(1/4)*sqrt(x^2 + x)*(sqrt(2) - 1) - 8^(1/4)*(sqrt(2)*x - x - 1))*s
qrt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4) + 1/8*8^(1/4)*sqrt(2*sqrt(2) + 4)
*(sqrt(2) - 2)*log(8*x^2 - 8*sqrt(x^2 + x)*x - 2*(8^(1/4)*sqrt(x^2 + x)*(sq
rt(2) - 1) - 8^(1/4)*(sqrt(2)*x - x - 1))*sqrt(2*sqrt(2) + 4) + 4*x + 4*sqr
t(2) + 4) + 1/2*8^(1/4)*sqrt(2)*sqrt(2*sqrt(2) + 4)*arctan(1/7*sqrt(2)*(sqr
t(2)*(5*x + 1) + 6*x + 4) + 1/112*sqrt(8*x^2 - 8*sqrt(x^2 + x)*x - 2*(8^(1/
4)*sqrt(x^2 + x)*(sqrt(2) - 1) - 8^(1/4)*(sqrt(2)*x - x - 1))*sqrt(2*sqrt(2
) + 4) + 4*x + 4*sqrt(2) + 4)*(8*sqrt(2)*(5*sqrt(2) + 6) + (8^(3/4)*(5*sqrt
(2) + 6) + 8*8^(1/4)*(2*sqrt(2) + 1))*sqrt(2*sqrt(2) + 4) + 64*sqrt(2) + 32
) - 1/7*sqrt(x^2 + x)*(sqrt(2)*(5*sqrt(2) + 6) + 8*sqrt(2) + 4) + 1/7*sqrt(
2)*(8*x + 3) + 1/56*(8^(3/4)*(sqrt(2)*(5*x + 1) + 6*x + 4) - sqrt(x^2 + x)*
(8^(3/4)*(5*sqrt(2) + 6) + 8*8^(1/4)*(2*sqrt(2) + 1)) + 8*8^(1/4)*(sqrt(2)*
(2*x - 1) + x + 3))*sqrt(2*sqrt(2) + 4) + 4/7*x + 5/7) + 1/2*8^(1/4)*sqrt(2
)*sqrt(2*sqrt(2) + 4)*arctan(-1/7*sqrt(2)*(sqrt(2)*(5*x + 1) + 6*x + 4) - 1
/112*sqrt(8*x^2 - 8*sqrt(x^2 + x)*x + 2*(8^(1/4)*sqrt(x^2 + x)*(sqrt(2) - 1
) - 8^(1/4)*(sqrt(2)*x - x - 1))*sqrt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4)
*(8*sqrt(2)*(5*sqrt(2) + 6) - (8^(3/4)*(5*sqrt(2) + 6) + 8*8^(1/4)*(2*sqrt(
2) + 1))*sqrt(2*sqrt(2) + 4) + 64*sqrt(2) + 32) + 1/7*sqrt(x^2 + x)*(sqrt(2
)*(5*sqrt(2) + 6) + 8*sqrt(2) + 4) - 1/7*sqrt(2)*(8*x + 3) + 1/56*(8^(3/4)*
(sqrt(2)*(5*x + 1) + 6*x + 4) - sqrt(x^2 + x)*(8^(3/4)*(5*sqrt(2) + 6) + 8*
8^(1/4)*(2*sqrt(2) + 1)) + 8*8^(1/4)*(sqrt(2)*(2*x - 1) + x + 3))*sqrt(2*sq
rt(2) + 4) - 4/7*x - 5/7) + 1/4*sqrt(x^2 + x)*(2*x + 5) + 5/8*log(-2*x + 2*
sqrt(x^2 + x) - 1)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)^(3/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
```

ror%%%{%%{poly1[29378258633931653019799718485334848549113596143372369654301
4566614744867108514065050877682658325195312500,-181983623107889624631149356
157406719337484171310120143823570373289503707930369037595766180522699542236
3281250,-457037569568074726029024220476354238878560215202443954711967761282
5585897607153309996504109115565063476562500,5657088028436513895060368238114
045087528706221186372034279204932374221645044901876954909719232468137744140
6250000]:[1,0,-31106,-99112,-65203241]%%},[6]%%%}+%%%{%%{poly1[563214846686
808121989333072018490142234944998851250120714869543444162112888787695442685
25719225646972656250,-69671463056492042794032844569028505467609156013361581
33975389261008891579718157298328760901400260223388671875,-18501639301691189
171876668611027555220095054439393285921280605772669698963742122921653330793
61994541992187500000,211543560165990204169085849967411852854582551443377700
410708031605115841400857697288402456445542251729797363281250]:[1,0,-31106,-
99112,-65203241]%%},[5]%%%}+%%%{%%{poly1[2497527834892881694485372868168218
93432606364903446504002320498492700674161652642483438265732405822753906250,
102899415255489529312024941498712410711762889441826371792182609371037315453
9467963565591544030347656250000000,-829533110278394819601873642487583961356
9744462926182727988533904129604785618275766584587664596969303680419921875,-
621314982897707824179256504166985600883550135798254936235154038672746633126
77694266518451768241345782073974609375]:[1,0,-31106,-99112,-65203241]%%},[4
]%%%}+%%%{%%{poly1[13406167943869060069832903395837439547234692769525032921
5956874109101434753056577258503485208281677246093750,1448223635470015710411
105400521262660890267467474254168325112744678537043998787547620879624916147
4060058593750,-450743946228316845834503722080596923970970618377104928698371
4907113445787303583904941659518660699478698730468750,-501259088879939665231
933655971301952587189466470689047006275328975844492780545187141383181367765
715754302978515625]:[1,0,-31106,-99112,-65203241]%%},[3]%%%}+%%%{%%{poly1[-
381381386714161407348746082633738466949446345681605193953880120440269250489
670513355644056050692291259765625,-9280904840184584672813286675892310136426
54084462078716361857082560926680726081331041009281187046966552734375,125754
762806389252665809919975988058534449424198247881435471309616603663322695610
14277652901709172789611816406250,163580374369001064797328406125711787375406
38352696122276658871622820423281266441048684041183868245192871093750000]:[1
,0,-31106,-99112,-65203241]%%},[2]%%%}+%%%{%%{poly1[-2829165046514258991518
864770923875171856122508474820224931553878066663155128213074011486795035416
25976562500,-11047522024183640405812419383946846813110461930170928876192826
646859750070859837003681504660562347747802734375,92622410573620422001390283
280266603413021660935063007466600699398333831523463749530142976259339105900
26855468750,379944404413115369887516401121805461986384147356195938788669206
085547964391733575642579455121057385054046630859375]:[1,0,-31106,-99112,-65
203241]%%},[1]%%%}+%%%{%%{poly1[1457442570281580782727937412636382766061047
29476650851683626206635230909249444607603805733373105438232421875,451515992
466773368261871056920647955432794275385491983650499399141514582936757420365
8363698710825103759765625,-494995282225192497001838548818146861751126606116
9463951678445919411830447649751834787235187730954876190185546875,-118464124

```
267461539474328024609355477174113903628048164370055623880000124338618560214
72161850558368016043090820312 5]:[1,0,-31106,-99112,-65203241]%%},[0]%%%} /
%%%{%%{poly1[-44900316662769750,278135011567527216375,698514226322709000750
,10286648666722750192 50]:[1,0,-31106,-99112,-65203241]%%},[1]%%%}+%%%{%%{[2
929989585103994875,368557731538442832750,907666859350191995250,-45807860415
265619064625]:[1,0,-31106,-99112,-65203241]%%},[0]%%%} Error: Bad Argument
Value
```

**maple [B]** time = 0.14, size = 789, normalized size = 6.07

$$\frac{\sqrt{x^2+x}\,x}{2}-\frac{5\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{8}+\frac{5\sqrt{x^2+x}}{4}+\frac{\sqrt{\frac{4\left(x-\sqrt{2}-1\right)^2}{\left(-x+1-\sqrt{2}\right)^2}-\frac{3\sqrt{2}\left(x-\sqrt{2}-1\right)^2}{\left(-x+1-\sqrt{2}\right)^2}+4+3\sqrt{2}}\,\sqrt{2}\left(-4\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{\frac{4\left(x-\sqrt{2}-1\right)^2}{\left(-x+1-\sqrt{2}\right)^2}-\frac{3\sqrt{2}\left(x-\sqrt{2}-1\right)^2}{\left(-x+1-\sqrt{2}\right)^2}+4+3\sqrt{2}}}{2\sqrt{1+\sqrt{2}}}\right)+6\operatorname{arctanh}\left(\frac{\sqrt{\frac{4\left(x-\sqrt{2}-1\right)^2}{\left(-x+1-\sqrt{2}\right)^2}-\frac{3\sqrt{2}\left(x-\sqrt{2}-1\right)^2}{\left(-x+1-\sqrt{2}\right)^2}+4+3\sqrt{2}}}{2\sqrt{1+\sqrt{2}}}\right)+\sqrt{-2+2\sqrt{2}}\,\sqrt{1+\sqrt{2}}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{-2+2\sqrt{2}}\,\sqrt{\left(3\sqrt{2}-4\right)\left(-\frac{\left(x-\sqrt{2}-1\right)^2}{\left(-x+1-\sqrt{2}\right)^2}+12\sqrt{2}+17\right)}\left(\frac{24\left(x-\sqrt{2}-1\right)^2}{\left(-x+1-\sqrt{2}\right)^2}+\frac{17\sqrt{2}\left(x-\sqrt{2}-1\right)^2}{\left(-x+1-\sqrt{2}\right)^2}-\sqrt{2}\right)\left(x-\sqrt{2}-1\right)\left(3\sqrt{2}-4\right)}{2\left(-x+1-\sqrt{2}\right)\left(\frac{\left(x-\sqrt{2}-1\right)^4}{\left(-x+1-\sqrt{2}\right)^4}-\frac{34\left(x-\sqrt{2}-1\right)^2}{\left(-x+1-\sqrt{2}\right)^2}+1\right)}\right)-2\sqrt{-2+2\sqrt{2}}\,\sqrt{1+\sqrt{2}}\,\arctan\left(\frac{\sqrt{-2+2\sqrt{2}}\,\sqrt{\left(3\sqrt{2}-4\right)\left(-\frac{\left(x-\sqrt{2}-1\right)^2}{\left(-x+1-\sqrt{2}\right)^2}+12\sqrt{2}+17\right)}\left(\frac{24\left(x-\sqrt{2}-1\right)^2}{\left(-x+1-\sqrt{2}\right)^2}+\frac{17\sqrt{2}\left(x-\sqrt{2}-1\right)^2}{\left(-x+1-\sqrt{2}\right)^2}-\sqrt{2}\right)\left(x-\sqrt{2}-1\right)\left(3\sqrt{2}-4\right)}{2\left(-x+1-\sqrt{2}\right)\left(\frac{\left(x-\sqrt{2}-1\right)^4}{\left(-x+1-\sqrt{2}\right)^4}-\frac{34\left(x-\sqrt{2}-1\right)^2}{\left(-x+1-\sqrt{2}\right)^2}+1\right)}\right)\right)}{2\sqrt{-\frac{\frac{3\sqrt{2}\left(x-\sqrt{2}-1\right)^2}{\left(-x+1-\sqrt{2}\right)^2}-\frac{4\left(x-\sqrt{2}-1\right)^2}{\left(-x+1-\sqrt{2}\right)^2}-3\sqrt{2}-4}{\left(\frac{x-\sqrt{2}-1}{-x+1-\sqrt{2}}+1\right)^2}}\left(\frac{x-\sqrt{2}-1}{-x+1-\sqrt{2}}+1\right)\left(3\sqrt{2}-4\right)\sqrt{1+\sqrt{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x)^(3/2)/(x^2+1),x)
```

```
[Out] 1/2*x*(x^2+x)^(1/2)+5/4*(x^2+x)^(1/2)-5/8*ln(1/2+x+(x^2+x)^(1/2))+1/2*(4*(-
2^(1/2)-1+x)^2/(1-x-2^(1/2))^2-3*2^(1/2)*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2+4
+3*2^(1/2))^(1/2)*2^(1/2)*((-2+2*2^(1/2))^(1/2)*arctan(1/2*(-2+2*2^(1/2))^(
1/2)*((3*2^(1/2)-4)*(-(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2+12*2^(1/2)+17))^(1/2
)*(24*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2+17*2^(1/2)*(-2^(1/2)-1+x)^2/(1-x-2^(
1/2))^2-2^(1/2))*(-2^(1/2)-1+x)/(1-x-2^(1/2))*(3*2^(1/2)-4)/((-2^(1/2)-1+x)
^4/(1-x-2^(1/2))^4-34*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2+1))*(1+2^(1/2))^(1/2
)*2^(1/2)-2*(-2+2*2^(1/2))^(1/2)*arctan(1/2*(-2+2*2^(1/2))^(1/2)*((3*2^(1/2
)-4)*(-(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2+12*2^(1/2)+17))^(1/2)*(24*(-2^(1/2)
-1+x)^2/(1-x-2^(1/2))^2+17*2^(1/2)*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2-2^(1/2)
)*(-2^(1/2)-1+x)/(1-x-2^(1/2))*(3*2^(1/2)-4)/((-2^(1/2)-1+x)^4/(1-x-2^(1/2)
)^4-34*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2+1))*(1+2^(1/2))^(1/2)-4*arctanh(1/2
*(4*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2-3*2^(1/2)*(-2^(1/2)-1+x)^2/(1-x-2^(1/2
))^2+4+3*2^(1/2))^(1/2)/(1+2^(1/2))^(1/2))*2^(1/2)+6*arctanh(1/2*(4*(-2^(1/
2)-1+x)^2/(1-x-2^(1/2))^2-3*2^(1/2)*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2+4+3*2^
(1/2))^(1/2)/(1+2^(1/2))^(1/2)))/(-(3*2^(1/2)*(-2^(1/2)-1+x)^2/(1-x-2^(1/2)
)^2-4*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2-3*2^(1/2)-4)/((-2^(1/2)-1+x)/(1-x-2^
(1/2))+1)^2)^(1/2)/((-2^(1/2)-1+x)/(1-x-2^(1/2))+1)/(3*2^(1/2)-4)/(1+2^(1/2
))^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+x\right)^{\frac{3}{2}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)^(3/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + x)^(3/2)/(x^2 + 1), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2+x\right)^{3/2}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2)^(3/2)/(x^2 + 1),x)
[Out] int((x + x^2)^(3/2)/(x^2 + 1), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x(x+1))^{\frac{3}{2}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x)**(3/2)/(x**2+1),x)
[Out] Integral((x*(x + 1))**(3/2)/(x**2 + 1), x)
```

## 3.94 $\int \frac{x^4}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx$

**Optimal**. Leaf size=369

$$-\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}f}+\frac{3b\sqrt{a+bx+cx^2}}{4c^2f}+\frac{d^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^2\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{d^{3/2}\tanh^{-1}}{2f}$$

**Rubi [A]** time = 0.81, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6725, 621, 206, 742, 640, 984, 724}

$$-\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}f}+\frac{3b\sqrt{a+bx+cx^2}}{4c^2f}+\frac{d^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^2\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{d^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^2\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{d\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2}-\frac{x\sqrt{a+bx+cx^2}}{2cf}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] (3*b*Sqrt[a + b*x + c*x^2])/(4*c^2*f) - (x*Sqrt[a + b*x + c*x^2])/(2*c*f) -
 (d*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*f^2) -
 ((3*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(
8*c^(5/2)*f) + (d^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b
*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]
)/(2*f^2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + (d^(3/2)*ArcTanh[(b*Sqrt[d]
 + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[
f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*
f])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 984

```
Int[1/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Sy
mbol] :> Dist[1/2, Int[1/((a - Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x],
 x] + Dist[1/2, Int[1/((a + Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx = \int\left(-\frac{d}{f^2\sqrt{a+bx+cx^2}}-\frac{x^2}{f\sqrt{a+bx+cx^2}}+\frac{d^2}{f^2\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\right)dx$$

$$=-\frac{d\int\frac{1}{\sqrt{a+bx+cx^2}}\,dx}{f^2}+\frac{d^2\int\frac{1}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{f^2}-\frac{\int\frac{x^2}{\sqrt{a+bx+cx^2}}\,dx}{f}$$

$$=-\frac{x\sqrt{a+bx+cx^2}}{2cf}-\frac{(2d)\,\mathrm{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f^2}+\frac{d^2\int\frac{1}{\left(d-\sqrt{d}\sqrt{f}x\right)\sqrt{a}}}{2f^2}$$

$$=\frac{3b\sqrt{a+bx+cx^2}}{4c^2f}-\frac{x\sqrt{a+bx+cx^2}}{2cf}-\frac{d\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2}-\frac{d^2\,\mathrm{Subst}\left(\right.}{}$$

$$=\frac{3b\sqrt{a+bx+cx^2}}{4c^2f}-\frac{x\sqrt{a+bx+cx^2}}{2cf}-\frac{d\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2}+\frac{d^{3/2}\tanh^{}}{2f^{}}$$

$$=\frac{3b\sqrt{a+bx+cx^2}}{4c^2f}-\frac{x\sqrt{a+bx+cx^2}}{2cf}-\frac{d\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2}-\frac{\left(3b^2-4ac\right.}{}$$

**Mathematica** [A] time = 1.71, size = 300, normalized size = 0.81

$$\frac{-\frac{\left(-4acf+3b^2f+8c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{c^{5/2}}-\frac{2f(2cx-3b)\sqrt{a+x(b+cx)}}{c^2}+\frac{4d^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}+\frac{4d^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}}{8f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] ((-2*f*(-3*b + 2*c*x)*Sqrt[a + x*(b + c*x)])/c^2 - ((8*c^2*d + 3*b^2*f - 4*
a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/c^(5/2) + (4
*d^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2
*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d + b*
Sqrt[d]*Sqrt[f] + a*f] + (4*d^(3/2)*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x +
 b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x
*(b + c*x)])])/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f])/(8*f^2)
```

**IntegrateAlgebraic [C]** time = 0.79, size = 263, normalized size = 0.71

$$-\frac{d^2\text{RootSum}\left[\#1^4(-f)+2\#1^2af+4\#1^2cd-4\#1b\sqrt{c}\,d-a^2f+b^2d\&,\frac{b\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)-2\#1\sqrt{c}\,\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)}{\#1^3f-\#1af-2\#1cd+b\sqrt{c}\,d}\&\right]}{2f^2}+\frac{\left(-4acf+3b^2f+8c^2d\right)\log\left(-2c^{5/2}f^2\sqrt{a+bx+cx^2}+bc^2f^2+2c^3f^2x\right)}{8c^{5/2}f^2}+\frac{(3b-2cx)\sqrt{a+bx+cx^2}}{4c^2f}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] ((3*b - 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c^2*f) + ((8*c^2*d + 3*b^2*f - 4*a
*c*f)*Log[b*c^2*f^2 + 2*c^3*f^2*x - 2*c^(5/2)*f^2*Sqrt[a + b*x + c*x^2]])/(
8*c^(5/2)*f^2) - (d^2*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2
 + 2*a*f*#1^2 - f*#1^4 & , (b*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1
] - 2*Sqrt[c]*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1)/(b*Sqrt[c]
*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ])/(2*f^2)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

**maple [A]** time = 0.03, size = 516, normalized size = 1.40

$$-\frac{d^2\ln\left(\frac{\frac{2af+2cd-2\sqrt{df}\,b}{f}+\frac{\left(bf-2\sqrt{df}\,c\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{af+cd-\sqrt{df}\,b}{f}}\,\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2c+\frac{\left(bf-2\sqrt{df}\,c\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+\frac{af+cd-\sqrt{df}\,b}{f}}}{x+\frac{\sqrt{df}}{f}}\right)}{2\sqrt{df}\,\sqrt{\frac{af+cd-\sqrt{df}\,b}{f}}\,f^2}+\frac{d^2\ln\left(\frac{\frac{2af+2cd+2\sqrt{df}\,b}{f}+\frac{\left(bf+2\sqrt{df}\,c\right)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{af+cd+\sqrt{df}\,b}{f}}\,\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^2c+\frac{\left(bf+2\sqrt{df}\,c\right)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+\frac{af+cd+\sqrt{df}\,b}{f}}}{x-\frac{\sqrt{df}}{f}}\right)}{2\sqrt{df}\,\sqrt{\frac{af+cd+\sqrt{df}\,b}{f}}\,f^2}+\frac{a\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}f}-\frac{3b^2\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{5}{2}}f}-\frac{d\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{\sqrt{c}\,f^2}-\frac{\sqrt{c\,x^2+bx+a}\,x}{2cf}+\frac{3\sqrt{c\,x^2+bx+a}\,b}{4c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -1/2*x*(c*x^2+b*x+a)^(1/2)/c/f+3/4*b*(c*x^2+b*x+a)^(1/2)/c^2/f-3/8/f*b^2/c^
(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/2/f*a/c^(3/2)*ln((c*x+1
```

```
/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/f^2*d*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*
x+a)^(1/2))/c^(1/2)-1/2/f^2*d^2/(d*f)^(1/2)/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/
2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/
f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(
1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2
)/f))+1/2/f^2*d^2/(d*f)^(1/2)/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+
c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+
(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*
f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))
```

**maxima** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                    /(2*f)                   +(c*d)/f+a))
     /f^2 zero or nonzero?
```

**mupad** [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(d-f\,x^2\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int(x^4/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)
```

**sympy** [**F**] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^4}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x**4/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)),
 x)
```

## 3.95 $\int \frac{x^3}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx$

**Optimal**. Leaf size=287

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f}-\frac{d\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^{3/2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{d\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^{3/2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\sqrt{a+b}}{c}$$

**Rubi** [A] time = 0.63, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6725, 640, 621, 206, 1033, 724}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f}-\frac{d\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^{3/2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{d\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^{3/2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\sqrt{a+bx+cx^2}}{cf}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -(Sqrt[a + b*x + c*x^2]/(c*f)) + (b*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a +
 b*x + c*x^2])])/(2*c^(3/2)*f) - (d*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c
*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*
x + c*x^2])])/(2*f^(3/2)*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + (d*ArcTanh[
(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqr
t[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(3/2)*Sqrt[c*d + b*Sqrt[d
]*Sqrt[f] + a*f])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \int \left(-\frac{x}{f\sqrt{a+bx+cx^2}} + \frac{dx}{f\sqrt{a+bx+cx^2}\,(d-fx^2)}\right)dx$$

$$= -\frac{\int \frac{x}{\sqrt{a+bx+cx^2}}\,dx}{f} + \frac{d\int \frac{x}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx}{f}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{cf} + \frac{b\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2cf} + \frac{d\int \frac{1}{(-\sqrt{d}\,\sqrt{f}-fx)\sqrt{a+bx+cx^2}}\,dx}{2f} + \frac{d\int \frac{}{(\sqrt{d}}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{cf} + \frac{b\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{cf} - \frac{d\,\mathrm{Subst}(\int \frac{1}{4cdf-4b\sqrt{d}\,f^3}}{}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{cf} + \frac{b\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f} - \frac{d\tanh^{-1}(\frac{b\sqrt{d}-2a\sqrt{f}+(2c\sqrt{d}-b\sqrt{f}}{2\sqrt{cd-b\sqrt{d}\,\sqrt{f}+af}\,\sqrt{a+bx}}}{2f^{3/2}\sqrt{cd-b\sqrt{d}\,\sqrt{f}+af}}$$

**Mathematica [A]** time = 1.16, size = 325, normalized size = 1.13

$$\frac{\frac{b\sqrt{f}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{c^{3/2}} + \frac{d\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{d\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}} - \frac{2\sqrt{f}x^2}{\sqrt{a+x(b+cx)}} - \frac{2b\sqrt{f}x}{c\sqrt{a+x(b+cx)}} - \frac{2a\sqrt{f}}{c\sqrt{a+x(b+cx)}}}{2f^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] ((-2*a*Sqrt[f])/(c*Sqrt[a + x*(b + c*x)]) - (2*b*Sqrt[f]*x)/(c*Sqrt[a + x*(
b + c*x)]) - (2*Sqrt[f]*x^2)/Sqrt[a + x*(b + c*x)] + (b*Sqrt[f]*ArcTanh[(b
+ 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/c^(3/2) + (d*ArcTanh[(b*Sqrt[d
] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt
[f] + a*f]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f] - (
d*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[
c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d - b*Sqrt[d
]*Sqrt[f] + a*f])/(2*f^(3/2))
```

**IntegrateAlgebraic [C]** time = 0.50, size = 224, normalized size = 0.78

$$-\frac{d\,\mathrm{RootSum}\left[\#1^4(-f)+2\#1^2af+4\#1^2cd-4\#1b\sqrt{c}\,d-a^2f+b^2d\&, \frac{a\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-\#1^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)}{\#1^3(-f)+\#1af+2\#1cd-b\sqrt{c}\,d}\&\right]}{2f} - \frac{b\log\left(-2c^{3/2}f\sqrt{a+bx+cx^2}+bcf+2c^2fx\right)}{2c^{3/2}f} - \frac{\sqrt{a+bx+cx^2}}{cf}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -(Sqrt[a + b*x + c*x^2]/(c*f)) - (b*Log[b*c*f + 2*c^2*f*x - 2*c^(3/2)*f*Sqr
t[a + b*x + c*x^2]])/(2*c^(3/2)*f) - (d*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]
*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (a*Log[-(Sqrt[c]*x) + Sqrt[a +
 b*x + c*x^2] - #1] - Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/
(-(b*Sqrt[c]*d) + 2*c*d*#1 + a*f*#1 - f*#1^3) & ])/(2*f)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

**maple [A]** time = 0.02, size = 410, normalized size = 1.43

$$\frac{d\ln\left(\frac{\frac{2af+2cd-2\sqrt{df}\,b}{f}+\frac{\left(bf-2\sqrt{df}\,c\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{af+cd-\sqrt{df}\,b}{f}}\,\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^{2}c+\frac{\left(bf-2\sqrt{df}\,c\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+\frac{af+cd-\sqrt{df}\,b}{f}}}{x+\frac{\sqrt{df}}{f}}\right)}{2\sqrt{\frac{af+cd-\sqrt{df}\,b}{f}}\,f^{2}}+\frac{d\ln\left(\frac{\frac{2af+2cd+2\sqrt{df}\,b}{f}+\frac{\left(bf+2\sqrt{df}\,c\right)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{af+cd+\sqrt{df}\,b}{f}}\,\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^{2}c+\frac{\left(bf+2\sqrt{df}\,c\right)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+\frac{af+cd+\sqrt{df}\,b}{f}}}{x-\frac{\sqrt{df}}{f}}\right)}{2\sqrt{\frac{af+cd+\sqrt{df}\,b}{f}}\,f^{2}}+\frac{b\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{2c^{\frac{3}{2}}f}-\frac{\sqrt{c\,x^{2}+bx+a}}{cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -(c*x^2+b*x+a)^(1/2)/c/f+1/2/f*b/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+
a)^(1/2))+1/2/f^2*d/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^
(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2
)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f
)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))+1/2/f^2*d/((a*f+c*
d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2
)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2
```

```
)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/
f)^(1/2))/(x-(d*f)^(1/2)/f))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                  /(2*f)                  +(c*d)/f+a))
     /f^2 zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\left(d-f\,x^2\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int(x^3/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^3}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x**3/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)),
 x)
```

## 3.96 $\int \frac{x^2}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx$

**Optimal**. Leaf size=266

$$\frac{\sqrt{d}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f}$$

**Rubi [A]** time = 0.22, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1079, 621, 206, 984, 724}

$$\frac{\sqrt{d}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -(ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]/(Sqrt[c]*f)) + (Sq
rt[d]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sq
rt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f*Sqrt[c*d -
b*Sqrt[d]*Sqrt[f] + a*f]) + (Sqrt[d]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*
c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b
*x + c*x^2])])/(2*f*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 984

```
Int[1/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Sy
mbol] :> Dist[1/2, Int[1/((a - Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x],
 x] + Dist[1/2, Int[1/((a + Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1079

```
Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) +
 (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x]
 + Dist[(A*c - a*C)/c, Int[1/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /;
 FreeQ[{a, c, d, e, f, A, C}, x] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx &= -\frac{\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{f}+\frac{d\int \frac{1}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{f} \\
&= -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f}+\frac{d\int \frac{1}{\left(d-\sqrt{d}\sqrt{f}x\right)\sqrt{a+bx+cx^2}}\,dx}{2f}+\cdots \\
&= -\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f}-\frac{d\,\mathrm{Subst}\left(\int \frac{1}{4cd^2-4bd^{3/2}\sqrt{f}+4adf-x^2}\,dx, x, \frac{-bd+2a\sqrt{d}\sqrt{f}-\cdots}{\sqrt{a+bx\cdots}}\cdots\right)}{f}\cdots \\
&= -\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f\sqrt{cd-b\sqrt{d}\sqrt{f}+af}}+\cdots
\end{aligned}
$$

**Mathematica [A]** time = 0.43, size = 250, normalized size = 0.94

$$
\frac{\sqrt{d}\left(\frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\sqrt{d}-b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)-\frac{2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}}}{2f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] ((-2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + Sqrt
[d]*(ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + 2*c*Sqrt[d]*x - b*Sqrt[f]*x)/(2*Sqr
t[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/Sqrt[c*d - b*Sqrt[
d]*Sqrt[f] + a*f] + ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sq
rt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/Sqr
t[c*d + b*Sqrt[d]*Sqrt[f] + a*f]))/(2*f)
```

**IntegrateAlgebraic [C]** time = 0.39, size = 197, normalized size = 0.74

$$\frac{\log\left(-2\sqrt{c}\,f\sqrt{a+bx+cx^2}+bf+2cfx\right)}{\sqrt{c}\,f}-\frac{d\text{RootSum}\left[\#1^4(-f)+2\#1^2af+4\#1^2cd-4\#1b\sqrt{c}\,d-a^2f+b^2d\&,\frac{b\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)-2\#1\sqrt{c}\,\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)}{\#1^3f-\#1af-2\#1cd+b\sqrt{c}\,d}\&\right]}{2f}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] Log[b*f + 2*c*f*x - 2*Sqrt[c]*f*Sqrt[a + b*x + c*x^2]]/(Sqrt[c]*f) - (d*Roo
tSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 &
, (b*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*Sqrt[c]*Log[-(Sqrt[
c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 +
f*#1^3) & ])/(2*f)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

**maple [A]** time = 0.02, size = 399, normalized size = 1.50

$$-\frac{d\ln\left(\frac{\frac{2af+2cd-2\sqrt{df}\,b}{f}+\frac{\left(bf-2\sqrt{df}\,c\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{af+cd-\sqrt{df}\,b}{f}}\,\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^{2}c+\frac{\left(bf-2\sqrt{df}\,c\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+\frac{af+cd-\sqrt{df}\,b}{f}}}{x+\frac{\sqrt{df}}{f}}\right)}{2\sqrt{df}\,\sqrt{\frac{af+cd-\sqrt{df}\,b}{f}}\,f}+\frac{d\ln\left(\frac{\frac{2af+2cd+2\sqrt{df}\,b}{f}+\frac{\left(bf+2\sqrt{df}\,c\right)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{af+cd+\sqrt{df}\,b}{f}}\,\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^{2}c+\frac{\left(bf+2\sqrt{df}\,c\right)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+\frac{af+cd+\sqrt{df}\,b}{f}}}{x-\frac{\sqrt{df}}{f}}\right)}{2\sqrt{df}\,\sqrt{\frac{af+cd+\sqrt{df}\,b}{f}}\,f}-\frac{\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{\sqrt{c}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -1/f*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-1/2*d/(d*f)^(1/2)/
f/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*
(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+
(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)
^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))+1/2*d/(d*f)^(1/2)/f/((a*f+c*d+(d*f)^
(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-
(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c
+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)
)/(x-(d*f)^(1/2)/f))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                   /(2*f)                 +(c*d)/f+a))
     /f^2 zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(d-f\,x^2\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int(x^2/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x**2/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)),
 x)
```

## 3.97 $\int \frac{x}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx$

**Optimal**. Leaf size=220

$$\frac{\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}$$

**Rubi [A]** time = 0.13, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1033, 724, 206}

$$\frac{\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*
d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]/(2*Sqrt[f]*Sqrt[c*d -
b*Sqrt[d]*Sqrt[f] + a*f]) + ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d]
 + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^
2])]/(2*Sqrt[f]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
```

```
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx = \frac{1}{2}\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx + \frac{1}{2}\int \frac{1}{\left(\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}$$

$$= -\text{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{b\sqrt{d}\sqrt{f}-2af-\left(-2c\sqrt{d}\sqrt{\cdot}\right.}{\sqrt{a+bx+cx^2}}\right.$$

$$= -\frac{\tanh^{-1}\left(\frac{b\sqrt{d}\sqrt{f}-2af-\left(-2c\sqrt{d}\sqrt{f}+bf\right)x}{2\sqrt{f}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{f}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} - \frac{\tanh^{-1}\left(\frac{-b\sqrt{d}\sqrt{f}-2af-\left(2c\sqrt{d}\sqrt{f}+bf\right)x}{2\sqrt{f}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx}}\right.}{2\sqrt{f}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}}$$

**Mathematica [A]** time = 0.17, size = 211, normalized size = 0.96

$$-\frac{\frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\tanh^{-1}\left(\frac{-2\left(a\sqrt{f}+c\sqrt{d}x\right)-b\left(\sqrt{d}+\sqrt{f}x\right)}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}}{2\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -1/2*(ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*S
qrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/Sqrt[c*d - b*Sqr
t[d]*Sqrt[f] + a*f] + ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] +
Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/
Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])/Sqrt[f]
```

**IntegrateAlgebraic [C]** time = 0.35, size = 149, normalized size = 0.68

$$-\frac{1}{2}\text{RootSum}\left[\#1^4(-f)+2\#1^2af+4\#1^2cd-4\#1b\sqrt{c}d-a^2f+b^2d\&, \frac{a\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-\#1^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)}{\#1^3(-f)+\#1af+2\#1cd-b\sqrt{c}d}\&\right]$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -1/2*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f
*#1^4 & , (a*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - Log[-(Sqrt[c]
*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(-(b*Sqrt[c]*d) + 2*c*d*#1 + a*f*#1
 - f*#1^3) & ]
```

**fricas [B]** time = 1.05, size = 2753, normalized size = 12.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt((c*d + a*f + (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*sqrt(b^2*
d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c +
 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))/(c^2*d^2*f + a^2*f^3 -
 (b^2 - 2*a*c)*d*f^2))*log((2*b*c*d*x + b^2*d + 2*(b^2*d*f - (c^3*d^3*f + a
^3*f^4 - (b^2*c - 3*a*c^2)*d^2*f^2 - (a*b^2 - 3*a^2*c)*d*f^3)*sqrt(b^2*d/(c
^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a
^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))*sqrt(c*x^2 + b*x + a)*sqrt
((c*d + a*f + (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*sqrt(b^2*d/(c^4*d
^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c
^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))/(c^2*d^2*f + a^2*f^3 - (b^2 -
2*a*c)*d*f^2)) - (2*a*c^2*d^2*f + 2*a^3*f^3 - 2*(a*b^2 - 2*a^2*c)*d*f^2 + (
b*c^2*d^2*f + a^2*b*f^3 - (b^3 - 2*a*b*c)*d*f^2)*x)*sqrt(b^2*d/(c^4*d^4*f +
 a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^
2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))/x) - 1/4*sqrt((c*d + a*f + (c^2*d^2*
f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2
*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^
2 - 2*a^3*c)*d*f^4)))/(c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2))*log((2*b
*c*d*x + b^2*d - 2*(b^2*d*f - (c^3*d^3*f + a^3*f^4 - (b^2*c - 3*a*c^2)*d^2*
f^2 - (a*b^2 - 3*a^2*c)*d*f^3)*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2
 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 -
2*a^3*c)*d*f^4)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d + a*f + (c^2*d^2*f + a^2*
f^3 - (b^2 - 2*a*c)*d*f^2)*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2
*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^
3*c)*d*f^4)))/(c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)) - (2*a*c^2*d^2*f
 + 2*a^3*f^3 - 2*(a*b^2 - 2*a^2*c)*d*f^2 + (b*c^2*d^2*f + a^2*b*f^3 - (b^3
- 2*a*b*c)*d*f^2)*x)*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3
)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d
*f^4)))/x) + 1/4*sqrt((c*d + a*f - (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f
^2)*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4
- 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))/(c^2*d^2*
f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2))*log((2*b*c*d*x + b^2*d + 2*(b^2*d*f + (
c^3*d^3*f + a^3*f^4 - (b^2*c - 3*a*c^2)*d^2*f^2 - (a*b^2 - 3*a^2*c)*d*f^3)*
sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*
a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))*sqrt(c*x^2 +
```

```
b*x + a)*sqrt((c*d + a*f - (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*sqrt
(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^
2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))/(c^2*d^2*f + a^2*
f^3 - (b^2 - 2*a*c)*d*f^2)) + (2*a*c^2*d^2*f + 2*a^3*f^3 - 2*(a*b^2 - 2*a^2
*c)*d*f^2 + (b*c^2*d^2*f + a^2*b*f^3 - (b^3 - 2*a*b*c)*d*f^2)*x)*sqrt(b^2*d
/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c +
6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))/x) - 1/4*sqrt((c*d + a*
f - (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*sqrt(b^2*d/(c^4*d^4*f + a^4
*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^
3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))/(c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f
^2))*log((2*b*c*d*x + b^2*d - 2*(b^2*d*f + (c^3*d^3*f + a^3*f^4 - (b^2*c -
3*a*c^2)*d^2*f^2 - (a*b^2 - 3*a^2*c)*d*f^3)*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5
 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 -
2*(a^2*b^2 - 2*a^3*c)*d*f^4)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d + a*f - (c^2
*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2
*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a
^2*b^2 - 2*a^3*c)*d*f^4)))/(c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)) + (
2*a*c^2*d^2*f + 2*a^3*f^3 - 2*(a*b^2 - 2*a^2*c)*d*f^2 + (b*c^2*d^2*f + a^2*
b*f^3 - (b^3 - 2*a*b*c)*d*f^2)*x)*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*
c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2
 - 2*a^3*c)*d*f^4)))/x)
```

**giac** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 0.52sym2pol
y/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argum
ent Value
```

**maple** [**B**] time = 0.02, size = 354, normalized size = 1.61

$$\frac{\ln\left(\frac{\frac{2af+2cd-2\sqrt{df}\,b}{f}+\frac{\left(bf-2\sqrt{df}\,c\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{af+cd-\sqrt{df}\,b}{f}}\,\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^{2}c+\frac{\left(bf-2\sqrt{df}\,c\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+\frac{af+cd-\sqrt{df}\,b}{f}}}{x+\frac{\sqrt{df}}{f}}\right)}{2\sqrt{\frac{af+cd-\sqrt{df}\,b}{f}}\,f}+\frac{\ln\left(\frac{\frac{2af+2cd+2\sqrt{df}\,b}{f}+\frac{\left(bf+2\sqrt{df}\,c\right)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{af+cd+\sqrt{df}\,b}{f}}\,\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^{2}c+\frac{\left(bf+2\sqrt{df}\,c\right)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+\frac{af+cd+\sqrt{df}\,b}{f}}}{x-\frac{\sqrt{df}}{f}}\right)}{2\sqrt{\frac{af+cd+\sqrt{df}\,b}{f}}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

[Out] 1/2/f/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))+1/2/f/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2) +b/(2*f)) ^2 -(c*((b*sqrt(4*d*f)) /(2*f) +(c*d)/f+a)) /f^2 zero or nonzero?

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(d-f\,x^2\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)

[Out] int(x/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)

[Out] -Integral(x/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)), x)

## 3.98 $\int \frac{1}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx$

**Optimal**. Leaf size=220

$$\frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}$$

**Rubi** [A] time = 0.12, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {984, 724, 206}

$$\frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d
 - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]/(2*Sqrt[d]*Sqrt[c*d - b
*Sqrt[d]*Sqrt[f] + a*f]) + ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d]
+ b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2
])]/(2*Sqrt[d]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 984

```
Int[1/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Sy
mbol] :> Dist[1/2, Int[1/((a - Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x],
```

```
x] + Dist[1/2, Int[1/((a + Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x], x]
/; FreeQ[{a, c, d, e, f}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx = \frac{1}{2}\int \frac{1}{\left(d-\sqrt{d}\sqrt{f}\,x\right)\sqrt{a+bx+cx^2}}\,dx + \frac{1}{2}\int \frac{1}{\left(d+\sqrt{d}\sqrt{f}\,x\right)\sqrt{a+bx+cx^2}}\,dx$$

$$= -\text{Subst}\left(\int \frac{1}{4cd^2-4bd^{3/2}\sqrt{f}+4adf-x^2}\,dx, x, \frac{-bd+2a\sqrt{d}\sqrt{f}-\left(2cd-b\sqrt{d}\right.}{\sqrt{a+bx+cx^2}}\right.$$

$$= -\frac{\tanh^{-1}\left(\frac{-bd+2a\sqrt{d}\sqrt{f}-\left(2cd-b\sqrt{d}\sqrt{f}\right)x}{2\sqrt{d}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} - \frac{\tanh^{-1}\left(\frac{-bd-2a\sqrt{d}\sqrt{f}-\left(2cd+b\sqrt{d}\sqrt{f}\right)x}{2\sqrt{d}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right.}{2\sqrt{d}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}}$$

**Mathematica [A]** time = 0.10, size = 209, normalized size = 0.95

$$\frac{\frac{\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}\,x+2c\sqrt{d}\,x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}+\frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}\,x\right)+2c\sqrt{d}\,x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}}{2\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] (ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*
d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/Sqrt[c*d + b*Sqrt[d]*S
qrt[f] + a*f] + ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f
]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/Sqrt[c
*d - b*Sqrt[d]*Sqrt[f] + a*f])/(2*Sqrt[d])
```

**IntegrateAlgebraic [C]** time = 0.35, size = 151, normalized size = 0.69

$$-\frac{1}{2}\text{RootSum}\left[\#1^4(-f)+2\#1^2af+4\#1^2cd-4\#1b\sqrt{c}\,d-a^2f+b^2d\&, \frac{b\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)-2\#1\sqrt{c}\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)}{\#1^3f-\#1af-2\#1cd+b\sqrt{c}\,d}\&\right]$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -1/2*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f
*#1^4 & , (b*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*Sqrt[c]*Log
[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1)/(b*Sqrt[c]*d - 2*c*d*#1 - a
*f*#1 + f*#1^3) & ]
```

**fricas [B]** time = 1.24, size = 2641, normalized size = 12.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt((c*d + a*f + (c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)*sqrt(b^2*
f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6
*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f^2 -
 (b^2 - 2*a*c)*d^2*f))*log((2*b*c*x + b^2 + 2*(b*c*d + a*b*f - (b*c^2*d^3 +
 a^2*b*d*f^2 - (b^3 - 2*a*b*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(
b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b
^2 - 2*a^3*c)*d^2*f^3)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d + a*f + (c^2*d^3 +
 a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*
c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 -
 2*a^3*c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)) - (2*a*c^
2*d^2 + 2*a^3*f^2 - 2*(a*b^2 - 2*a^2*c)*d*f + (b*c^2*d^2 + a^2*b*f^2 - (b^3
 - 2*a*b*c)*d*f)*x)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)
*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*
f^3)))/x) - 1/4*sqrt((c*d + a*f + (c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*
f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4
*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/(c^2*d^3 +
 a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f))*log((2*b*c*x + b^2 - 2*(b*c*d + a*b*f -
(b*c^2*d^3 + a^2*b*d*f^2 - (b^3 - 2*a*b*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4
*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^
2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d + a*f
+ (c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f
^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 -
2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f
)) - (2*a*c^2*d^2 + 2*a^3*f^2 - 2*(a*b^2 - 2*a^2*c)*d*f + (b*c^2*d^2 + a^2*
b*f^2 - (b^3 - 2*a*b*c)*d*f)*x)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^
2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2
*a^3*c)*d^2*f^3)))/x) + 1/4*sqrt((c*d + a*f - (c^2*d^3 + a^2*d*f^2 - (b^2 -
 2*a*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*
f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3))
)/(c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f))*log((2*b*c*x + b^2 + 2*(b*c*
d + a*b*f + (b*c^2*d^3 + a^2*b*d*f^2 - (b^3 - 2*a*b*c)*d^2*f)*sqrt(b^2*f/(c
^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2
*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))*sqrt(c*x^2 + b*x + a)*sqrt
((c*d + a*f - (c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)*sqrt(b^2*f/(c^4*d
```

```
^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2
)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f^2 - (b^2 -
2*a*c)*d^2*f)) + (2*a*c^2*d^2 + 2*a^3*f^2 - 2*(a*b^2 - 2*a^2*c)*d*f + (b*c^
2*d^2 + a^2*b*f^2 - (b^3 - 2*a*b*c)*d*f)*x)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4
 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*
(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/x) - 1/4*sqrt((c*d + a*f - (c^2*d^3 + a^2*d*
f^2 - (b^2 - 2*a*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2
*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*
c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f))*log((2*b*c*x + b
^2 - 2*(b*c*d + a*b*f + (b*c^2*d^3 + a^2*b*d*f^2 - (b^3 - 2*a*b*c)*d^2*f)*s
qrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b
^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))*sqrt(c*x^2 + b
*x + a)*sqrt((c*d + a*f - (c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)*sqrt(
b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c
 + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f
^2 - (b^2 - 2*a*c)*d^2*f)) + (2*a*c^2*d^2 + 2*a^3*f^2 - 2*(a*b^2 - 2*a^2*c)
*d*f + (b*c^2*d^2 + a^2*b*f^2 - (b^3 - 2*a*b*c)*d*f)*x)*sqrt(b^2*f/(c^4*d^5
 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*
d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/x)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 0.58sym2pol
y/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argum
ent Value
```

**maple [B]** time = 0.02, size = 358, normalized size = 1.63

$$-\frac{\ln\left(\frac{\frac{2af+2cd-2\sqrt{df}\,b}{f}+\frac{\left(bf-2\sqrt{df}\,c\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{af+cd-\sqrt{df}\,b}{f}}\,\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^{2}c+\frac{\left(bf-2\sqrt{df}\,c\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+\frac{af+cd-\sqrt{df}\,b}{f}}}{x+\frac{\sqrt{df}}{f}}\right)}{2\sqrt{df}\,\sqrt{\frac{af+cd-\sqrt{df}\,b}{f}}}+\frac{\ln\left(\frac{\frac{2af+2cd+2\sqrt{df}\,b}{f}+\frac{\left(bf+2\sqrt{df}\,c\right)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{af+cd+\sqrt{df}\,b}{f}}\,\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^{2}c+\frac{\left(bf+2\sqrt{df}\,c\right)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+\frac{af+cd+\sqrt{df}\,b}{f}}}{x-\frac{\sqrt{df}}{f}}\right)}{2\sqrt{df}\,\sqrt{\frac{af+cd+\sqrt{df}\,b}{f}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -1/2/(d*f)^(1/2)/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/
2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b
```

```
)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f
+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))+1/2/(d*f)^(1/2)/((a*f
+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(
1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(
1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*
b)/f)^(1/2))/(x-(d*f)^(1/2)/f))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                    /(2*f)                 +(c*d)/f+a))
     /f^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(d-f\,x^2\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int(1/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(1/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)), x)
```

## 3.99 $\int \frac{1}{x\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx$

**Optimal**. Leaf size=267

$$-\frac{\sqrt{f}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\sqrt{f}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\,d}$$

**Rubi [A]** time = 0.66, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6725, 724, 206, 1033}

$$-\frac{\sqrt{f}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\sqrt{f}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -(ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]/(Sqrt[a]*d)) - (Sq
rt[f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sq
rt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d*Sqrt[c*d -
b*Sqrt[d]*Sqrt[f] + a*f]) + (Sqrt[f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*
c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b
*x + c*x^2])])/(2*d*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx = \int \left(\frac{1}{dx\sqrt{a+bx+cx^2}} - \frac{fx}{d\sqrt{a+bx+cx^2}\left(-d+fx^2\right)}\right)dx$$

$$= \frac{\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{d} - \frac{f\int \frac{x}{\sqrt{a+bx+cx^2}\left(-d+fx^2\right)}\,dx}{d}$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d} - \frac{f\int \frac{1}{\left(-\sqrt{d}\,\sqrt{f}+fx\right)\sqrt{a+bx+cx^2}}\,dx}{2d} - \frac{f\int \frac{}{(\sqrt{}}$$

$$= -\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\,\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\,d} + \frac{f\,\text{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}\,f^{3/2}+4af^2-x^2}\,dx, x, \frac{-b\sqrt{d}\,\sqrt{f}+2}{\sqrt{}}\right)}{d}$$

$$= -\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\,\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{f}\,\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\,\sqrt{f}+af}\,\sqrt{a+bx+cx^2}}\right)}{2d\sqrt{cd-b\sqrt{d}\,\sqrt{f}+af}} + \frac{\sqrt{f}\,\text{ta}}{}$$

**Mathematica [A]** time = 0.44, size = 252, normalized size = 0.94

$$\frac{\sqrt{f}\left(\frac{\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}\,x+2c\sqrt{d}\,x}{2\sqrt{a+x(b+cx)}\,\sqrt{af+b\sqrt{d}\,\sqrt{f}+cd}}\right)}{\sqrt{af+b\sqrt{d}\,\sqrt{f}+cd}} - \frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\sqrt{d}-b\sqrt{f}\,x+2c\sqrt{d}\,x}{2\sqrt{a+x(b+cx)}\,\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right) - \frac{2\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\,\sqrt{a+x(b+cx)}}\right)}{\sqrt{a}}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] ((-2*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/Sqrt[a] + Sqrt
[f]*(-(ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + 2*c*Sqrt[d]*x - b*Sqrt[f]*x)/(2*S
qrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/Sqrt[c*d - b*Sqr
t[d]*Sqrt[f] + a*f]) + ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b
*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/
Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]))/(2*d)
```

**IntegrateAlgebraic [C]** time = 0.39, size = 199, normalized size = 0.75

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}-\frac{\sqrt{a+bx+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d}-\frac{f\text{RootSum}\left[\#1^4(-f)+2\#1^2af+4\#1^2cd-4\#1b\sqrt{c}\,d-a^2f+b^2d\&,\frac{a\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)-\#1^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)}{\#1^3(-f)+\#1af+2\#1cd-b\sqrt{c}\,d}\&\right]}{2d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] (2*ArcTanh[(Sqrt[c]*x)/Sqrt[a] - Sqrt[a + b*x + c*x^2]/Sqrt[a]])/(Sqrt[a]*d
) - (f*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 -
 f*#1^4 & , (a*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - Log[-(Sqrt[
c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(-(b*Sqrt[c]*d) + 2*c*d*#1 + a*f*
#1 - f*#1^3) & ])/(2*d)
```

**fricas [B]** time = 70.19, size = 5995, normalized size = 22.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/4*(a*d*sqrt((c*d*f + a*f^2 + (c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*
f)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4
 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/(c^2*d
^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f))*log((2*b*c*f^2*x + b^2*f^2 + 2*(b^
2*d*f^2 - (c^3*d^5 + a^3*d^2*f^3 - (b^2*c - 3*a*c^2)*d^4*f - (a*b^2 - 3*a^2
*c)*d^3*f^2)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^
6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3
)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d*f + a*f^2 + (c^2*d^4 + a^2*d^2*f^2 - (b
^2 - 2*a*c)*d^3*f)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c
^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d
^4*f^3)))/(c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)) - (2*a*c^2*d^3*f +
 2*a^3*d*f^3 - 2*(a*b^2 - 2*a^2*c)*d^2*f^2 + (b*c^2*d^3*f + a^2*b*d*f^3 - (
b^3 - 2*a*b*c)*d^2*f^2)*x)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2
 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*
a^3*c)*d^4*f^3)))/x) - a*d*sqrt((c*d*f + a*f^2 + (c^2*d^4 + a^2*d^2*f^2 - (
b^2 - 2*a*c)*d^3*f)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*
```

```
c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*
d^4*f^3)))/(c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f))*log((2*b*c*f^2*x
+ b^2*f^2 - 2*(b^2*d*f^2 - (c^3*d^5 + a^3*d^2*f^3 - (b^2*c - 3*a*c^2)*d^4*f
 - (a*b^2 - 3*a^2*c)*d^3*f^2)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*
c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 -
 2*a^3*c)*d^4*f^3)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d*f + a*f^2 + (c^2*d^4 +
 a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2
*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2
*b^2 - 2*a^3*c)*d^4*f^3)))/(c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)) -
 (2*a*c^2*d^3*f + 2*a^3*d*f^3 - 2*(a*b^2 - 2*a^2*c)*d^2*f^2 + (b*c^2*d^3*f
+ a^2*b*d*f^3 - (b^3 - 2*a*b*c)*d^2*f^2)*x)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3
*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2
- 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/x) + a*d*sqrt((c*d*f + a*f^2 - (c^2*d^4
+ a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 -
2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^
2*b^2 - 2*a^3*c)*d^4*f^3)))/(c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f))*
log((2*b*c*f^2*x + b^2*f^2 + 2*(b^2*d*f^2 + (c^3*d^5 + a^3*d^2*f^3 - (b^2*c
 - 3*a*c^2)*d^4*f - (a*b^2 - 3*a^2*c)*d^3*f^2)*sqrt(b^2*f^3/(c^4*d^7 + a^4*
d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f
^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d*f + a
*f^2 - (c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)*sqrt(b^2*f^3/(c^4*d^7
+ a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)
*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/(c^2*d^4 + a^2*d^2*f^2 - (b^2 -
 2*a*c)*d^3*f)) + (2*a*c^2*d^3*f + 2*a^3*d*f^3 - 2*(a*b^2 - 2*a^2*c)*d^2*f^
2 + (b*c^2*d^3*f + a^2*b*d*f^3 - (b^3 - 2*a*b*c)*d^2*f^2)*x)*sqrt(b^2*f^3/(
c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*
a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/x) - a*d*sqrt((c*d*f +
a*f^2 - (c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)*sqrt(b^2*f^3/(c^4*d^7
 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2
)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/(c^2*d^4 + a^2*d^2*f^2 - (b^2
- 2*a*c)*d^3*f))*log((2*b*c*f^2*x + b^2*f^2 - 2*(b^2*d*f^2 + (c^3*d^5 + a^3
*d^2*f^3 - (b^2*c - 3*a*c^2)*d^4*f - (a*b^2 - 3*a^2*c)*d^3*f^2)*sqrt(b^2*f^
3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c +
 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))*sqrt(c*x^2 + b*x + a
)*sqrt((c*d*f + a*f^2 - (c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)*sqrt(
b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b
^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/(c^2*d^4 + a^2
*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)) + (2*a*c^2*d^3*f + 2*a^3*d*f^3 - 2*(a*b^2
- 2*a^2*c)*d^2*f^2 + (b*c^2*d^3*f + a^2*b*d*f^3 - (b^3 - 2*a*b*c)*d^2*f^2)*
x)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4
 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/x) + 2
*sqrt(a)*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x +
 2*a)*sqrt(a) + 8*a^2)/x^2))/(a*d), 1/4*(a*d*sqrt((c*d*f + a*f^2 + (c^2*d^4
 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 -
 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a
```

^2*b^2 - 2*a^3*c)*d^4*f^3)))/(c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f))
*log((2*b*c*f^2*x + b^2*f^2 + 2*(b^2*d*f^2 - (c^3*d^5 + a^3*d^2*f^3 - (b^2*
c - 3*a*c^2)*d^4*f - (a*b^2 - 3*a^2*c)*d^3*f^2)*sqrt(b^2*f^3/(c^4*d^7 + a^4
*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*
f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d*f +
a*f^2 + (c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)*sqrt(b^2*f^3/(c^4*d^7
 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2
)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/(c^2*d^4 + a^2*d^2*f^2 - (b^2
- 2*a*c)*d^3*f)) - (2*a*c^2*d^3*f + 2*a^3*d*f^3 - 2*(a*b^2 - 2*a^2*c)*d^2*f
^2 + (b*c^2*d^3*f + a^2*b*d*f^3 - (b^3 - 2*a*b*c)*d^2*f^2)*x)*sqrt(b^2*f^3/
(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6
*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/x) - a*d*sqrt((c*d*f +
 a*f^2 + (c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)*sqrt(b^2*f^3/(c^4*d^
7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^
2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/(c^2*d^4 + a^2*d^2*f^2 - (b^2
 - 2*a*c)*d^3*f))*log((2*b*c*f^2*x + b^2*f^2 - 2*(b^2*d*f^2 - (c^3*d^5 + a^
3*d^2*f^3 - (b^2*c - 3*a*c^2)*d^4*f - (a*b^2 - 3*a^2*c)*d^3*f^2)*sqrt(b^2*f
^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c
+ 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))*sqrt(c*x^2 + b*x +
a)*sqrt((c*d*f + a*f^2 + (c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)*sqrt
(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*
b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/(c^2*d^4 + a^
2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)) - (2*a*c^2*d^3*f + 2*a^3*d*f^3 - 2*(a*b^2
 - 2*a^2*c)*d^2*f^2 + (b*c^2*d^3*f + a^2*b*d*f^3 - (b^3 - 2*a*b*c)*d^2*f^2)
*x)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^
4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/x) +
a*d*sqrt((c*d*f + a*f^2 - (c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)*sqr
t(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a
*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/(c^2*d^4 + a
^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f))*log((2*b*c*f^2*x + b^2*f^2 + 2*(b^2*d*f^
2 + (c^3*d^5 + a^3*d^2*f^3 - (b^2*c - 3*a*c^2)*d^4*f - (a*b^2 - 3*a^2*c)*d^
3*f^2)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f +
(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))*sq
rt(c*x^2 + b*x + a)*sqrt((c*d*f + a*f^2 - (c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2
*a*c)*d^3*f)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^
6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3
)))/(c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)) + (2*a*c^2*d^3*f + 2*a^3
*d*f^3 - 2*(a*b^2 - 2*a^2*c)*d^2*f^2 + (b*c^2*d^3*f + a^2*b*d*f^3 - (b^3 -
2*a*b*c)*d^2*f^2)*x)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a
*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)
*d^4*f^3)))/x) - a*d*sqrt((c*d*f + a*f^2 - (c^2*d^4 + a^2*d^2*f^2 - (b^2 -
2*a*c)*d^3*f)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d
^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^
3)))/(c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f))*log((2*b*c*f^2*x + b^2*
f^2 - 2*(b^2*d*f^2 + (c^3*d^5 + a^3*d^2*f^3 - (b^2*c - 3*a*c^2)*d^4*f - (a*

```
b^2 - 3*a^2*c)*d^3*f^2)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 -
2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3
*c)*d^4*f^3)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d*f + a*f^2 - (c^2*d^4 + a^2*d
^2*f^2 - (b^2 - 2*a*c)*d^3*f)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*
c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 -
 2*a^3*c)*d^4*f^3)))/(c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)) + (2*a*
c^2*d^3*f + 2*a^3*d*f^3 - 2*(a*b^2 - 2*a^2*c)*d^2*f^2 + (b*c^2*d^3*f + a^2*
b*d*f^3 - (b^3 - 2*a*b*c)*d^2*f^2)*x)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 -
 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a
^2*b^2 - 2*a^3*c)*d^4*f^3)))/x) + 4*sqrt(-a)*arctan(1/2*sqrt(c*x^2 + b*x +
a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)))/(a*d)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$sage_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] sage2
```

**maple [A]** time = 0.02, size = 391, normalized size = 1.46

$$\frac{\ln\left(\frac{\frac{2af+2cd-2\sqrt{df}\,b}{f}+\frac{\left(bf-2\sqrt{df}\,c\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{af+cd-\sqrt{df}\,b}{f}}\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2 c+\frac{\left(bf-2\sqrt{df}\,c\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+\frac{af+cd-\sqrt{df}\,b}{f}}}{x+\frac{\sqrt{df}}{f}}\right)}{2\sqrt{\frac{af+cd-\sqrt{df}\,b}{f}}\,d}+\frac{\ln\left(\frac{\frac{2af+2cd+2\sqrt{df}\,b}{f}+\frac{\left(bf+2\sqrt{df}\,c\right)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{af+cd+\sqrt{df}\,b}{f}}\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^2 c+\frac{\left(bf+2\sqrt{df}\,c\right)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+\frac{af+cd+\sqrt{df}\,b}{f}}}{x-\frac{\sqrt{df}}{f}}\right)}{2\sqrt{\frac{af+cd+\sqrt{df}\,b}{f}}\,d}-\frac{\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\,\sqrt{a}}{x}\right)}{\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] 1/2/d/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*
f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*
((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(
d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))+1/2/d/((a*f+c*d+(d*f)^(1/2)*b)/f
)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2
)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d
*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)
^(1/2)/f))-1/d/a^(1/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{cx^2+bx+a}\left(fx^2-d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)*x), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\left(d-f\,x^2\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int(1/(x*(d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-dx\sqrt{a+bx+cx^2}+fx^3\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(1/(-d*x*sqrt(a + b*x + c*x**2) + f*x**3*sqrt(a + b*x + c*x**2)),
x)
```

# 3.100 $\int \frac{1}{x^2\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx$

**Optimal**. Leaf size=291

$$\frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d}+\frac{f\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^{3/2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{f\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^{3/2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\sqrt{a}$$

**Rubi** [A] time = 0.66, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {6725, 730, 724, 206, 984}

$$\frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d}+\frac{f\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^{3/2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{f\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^{3/2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\sqrt{a+bx+cx^2}}{adx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -(Sqrt[a + b*x + c*x^2]/(a*d*x)) + (b*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a
 + b*x + c*x^2])])/(2*a^(3/2)*d) + (f*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2
*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a +
b*x + c*x^2])])/(2*d^(3/2)*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + (f*ArcTan
h[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*S
qrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^(3/2)*Sqrt[c*d + b*Sqrt
[d]*Sqrt[f] + a*f])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 984

```
Int[1/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Sy
mbol] :> Dist[1/2, Int[1/((a - Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x],
 x] + Dist[1/2, Int[1/((a + Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx = \int\left(\frac{1}{dx^2\sqrt{a+bx+cx^2}}+\frac{f}{d\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\right)dx$$

$$= \frac{\int \frac{1}{x^2\sqrt{a+bx+cx^2}}\,dx}{d}+\frac{f\int \frac{1}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{d}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{adx}-\frac{b\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{2ad}+\frac{f\int \frac{1}{\left(d-\sqrt{d}\sqrt{f}x\right)\sqrt{a+bx+cx^2}}\,dx}{2d}+\frac{f\int \frac{\phantom{1}}{(d\cdots}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{adx}+\frac{b\,\mathrm{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{ad}-\frac{f\,\mathrm{Subst}\left(\int \frac{\phantom{1}}{4cd^2-4bd^{3/\cdots}}\right.}{\phantom{1}}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{adx}+\frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d}+\frac{f\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-\cdots\right.}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+\cdots}}\right.}{2d^{3/2}\sqrt{cd-b\sqrt{d}\sqrt{f}+a\cdots}}$$

**Mathematica [A]** time = 1.03, size = 325, normalized size = 1.12

$$\frac{\frac{b\sqrt{d}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{a^{3/2}}+\frac{f\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}+\frac{f\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}-\frac{2b\sqrt{d}}{a\sqrt{a+x(b+cx)}}-\frac{2c\sqrt{d}x}{a\sqrt{a+x(b+cx)}}-\frac{2\sqrt{d}}{x\sqrt{a+x(b+cx)}}}{2d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] ((-2*b*Sqrt[d])/(a*Sqrt[a + x*(b + c*x)]) - (2*Sqrt[d])/(x*Sqrt[a + x*(b +
c*x)]) - (2*c*Sqrt[d]*x)/(a*Sqrt[a + x*(b + c*x)]) + (b*Sqrt[d]*ArcTanh[(2*
a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/a^(3/2) + (f*ArcTanh[(b*Sqrt[d
] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt
[f] + a*f]*Sqrt[a + x*(b + c*x)])]])/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f] + (
f*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[
c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]])/Sqrt[c*d - b*Sqrt[d
]*Sqrt[f] + a*f])/(2*d^(3/2))
```

**IntegrateAlgebraic [C]** time = 0.52, size = 227, normalized size = 0.78

$$-\frac{f\text{RootSum}\left[\#1^4(-f)+2\#1^2af+4\#1^2cd-4\#1b\sqrt{c}d-a^2f+b^2d\&,\frac{b\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2\#1\sqrt{c}\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)}{\#1^3f-\#1af-2\#1cd+b\sqrt{c}d}\&\right]}{2d}-\frac{b\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}-\frac{\sqrt{a+bx+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{\sqrt{a+bx+cx^2}}{adx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -(Sqrt[a + b*x + c*x^2]/(a*d*x)) - (b*ArcTanh[(Sqrt[c]*x)/Sqrt[a] - Sqrt[a
+ b*x + c*x^2]/Sqrt[a]])/(a^(3/2)*d) - (f*RootSum[b^2*d - a^2*f - 4*b*Sqrt[
c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (b*Log[-(Sqrt[c]*x) + Sqrt[a
 + b*x + c*x^2] - #1] - 2*Sqrt[c]*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2]
- #1]*#1)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ])/(2*d)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 1.11sym2pol
y/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argum
ent Value
```

**maple [A]** time = 0.02, size = 427, normalized size = 1.47

$$-\frac{f \ln\left(\frac{\frac{2af+2cd-2\sqrt{df}\,b}{f}+\frac{\left(bf-2\sqrt{df}\,c\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{af+cd-\sqrt{df}\,b}{f}}\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2 c+\frac{\left(bf-2\sqrt{df}\,c\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+\frac{af+cd-\sqrt{df}\,b}{f}}}{x+\frac{\sqrt{df}}{f}}\right)}{2\sqrt{df}\,\sqrt{\frac{af+cd-\sqrt{df}\,b}{f}}\,d}+\frac{f \ln\left(\frac{\frac{2af+2cd+2\sqrt{df}\,b}{f}+\frac{\left(bf+2\sqrt{df}\,c\right)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{af+cd+\sqrt{df}\,b}{f}}\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^2 c+\frac{\left(bf+2\sqrt{df}\,c\right)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+\frac{af+cd+\sqrt{df}\,b}{f}}}{x-\frac{\sqrt{df}}{f}}\right)}{2\sqrt{df}\,\sqrt{\frac{af+cd+\sqrt{df}\,b}{f}}\,d}+\frac{b\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}d}-\frac{\sqrt{c\,x^2+bx+a}}{adx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -1/2*f/d/(d*f)^(1/2)/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)
^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/
2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/
f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))-(c*x^2+b*x+a)^(1/
2)/a/d/x+1/2/d*b/a^(3/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)+1/2*
f/d/(d*f)^(1/2)/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2
)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)
/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+
(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{cx^2+bx+a}\left(fx^2-d\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)*x^2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\left(d-f\,x^2\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int(1/(x^2*(d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-dx^2\sqrt{a+bx+cx^2}+fx^4\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(1/(-d*x**2*sqrt(a + b*x + c*x**2) + f*x**4*sqrt(a + b*x + c*x**2)
), x)
```

## 3.101 $\int \frac{1}{x^3\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx$

**Optimal**. Leaf size=376

$$-\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}d}+\frac{3b\sqrt{a+bx+cx^2}}{4a^2dx}-\frac{f^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^2\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{f^{3/2}\tanh^{-1}}{2d^2}$$

**Rubi [A]** time = 0.73, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6725, 744, 806, 724, 206, 1033}

$$-\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}d}+\frac{3b\sqrt{a+bx+cx^2}}{4a^2dx}-\frac{f^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^2\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{f^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^2\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{f\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\,d^2}-\frac{\sqrt{a+bx+cx^2}}{2adx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -Sqrt[a + b*x + c*x^2]/(2*a*d*x^2) + (3*b*Sqrt[a + b*x + c*x^2])/(4*a^2*d*x
) - ((3*b^2 - 4*a*c)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]
)/(8*a^(5/2)*d) - (f*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]
)/(Sqrt[a]*d^2) - (f^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d]
- b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2
])])/(2*d^2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + (f^(3/2)*ArcTanh[(b*Sqrt
[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sq
rt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] +
 a*f])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx = \int\left(\frac{1}{dx^3\sqrt{a+bx+cx^2}} + \frac{f}{d^2x\sqrt{a+bx+cx^2}} + \frac{f^2x}{d^2\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\right)$$

$$= \frac{\int \frac{1}{x^3\sqrt{a+bx+cx^2}}\,dx}{d} + \frac{f\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{d^2} + \frac{f^2\int \frac{x}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{d^2}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{2adx^2} - \frac{\int \frac{\frac{3b}{2}+cx}{x^2\sqrt{a+bx+cx^2}}\,dx}{2ad} - \frac{(2f)\,\mathrm{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d^2}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{2adx^2} + \frac{3b\sqrt{a+bx+cx^2}}{4a^2dx} - \frac{f\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\,d^2} + \frac{\left(3b^2-4\right.}{}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{2adx^2} + \frac{3b\sqrt{a+bx+cx^2}}{4a^2dx} - \frac{f\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\,d^2} - \frac{f^{3/2}\tanh}{2}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{2adx^2} + \frac{3b\sqrt{a+bx+cx^2}}{4a^2dx} - \frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}d}$$

**Mathematica [A]** time = 2.01, size = 314, normalized size = 0.84

$$\frac{2\sqrt{a}\left(\frac{2a^2f^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{2a^2f^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}} - \frac{d(2a-3bx)\sqrt{a+x(b+cx)}}{x^2}\right) + \left(4a(cd-2af)-3b^2d\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{8a^{5/2}d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] ((-3*b^2*d + 4*a*(c*d - 2*a*f))*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(
b + c*x)])] + 2*Sqrt[a]*(-((d*(2*a - 3*b*x)*Sqrt[a + x*(b + c*x)])/x^2) + (
2*a^2*f^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*
x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]])/Sqrt[c*d
 + b*Sqrt[d]*Sqrt[f] + a*f] - (2*a^2*f^(3/2)*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sq
rt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*S
qrt[a + x*(b + c*x)])]])/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]))/(8*a^(5/2)*d^
2)
```

**IntegrateAlgebraic [C]** time = 0.84, size = 251, normalized size = 0.67

$$-\frac{f^2\text{RootSum}\left[\#1^4(-f)+2\#1^2af+4\#1^2cd-4\#1b\sqrt{c}\,d-a^2f+b^2d\&,\frac{a\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)-\#1^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)}{\#1^3(-f)+\#1af+2\#1cd-b\sqrt{c}\,d}\&\right]}{2d^2}+\frac{(3bx-2a)\sqrt{a+bx+cx^2}}{4a^2dx^2}+\frac{\left(-8a^2f+4acd-3b^2d\right)\tanh^{-1}\left(\frac{\sqrt{a+bx+cx^2}-\sqrt{c}\,x}{\sqrt{a}}\right)}{4a^{5/2}d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] ((-2*a + 3*b*x)*Sqrt[a + b*x + c*x^2])/(4*a^2*d*x^2) + ((-3*b^2*d + 4*a*c*d
 - 8*a^2*f)*ArcTanh[(-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2])/Sqrt[a]])/(4*a^(
5/2)*d^2) - (f^2*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*
a*f*#1^2 - f*#1^4 & , (a*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - L
og[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(-(b*Sqrt[c]*d) + 2*c*d
*#1 + a*f*#1 - f*#1^3) & ])/(2*d^2)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 1.43sym2pol
y/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argum
ent Value
```

**maple [A]** time = 0.02, size = 519, normalized size = 1.38

$$\frac{f\ln\left(\frac{\frac{2af+2cd-2\sqrt{df}\,b}{f}+\frac{\left(bf-2\sqrt{df}\,c\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{af+cd-\sqrt{df}\,b}{f}}\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2c+\frac{\left(bf-2\sqrt{df}\,c\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+\frac{af+cd-\sqrt{df}\,b}{f}}}{x+\frac{\sqrt{df}}{f}}\right)}{2\sqrt{\frac{af+cd-\sqrt{df}\,b}{f}}\,d^2}+\frac{f\ln\left(\frac{\frac{2af+2cd+2\sqrt{df}\,b}{f}+\frac{\left(bf+2\sqrt{df}\,c\right)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{af+cd+\sqrt{df}\,b}{f}}\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^2c+\frac{\left(bf+2\sqrt{df}\,c\right)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+\frac{af+cd+\sqrt{df}\,b}{f}}}{x-\frac{\sqrt{df}}{f}}\right)}{2\sqrt{\frac{af+cd+\sqrt{df}\,b}{f}}\,d^2}-\frac{f\ln\left(\frac{bx+2a+2\sqrt{cx^2+bx+a}\,\sqrt{a}}{x}\right)}{\sqrt{a}\,d^2}+\frac{c\ln\left(\frac{bx+2a+2\sqrt{cx^2+bx+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}d}-\frac{3b^2\ln\left(\frac{bx+2a+2\sqrt{cx^2+bx+a}\,\sqrt{a}}{x}\right)}{8a^{\frac{5}{2}}d}+\frac{3\sqrt{cx^2+bx+a}\,b}{4a^2dx}-\frac{\sqrt{cx^2+bx+a}}{2adx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] 1/2*f/d^2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f
+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1
/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c
*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))+1/2*f/d^2/((a*f+c*d+(d*f)^(1
/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d
*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(
b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/
(x-(d*f)^(1/2)/f))-1/2*(c*x^2+b*x+a)^(1/2)/a/d/x^2+3/4*b*(c*x^2+b*x+a)^(1/2
)/a^2/d/x-3/8/d*b^2/a^(5/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)+1
/2/d*c/a^(3/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-f/d^2/a^(1/2)*
ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{cx^2+bx+a}\left(fx^2-d\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)*x^3), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,\left(d-f\,x^2\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int(1/(x^3*(d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-dx^3\sqrt{a+bx+cx^2}+fx^5\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(1/(-d*x**3*sqrt(a + b*x + c*x**2) + f*x**5*sqrt(a + b*x + c*x**2)
), x)
```

## 3.102 $\int \frac{x^4}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx$

**Optimal**. Leaf size=466

$$-\frac{2d^2\left(b\left(b^2f-c(3af+cd)\right)-cx\left(2acf+b^2(-f)+2c^2d\right)\right)}{f^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}+\frac{2d(b+2cx)}{f^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{2b\sqrt{a+bx+cx^2}}{cf\left(b^2-4ac\right)}-\frac{\ }{f}$$

**Rubi [A]** time = 1.35, antiderivative size = 466, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {6725, 613, 738, 640, 621, 206, 975, 1033, 724}

$$-\frac{2d^2\left(b\left(b^2f-c(3af+cd)\right)-cx\left(2acf+b^2(-f)+2c^2d\right)\right)}{f^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}+\frac{2d(b+2cx)}{f^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{2b\sqrt{a+bx+cx^2}}{cf\left(b^2-4ac\right)}-\frac{2x(2a+bx)}{f\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}f}+\frac{d^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{d^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (-2*x*(2*a + b*x))/((b^2 - 4*a*c)*f*Sqrt[a + b*x + c*x^2]) + (2*d*(b + 2*c*
x))/((b^2 - 4*a*c)*f^2*Sqrt[a + b*x + c*x^2]) - (2*d^2*(b*(b^2*f - c*(c*d +
 3*a*f)) - c*(2*c^2*d - b^2*f + 2*a*c*f)*x))/((b^2 - 4*a*c)*f^2*(b^2*d*f -
(c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) + (2*b*Sqrt[a + b*x + c*x^2])/(c*(b^2
 - 4*a*c)*f) - ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]/(c^(3
/2)*f) + (d^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[
f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f
*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (d^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a
*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a
*f]*Sqrt[a + b*x + c*x^2])])/(2*f*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 975

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(2*a*
f))*x)*(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(q + 1))/((b^2 - 4*a*c)*(b^2*d
*f + (c*d - a*f)^2)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d -
a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[2*c*(b^2
*d*f + (c*d - a*f)^2)*(p + 1) - (2*c^2*d + b^2*f - c*(2*a*f))*(a*f*(p + 1)
- c*d*(p + 2)) + (2*f*(b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d +
b^2*f - c*(2*a*f))*(b*f*(p + 1)))*x + c*f*(2*c^2*d + b^2*f - c*(2*a*f))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[
p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx = \int \left(-\frac{d}{f^2\left(a+bx+cx^2\right)^{3/2}}-\frac{x^2}{f\left(a+bx+cx^2\right)^{3/2}}+\frac{d^2}{f^2\left(a+bx+cx^2\right)^{3/2}\left(d-\cdots\right.}\right.$$

$$=-\frac{d\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}}\,dx}{f^2}+\frac{d^2\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx}{f^2}-\frac{\int \frac{x^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx}{f}$$

$$=-\frac{2x(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}}+\frac{2d(b+2cx)}{\left(b^2-4ac\right)f^2\sqrt{a+bx+cx^2}}-\frac{2d^2\left(b\left(b^2f-\cdots\right.\right.}{\left(b^2-4ac\right.\cdots}$$

$$=-\frac{2x(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}}+\frac{2d(b+2cx)}{\left(b^2-4ac\right)f^2\sqrt{a+bx+cx^2}}-\frac{2d^2\left(b\left(b^2f-\cdots\right.\right.}{\left(b^2-4ac\right.\cdots}$$

$$=-\frac{2x(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}}+\frac{2d(b+2cx)}{\left(b^2-4ac\right)f^2\sqrt{a+bx+cx^2}}-\frac{2d^2\left(b\left(b^2f-\cdots\right.\right.}{\left(b^2-4ac\right.\cdots}$$

$$=-\frac{2x(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}}+\frac{2d(b+2cx)}{\left(b^2-4ac\right)f^2\sqrt{a+bx+cx^2}}-\frac{2d^2\left(b\left(b^2f-\cdots\right.\right.}{\left(b^2-4ac\right.\cdots}$$

**Mathematica** [A] time = 1.41, size = 562, normalized size = 1.21

$$\frac{\frac{f\left(a\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-2\sqrt{c}\left(a(b-2cx)+bcx^2\right)\sqrt{a+x(b+cx)}\right)}{ac^{3/2}\left(4ac-b^2\right)}+\frac{d^{3/2}f\left(\frac{\left(b^2-4ac\right)\left(af+b\sqrt{d}\sqrt{f}+cd\right)\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\left(4ac-b^2\right)\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)\tanh^{-1}\left(\frac{-2\left(a\sqrt{f}+c\sqrt{d}x\right)-b\left(\sqrt{d}+\sqrt{f}x\right)}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\left(b^2-4ac\right)\left((af+cd)^2-b^2df\right)}+\frac{2d(b+2cx)}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}}-\frac{2fx^3\left(-2ac+b^2+bcx\right)}{a\left(b^2-4ac\right)\sqrt{a+x(b+cx)}}-\frac{2d^2\left(-bc(3af+cd)-2c^2x(af+cd)+b^3f+b^2cfx\right)}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(b^2df-(af+cd)^2\right)}}{f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] ((2*d*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]) - (2*f*x^3*(b^2 -
2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]) - (2*d^2*(b^3*f - b
*c*(c*d + 3*a*f) + b^2*c*f*x - 2*c^2*(c*d + a*f)*x))/((b^2 - 4*a*c)*(b^2*d*
f - (c*d + a*f)^2)*Sqrt[a + x*(b + c*x)]) + (f*(-2*Sqrt[c]*(b*c*x^2 + a*(b
- 2*c*x))*Sqrt[a + x*(b + c*x)] + a*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + x*(b + c*x)])]))/(a*c^(3/2)*(-b^2 + 4*a*c)) + (d^(3/2)*f*(((
b^2 - 4*a*c)*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sq
rt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*S
qrt[a + x*(b + c*x)])])/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f] + ((-b^2 + 4*a*
c)*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) -
b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*
(b + c*x)])])/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]))/(2*(b^2 - 4*a*c)*(-(b^2
*d*f) + (c*d + a*f)^2)))/f^2
```

**IntegrateAlgebraic [C]** time = 1.54, size = 469, normalized size = 1.01

$$-\frac{d^2\text{RootSum}\left[\#1^4(-f)+2\#1^2af+4\#1^2cd-4\#1b\sqrt{c}\,d-a^2f+b^2d\&,\frac{\#1^2(-b)f\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)-2\#1c^{3/2}d\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)+bcd\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)-2\#1a\sqrt{c}\,f\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)+2abf\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)}{\#1^3f-\#1af-2\#1cd+b\sqrt{c}\,d}\&\right]}{2f\left(a^2f^2+2acdf+b^2(-d)f+c^2d^2\right)}-\frac{2\left(a^3bf-2a^3cfx+a^2b^2fx+3a^2bcd-2a^2c^2dx-ab^3d+4ab^2cdx+b^4(-d)x\right)}{c\left(4ac-b^2\right)\sqrt{a+bx+cx^2}\left(a^2f^2+2acdf+b^2(-d)f+c^2d^2\right)}+\frac{\log\left(-2c^{3/2}f\sqrt{a+bx+cx^2}+bcf+2c^2fx\right)}{c^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (-2*(-(a*b^3*d) + 3*a^2*b*c*d + a^3*b*f - b^4*d*x + 4*a*b^2*c*d*x - 2*a^2*c
^2*d*x + a^2*b^2*f*x - 2*a^3*c*f*x))/(c*(-b^2 + 4*a*c)*(c^2*d^2 - b^2*d*f +
 2*a*c*d*f + a^2*f^2)*Sqrt[a + b*x + c*x^2]) + Log[b*c*f + 2*c^2*f*x - 2*c^
(3/2)*f*Sqrt[a + b*x + c*x^2]]/(c^(3/2)*f) - (d^2*RootSum[b^2*d - a^2*f - 4
*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (b*c*d*Log[-(Sqrt[c]
*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*a*b*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x
 + c*x^2] - #1] - 2*c^(3/2)*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1
]*#1 - 2*a*Sqrt[c]*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - b*
f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d
*#1 - a*f*#1 + f*#1^3) & ])/(2*f*(c^2*d^2 - b^2*d*f + 2*a*c*d*f + a^2*f^2))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

**maple [B]** time = 0.02, size = 1648, normalized size = 3.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] 1/f*x/c/(c*x^2+b*x+a)^(1/2)-1/2/f*b/c^2/(c*x^2+b*x+a)^(1/2)-1/f*b^2/c/(4*a*
c-b^2)/(c*x^2+b*x+a)^(1/2)*x-1/2/f*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-
1/f/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-4/f^2*d/(4*a*c-b^2)
/(c*x^2+b*x+a)^(1/2)*c*x-2/f^2*d/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*b+1/2/f*d^
2/(d*f)^(1/2)/(a*f+c*d-(d*f)^(1/2)*b)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(
1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)+2/f^2*d^2/(a*f
+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c
)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*c^2-1/f*d^2/(d*f)^
(1/2)/(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*
f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*b*c+1/f^
2*d^2/(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*
f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b*c-1/2/f*
d^2/(d*f)^(1/2)/(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+
(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*
b^2-1/2/f*d^2/(d*f)^(1/2)/(a*f+c*d-(d*f)^(1/2)*b)/((a*f+c*d-(d*f)^(1/2)*b)/
f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/
2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(
d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f
)^(1/2)/f))-1/2/f*d^2/(d*f)^(1/2)/(a*f+c*d+(d*f)^(1/2)*b)/((x-(d*f)^(1/2)/f
)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^
(1/2)+2/f^2*d^2/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+
(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*
x*c^2+1/f*d^2/(d*f)^(1/2)/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/
2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)
/f)^(1/2)*x*b*c+1/f^2*d^2/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/
2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)
/f)^(1/2)*b*c+1/2/f*d^2/(d*f)^(1/2)/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x
-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f
```

```
)^(1/2)*b)/f)^(1/2)*b^2+1/2/f*d^2/(d*f)^(1/2)/(a*f+c*d+(d*f)^(1/2)*b)/((a*f
+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(
1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(
1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*
b)/f)^(1/2))/(x-(d*f)^(1/2)/f))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                   /(2*f)                 +(c*d)/f+a))
     /f^2 zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(d-f\,x^2\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int(x^4/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^{4}}{- a d \sqrt{a + b x + c x^{2}} + a f x^{2} \sqrt{a + b x + c x^{2}} - b d x \sqrt{a + b x + c x^{2}} + b f x^{3} \sqrt{a + b x + c x^{2}} - c d x^{2} \sqrt{a + b x + c x^{2}} + c f x^{4} \sqrt{a + b x + c x^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x**4/(-a*d*sqrt(a + b*x + c*x**2) + a*f*x**2*sqrt(a + b*x + c*x**
2) - b*d*x*sqrt(a + b*x + c*x**2) + b*f*x**3*sqrt(a + b*x + c*x**2) - c*d*x
**2*sqrt(a + b*x + c*x**2) + c*f*x**4*sqrt(a + b*x + c*x**2)), x)
```

## 3.103 $\int \frac{x^3}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx$

**Optimal**. Leaf size=341

$$-\frac{2d\left(a\left(2acf+b^2(-f)+2c^2d\right)+bcx(cd-af)\right)}{f\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}-\frac{2(2a+bx)}{f\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{d\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right.}}\right.}{2\sqrt{f}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+c\right.}$$

**Rubi** [A] time = 1.04, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6725, 636, 1018, 1033, 724, 206}

$$-\frac{2d\left(a\left(2acf+b^2(-f)+2c^2d\right)+bcx(cd-af)\right)}{f\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}-\frac{2(2a+bx)}{f\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{d\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{f}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{d\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{f}\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (-2*(2*a + b*x))/((b^2 - 4*a*c)*f*Sqrt[a + b*x + c*x^2]) - (2*d*(a*(2*c^2*d
 - b^2*f + 2*a*c*f) + b*c*(c*d - a*f)*x))/((b^2 - 4*a*c)*f*(b^2*d*f - (c*d
+ a*f)^2)*Sqrt[a + b*x + c*x^2]) - (d*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2
*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a +
b*x + c*x^2])]])/(2*Sqrt[f]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (d*ArcT
anh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b
*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]])/(2*Sqrt[f]*(c*d + b*Sqrt[d
]*Sqrt[f] + a*f)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1018

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^
(q + 1)*((g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f
)) + c*(g*(2*c^2*d + b^2*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x))/((b^2 - 4*
a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx=\int \left(-\frac{x}{f\left(a+bx+cx^2\right)^{3/2}}+\frac{dx}{f\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\right)dx$$

$$=-\frac{\int \frac{x}{\left(a+bx+cx^2\right)^{3/2}}\,dx}{f}+\frac{d\int \frac{x}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx}{f}$$

$$=-\frac{2(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}}-\frac{2d\left(a\left(2c^2d-b^2f+2acf\right)+bc(cd-af)x\right.}{\left(b^2-4ac\right)f\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+\right.}}$$

$$=-\frac{2(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}}-\frac{2d\left(a\left(2c^2d-b^2f+2acf\right)+bc(cd-af)x\right.}{\left(b^2-4ac\right)f\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+\right.}}$$

$$=-\frac{2(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}}-\frac{2d\left(a\left(2c^2d-b^2f+2acf\right)+bc(cd-af)x\right.}{\left(b^2-4ac\right)f\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+\right.}}$$

$$=-\frac{2(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}}-\frac{2d\left(a\left(2c^2d-b^2f+2acf\right)+bc(cd-af)x\right.}{\left(b^2-4ac\right)f\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+\right.}}$$

**Mathematica** [A] time = 1.20, size = 414, normalized size = 1.21

$$\frac{1}{2}\left(\frac{8a^3f+4a^2(bfx+2cd)-4abd(b-3cx)-4b^3dx}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(f\left(b^2d-a^2f\right)-2acdf-c^2d^2\right)}-\frac{d\log\left(\sqrt{d}\sqrt{f}-fx\right)}{\sqrt{f}\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}-\frac{d\log\left(\sqrt{d}\sqrt{f}+fx\right)}{\sqrt{f}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{d\log\left(\sqrt{d}\left(2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}+2a\sqrt{f}-b\sqrt{d}+b\sqrt{f}x-2c\sqrt{d}x\right)\right)}{\sqrt{f}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{d\log\left(\sqrt{d}\left(2\left(\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}+a\sqrt{f}+c\sqrt{d}x\right)+b\left(\sqrt{d}+\sqrt{f}x\right)\right)\right)}{\sqrt{f}\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] ((8*a^3*f - 4*b^3*d*x - 4*a*b*d*(b - 3*c*x) + 4*a^2*(2*c*d + b*f*x))/((b^2
- 4*a*c)*(-(c^2*d^2) - 2*a*c*d*f + f*(b^2*d - a^2*f))*Sqrt[a + x*(b + c*x)]
) - (d*Log[Sqrt[d]*Sqrt[f] - f*x])/(Sqrt[f]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)
^(3/2)) - (d*Log[Sqrt[d]*Sqrt[f] + f*x])/(Sqrt[f]*(c*d - b*Sqrt[d]*Sqrt[f]
+ a*f)^(3/2)) + (d*Log[Sqrt[d]*(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x
+ b*Sqrt[f]*x + 2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)]
)])/(Sqrt[f]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (d*Log[Sqrt[d]*(b*(Sq
rt[d] + Sqrt[f]*x) + 2*(a*Sqrt[f] + c*Sqrt[d]*x + Sqrt[c*d + b*Sqrt[d]*Sqrt
[f] + a*f]*Sqrt[a + x*(b + c*x)]))])/(Sqrt[f]*(c*d + b*Sqrt[d]*Sqrt[f] + a*
f)^(3/2)))/2
```

**IntegrateAlgebraic [C]** time = 0.96, size = 424, normalized size = 1.24

$$\frac{d\text{RootSum}\left[\#1^4(-f)+2\#1^2af+4\#1^2cd-4\#1b\sqrt{c}\,d-a^2f+b^2d\&,\frac{-\#1^2cd\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)-\#1^2af\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)+a^2f\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)+b^2d\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)+acd\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)-2\#1b\sqrt{c}\,d\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)}{\#1^3f-\#1af-2\#1cd+b\sqrt{c}\,d}\&\right]}{2\left(a^2f^2+2acdf+b^2(-d)f+c^2d^2\right)}+\frac{2\left(2a^3f+a^2bfx+2a^2cd-ab^2d+3abcdx-b^3dx\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(-a^2f^2-2acdf+b^2df-c^2d^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (2*(-(a*b^2*d) + 2*a^2*c*d + 2*a^3*f - b^3*d*x + 3*a*b*c*d*x + a^2*b*f*x))/
((b^2 - 4*a*c)*(-(c^2*d^2) + b^2*d*f - 2*a*c*d*f - a^2*f^2)*Sqrt[a + b*x +
c*x^2]) + (d*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*
#1^2 - f*#1^4 & , (b^2*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a
*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a^2*f*Log[-(Sqrt[c]*x
) + Sqrt[a + b*x + c*x^2] - #1] - 2*b*Sqrt[c]*d*Log[-(Sqrt[c]*x) + Sqrt[a +
 b*x + c*x^2] - #1]*#1 - c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]
*#1^2 - a*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]
*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ])/(2*(c^2*d^2 - b^2*d*f + 2*a*c*d*f + a
^2*f^2))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valueint()  Bad Argument Type
```

**maple [B]** time = 0.02, size = 1480, normalized size = 4.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] 1/f/c/(c*x^2+b*x+a)^(1/2)+2/f*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+1/f*b^2/c
/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-1/2/f*d/(a*f+c*d-(d*f)^(1/2)*b)/((x+(d*f)^
(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)
*b)/f)^(1/2)-2/f^2*d/(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)
^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(
1/2)*(d*f)^(1/2)*x*c^2+1/f*d/(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^
(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)
*b)/f)^(1/2)*x*b*c-1/f^2*d/(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1
/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b
)/f)^(1/2)*(d*f)^(1/2)*b*c+1/2/f*d/(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+
(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)
^(1/2)*b)/f)^(1/2)*b^2+1/2/f*d/(a*f+c*d-(d*f)^(1/2)*b)/((a*f+c*d-(d*f)^(1/2
)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f
)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*
f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x
+(d*f)^(1/2)/f))-1/2/f*d/(a*f+c*d+(d*f)^(1/2)*b)/((x-(d*f)^(1/2)/f)^2*c+(b*
f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)+2/f
^2*d/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f
)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*(d*f)^(1/2)
*x*c^2+1/f*d/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(b*
f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*x*b
*c+1/f^2*d/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(b*f+
2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*(d*f)
^(1/2)*b*c+1/2/f*d/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2
*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/
2)*b^2+1/2/f*d/(a*f+c*d+(d*f)^(1/2)*b)/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln
((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*(
(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*
c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                /(2*f)               +(c*d)/f+a))
    /f^2 zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\left(d-f\,x^2\right)\,\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int(x^3/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^3}{-ad\sqrt{a+bx+cx^2}+afx^2\sqrt{a+bx+cx^2}-bdx\sqrt{a+bx+cx^2}+bfx^3\sqrt{a+bx+cx^2}-cdx^2\sqrt{a+bx+cx^2}+cfx^4\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x**3/(-a*d*sqrt(a + b*x + c*x**2) + a*f*x**2*sqrt(a + b*x + c*x**
2) - b*d*x*sqrt(a + b*x + c*x**2) + b*f*x**3*sqrt(a + b*x + c*x**2) - c*d*x
**2*sqrt(a + b*x + c*x**2) + c*f*x**4*sqrt(a + b*x + c*x**2)), x)
```

## 3.104 $\int \frac{x^2}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx$

**Optimal**. Leaf size=297

$$\frac{2\left(cx\left(b^2d-2a(af+cd)\right)+ab(cd-af)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{2a\sqrt{f}+\ldots}{2\sqrt{a+bx+\ldots}}\right)}{2\left(af+b\sqrt{d}\ldots\right.}$$

**Rubi** [A] time = 0.45, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1065, 1033, 724, 206}

$$\frac{2\left(cx\left(b^2d-2a(af+cd)\right)+ab(cd-af)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (2*(a*b*(c*d - a*f) + c*(b^2*d - 2*a*(c*d + a*f))*x))/((b^2 - 4*a*c)*(b^2*d
*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) + (Sqrt[d]*ArcTanh[(b*Sqrt[d] -
2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f]
+ a*f]*Sqrt[a + b*x + c*x^2])]])/(2*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) +
 (Sqrt[d]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(
2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]])/(2*(c*d + b*
Sqrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1065

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) +
(f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2
)^(q + 1)*((A*c - a*C)*(-(b*(c*d + a*f))) + (A*b)*(2*c^2*d + b^2*f - c*(2*a
*f)) + c*(A*(2*c^2*d + b^2*f - c*(2*a*f)) + C*(b^2*d - 2*a*(c*d - a*f)))*x)
)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*
a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d +
 f*x^2)^q*Simp[(-2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) +
(b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(a*f*(p + 1) -
c*d*(p + 2)) - (2*f*((A*c - a*C)*(-(b*(c*d + a*f))) + (A*b)*(2*c^2*d + b^2*
f - c*(2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f) - a*(c*
C*d - a*C*f)))*(b*f*(p + 1)))*x - c*f*(b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f)
 - a*(c*C*d - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d
, f, A, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*
d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx = \frac{2\left(ab(cd-af)+c\left(b^2d-2a(cd+af)\right)x\right)}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} + \frac{2\int \frac{-\frac{1}{2}\left(b^2-4ac\right)d(cd+af)+\frac{1}{2}b\left(b^2-4\cdots\right.}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}}{\left(b^2-4ac\right)\left(b^2df-(cd+\cdots\right.}$$

$$= \frac{2\left(ab(cd-af)+c\left(b^2d-2a(cd+af)\right)x\right)}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} - \frac{\left(\sqrt{d}\sqrt{f}\right)\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+b\cdots}}}{2\left(cd-b\sqrt{d}\sqrt{f}+af\cdots\right.}$$

$$= \frac{2\left(ab(cd-af)+c\left(b^2d-2a(cd+af)\right)x\right)}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} + \frac{\left(\sqrt{d}\sqrt{f}\right)\operatorname{Subst}\left(\int \frac{\cdots}{4cdf-4b\sqrt{\cdots}}\right.}{c\cdots}$$

$$= \frac{2\left(ab(cd-af)+c\left(b^2d-2a(cd+af)\right)x\right)}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} + \frac{\sqrt{d}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\cdots\right.}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af\cdots}}\right.}{2\left(cd-b\sqrt{d}\sqrt{f}+a\cdots\right.}$$

**Mathematica [A]** time = 0.40, size = 352, normalized size = 1.19

$$\frac{2\left(\frac{a^2f(b+2cx)+acd(2cx-b)-b^2cdx}{\sqrt{a+x(b+cx)}}+\frac{\sqrt{d}\left(b^2-4ac\right)\left(af+b\sqrt{d}\sqrt{f}+cd\right)\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{4\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\sqrt{d}\left(4ac-b^2\right)\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)\tanh^{-1}\left(\frac{-2\left(a\sqrt{f}+c\sqrt{d}x\right)-b\left(\sqrt{d}+\sqrt{f}x\right)}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{4\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\left(b^2-4ac\right)\left((af+cd)^2-b^2df\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (2*((-(b^2*c*d*x) + a*c*d*(-b + 2*c*x) + a^2*f*(b + 2*c*x))/Sqrt[a + x*(b +
 c*x)] + ((b^2 - 4*a*c)*Sqrt[d]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(-2
*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[
d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]])/(4*Sqrt[c*d - b*Sqrt[d]*Sqrt[f]
+ a*f]) + ((-b^2 + 4*a*c)*Sqrt[d]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(
-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqr
t[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]])/(4*Sqrt[c*d + b*Sqrt[d]*Sqrt[f
] + a*f])))/((b^2 - 4*a*c)*(-(b^2*d*f) + (c*d + a*f)^2))
```

**IntegrateAlgebraic [C]** time = 0.87, size = 388, normalized size = 1.31

$$-\frac{d\,\mathrm{RootSum}\left[\#1^4(-f)+2\#1^2af+4\#1^2cd-4\#1b\sqrt{c}\,d-a^2f+b^2d\&,\frac{\#1^2(-b)f\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2\#1c^{3/2}d\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+bcd\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2\#1a\sqrt{c}f\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+2abf\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)}{\#1^3f-\#1af-2\#1cd+b\sqrt{c}\,d}\&\right]}{2\left(a^2f^2+2acdf+b^2(-d)f+c^2d^2\right)}-\frac{2\left(a^2bf+2a^2cfx-abcd+2ac^2dx-b^2cdx\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(-a^2f^2-2acdf+b^2df-c^2d^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (-2*(-(a*b*c*d) + a^2*b*f - b^2*c*d*x + 2*a*c^2*d*x + 2*a^2*c*f*x))/((b^2 -
 4*a*c)*(-(c^2*d^2) + b^2*d*f - 2*a*c*d*f - a^2*f^2)*Sqrt[a + b*x + c*x^2])
 - (d*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 -
f*#1^4 & , (b*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*a*b*f*
Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*c^(3/2)*d*Log[-(Sqrt[c]*
x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*a*Sqrt[c]*f*Log[-(Sqrt[c]*x) + Sqrt
[a + b*x + c*x^2] - #1]*#1 - b*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] -
 #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ])/(2*(c^2*d^2 - b^
2*d*f + 2*a*c*d*f + a^2*f^2))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valueint()  Bad Argument Type
```

**maple [B]** time = 0.02, size = 1427, normalized size = 4.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] -2/f*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+1/2*d/(d*f)^(1/2)/(a*f+c*d-(
d*f)^(1/2)*b)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f
)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)+2*d/f/(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b
^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c
*d-(d*f)^(1/2)*b)/f)^(1/2)*x*c^2-d/(d*f)^(1/2)/(a*f+c*d-(d*f)^(1/2)*b)/(4*a
*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a
*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*b*c+d/f/(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2
)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d
-(d*f)^(1/2)*b)/f)^(1/2)*b*c-1/2*d/(d*f)^(1/2)/(a*f+c*d-(d*f)^(1/2)*b)/(4*a
*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a
*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b^2-1/2*d/(d*f)^(1/2)/(a*f+c*d-(d*f)^(1/2)*b
)/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*
(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+
(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)
^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))-1/2*d/(d*f)^(1/2)/(a*f+c*d+(d*f)^(1/
2)*b)/((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f
+c*d+(d*f)^(1/2)*b)/f)^(1/2)+2*d/f/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-
(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)
^(1/2)*b)/f)^(1/2)*x*c^2+d/(d*f)^(1/2)/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/
((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(
d*f)^(1/2)*b)/f)^(1/2)*x*b*c+d/f/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d
*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(
1/2)*b)/f)^(1/2)*b*c+1/2*d/(d*f)^(1/2)/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/
((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(
d*f)^(1/2)*b)/f)^(1/2)*b^2+1/2*d/(d*f)^(1/2)/(a*f+c*d+(d*f)^(1/2)*b)/((a*f+
c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1
/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1
```

```
/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b
)/f)^(1/2))/(x-(d*f)^(1/2)/f))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                  /(2*f)                  +(c*d)/f+a))
     /f^2 zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(d-f\,x^2\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int(x^2/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^{2}}{-ad\sqrt{a+bx+cx^{2}}+afx^{2}\sqrt{a+bx+cx^{2}}-bdx\sqrt{a+bx+cx^{2}}+bfx^{3}\sqrt{a+bx+cx^{2}}-cdx^{2}\sqrt{a+bx+cx^{2}}+cfx^{4}\sqrt{a+bx+cx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x**2/(-a*d*sqrt(a + b*x + c*x**2) + a*f*x**2*sqrt(a + b*x + c*x**
2) - b*d*x*sqrt(a + b*x + c*x**2) + b*f*x**3*sqrt(a + b*x + c*x**2) - c*d*x
**2*sqrt(a + b*x + c*x**2) + c*f*x**4*sqrt(a + b*x + c*x**2)), x)
```

## 3.105 $\int \frac{x}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx$

**Optimal**. Leaf size=299

$$-\frac{2\left(a\left(2acf+b^2(-f)+2c^2d\right)+bcx(cd-af)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\,\left(b^2df-(af+cd)^2\right)}-\frac{\sqrt{f}\,\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\,\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\left(af+b\left(-\sqrt{d}\right)\sqrt{f}\,+cd\right)^{3/2}}+\frac{\sqrt{f}\,\tanh^{-1}\left(\frac{2a\sqrt{f}+}{2\sqrt{a+bx+}}\right.}{2\left(af+b\sqrt{d}\right.}$$

**Rubi** [A] time = 0.40, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1018, 1033, 724, 206}

$$-\frac{2\left(a\left(2acf+b^2(-f)+2c^2d\right)+bcx(cd-af)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\,\left(b^2df-(af+cd)^2\right)}-\frac{\sqrt{f}\,\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\,\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\left(af+b\left(-\sqrt{d}\right)\sqrt{f}\,+cd\right)^{3/2}}+\frac{\sqrt{f}\,\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\,\sqrt{af+b\sqrt{d}\,\sqrt{f}+cd}}\right)}{2\left(af+b\sqrt{d}\,\sqrt{f}\,+cd\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (-2*(a*(2*c^2*d - b^2*f + 2*a*c*f) + b*c*(c*d - a*f)*x))/((b^2 - 4*a*c)*(b^
2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) - (Sqrt[f]*ArcTanh[(b*Sqrt[d]
 - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[
f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)
) + (Sqrt[f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x
)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*(c*d +
 b*Sqrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1018

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^
(q + 1)*((g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f
)) + c*(g*(2*c^2*d + b^2*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x))/((b^2 - 4*
a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\int \frac{x}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx = -\frac{2\left(a\left(2c^2d-b^2f+2acf\right)+bc(cd-af)x\right)}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} + \frac{2\int \frac{\frac{1}{2}b\left(b^2-4ac\right)df-\frac{1}{2}\left(b^2-4ac\right)f \cdots}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)\cdots}}{\left(b^2-4ac\right)\left(b^2df-(cd \cdots\right.}$$

$$= -\frac{2\left(a\left(2c^2d-b^2f+2acf\right)+bc(cd-af)x\right)}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} + \frac{f\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\cdots}{2\left(cd-b\sqrt{d}\sqrt{f}+af\cdots\right.}$$

$$= -\frac{2\left(a\left(2c^2d-b^2f+2acf\right)+bc(cd-af)x\right)}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} - \frac{f\,\text{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+\cdots}\cdots\right.}{cd\cdots}$$

$$= -\frac{2\left(a\left(2c^2d-b^2f+2acf\right)+bc(cd-af)x\right)}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} - \frac{\sqrt{f}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\cdots}{2\sqrt{cd-b\sqrt{d}\sqrt{f}\cdots}}\cdots\right.}{2\left(cd-b\sqrt{d}\sqrt{f}+\cdots\right.}$$

**Mathematica [A]** time = 0.35, size = 356, normalized size = 1.19

$$\frac{2\left(\frac{2a^2cf+a\left(b^2(-f)-bcfx+2c^2d\right)+bc^2dx}{\sqrt{a+x(b+cx)}}-\frac{\sqrt{f}\left(b^2-4ac\right)\left(af+b\sqrt{d}\sqrt{f}+cd\right)\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{4\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\sqrt{f}\left(4ac-b^2\right)\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)\tanh^{-1}\left(\frac{-2\left(a\sqrt{f}+c\sqrt{d}x\right)-b\left(\sqrt{d}+\sqrt{f}x\right)}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{4\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\left(b^2-4ac\right)\left((af+cd)^2-b^2df\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (2*((2*a^2*c*f + b*c^2*d*x + a*(2*c^2*d - b^2*f - b*c*f*x))/Sqrt[a + x*(b +
 c*x)] - ((b^2 - 4*a*c)*Sqrt[f]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(-2
*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[
d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]])/(4*Sqrt[c*d - b*Sqrt[d]*Sqrt[f]
+ a*f]) + ((-b^2 + 4*a*c)*Sqrt[f]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(
-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqr
t[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]])/(4*Sqrt[c*d + b*Sqrt[d]*Sqrt[f
] + a*f])))/((b^2 - 4*a*c)*(-(b^2*d*f) + (c*d + a*f)^2))
```

**IntegrateAlgebraic [C]** time = 0.91, size = 419, normalized size = 1.40

$$\frac{f\text{RootSum}\left[\#1^4(-f)+2\#1^2af+4\#1^2cd-4\#1b\sqrt{c}d-a^2f+b^2d\&,\frac{-\#1^2cd\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-\#1^2af\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+a^2f\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+b^2d\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+acd\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2\#1b\sqrt{c}d\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)}{\#1^3f-\#1af-2\#1cd+b\sqrt{c}d}\&\right]}{2\left(a^2f^2+2acdf+b^2(-d)f+c^2d^2\right)}-\frac{2\left(2a^2cf-ab^2f-abcfx+2ac^2d+bc^2dx\right)}{\left(4ac-b^2\right)\sqrt{a+bx+cx^2}\left(a^2f^2+2acdf+b^2(-d)f+c^2d^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (-2*(2*a*c^2*d - a*b^2*f + 2*a^2*c*f + b*c^2*d*x - a*b*c*f*x))/((-b^2 + 4*a
*c)*(c^2*d^2 - b^2*d*f + 2*a*c*d*f + a^2*f^2)*Sqrt[a + b*x + c*x^2]) + (f*R
ootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4
& , (b^2*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a*c*d*Log[-(Sqr
t[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b
*x + c*x^2] - #1] - 2*b*Sqrt[c]*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2]
- #1]*#1 - c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 - a*f*Lo
g[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1
- a*f*#1 + f*#1^3) & ])/(2*(c^2*d^2 - b^2*d*f + 2*a*c*d*f + a^2*f^2))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valueint()  Bad Argument Type
```

**maple [B]** time = 0.02, size = 1360, normalized size = 4.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] -1/2/(a*f+c*d-(d*f)^(1/2)*b)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(
x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)-2/f/(a*f+c*d-(d*f)^(1/2
)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2
)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*(d*f)^(1/2)*x*c^2+1/(a*f+c*d-(d*f)^
(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^
(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*b*c-1/f/(a*f+c*d-(d*f)^(1/2)*
b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/
f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*(d*f)^(1/2)*b*c+1/2/(a*f+c*d-(d*f)^(1
/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1
/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b^2+1/2/(a*f+c*d-(d*f)^(1/2)*b)/(
(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*
f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*
f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1
/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))-1/2/(a*f+c*d+(d*f)^(1/2)*b)/((x-(d*f)^(
1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*
b)/f)^(1/2)+2/f/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+
(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*
(d*f)^(1/2)*x*c^2+1/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^
2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1
/2)*x*b*c+1/f/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(b
*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*(d
*f)^(1/2)*b*c+1/2/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*
c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2
)*b^2+1/2/(a*f+c*d+(d*f)^(1/2)*b)/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(
a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+
c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x
-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                  /(2*f)                 +(c*d)/f+a))
     /f^2 zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(d-f\,x^2\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int(x/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-ad\sqrt{a+bx+cx^2}+afx^2\sqrt{a+bx+cx^2}-bdx\sqrt{a+bx+cx^2}+bfx^3\sqrt{a+bx+cx^2}-cdx^2\sqrt{a+bx+cx^2}+cfx^4\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x/(-a*d*sqrt(a + b*x + c*x**2) + a*f*x**2*sqrt(a + b*x + c*x**2)
- b*d*x*sqrt(a + b*x + c*x**2) + b*f*x**3*sqrt(a + b*x + c*x**2) - c*d*x**2
*sqrt(a + b*x + c*x**2) + c*f*x**4*sqrt(a + b*x + c*x**2)), x)
```

## 3.106 $\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx$

**Optimal**. Leaf size=310

$$-\frac{2\left(b\left(b^2f-c(3af+cd)\right)-cx\left(2acf+b^2(-f)+2c^2d\right)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}+\frac{f\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\cdots$$

**Rubi** [A] time = 0.41, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {975, 1033, 724, 206}

$$-\frac{2\left(b\left(b^2f-c(3af+cd)\right)-cx\left(2acf+b^2(-f)+2c^2d\right)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}+\frac{f\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{f\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (-2*(b*(b^2*f - c*(c*d + 3*a*f)) - c*(2*c^2*d - b^2*f + 2*a*c*f)*x))/((b^2
- 4*a*c)*(b^2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) + (f*ArcTanh[(b*S
qrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]
*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]])/(2*Sqrt[d]*(c*d - b*Sqrt[d]*Sqrt[f
] + a*f)^(3/2)) + (f*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sq
rt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]])/(
2*Sqrt[d]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 975

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(2*a*
f))*x)*(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(q + 1))/((b^2 - 4*a*c)*(b^2*d
*f + (c*d - a*f)^2)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d -
a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[2*c*(b^2
*d*f + (c*d - a*f)^2)*(p + 1) - (2*c^2*d + b^2*f - c*(2*a*f))*(a*f*(p + 1)
- c*d*(p + 2)) + (2*f*(b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d +
b^2*f - c*(2*a*f))*(b*f*(p + 1)))*x + c*f*(2*c^2*d + b^2*f - c*(2*a*f))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[
p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx = -\frac{2\left(b\left(b^2f-c(cd+3af)\right)-c\left(2c^2d-b^2f+2acf\right)x\right)}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} - \frac{2\int \frac{\frac{1}{2}\left(b^2-4ac\right)f(cd+af)-\ldots}{\sqrt{a+bx+cx^2}\left(\ldots\right.}}{\left(b^2-4ac\right)\left(b^2df-\ldots\right.}$$

$$= -\frac{2\left(b\left(b^2f-c(cd+3af)\right)-c\left(2c^2d-b^2f+2acf\right)x\right)}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} - \frac{f^{3/2}\int \frac{1}{\left(-\sqrt{d}\,\sqrt{f}-fx\right)\sqrt{a\ldots}}}{2\sqrt{d}\,\left(cd-b\sqrt{d}\,\sqrt{f}\ldots\right.}$$

$$= -\frac{2\left(b\left(b^2f-c(cd+3af)\right)-c\left(2c^2d-b^2f+2acf\right)x\right)}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} + \frac{f^{3/2}\,\mathrm{Subst}\left(\int \frac{\ldots}{4cdf-4b\ldots}\right.}{\ldots}$$

$$= -\frac{2\left(b\left(b^2f-c(cd+3af)\right)-c\left(2c^2d-b^2f+2acf\right)x\right)}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} + \frac{f\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}\ldots}{2\sqrt{cd-b\sqrt{d}}\ldots}\right.}{2\sqrt{d}\,\left(cd-b\sqrt{d}\ldots\right.}$$

**Mathematica [A]** time = 0.40, size = 360, normalized size = 1.16

$$\frac{2\left(\frac{f\left(b^2-4ac\right)\left(af+b\sqrt{d}\sqrt{f}+cd\right)\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{4\sqrt{d}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{f\left(4ac-b^2\right)\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)\tanh^{-1}\left(\frac{-2\left(a\sqrt{f}+c\sqrt{d}x\right)-b\left(\sqrt{d}+\sqrt{f}x\right)}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{4\sqrt{d}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}+\frac{-bc(3af+cd)-2c^2x(af+cd)+b^3f+b^2cfx}{\sqrt{a+x(b+cx)}}\right)}{\left(b^2-4ac\right)\left((af+cd)^2-b^2df\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (2*((b^3*f - b*c*(c*d + 3*a*f) + b^2*c*f*x - 2*c^2*(c*d + a*f)*x)/Sqrt[a +
x*(b + c*x)] + ((b^2 - 4*a*c)*f*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(-2
*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[
d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(4*Sqrt[d]*Sqrt[c*d - b*Sqrt[d]*
Sqrt[f] + a*f]) + ((-b^2 + 4*a*c)*f*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh
[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*S
qrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(4*Sqrt[d]*Sqrt[c*d + b*Sqrt
[d]*Sqrt[f] + a*f])))/((b^2 - 4*a*c)*(-(b^2*d*f) + (c*d + a*f)^2))
```

**IntegrateAlgebraic [C]** time = 0.88, size = 392, normalized size = 1.26

$$-\frac{f\,\mathrm{RootSum}\left[\#1^4(-f)+2\#1^2af+4\#1^2cd-4\#1b\sqrt{c}\,d-a^2f+b^2d\&,\frac{\#1^2(-b)f\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)-2\#1c^{3/2}d\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)+bcd\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)-2\#1a\sqrt{c}\,f\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)+2abf\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)}{\#1^3f-\#1af-2\#1cd+b\sqrt{c}\,d}\&\right]}{2\left(a^2f^2+2acdf+b^2(-d)f+c^2d^2\right)}-\frac{2\left(-3abcf-2ac^2fx+b^3f+b^2cfx-bc^2d-2c^3dx\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(-a^2f^2-2acdf+b^2df-c^2d^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (-2*(-(b*c^2*d) + b^3*f - 3*a*b*c*f - 2*c^3*d*x + b^2*c*f*x - 2*a*c^2*f*x))
/((b^2 - 4*a*c)*(-(c^2*d^2) + b^2*d*f - 2*a*c*d*f - a^2*f^2)*Sqrt[a + b*x +
 c*x^2]) - (f*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f
*#1^2 - f*#1^4 & , (b*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] +
2*a*b*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*c^(3/2)*d*Log[-(
Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*a*Sqrt[c]*f*Log[-(Sqrt[c]*x
) + Sqrt[a + b*x + c*x^2] - #1]*#1 - b*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x +
c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ])/(2*(c^2*
d^2 - b^2*d*f + 2*a*c*d*f + a^2*f^2))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valueint()  Bad Argument Type
```

**maple [B]** time = 0.02, size = 1376, normalized size = 4.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] 1/2/(d*f)^(1/2)/(a*f+c*d-(d*f)^(1/2)*b)*f/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*
f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)+2/(a*f+c*d
-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x
+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*c^2-1/(d*f)^(1/2)/(a*f
+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c
)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*b*c*f+1/(a*f+c*d-(
d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(
d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b*c-1/2/(d*f)^(1/2)/(a*f+c
*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*
(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b^2*f-1/2/(d*f)^(1/2)/
(a*f+c*d-(d*f)^(1/2)*b)*f/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-
(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f
)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(
1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))-1/2/(d*f)^(1
/2)/(a*f+c*d+(d*f)^(1/2)*b)*f/((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*
(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)+2/(a*f+c*d+(d*f)^(1/2)
*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)
/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*x*c^2+1/(d*f)^(1/2)/(a*f+c*d+(d*f)^(
1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(
1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*x*b*c*f+1/(a*f+c*d+(d*f)^(1/2)*b
)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f
)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*b*c+1/2/(d*f)^(1/2)/(a*f+c*d+(d*f)^(1/
2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/
2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*b^2*f+1/2/(d*f)^(1/2)/(a*f+c*d+(d*
f)^(1/2)*b)*f/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*
b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f
```

```
)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a
*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                   /(2*f)                 +(c*d)/f+a))
     /f^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(d-f\,x^2\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int(1/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-ad\sqrt{a+bx+cx^2}+afx^2\sqrt{a+bx+cx^2}-bdx\sqrt{a+bx+cx^2}+bfx^3\sqrt{a+bx+cx^2}-cdx^2\sqrt{a+bx+cx^2}+cfx^4\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(1/(-a*d*sqrt(a + b*x + c*x**2) + a*f*x**2*sqrt(a + b*x + c*x**2)
- b*d*x*sqrt(a + b*x + c*x**2) + b*f*x**3*sqrt(a + b*x + c*x**2) - c*d*x**2
*sqrt(a + b*x + c*x**2) + c*f*x**4*sqrt(a + b*x + c*x**2)), x)
```

## 3.107 $\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx$

**Optimal**. Leaf size=394

$$-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{a^{3/2}d}-\frac{2f\left(a\left(2acf+b^2(-f)+2c^2d\right)+bcx(cd-af)\right)}{d\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ad\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{f^{3/2}\tanh^{-1}\ldots}{2\ldots}$$

**Rubi** [A] time = 1.18, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6725, 740, 12, 724, 206, 1018, 1033}

$$-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{a^{3/2}d}-\frac{2f\left(a\left(2acf+b^2(-f)+2c^2d\right)+bcx(cd-af)\right)}{d\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ad\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{f^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{f^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*d*Sqrt[a + b*x + c*x^2]) - (2*f*
(a*(2*c^2*d - b^2*f + 2*a*c*f) + b*c*(c*d - a*f)*x))/((b^2 - 4*a*c)*d*(b^2*
d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) - ArcTanh[(2*a + b*x)/(2*Sqrt[a
]*Sqrt[a + b*x + c*x^2])]/(a^(3/2)*d) - (f^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*S
qrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f
]*Sqrt[a + b*x + c*x^2])]])/(2*d*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (f
^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*S
qrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]])/(2*d*(c*d + b*S
qrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1018

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^
(q + 1)*((g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f
)) + c*(g*(2*c^2*d + b^2*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x))/((b^2 - 4*
a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
```

```
[n, 0]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx = \int \left(\frac{1}{dx\left(a+bx+cx^2\right)^{3/2}} - \frac{fx}{d\left(a+bx+cx^2\right)^{3/2}\left(-d+fx^2\right)}\right) dx$$

$$= \frac{\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}}\,dx}{d} - \frac{f\int \frac{x}{\left(a+bx+cx^2\right)^{3/2}\left(-d+fx^2\right)}\,dx}{d}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}} - \frac{2f\left(a\left(2c^2d-b^2f+2acf\right)+bc(cd-af)x\right.}{\left(b^2-4ac\right)d\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+}}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}} - \frac{2f\left(a\left(2c^2d-b^2f+2acf\right)+bc(cd-af)x\right.}{\left(b^2-4ac\right)d\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+}}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}} - \frac{2f\left(a\left(2c^2d-b^2f+2acf\right)+bc(cd-af)x\right.}{\left(b^2-4ac\right)d\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+}}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}} - \frac{2f\left(a\left(2c^2d-b^2f+2acf\right)+bc(cd-af)x\right.}{\left(b^2-4ac\right)d\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+}}$$

**Mathematica [A]** time = 0.87, size = 436, normalized size = 1.11

$$\frac{-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{a^{3/2}} - \frac{2f\left(a\left(2acf+b^2(-f)+2c^2d\right)+bcx(cd-af)\right)}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(b^2df-(af+cd)^2\right)} + \frac{f^{3/2}\left(\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)+\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}-b\sqrt{d}+b\sqrt{f}x-2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)\right)}{2\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\left((af+cd)^2-b^2df\right)} + \frac{2\left(-2ac+b^2+bcx\right)}{a\left(b^2-4ac\right)\sqrt{a+x(b+cx)}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] ((2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]) - (2*f*(
a*(2*c^2*d - b^2*f + 2*a*c*f) + b*c*(c*d - a*f)*x))/((b^2 - 4*a*c)*(b^2*d*f
 - (c*d + a*f)^2)*Sqrt[a + x*(b + c*x)]) - ArcTanh[(2*a + b*x)/(2*Sqrt[a]*S
qrt[a + x*(b + c*x)])]/a^(3/2) + (f^(3/2)*((c*d + b*Sqrt[d]*Sqrt[f] + a*f)^
(3/2)*ArcTanh[(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2
```

```
*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] + (c*d - b*Sqr
t[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x
+ b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)]
)]))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] +
a*f]*(-(b^2*d*f) + (c*d + a*f)^2)))/d
```

**IntegrateAlgebraic** [C]     time = 1.59, size = 498, normalized size = 1.26

$$\frac{f^2\text{RootSum}\left[\#1^4(-f)+2\#1^2af+4\#1^2cd-4\#1b\sqrt{c}\,d-a^2f+b^2d\&,\frac{-\#1^2cd\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)-\#1^2af\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)+a^2f\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)+b^2d\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)+acd\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)-2\#1b\sqrt{c}\,d\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)}{\#1^3f-\#1af-2\#1cd+b\sqrt{c}\,d}\&\right]}{2d\left(a^2f^2+2acdf+b^2(-d)f+c^2d^2\right)}+\frac{2\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}-\frac{\sqrt{a+bx+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2\left(2a^2c^2f-4ab^2cf-3abc^2fx+2ac^3d+b^4f+b^3cfx-b^2c^2d-bc^3dx\right)}{a\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(-a^2f^2-2acdf+b^2df-c^2d^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (2*(-(b^2*c^2*d) + 2*a*c^3*d + b^4*f - 4*a*b^2*c*f + 2*a^2*c^2*f - b*c^3*d*
x + b^3*c*f*x - 3*a*b*c^2*f*x))/(a*(b^2 - 4*a*c)*(-(c^2*d^2) + b^2*d*f - 2*
a*c*d*f - a^2*f^2)*Sqrt[a + b*x + c*x^2]) + (2*ArcTanh[(Sqrt[c]*x)/Sqrt[a]
- Sqrt[a + b*x + c*x^2]/Sqrt[a]])/(a^(3/2)*d) + (f^2*RootSum[b^2*d - a^2*f
- 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (b^2*d*Log[-(Sqrt
[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*
x + c*x^2] - #1] + a^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2
*b*Sqrt[c]*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - c*d*Log[-(
Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 - a*f*Log[-(Sqrt[c]*x) + Sqrt
[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) &
])/(2*d*(c^2*d^2 - b^2*d*f + 2*a*c*d*f + a^2*f^2))
```

**fricas** [F(-1)]     time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac** [F]     time = 0.00, size = 0, normalized size = 0.00

$$sage_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] sage2
```

**maple** [B]     time = 0.02, size = 1518, normalized size = 3.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] -1/2/d/(a*f+c*d-(d*f)^(1/2)*b)*f/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*
c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)-2/d/(a*f+c*d-(d*f)^
(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^
(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*(d*f)^(1/2)*x*c^2+1/d/(a*f+c*d-
(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+
(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*b*c*f-1/d/(a*f+c*d-(d*f
)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f
)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*(d*f)^(1/2)*b*c+1/2/d/(a*f+c*
d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(
x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b^2*f+1/2/d/(a*f+c*d-(d
*f)^(1/2)*b)*f/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)
*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/
f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(
a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))-1/2/d/(a*f+c*d+(d*f)^(1
/2)*b)*f/((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(
a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)+2/d/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x
-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f
)^(1/2)*b)/f)^(1/2)*(d*f)^(1/2)*x*c^2+1/d/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^
2)/((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*
d+(d*f)^(1/2)*b)/f)^(1/2)*x*b*c*f+1/d/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/(
(x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d
*f)^(1/2)*b)/f)^(1/2)*(d*f)^(1/2)*b*c+1/2/d/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-
b^2)/((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+
c*d+(d*f)^(1/2)*b)/f)^(1/2)*b^2*f+1/2/d/(a*f+c*d+(d*f)^(1/2)*b)*f/((a*f+c*d
+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)
*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)
/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f
)^(1/2))/(x-(d*f)^(1/2)/f))+1/d/a/(c*x^2+b*x+a)^(1/2)-2/d*b/a/(4*a*c-b^2)/(
c*x^2+b*x+a)^(1/2)*c*x-1/d*b^2/a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-1/d/a^(3/2
)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(cx^2 + bx + a\right)^{\frac{3}{2}}\left(fx^2 - d\right)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] -integrate(1/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)*x), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\,\left(d-f\,x^2\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int(1/(x*(d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-adx\sqrt{a+bx+cx^2}+afx^3\sqrt{a+bx+cx^2}-bdx^2\sqrt{a+bx+cx^2}+bfx^4\sqrt{a+bx+cx^2}-cdx^3\sqrt{a+bx+cx^2}+cfx^5\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(1/(-a*d*x*sqrt(a + b*x + c*x**2) + a*f*x**3*sqrt(a + b*x + c*x**2
) - b*d*x**2*sqrt(a + b*x + c*x**2) + b*f*x**4*sqrt(a + b*x + c*x**2) - c*d
*x**3*sqrt(a + b*x + c*x**2) + c*f*x**5*sqrt(a + b*x + c*x**2)), x)
```

## 3.108 $\int \frac{1}{x^2\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx$

**Optimal**. Leaf size=454

$$\frac{3b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{5/2}d}-\frac{\left(3b^2-8ac\right)\sqrt{a+bx+cx^2}}{a^2dx\left(b^2-4ac\right)}-\frac{2f\left(b\left(b^2f-c(3af+cd)\right)-cx\left(2acf+b^2(-f)+2c^2d\right)\right)}{d\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}\cdot$$

**Rubi [A]** time = 1.19, antiderivative size = 454, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6725, 740, 806, 724, 206, 975, 1033}

$$-\frac{\left(3b^2-8ac\right)\sqrt{a+bx+cx^2}}{a^2dx\left(b^2-4ac\right)}+\frac{3b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{5/2}d}-\frac{2f\left(b\left(b^2f-c(3af+cd)\right)-cx\left(2acf+b^2(-f)+2c^2d\right)\right)}{d\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{adx\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{f^2\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^{3/2}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{f^2\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^{3/2}\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*d*x*Sqrt[a + b*x + c*x^2]) - (2*
f*(b*(b^2*f - c*(c*d + 3*a*f)) - c*(2*c^2*d - b^2*f + 2*a*c*f)*x))/((b^2 -
4*a*c)*d*(b^2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) - ((3*b^2 - 8*a*c
)*Sqrt[a + b*x + c*x^2])/(a^2*(b^2 - 4*a*c)*d*x) + (3*b*ArcTanh[(2*a + b*x)
/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*a^(5/2)*d) + (f^2*ArcTanh[(b*Sqrt[d
] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt
[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^(3/2)*(c*d - b*Sqrt[d]*Sqrt[f] + a
*f)^(3/2)) + (f^2*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[
f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d
^(3/2)*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 975

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(2*a*
f))*x)*(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(q + 1))/((b^2 - 4*a*c)*(b^2*d
*f + (c*d - a*f)^2)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d -
a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[2*c*(b^2
*d*f + (c*d - a*f)^2)*(p + 1) - (2*c^2*d + b^2*f - c*(2*a*f))*(a*f*(p + 1)
- c*d*(p + 2)) + (2*f*(b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d +
b^2*f - c*(2*a*f))*(b*f*(p + 1)))*x + c*f*(2*c^2*d + b^2*f - c*(2*a*f))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[
p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx = \int \left(\frac{1}{dx^2\left(a+bx+cx^2\right)^{3/2}} + \frac{f}{d\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\right)dx$$

$$= \frac{\int \frac{1}{x^2\left(a+bx+cx^2\right)^{3/2}}\,dx}{d} + \frac{f\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx}{d}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)dx\sqrt{a+bx+cx^2}} - \frac{2f\left(b\left(b^2f-c(cd+3af)\right)-c\left(2c^2d-b^2f\right.\right.}{\left(b^2-4ac\right)d\left(b^2df-(cd+af)^2\right)\sqrt{a+}}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)dx\sqrt{a+bx+cx^2}} - \frac{2f\left(b\left(b^2f-c(cd+3af)\right)-c\left(2c^2d-b^2f\right.\right.}{\left(b^2-4ac\right)d\left(b^2df-(cd+af)^2\right)\sqrt{a+}}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)dx\sqrt{a+bx+cx^2}} - \frac{2f\left(b\left(b^2f-c(cd+3af)\right)-c\left(2c^2d-b^2f\right.\right.}{\left(b^2-4ac\right)d\left(b^2df-(cd+af)^2\right)\sqrt{a+}}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)dx\sqrt{a+bx+cx^2}} - \frac{2f\left(b\left(b^2f-c(cd+3af)\right)-c\left(2c^2d-b^2f\right.\right.}{\left(b^2-4ac\right)d\left(b^2df-(cd+af)^2\right)\sqrt{a+}}$$

**Mathematica** [**A**] time = 1.19, size = 488, normalized size = 1.07

$$\frac{\frac{3b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{a^{5/2}}+\frac{2\left(8ac-3b^2\right)\sqrt{a+x(b+cx)}}{a^2x}-\frac{f^2\left(\frac{\left(b^2-4ac\right)\left(af+b\sqrt{d}\sqrt{f}+cd\right)\tanh^{-1}\left(\frac{2a\sqrt{f}-b\sqrt{d}+b\sqrt{f}x-2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\left(4ac-b^2\right)\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{d}\left((af+cd)^2-b^2df\right)}+\frac{4\left(-2ac+b^2+bcx\right)}{ax\sqrt{a+x(b+cx)}}-\frac{4f\left(-bc(3af+cd)-2c^2x(af+cd)+b^3f+b^2cfx\right)}{\sqrt{a+x(b+cx)}\left(b^2df-(af+cd)^2\right)}}{2d\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] ((4*(b^2 - 2*a*c + b*c*x))/(a*x*Sqrt[a + x*(b + c*x)]) - (4*f*(b^3*f - b*c*
(c*d + 3*a*f) + b^2*c*f*x - 2*c^2*(c*d + a*f)*x))/((b^2*d*f - (c*d + a*f)^2
)*Sqrt[a + x*(b + c*x)]) + (2*(-3*b^2 + 8*a*c)*Sqrt[a + x*(b + c*x)])/(a^2*
```

```
x) + (3*b*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)
])])/a^(5/2) - (f^2*(((b^2 - 4*a*c)*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh
[(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d - b
*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d - b*Sqrt[d]*Sqrt[
f] + a*f] + ((-b^2 + 4*a*c)*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(b*Sqrt
[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sq
rt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]))
/(Sqrt[d]*(-(b^2*d*f) + (c*d + a*f)^2)))/(2*(b^2 - 4*a*c)*d)
```

**IntegrateAlgebraic [C]** time = 2.92, size = 649, normalized size = 1.43

$$-\frac{f^2\text{RootSum}\left[\#1^4(-f)+2\#1^2af+4\#1^2cd-4\#1b\sqrt{c}d-a^2f+b^2d\&,\frac{\#1^2(-b)f\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2\#1c^{3/2}d\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+bcd\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2\#1a\sqrt{c}f\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+2abf\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)}{\#1^3f-\#1af-2\#1cd+b\sqrt{c}d}\&\right]}{2d\left(a^2f^2+2acdf+b^2(-d)f+c^2d^2\right)}-\frac{3b\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}-\frac{\sqrt{a+bx+cx^2}}{\sqrt{a}}\right)}{a^{5/2}d}+\frac{-4a^4cf^2+a^3b^2f^2-4a^3bcf^2x-8a^3c^2df-4a^3c^2f^2x^2+a^2b^3f^2x+6a^2b^2cdf+a^2b^2cf^2x^2-18a^2bc^2dfx-4a^2c^3d^2-12a^2c^3dfx^2-ab^4df+16ab^3cdfx+ab^2c^2d^2+14ab^2c^2dfx^2-10abc^3d^2x-8ac^4d^2x^2-3b^5dfx-3b^4cdfx^2+3b^3c^2d^2x+3b^2c^3d^2x^2}{a^2dx\left(4ac-b^2\right)\sqrt{a+bx+cx^2}\left(a^2f^2+2acdf+b^2(-d)f+c^2d^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (a*b^2*c^2*d^2 - 4*a^2*c^3*d^2 - a*b^4*d*f + 6*a^2*b^2*c*d*f - 8*a^3*c^2*d*
f + a^3*b^2*f^2 - 4*a^4*c*f^2 + 3*b^3*c^2*d^2*x - 10*a*b*c^3*d^2*x - 3*b^5*
d*f*x + 16*a*b^3*c*d*f*x - 18*a^2*b*c^2*d*f*x + a^2*b^3*f^2*x - 4*a^3*b*c*f
^2*x + 3*b^2*c^3*d^2*x^2 - 8*a*c^4*d^2*x^2 - 3*b^4*c*d*f*x^2 + 14*a*b^2*c^2
*d*f*x^2 - 12*a^2*c^3*d*f*x^2 + a^2*b^2*c*f^2*x^2 - 4*a^3*c^2*f^2*x^2)/(a^2
*(-b^2 + 4*a*c)*d*(c^2*d^2 - b^2*d*f + 2*a*c*d*f + a^2*f^2)*x*Sqrt[a + b*x
+ c*x^2]) - (3*b*ArcTanh[(Sqrt[c]*x)/Sqrt[a] - Sqrt[a + b*x + c*x^2]/Sqrt[a
]])/(a^(5/2)*d) - (f^2*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^
2 + 2*a*f*#1^2 - f*#1^4 & , (b*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2]
 - #1] + 2*a*b*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*c^(3/2)
*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*a*Sqrt[c]*f*Log[-(
Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - b*f*Log[-(Sqrt[c]*x) + Sqrt[a
 + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ])
/(2*d*(c^2*d^2 - b^2*d*f + 2*a*c*d*f + a^2*f^2))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 1.84int()
Bad Argument Type
```

**maple [B]** time = 0.02, size = 1656, normalized size = 3.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] 1/2*f^2/d/(d*f)^(1/2)/(a*f+c*d-(d*f)^(1/2)*b)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2
*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)+2*f/d/
(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/
2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*c^2-f^2/d/(d*f
)^(1/2)/(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(
d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*b*c+f/
d/(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(
1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b*c-1/2*f^2/d/
(d*f)^(1/2)/(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f
-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b^2-
1/2*f^2/d/(d*f)^(1/2)/(a*f+c*d-(d*f)^(1/2)*b)/((a*f+c*d-(d*f)^(1/2)*b)/f)^(
1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f
)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)
^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1
/2)/f))-1/d/a/x/(c*x^2+b*x+a)^(1/2)-3/2/d*b/a^2/(c*x^2+b*x+a)^(1/2)+3/d*b^2
/a^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*c*x+3/2/d*b^3/a^2/(4*a*c-b^2)/(c*x^2+b
*x+a)^(1/2)+3/2/d*b/a^(5/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-8
/d*c^2/a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-4/d*c/a/(4*a*c-b^2)/(c*x^2+b*x+a
)^(1/2)*b-1/2*f^2/d/(d*f)^(1/2)/(a*f+c*d+(d*f)^(1/2)*b)/((x-(d*f)^(1/2)/f)^
2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1
/2)+2*f/d/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(b*f+2
*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*x*c^2+
f^2/d/(d*f)^(1/2)/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*
c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2
)*x*b*c+f/d/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(b*f
+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*b*c+
1/2*f^2/d/(d*f)^(1/2)/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f
)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^
(1/2)*b^2+1/2*f^2/d/(d*f)^(1/2)/(a*f+c*d+(d*f)^(1/2)*b)/((a*f+c*d+(d*f)^(1/
2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*
f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b
*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(
```

```
x-(d*f)^(1/2)/f))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{3}{2}}\left(fx^2-d\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] -integrate(1/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)*x^2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\left(d-f\,x^2\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int(1/(x^2*(d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-adx^2\sqrt{a+bx+cx^2}+afx^4\sqrt{a+bx+cx^2}-bdx^3\sqrt{a+bx+cx^2}+bfx^5\sqrt{a+bx+cx^2}-cdx^4\sqrt{a+bx+cx^2}+cfx^6\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(1/(-a*d*x**2*sqrt(a + b*x + c*x**2) + a*f*x**4*sqrt(a + b*x + c*x
**2) - b*d*x**3*sqrt(a + b*x + c*x**2) + b*f*x**5*sqrt(a + b*x + c*x**2) -
c*d*x**4*sqrt(a + b*x + c*x**2) + c*f*x**6*sqrt(a + b*x + c*x**2)), x)
```

## 3.109 $\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$

**Optimal**. Leaf size=761

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4cf(be-af)+b^2f^2-8c^2\left(e^2-df\right)\right)}{8c^{3/2}f^3}-\frac{\left(f\left(af\left(-e\sqrt{e^2-4df}-2df+e^2\right)-b\left(-e^2\sqrt{e^2-\right.\right.\right.}{}$$

**Rubi [A]** time = 3.14, antiderivative size = 761, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1067, 1076, 621, 206, 1032, 724}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4cf(be-af)+b^2f^2-8c^2\left(e^2-df\right)\right)}{8c^{3/2}f^3}-\frac{\left(f\left(af\left(-e\sqrt{e^2-4df}-2df+e^2\right)-b\left(-e^2\sqrt{e^2-4df}+df\sqrt{e^2-4df}-3def+e^3\right)\right)+c\left(2d^2f^2-e^3\sqrt{e^2-4df}-4de^2f+2def\sqrt{e^2-4df}+e^4\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(f\left(af\left(e\sqrt{e^2-4df}-2df+e^2\right)-b\left(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3\right)\right)+c\left(2d^2f^2+e^3\sqrt{e^2-4df}-4de^2f-2def\sqrt{e^2-4df}+e^4\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\sqrt{a+bx+cx^2}(-bf+4ce-2cfx)}{4cf^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] -((4*c*e - b*f - 2*c*f*x)*Sqrt[a + b*x + c*x^2])/(4*c*f^2) - ((b^2*f^2 + 4*
c*f*(b*e - a*f) - 8*c^2*(e^2 - d*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
+ b*x + c*x^2])])/(8*c^(3/2)*f^3) - ((c*(e^4 - 4*d*e^2*f + 2*d^2*f^2 - e^3*
Sqrt[e^2 - 4*d*f] + 2*d*e*f*Sqrt[e^2 - 4*d*f]) + f*(a*f*(e^2 - 2*d*f - e*Sq
rt[e^2 - 4*d*f]) - b*(e^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*d*f] + d*f*Sqrt[e^2
- 4*d*f])))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sq
rt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c
*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f^3*Sqrt[e^2
 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sq
rt[e^2 - 4*d*f]))]) + ((c*(e^4 - 4*d*e^2*f + 2*d^2*f^2 + e^3*Sqrt[e^2 - 4*d
*f] - 2*d*e*f*Sqrt[e^2 - 4*d*f]) + f*(a*f*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f
]) - b*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f])))*Ar
cTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f
]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt
[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt
[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f
]))])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1067

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) +
(e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((C*(b*f*p - c*e*(2*p +
q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + e*x + f*x^2)^(q +
 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1
)*(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[
p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) - c*(C*e)*(2*p + 2*q + 3)) + (p + q +
1)*(b^2*C*d*f*p + a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(-2*A*f)*(2*p + 2*
q + 3))) + (2*p*(c*d - a*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e)*(2*p + 2*q + 3
)) + (p + q + 1)*(C*e*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2
) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*
(q + 1) - c*(C*e)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c
^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3))))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
```

```
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = -\frac{(4ce-bf-2cfx)\sqrt{a+bx+cx^2}}{4cf^2} - \frac{\int \frac{-\frac{1}{4}d\left(4bce-b^2f-4acf\right)-\frac{1}{4}\left(8c^2de-b^2ef-4acef+4bc\left(e^2-2df\right)\right)x+}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}}{2cf^2}$$

$$= -\frac{(4ce-bf-2cfx)\sqrt{a+bx+cx^2}}{4cf^2} - \frac{\int \frac{-\frac{1}{4}df\left(4bce-b^2f-4acf\right)-\frac{1}{4}d\left(b^2f^2+4cf(be-af)-8c^2\left(e^2-df\right)\right)+(}{\sqrt{a+}}}{}$$

$$= -\frac{(4ce-bf-2cfx)\sqrt{a+bx+cx^2}}{4cf^2} - \frac{\left(b^2f^2+4cf(be-af)-8c^2\left(e^2-df\right)\right)\text{Subst}\left(\int \frac{}{4c}\right.}{4cf^3}$$

$$= -\frac{(4ce-bf-2cfx)\sqrt{a+bx+cx^2}}{4cf^2} - \frac{\left(b^2f^2+4cf(be-af)-8c^2\left(e^2-df\right)\right)\tanh^{-1}\left(\frac{}{2\sqrt{}}\right.}{8c^{3/2}f^3}$$

$$= -\frac{(4ce-bf-2cfx)\sqrt{a+bx+cx^2}}{4cf^2} - \frac{\left(b^2f^2+4cf(be-af)-8c^2\left(e^2-df\right)\right)\tanh^{-1}\left(\frac{}{2\sqrt{}}\right.}{8c^{3/2}f^3}$$

**Mathematica [A]** time = 2.31, size = 552, normalized size = 0.73

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(4cf(af-be)-b^2f^2+8c^2\left(e^2-df\right)\right)}{8c^{3/2}f^3}+\frac{f\sqrt{e^2-4df}\sqrt{a+x(b+cx)}(bf-4ce+2cfx)+\sqrt{2}c\left(e\sqrt{e^2-4df}-2df+e^2\right)\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af-b\left(\sqrt{e^2-4df}+e-2fx\right)-2cx\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)+\sqrt{2}c\left(e\sqrt{e^2-4df}+2df-e^2\right)\sqrt{f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af+b\left(\sqrt{e^2-4df}-e+2fx\right)+2cx\left(\sqrt{e^2-4df}-e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{4cf^3\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] ((-(b^2*f^2) + 4*c*f*(-(b*e) + a*f) + 8*c^2*(e^2 - d*f))*ArcTanh[(b + 2*c*x
)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(8*c^(3/2)*f^3) + (f*Sqrt[e^2 - 4*d*f
]*(-4*c*e + b*f + 2*c*f*x)*Sqrt[a + x*(b + c*x)] + Sqrt[2]*c*(e^2 - 2*d*f +
 e*Sqrt[e^2 - 4*d*f])*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f
 - b*(e + Sqrt[e^2 - 4*d*f]))]*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])
*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*
Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b +
 c*x)])] + Sqrt[2]*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])*Sqrt[c*(e^2 - 2*d
*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]*ArcTanh
[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*
```

```
x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-
e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/(4*c*f^3*Sqrt[e^2 - 4*d*f
])
```

**IntegrateAlgebraic [C]** time = 1.45, size = 869, normalized size = 1.14

$$\frac{\sqrt{cx^2+bx+a}\,(-4ce+bf+2cfx)}{4cf^2}+\frac{\left(-8e^2c^2+8dfc^2-4af^2c+4befc+b^2f^2\right)\log\left(2xc^2-2\sqrt{cx^2+bx+a}\,c^{3/2}+bc\right)}{8c^{3/2}f^3}-\frac{\mathrm{RootSum}\left[f\#1^4-2\sqrt{c}\,e\#1^3+4cd\#1^2+be\#1^2-2af\#1^2-4b\sqrt{c}\,d\#1+2a\sqrt{c}\,e\#1+b^2d-abe+a^2f\&,\ \frac{-c\log\left(-\sqrt{c}\,x-\#1+\sqrt{cx^2+bx+a}\right)\#1^2e^3+ac\log\left(-\sqrt{c}\,x-\#1+\sqrt{cx^2+bx+a}\right)e^3+bf\log\left(-\sqrt{c}\,x-\#1+\sqrt{cx^2+bx+a}\right)\#1^2e^2-bcd\log\left(-\sqrt{c}\,x-\#1+\sqrt{cx^2+bx+a}\right)e^2-abf\log\left(-\sqrt{c}\,x-\#1+\sqrt{cx^2+bx+a}\right)e^2+2c^{3/2}d\log\left(-\sqrt{c}\,x-\#1+\sqrt{cx^2+bx+a}\right)\#1e^2-af^2\log\left(-\sqrt{c}\,x-\#1+\sqrt{cx^2+bx+a}\right)\#1^2e+2cdf\log\left(-\sqrt{c}\,x-\#1+\sqrt{cx^2+bx+a}\right)\#1^2e+a^2f^2\log\left(-\sqrt{c}\,x-\#1+\sqrt{cx^2+bx+a}\right)e+b^2df\log\left(-\sqrt{c}\,x-\#1+\sqrt{cx^2+bx+a}\right)e-2acdf\log\left(-\sqrt{c}\,x-\#1+\sqrt{cx^2+bx+a}\right)e-2b\sqrt{c}\,df\log\left(-\sqrt{c}\,x-\#1+\sqrt{cx^2+bx+a}\right)\#1e-bdf^2\log\left(-\sqrt{c}\,x-\#1+\sqrt{cx^2+bx+a}\right)\#1^2+bcd^2f\log\left(-\sqrt{c}\,x-\#1+\sqrt{cx^2+bx+a}\right)+2a\sqrt{c}\,df^2\log\left(-\sqrt{c}\,x-\#1+\sqrt{cx^2+bx+a}\right)\#1-2c^{3/2}d^2f\log\left(-\sqrt{c}\,x-\#1+\sqrt{cx^2+bx+a}\right)\#1}{-2f\#1^3+3\sqrt{c}\,e\#1^2-4cd\#1-be\#1+2af\#1+2b\sqrt{c}\,d-a\sqrt{c}\,e}\&\right]}{f^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] ((-4*c*e + b*f + 2*c*f*x)*Sqrt[a + b*x + c*x^2])/(4*c*f^2) + ((-8*c^2*e^2 +
 8*c^2*d*f + 4*b*c*e*f + b^2*f^2 - 4*a*c*f^2)*Log[b*c + 2*c^2*x - 2*c^(3/2)
*Sqrt[a + b*x + c*x^2]])/(8*c^(3/2)*f^3) - RootSum[b^2*d - a*b*e + a^2*f -
4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#1^2 -
2*Sqrt[c]*e*#1^3 + f*#1^4 & , (-(b*c*d*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x
+ c*x^2] - #1]) + a*c*e^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] +
b*c*d^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + b^2*d*e*f*Log[-(
Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*a*c*d*e*f*Log[-(Sqrt[c]*x) + S
qrt[a + b*x + c*x^2] - #1] - a*b*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*
x^2] - #1] + a^2*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*c
^(3/2)*d*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*c^(3/2)*
d^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*b*Sqrt[c]*d*e*f
*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*a*Sqrt[c]*d*f^2*Log[
-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - c*e^3*Log[-(Sqrt[c]*x) + Sq
rt[a + b*x + c*x^2] - #1]*#1^2 + 2*c*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x
+ c*x^2] - #1]*#1^2 + b*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1
]*#1^2 - b*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 - a*e*
f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(2*b*Sqrt[c]*d - a
*Sqrt[c]*e - 4*c*d*#1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*#1^2 - 2*f*#1^3) &
]/f^3
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

**maple [B]** time = 0.02, size = 14815, normalized size = 19.47

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \sqrt{c\,x^2+b\,x+a}}{f\,x^2+e\,x+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x + c*x^2)^(1/2))/(d + e*x + f*x^2),x)
```

```
[Out] int((x^2*(a + b*x + c*x^2)^(1/2))/(d + e*x + f*x^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x**2*sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)
```

## 3.110 $\int \frac{x\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$

**Optimal**. Leaf size=549

$$-\frac{\left(\left(e-\sqrt{e^2-4df}\right)\left(f(be-af)-c\left(e^2-df\right)\right)+2df(ce-bf)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

**Rubi** [A] time = 7.03, antiderivative size = 549, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1019, 1076, 621, 206, 1032, 724}

$$-\frac{\left(\left(e-\sqrt{e^2-4df}\right)\left(f(be-af)-c\left(e^2-df\right)\right)+2df(ce-bf)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}+\frac{\left(\left(\sqrt{e^2-4df}+e\right)\left(f(be-af)-c\left(e^2-df\right)\right)+2df(ce-bf)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{(2ce-bf)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2}+\frac{\sqrt{a+bx+cx^2}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] Sqrt[a + b*x + c*x^2]/f - ((2*c*e - b*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqr
t[a + b*x + c*x^2])])/(2*Sqrt[c]*f^2) - ((2*d*f*(c*e - b*f) + (e - Sqrt[e^2
 - 4*d*f])*(f*(b*e - a*f) - c*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^
2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2
- 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x +
 c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*
a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) + ((2*d*f*(c*e - b*f) + (e + Sqrt[e
^2 - 4*d*f])*(f*(b*e - a*f) - c*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e + Sqrt[
e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^
2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x
 + c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f +
2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1019

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d
+ e*x + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), I
nt[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[h*p*(b*d - a*e) + a*(
h*e - 2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(h*e - 2*g*f)*(p + q + 1)
)*x + (h*p*(c*e - b*f) + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d
*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{x\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \frac{\sqrt{a+bx+cx^2}}{f} - \frac{\int \frac{\frac{bd}{2}+\frac{1}{2}(2cd+be-2af)x+\frac{1}{2}(2ce-bf)x^2}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx}{f}$$

$$= \frac{\sqrt{a+bx+cx^2}}{f} - \frac{\int \frac{\frac{bdf}{2}-\frac{1}{2}d(2ce-bf)+\left(\frac{1}{2}f(2cd+be-2af)-\frac{1}{2}e(2ce-bf)\right)x}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx}{f^2} - \frac{(2ce-bf)\int \frac{1}{\sqrt{a+bx+c}}}{2f^2}$$

$$= \frac{\sqrt{a+bx+cx^2}}{f} - \frac{(2ce-bf)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f^2} - \frac{\left(2f\left(\frac{bdf}{2}-\frac{1}{2}d(2ce-bf\right.\right.}{}$$

$$= \frac{\sqrt{a+bx+cx^2}}{f} - \frac{(2ce-bf)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\,f^2} + \frac{\left(2\left(2f\left(\frac{bdf}{2}-\frac{1}{2}d(2ce-bf)\right)-\left(\right.\right.\right.}{}$$

$$= \frac{\sqrt{a+bx+cx^2}}{f} - \frac{(2ce-bf)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\,f^2} - \frac{\left(2f(cde-bdf)+\left(e-\sqrt{e^2-4d}\right.\right.}{\sqrt{2}\,f^2}$$

**Mathematica** [A] time = 1.78, size = 496, normalized size = 0.90

$$\frac{4f\sqrt{e^2-4df}\sqrt{a+x(b+cx)} - \sqrt{2}\left(\sqrt{e^2-4df}+e\right)\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af-b\left(\sqrt{e^2-4df}+e-2fx\right)-2cx\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right) - \sqrt{2}\left(\sqrt{e^2-4df}-e\right)\sqrt{f\left(2af+b\sqrt{e^2-4df}+b(-e)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af+b\left(\sqrt{e^2-4df}-e+2fx\right)+2cx\left(\sqrt{e^2-4df}-e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af+b\sqrt{e^2-4df}+b(-e)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{4f^2\sqrt{e^2-4df}} - \frac{(2ce-bf)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{2\sqrt{c}\,f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] -1/2*((2*c*e - b*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])
/(Sqrt[c]*f^2) + (4*f*Sqrt[e^2 - 4*d*f]*Sqrt[a + x*(b + c*x)] - Sqrt[2]*(e
+ Sqrt[e^2 - 4*d*f])*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f
- b*(e + Sqrt[e^2 - 4*d*f]))]*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*
x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*S
qrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b +
c*x)])] - Sqrt[2]*(-e + Sqrt[e^2 - 4*d*f])*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[
e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(4*a*f + 2*c
*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*Sqrt[2
]*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e
^2 - 4*d*f])]*Sqrt[a + x*(b + c*x)])])/(4*f^2*Sqrt[e^2 - 4*d*f])
```

**IntegrateAlgebraic** [C] time = 0.86, size = 646, normalized size = 1.18

$$\frac{\text{RootSum}\left[\#1^4 f - 2\#1^3\sqrt{c}e - 2\#1^2 af + \#1^2 be + 4\#1^2 cd + 2\#1 a\sqrt{c}e - 4\#1 b\sqrt{c}d + a^2 f - abe + b^2 d\&, \frac{\#1^2 cdf\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right) - \#1^2 ce^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right) + \#1^2 bef\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right) - \#1^2 af^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right) + a^2f^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right) + b^2df\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right) + 2\#1 c^{3/2}de\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right) - bcde\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right) - 2\#1 b\sqrt{c}df\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right) - acdf\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right) + ace^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right) - abef\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)}{-2\#1^3 f + 3\#1^2\sqrt{c}e + 2\#1 af - \#1 be - 4\#1 cd - a\sqrt{c}e + 2b\sqrt{c}d}\&\right]}{f^2} + \frac{(2ce-bf)\log\left(-2\sqrt{c}\sqrt{a+bx+cx^2}+b+2cx\right)}{2\sqrt{c}\,f^2} + \frac{\sqrt{a+bx+cx^2}}{f}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] Sqrt[a + b*x + c*x^2]/f + ((2*c*e - b*f)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a +
 b*x + c*x^2]])/(2*Sqrt[c]*f^2) + RootSum[b^2*d - a*b*e + a^2*f - 4*b*Sqrt[
c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]
*e*#1^3 + f*#1^4 & , (-(b*c*d*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] -
#1]) + a*c*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + b^2*d*f*Log
[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*c*d*f*Log[-(Sqrt[c]*x) + Sq
rt[a + b*x + c*x^2] - #1] - a*b*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2
] - #1] + a^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*c^(3/2
)*d*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*b*Sqrt[c]*d*f*L
og[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - c*e^2*Log[-(Sqrt[c]*x) +
 Sqrt[a + b*x + c*x^2] - #1]*#1^2 + c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x +
 c*x^2] - #1]*#1^2 + b*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#
1^2 - a*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(2*b*Sqrt[
c]*d - a*Sqrt[c]*e - 4*c*d*#1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*#1^2 - 2*f*
#1^3) & ]/f^2
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

**maple [B]** time = 0.02, size = 10138, normalized size = 18.47

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\sqrt{c\,x^2+b\,x+a}}{f\,x^2+e\,x+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x + c*x^2)^(1/2))/(d + e*x + f*x^2),x)
```

```
[Out] int((x*(a + b*x + c*x^2)^(1/2))/(d + e*x + f*x^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x*sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)
```

## 3.111 $\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2} \, dx$

**Optimal**. Leaf size=431

$$-\frac{\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf}}\right.}{\sqrt{2}f\sqrt{e^2-4df}}$$

**Rubi [A]** time = 0.65, antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {989, 621, 206, 1032, 724}

$$-\frac{\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}}+\frac{\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f - (Sqrt[
c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]
))]*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 -
 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f
)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f])
 + (Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2
 - 4*d*f]))]*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + S
qrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (
c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f*Sqrt[e^2
- 4*d*f])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 989

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (e_.)*(x_) + (f_.)*(x_)^
2), x_Symbol] :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f,
Int[(c*d - a*f + (c*e - b*f)*x)/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 -
4*d*f, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = -\frac{\int \frac{cd-af+(ce-bf)x}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx}{f} + \frac{c\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{f}$$

$$= \frac{(2c)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f} - \frac{\left(2f(cd-af)-(ce-bf)\left(e-\sqrt{e^2-4df}\right)\right)\int \frac{\quad}{(e-\cdots}}{f\sqrt{e^2-4df}\cdots}$$

$$= \frac{\sqrt{c}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{f} + \frac{\left(2\left(2f(cd-af)-(ce-bf)\left(e-\sqrt{e^2-4df}\right)\right)\right)\text{Subst}\left(\int \frac{\cdots}{1\cdots}\right.}{\cdots}$$

$$= \frac{\sqrt{c}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{f} - \frac{\sqrt{c\left(e^2-2df-e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e-\sqrt{e^2-4df}\cdots\right.\right.}}{\sqrt{2}\,f\sqrt{e\cdots}}$$

**Mathematica [A]** time = 0.74, size = 417, normalized size = 0.97

$$\frac{\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af-b\left(\sqrt{e^2-4df}+e-2fx\right)-2cx\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)-\sqrt{f\left(2af+b\sqrt{e^2-4df}+b(-e)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af+b\left(\sqrt{e^2-4df}-e+2fx\right)+2cx\left(\sqrt{e^2-4df}-e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af+b\sqrt{e^2-4df}+b(-e)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\,f\sqrt{e^2-4df}}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/f + (Sqrt[
c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]
))]*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*
f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a
*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])] - Sqrt[f*(-(b*e) +
 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*ArcT
anh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2
*f*x))/(2*Sqrt[2]*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 -
2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + x*(b + c*x)])])/(Sqrt[2]*f*Sqrt[e^2
- 4*d*f])
```

**IntegrateAlgebraic [C]** time = 0.00, size = 402, normalized size = 0.93

$$\frac{\text{RootSum}\left[\#1^4 f - 2\#1^3\sqrt{c}\,e - 2\#1^2 af + \#1^2 be + 4\#1^2 cd + 2\#1 a\sqrt{c}\,e - 4\#1 b\sqrt{c}\,d + a^2 f - abe + b^2 d\&, \frac{\#1^2 ce\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right) - \#1^2 bf\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right) - 2\#1 c^{3/2} d\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right) + bcd\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right) - ace\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right) + 2\#1 a\sqrt{c}\,f\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)}{-2\#1^3 f + 3\#1^2\sqrt{c}\,e + 2\#1 af - \#1 be - 4\#1 cd - a\sqrt{c}\,e + 2b\sqrt{c}\,d}\&\right]}{f} - \frac{\sqrt{c}\log\left(-2\sqrt{c}\,f\sqrt{a+bx+cx^2} + bf + 2cfx\right)}{f}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2),x]
```

```
[Out] -((Sqrt[c]*Log[b*f + 2*c*f*x - 2*Sqrt[c]*f*Sqrt[a + b*x + c*x^2]])/f) + Roo
tSum[b^2*d - a*b*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1
^2 + b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (b*c*d*Log[-(Sqr
t[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*c*e*Log[-(Sqrt[c]*x) + Sqrt[a + b
*x + c*x^2] - #1] - 2*c^(3/2)*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] -
#1]*#1 + 2*a*Sqrt[c]*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 +
c*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 - b*f*Log[-(Sqrt[c]
*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(2*b*Sqrt[c]*d - a*Sqrt[c]*e - 4*c*
d*#1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*#1^2 - 2*f*#1^3) & ]/f
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

**maple [B]** time = 0.02, size = 6019, normalized size = 13.97

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}}{f\,x^2+e\,x+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/(d + e*x + f*x^2),x)
```

```
[Out] int((a + b*x + c*x^2)^(1/2)/(d + e*x + f*x^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx + cx^2}}{d + ex + fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)
```

## 3.112 $\int \frac{\sqrt{a+bx+cx^2}}{x\left(d+ex+fx^2\right)}\,dx$

**Optimal**. Leaf size=523

$$\frac{\left(cd\left(e-\sqrt{e^2-4df}\right)-f\left(2bd-a\left(\sqrt{e^2-4df}+e\right)\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\,d\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{(cd}{}$$

**Rubi [A]** time = 3.70, antiderivative size = 521, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6728, 734, 843, 621, 206, 724, 1019, 1076, 1032}

$$-\frac{\left(-af\left(\sqrt{e^2-4df}+e\right)+2bdf-cd\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\,d\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(-af\left(e-\sqrt{e^2-4df}\right)+2bdf-cd\left(\sqrt{e^2-4df}+e\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\,d\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(x*(d + e*x + f*x^2)),x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/d) - ((2
*b*d*f - c*d*(e - Sqrt[e^2 - 4*d*f]) - a*f*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh
[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x
)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2
- 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2
 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]))]) +
 ((2*b*d*f - a*f*(e - Sqrt[e^2 - 4*d*f]) - c*d*(e + Sqrt[e^2 - 4*d*f]))*Arc
Tanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]
))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[
e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[c*
(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))
])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1019

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d
+ e*x + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), I
nt[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[h*p*(b*d - a*e) + a*(
h*e - 2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(h*e - 2*g*f)*(p + q + 1)
)*x + (h*p*(c*e - b*f) + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d
*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{x\left(d+ex+fx^2\right)}\,dx = \int \left(\frac{\sqrt{a+bx+cx^2}}{dx} + \frac{(-e-fx)\sqrt{a+bx+cx^2}}{d\left(d+ex+fx^2\right)}\right) dx$$

$$= \frac{\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx}{d} + \frac{\int \frac{(-e-fx)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx}{d}$$

$$= -\frac{\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx}{2d} - \frac{\int \frac{-\frac{1}{2}(bd-2ae)f-\frac{1}{2}f(2cd-be-2af)x+\frac{1}{2}bf^2x^2}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx}{df}$$

$$= \frac{a\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{d} - \frac{\int \frac{-\frac{1}{2}bdf^2-\frac{1}{2}(bd-2ae)f^2+\left(-\frac{1}{2}bef^2-\frac{1}{2}f^2(2cd-be-2af)\right)x}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx}{df^2}$$

$$= -\frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d} - \frac{\left(2bdf-af\left(e-\sqrt{e^2-4df}\right)-cd\left(e+\sqrt{e^2-4\right.\right.}}{d\sqrt{e^2-4df}}$$

$$= -\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d} + \frac{\left(2\left(2bdf-af\left(e-\sqrt{e^2-4df}\right)-cd\left(e+\sqrt{e^2-4df}\right)\right)\right)}{}$$

$$= -\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d} - \frac{\left(2bdf-cd\left(e-\sqrt{e^2-4df}\right)-af\left(e+\sqrt{e^2-4df}\right)\right)\mathrm{tanh}}{\sqrt{2}\,d\sqrt{e^2-4df}\,\sqrt{ce^2-2cdf-bef}}$$

**Mathematica [A]** time = 1.31, size = 454, normalized size = 0.87

$$\frac{\left(\sqrt{e^2-4df}-e\right)\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af-b\left(\sqrt{e^2-4df}+e-2fx\right)-2cx\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)+\left(\sqrt{e^2-4df}+e\right)\sqrt{f\left(2af+b\sqrt{e^2-4df}+b(-e)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af+b\left(\sqrt{e^2-4df}-e+2fx\right)+2cx\left(\sqrt{e^2-4df}-e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af+b\sqrt{e^2-4df}+b(-e)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{2\sqrt{2}df\sqrt{e^2-4df}}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(x*(d + e*x + f*x^2)),x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/d) + ((-
e + Sqrt[e^2 - 4*d*f])*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*
f - b*(e + Sqrt[e^2 - 4*d*f]))]*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f]
)*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e
*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b
+ c*x)])] + (e + Sqrt[e^2 - 4*d*f])*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4
*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(4*a*f + 2*c*(-e +
Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[
f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*
d*f])]*Sqrt[a + x*(b + c*x)])])/(2*Sqrt[2]*d*f*Sqrt[e^2 - 4*d*f])
```

**IntegrateAlgebraic [C]** time = 0.64, size = 474, normalized size = 0.91

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}-\frac{\sqrt{a+bx+cx^2}}{\sqrt{a}}\right)}{d}-\frac{\text{RootSum}\left[\#1^4f-2\#1^3\sqrt{c}e-2\#1^2af+\#1^2be+4\#1^2cd+2\#1a\sqrt{c}e-4\#1b\sqrt{c}d+a^2f-abe+b^2d\&,\frac{\#1^2cd\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-\#1^2af\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+a^2f\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+b^2d\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-acd\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2\#1b\sqrt{c}d\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-abe\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+2\#1a\sqrt{c}e\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)}{-2\#1^3f+3\#1^2\sqrt{c}e+2\#1af-\#1be-4\#1cd-a\sqrt{c}e+2b\sqrt{c}d}\&\right]}{d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x + c*x^2]/(x*(d + e*x + f*x^2)),x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[(Sqrt[c]*x)/Sqrt[a] - Sqrt[a + b*x + c*x^2]/Sqrt[a]])/d
- RootSum[b^2*d - a*b*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c
*d*#1^2 + b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (b^2*d*Log[
-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*c*d*Log[-(Sqrt[c]*x) + Sqrt[
a + b*x + c*x^2] - #1] - a*b*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #
1] + a^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*b*Sqrt[c]*d*L
og[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*a*Sqrt[c]*e*Log[-(Sqrt
[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b
*x + c*x^2] - #1]*#1^2 - a*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]
*#1^2)/(2*b*Sqrt[c]*d - a*Sqrt[c]*e - 4*c*d*#1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt
[c]*e*#1^2 - 2*f*#1^3) & ]/d
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 7.77sym2poly/r2sym(const ge
n & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

**maple [B]** time = 0.02, size = 6460, normalized size = 12.35

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/x/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + bx + a}}{(fx^2 + ex + d)x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 + e*x + d)*x), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2 + b\,x + a}}{x\,\left(f\,x^2 + e\,x + d\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/(x*(d + e*x + f*x^2)),x)
```

```
[Out] int((a + b*x + c*x^2)^(1/2)/(x*(d + e*x + f*x^2)), x)
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx+cx^2}}{x\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/x/(f*x**2+e*x+d),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/(x*(d + e*x + f*x**2)), x)
```

## 3.113 $\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex+fx^2)}\,dx$

**Optimal**. Leaf size=736

$$-\frac{f\left(a\left(e\sqrt{e^2-4df}-2df+e^2\right)-bd\left(\sqrt{e^2-4df}+e\right)+2cd^2\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+}}\right.}{\sqrt{2}\,d^2\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

**Rubi [A]** time = 3.48, antiderivative size = 736, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6728, 732, 843, 621, 206, 724, 734, 1019, 1076, 1032}

$$-\frac{f\left(a\left(e\sqrt{e^2-4df}-2df+e^2\right)-bd\left(\sqrt{e^2-4df}+e\right)+2cd^2\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\,d^2\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(a\left(-e\sqrt{e^2-4df}-2df+e^2\right)-bd\left(e-\sqrt{e^2-4df}\right)+2cd^2\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\,d^2\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\sqrt{a}\,e\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2}-\frac{be\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\,d^2}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\,d^2}-\frac{\sqrt{a+bx+cx^2}}{dx}-\frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}\,d}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(x^2*(d + e*x + f*x^2)),x]
```

```
[Out] -(Sqrt[a + b*x + c*x^2]/(d*x)) - (b*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a +
 b*x + c*x^2])])/(2*Sqrt[a]*d) + (Sqrt[a]*e*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*
Sqrt[a + b*x + c*x^2])])/d^2 + (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt
[a + b*x + c*x^2])])/d - (b*e*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x +
 c*x^2])])/(2*Sqrt[c]*d^2) - ((2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*d^2) - (f*(2*c*d^2 - b*d*(e + Sqrt[e^2
- 4*d*f]) + a*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e -
Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt
[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a
+ b*x + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f - e*Sq
rt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]))]) + (f*(2*c*d^2 -
b*d*(e - Sqrt[e^2 - 4*d*f]) + a*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))*ArcTan
h[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*
x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2
 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*(
e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1019

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d
+ e*x + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), I
nt[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[h*p*(b*d - a*e) + a*(
```

```
h*e - 2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(h*e - 2*g*f)*(p + q + 1)
)*x + (h*p*(c*e - b*f) + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d
*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2\left(d+ex+fx^2\right)}\,dx = \int \left(\frac{\sqrt{a+bx+cx^2}}{dx^2} - \frac{e\sqrt{a+bx+cx^2}}{d^2x} + \frac{\left(e^2-df+efx\right)\sqrt{a+bx+cx^2}}{d^2\left(d+ex+fx^2\right)}\right)dx$$

$$= \frac{\int \frac{\left(e^2-df+efx\right)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx}{d^2} + \frac{\int \frac{\sqrt{a+bx+cx^2}}{x^2}\,dx}{d} - \frac{e\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx}{d^2}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx} + \frac{\int \frac{b+2cx}{x\sqrt{a+bx+cx^2}}\,dx}{2d} + \frac{e\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx}{2d^2} - \frac{\int \frac{\frac{1}{2}f\left(bde-2ae^2+2adf\right)+\frac{1}{2}f\left(2cde}{\sqrt{a+bx+c}}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx} + \frac{b\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{2d} + \frac{c\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{d} - \frac{(ae)\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{d^2} - \frac{(b}{}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx} - \frac{b\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d} + \frac{(2c)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b}{\sqrt{a}}\right.}{d}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx} - \frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}\,d} + \frac{\sqrt{a}\,e\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2} + \frac{\sqrt{c}\ \text{ta}}{}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx} - \frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}\,d} + \frac{\sqrt{a}\,e\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2} + \frac{\sqrt{c}\ \text{ta}}{}$$

**<u>Mathematica</u> [A]** time = 1.68, size = 520, normalized size = 0.71

$$\frac{(2ae-bd)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{2\sqrt{a}\,d^2} - \frac{4df\sqrt{e^2-4df}\sqrt{a+x(b+cx)} + \sqrt{2}x\left(e\sqrt{e^2-4df}+2df-e^2\right)\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af-b\left(\sqrt{e^2-4df}+e-2fx\right)-2cx\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right) + \sqrt{2}x\left(e\sqrt{e^2-4df}-2df+e^2\right)\sqrt{f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af+b\left(\sqrt{e^2-4df}-e+2fx\right)+2cx\left(\sqrt{e^2-4df}-e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{4d^2fx\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(x^2*(d + e*x + f*x^2)),x]
```

```
[Out] ((-(b*d) + 2*a*e)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/(
2*Sqrt[a]*d^2) - (4*d*f*Sqrt[e^2 - 4*d*f]*Sqrt[a + x*(b + c*x)] + Sqrt[2]*(
-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*
f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*x*ArcTanh[(4*a*f - 2*c*(e + Sq
rt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(
e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]
*Sqrt[a + x*(b + c*x)])] + Sqrt[2]*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*Sqrt
[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*
```

```
f]))]*x*ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2
- 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) +
f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/(4*d^2*f*S
qrt[e^2 - 4*d*f]*x)
```

**IntegrateAlgebraic [C]** time = 0.88, size = 589, normalized size = 0.80

$$\frac{\text{RootSum}\left[\#1^4 f-2\#1^3\sqrt{c}e-2\#1^2af+\#1^2be+4\#1^2cd+2\#1a\sqrt{c}e-4\#1b\sqrt{c}d+a^2f-abe+b^2d\&,\frac{\#1^2bdf\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-\#1^2aef\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+a^2ef\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+b^2de\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+2\#1c^{3/2}d^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-bcd^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2\#1b\sqrt{c}de\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2\#1a\sqrt{c}df\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-abe^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+2\#1a\sqrt{c}e^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)}{-2\#1^3f+3\#1^2\sqrt{c}e+2\#1af-\#1be-4\#1cd-a\sqrt{c}e+2b\sqrt{c}d}\&\right]}{d^2}+\frac{(2ae-bd)\tanh^{-1}\left(\frac{\sqrt{a+bx+cx^2}-\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}d^2}-\frac{\sqrt{a+bx+cx^2}}{dx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x + c*x^2]/(x^2*(d + e*x + f*x^2)),x]
```

```
[Out] -(Sqrt[a + b*x + c*x^2]/(d*x)) + ((-(b*d) + 2*a*e)*ArcTanh[(-(Sqrt[c]*x) +
Sqrt[a + b*x + c*x^2])/Sqrt[a]])/(Sqrt[a]*d^2) + RootSum[b^2*d - a*b*e + a^
2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#
1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (-(b*c*d^2*Log[-(Sqrt[c]*x) + Sqrt[a +
b*x + c*x^2] - #1]) + b^2*d*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1
] - a*b*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a^2*e*f*Log[-(
Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*c^(3/2)*d^2*Log[-(Sqrt[c]*x) +
 Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*b*Sqrt[c]*d*e*Log[-(Sqrt[c]*x) + Sqrt[a
 + b*x + c*x^2] - #1]*#1 + 2*a*Sqrt[c]*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x
+ c*x^2] - #1]*#1 - 2*a*Sqrt[c]*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2
] - #1]*#1 + b*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 - a*
e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(2*b*Sqrt[c]*d - a
*Sqrt[c]*e - 4*c*d*#1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*#1^2 - 2*f*#1^3) &
]/d^2
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.02, size = 6765, normalized size = 9.19

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/x^2/(f*x^2+e*x+d),x)
[Out] result too large to display
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + bx + a}}{\left(fx^2 + ex + d\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 + e*x + d)*x^2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2 + b\,x + a}}{x^2\,\left(f\,x^2 + e\,x + d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/(x^2*(d + e*x + f*x^2)),x)
[Out] int((a + b*x + c*x^2)^(1/2)/(x^2*(d + e*x + f*x^2)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/x**2/(f*x**2+e*x+d),x)
[Out] Timed out
```

## 3.114 $\int \frac{x^3}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx$

**Optimal**. Leaf size=545

$$-\frac{b\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f}-\frac{\left(2def-\left(e^2-df\right)\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cd}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}$$

**Rubi [A]** time = 3.72, antiderivative size = 545, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6728, 621, 206, 640, 1032, 724}

$$-\frac{b\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f}-\frac{\left(2def-\left(e^2-df\right)\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}+\frac{\left(2def-\left(e^2-df\right)\left(\sqrt{e^2-4df}+e\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}-\frac{e\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2}+\frac{\sqrt{a+bx+cx^2}}{cf}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] Sqrt[a + b*x + c*x^2]/(c*f) - (e*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(Sqrt[c]*f^2) - (b*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(2*c^(3/2)*f) - ((2*d*e*f - (e^2 - d*f)*(e - Sqrt[e^2 - 4*d*f
]))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 -
 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f
)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f
]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
+ ((2*d*e*f - (e^2 - d*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e +
Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt
[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a
+ b*x + c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e
*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx = \int \left(-\frac{e}{f^2\sqrt{a+bx+cx^2}} + \frac{x}{f\sqrt{a+bx+cx^2}} + \frac{de+\left(e^2-df\right)x}{f^2\sqrt{a+bx+cx^2}\left(d+ex+\right.}\right.$$

$$= \frac{\int \frac{de+\left(e^2-df\right)x}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx}{f^2} - \frac{e\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{f^2} + \frac{\int \frac{x}{\sqrt{a+bx+cx^2}}\,dx}{f}$$

$$= \frac{\sqrt{a+bx+cx^2}}{cf} - \frac{(2e)\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f^2} - \frac{b\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2cf}$$

$$= \frac{\sqrt{a+bx+cx^2}}{cf} - \frac{e\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\,f^2} - \frac{b\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2}{\sqrt{a+bx}}\right.}{cf}$$

$$= \frac{\sqrt{a+bx+cx^2}}{cf} - \frac{e\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\,f^2} - \frac{b\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f} - \ldots$$

**Mathematica [A]** time = 2.20, size = 550, normalized size = 1.01

$$-\frac{\frac{bf\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{c^{3/2}} + \frac{\sqrt{2}\left(-\frac{e\left(e^2-3df\right)}{\sqrt{e^2-4df}}-df+e^2\right)\tanh^{-1}\left(\frac{4af+b\left(\sqrt{e^2-4df}-e+2fx\right)+2cx\left(\sqrt{e^2-4df}-e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}} + \frac{\sqrt{2}\left(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3\right)\tanh^{-1}\left(\frac{4af-b\left(\sqrt{e^2-4df}+e-2fx\right)-2cx\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}} + \frac{2e\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}} - \frac{2f\sqrt{a+x(b+cx)}}{c}}{2f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -1/2*((-2*f*Sqrt[a + x*(b + c*x)])/c + (2*e*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + x*(b + c*x)])])/Sqrt[c] + (b*f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt
[a + x*(b + c*x)])])/c^(3/2) + (Sqrt[2]*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d
*f] - d*f*Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x
 - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sq
rt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c
*x)])])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(
2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]) + (Sqrt[2]*(e^2 - d*f - (e*(e^2 - 3*d*
f))/Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*
(-e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e
^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)
])])/Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e
^2 - 4*d*f]))])/f^2
```

**IntegrateAlgebraic [C]** time = 0.74, size = 430, normalized size = 0.79

$$-\frac{\text{RootSum}\left[\#1^4 f-2\#1^3\sqrt{c}\,e-2\#1^2 af+\#1^2 be+4\#1^2 cd+2\#1 a\sqrt{c}\,e-4\#1 b\sqrt{c}\,d+a^2 f-abe+b^2 d\&,\frac{-\#1^2 df\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)+\#1^2 e^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)+bde\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)-2\#1\sqrt{c}\,de\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)+adf\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)-ae^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)}{-2\#1^3 f+3\#1^2\sqrt{c}\,e+2\#1 af-\#1 be-4\#1 cd-a\sqrt{c}\,e+2b\sqrt{c}\,d}\&\right]}{f^2}+\frac{(bf+2ce)\log\left(-2c^{3/2}\sqrt{a+bx+cx^2}+bc+2c^2x\right)}{2c^{3/2}f^2}+\frac{\sqrt{a+bx+cx^2}}{cf}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] Sqrt[a + b*x + c*x^2]/(c*f) + ((2*c*e + b*f)*Log[b*c + 2*c^2*x - 2*c^(3/2)*
Sqrt[a + b*x + c*x^2]])/(2*c^(3/2)*f^2) - RootSum[b^2*d - a*b*e + a^2*f - 4
*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#1^2 - 2
*Sqrt[c]*e*#1^3 + f*#1^4 & , (b*d*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2
] - #1] - a*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a*d*f*Log[
-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*Sqrt[c]*d*e*Log[-(Sqrt[c]*x)
 + Sqrt[a + b*x + c*x^2] - #1]*#1 + e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c
*x^2] - #1]*#1^2 - d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)
/(2*b*Sqrt[c]*d - a*Sqrt[c]*e - 4*c*d*#1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*
#1^2 - 2*f*#1^3) & ]/f^2
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

**maple [B]** time = 0.03, size = 3131, normalized size = 5.74

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] (c*x^2+b*x+a)^(1/2)/c/f-1/2*b/c^(3/2)/f*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a
)^(1/2))-1/f^2*e*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+1/2/f^
2*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*
c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+
e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^
2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*
f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*d-1/2/f^3*2^
(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*
f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e
+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)
^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(
1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*
c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*e^2+3/2/f^2/(-4*
d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b
*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*
b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*
(e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))
*d*e-1/2/f^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1
/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4
*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-
4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^
2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)
+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d
*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*
f+e^2)^(1/2))/f))*e^3+1/2/f^2*2^(1/2)/(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)
^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*
b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)
^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e
^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2
)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)
/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)
^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)
^(1/2))/f))*d-1/2/f^3*2^(1/2)/(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*b*f-(-4*
d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b
```

```
*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2
)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-
1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/
f))*e^2-3/2/f^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f
+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(
1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f
+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*
d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)
^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f
-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f
+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f
+e^2)^(1/2))/f))*d*e+1/2/f^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2
)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((
-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f
^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2
^(1/2)*(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*
f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e
^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2
)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f))*e^3
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(x^3/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}\,\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x**3/(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

## 3.115 $\int \frac{x^2}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx$

**Optimal**. Leaf size=463

$$-\frac{\left(-e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}-\frac{\left(2df-e\left(\sqrt{e^2-4df}+e\right)\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{}}$$

**Rubi** [A] time = 3.44, antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1077, 621, 206, 1032, 724}

$$-\frac{\left(-e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}-\frac{\left(2df-e\left(\sqrt{e^2-4df}+e\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}+\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]/(Sqrt[c]*f) - ((e^2
- 2*d*f - e*Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) +
 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b
*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(S
qrt[2]*f*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e -
b*f)*Sqrt[e^2 - 4*d*f]]) - ((2*d*f - e*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*
a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2
*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*
d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*
c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1077

```
Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C - b*C*x)/((a + b*x + c*x^2)*Sqrt[d
 + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx = \frac{\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{f} + \frac{\int \frac{-d-ex}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx}{f}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f} + \frac{\left(e^2-2df-e\sqrt{e^2-4df}\right)\int \frac{}{\left(e-\sqrt{e^2-4d\ldots}\right.}}{f\sqrt{e^2-4df}}$$

$$= \frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\,f} - \frac{\left(2\left(e^2-2df-e\sqrt{e^2-4df}\right)\right)\text{Subst}\left(\int \frac{}{16af^2-8bf\ldots}\right.}{}$$

$$= \frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\,f} - \frac{\left(e^2-2df-e\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{4af-b\left(e-\ldots\right.}{2\sqrt{2}\,\sqrt{ce^2-2cdf-\ldots}}\right.}{\sqrt{2}\,f\sqrt{e^2-4df}\,\sqrt{ce^2-2cdf-bef+2a\ldots}}$$

**Mathematica [A]** time = 0.95, size = 468, normalized size = 1.01

$$\frac{\frac{\sqrt{2}\left(e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{4af-b\left(\sqrt{e^2-4df}+e-2fx\right)-2cx\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\sqrt{2}\left(\frac{2df-e^2}{\sqrt{e^2-4df}}+e\right)\tanh^{-1}\left(\frac{4af+b\left(\sqrt{e^2-4df}-e+2fx\right)+2cx\left(\sqrt{e^2-4df}-e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ((2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + (Sqrt
[2]*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2
- 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2
*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a
 + x*(b + c*x)])])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*
d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]) + (Sqrt[2]*(e + (-e^2 + 2*d
*f)/Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*
(-e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e
^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)
])])/Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e
^2 - 4*d*f]))])/(2*f)
```

**IntegrateAlgebraic [C]** time = 0.53, size = 324, normalized size = 0.70

$$\frac{\text{RootSum}\left[\#1^4 f-2\#1^3\sqrt{c}e-2\#1^2af+\#1^2be+4\#1^2cd+2\#1a\sqrt{c}e-4\#1b\sqrt{c}d+a^2f-abe+b^2d\&,\frac{\#1^2e\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2\#1\sqrt{c}d\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+bd\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-ae\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)}{-2\#1^3f+3\#1^2\sqrt{c}e+2\#1af-\#1be-4\#1cd-a\sqrt{c}e+2b\sqrt{c}d}\&\right]}{f}-\frac{\log\left(-2\sqrt{c}f\sqrt{a+bx+cx^2}+bf+2cfx\right)}{\sqrt{c}f}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -(Log[b*f + 2*c*f*x - 2*Sqrt[c]*f*Sqrt[a + b*x + c*x^2]]/(Sqrt[c]*f)) + Roo
tSum[b^2*d - a*b*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1
^2 + b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (b*d*Log[-(Sqrt[
c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x +
 c*x^2] - #1] - 2*Sqrt[c]*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*
#1 + e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(2*b*Sqrt[c]*d
- a*Sqrt[c]*e - 4*c*d*#1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*#1^2 - 2*f*#1^3)
 & ]/f
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

**maple [B]** time = 0.02, size = 2321, normalized size = 5.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] 1/c^(1/2)/f*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/2/f^2*2^(1/2)/((-
(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/
f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*
f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+
e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f
)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+
2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e
^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*e-1/(-4*d*f+e^2)^(1/2)/f*
2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*
d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2
*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*
(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2
)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^
2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)
^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-
2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*d+1/2/(-4*d*f+
e^2)^(1/2)/f^2*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b
*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*
b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*
(e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))
```

```
*e^2+1/2/f^2*2^(1/2)/(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^
2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f
*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*b*f-(-4
*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(
-4*d*f+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4
*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^
2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*e+1/(
-4*d*f+e^2)^(1/2)/f*2^(1/2)/(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*b*f-(-4*d*
f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f
-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*
b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*
(-e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)
)*d-1/2/(-4*d*f+e^2)^(1/2)/f^2*2^(1/2)/(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2
)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)
*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2
)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+
e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/
2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e
)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2
)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2
)^(1/2))/f))*e^2
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
[Out] int(x^2/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

**sympy [F]**     time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
[Out] Integral(x**2/(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

## 3.116 $\int \frac{x}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx$

**Optimal**. Leaf size=402

$$\frac{\left(e-\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}-\frac{\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{4af+2x(}{2\sqrt{2}\sqrt{a+bx+cx}}\right.}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4d}}}$$

**Rubi [A]** time = 0.96, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1032, 724, 206}

$$\frac{\left(e-\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}-\frac{\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ((e - Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*
f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f +
 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]
*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqr
t[e^2 - 4*d*f]]) - ((e + Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e + Sqrt[e^
2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2
- 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x +
 c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^
2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx = -\left(\left(-1-\frac{e}{\sqrt{e^2-4df}}\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx\right)+\left(1\ \cdots\right.$$

$$= -\left(\left(2\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\right)\text{Subst}\left(\int \frac{1}{16af^2-8bf\left(e-\sqrt{e^2-4df}\right)+4c\left(e-\cdots\right.}\right.\right.$$

$$= -\frac{\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\tanh^{-1}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}}-\ \cdots$$

**Mathematica [A]** time = 0.97, size = 407, normalized size = 1.01

$$\frac{-\frac{\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{4af-b\left(\sqrt{e^2-4df}+e-2fx\right)-2cx\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\tanh^{-1}\left(\frac{4af+b\left(\sqrt{e^2-4df}-e+2fx\right)+2cx\left(\sqrt{e^2-4df}-e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] (-(((e + Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x
- b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqr
t[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*
x)])])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2
*a*f - b*(e + Sqrt[e^2 - 4*d*f]))])) - ((1 - e/Sqrt[e^2 - 4*d*f])*ArcTanh[(
4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x)
)/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e
+ Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*(e^2 - 2*d*f - e*Sqr
t[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))])/Sqrt[2]
```

**IntegrateAlgebraic [C]** time = 0.43, size = 204, normalized size = 0.51

$$\text{RootSum}\left[\#1^4 f-2\#1^3\sqrt{c}e-2\#1^2 af+\#1^2 be+4\#1^2 cd+2\#1a\sqrt{c}e-4\#1b\sqrt{c}d+a^2 f-abe+b^2 d\&,\frac{\#1^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-a\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)}{2\#1^3 f-3\#1^2\sqrt{c}e-2\#1af+\#1be+4\#1cd+a\sqrt{c}e-2b\sqrt{c}d}\&\right]$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] RootSum[b^2*d - a*b*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d
*#1^2 + b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (-(a*Log[-(Sq
rt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]) + Log[-(Sqrt[c]*x) + Sqrt[a + b*x +
 c*x^2] - #1]*#1^2)/(-2*b*Sqrt[c]*d + a*Sqrt[c]*e + 4*c*d*#1 + b*e*#1 - 2*a
*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]
```

**fricas [B]** time = 14.50, size = 11311, normalized size = 28.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*sqrt((2*c*d^2 - b*d*e + a*e^2 - 2*a*d*f + (c^2*d^2*e^2 - b*c*d*
e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f
^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((b^2
*d^2 - 2*a*b*d*e + a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^
5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e
+ a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^
2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^
2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20
*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2
*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 +
6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*
e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a
^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b
^2 - 6*a*c)*d*e^2)*f))*log(-(2*b^2*d^3 - 4*a*b*d^2*e + 2*a^2*d*e^2 + sqrt(2
)*(b^2*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4 - 4*(b^2*d^3 - 2*a*b*d^2*e + a^2*d*e
^2)*f - (2*c^3*d^4*e^2 - 3*b*c^2*d^3*e^3 - 2*a*b*c*d*e^5 + a^2*c*e^6 + 8*a^
3*d^2*f^4 + (b^2*c + 3*a*c^2)*d^2*e^4 - 2*(2*a^2*b*d^2*e + 3*a^3*d*e^2 - 4*
(a*b^2 - 3*a^2*c)*d^3)*f^3 + (5*a^2*b*d*e^3 + a^3*e^4 - 8*(b^2*c - 3*a*c^2)
*d^4 + 4*(b^3 - 2*a*b*c)*d^3*e - 2*(5*a*b^2 - 11*a^2*c)*d^2*e^2)*f^2 - (8*c
^3*d^5 - 12*b*c^2*d^4*e + a^2*b*e^5 + 2*(b^2*c + 9*a*c^2)*d^3*e^2 + (b^3 -
10*a*b*c)*d^2*e^3 - 2*(a*b^2 - 4*a^2*c)*d*e^4)*f)*sqrt((b^2*d^2 - 2*a*b*d*e
 + a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6
- 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a
^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)
```

```
*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2
*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^
2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2
 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2
 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c
*x^2 + b*x + a)*sqrt((2*c*d^2 - b*d*e + a*e^2 - 2*a*d*f + (c^2*d^2*e^2 - b*
c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^
2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt(
(b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*
d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*
d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*
a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*
d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4
- 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2
 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^
2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2
)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e
 + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3
- (b^2 - 6*a*c)*d*e^2)*f)) + (4*b*c*d^3 + a*b*d*e^2 - (b^2 + 4*a*c)*d^2*e)*
x - (2*a*c^2*d^3*e^2 - 2*a*b*c*d^2*e^3 + 2*a^2*c*d*e^4 - 8*a^3*d^2*f^3 + 2*
(4*a^2*b*d^2*e + a^3*d*e^2 - 4*(a*b^2 - 2*a^2*c)*d^3)*f^2 - 2*(4*a*c^2*d^4
- 4*a*b*c*d^3*e + a^2*b*d*e^3 - (a*b^2 - 6*a^2*c)*d^2*e^2)*f + (b*c^2*d^3*e
^2 - b^2*c*d^2*e^3 + a*b*c*d*e^4 - 4*a^2*b*d^2*f^3 + (4*a*b^2*d^2*e + a^2*b
*d*e^2 - 4*(b^3 - 2*a*b*c)*d^3)*f^2 - (4*b*c^2*d^4 - 4*b^2*c*d^3*e + a*b^2*
d*e^3 - (b^3 - 6*a*b*c)*d^2*e^2)*f)*x)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)
/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f
^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*
a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a
*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*
c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*
e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4
*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c -
 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x) - 1/4*sqrt(2)*
sqrt((2*c*d^2 - b*d*e + a*e^2 - 2*a*d*f + (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^
4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*
d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((b^2*d^2 - 2*a*b
*d*e + a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*
e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 -
8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*
c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*
(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 2
2*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)
*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3
*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c
^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(
```

b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*
d*e^2)*f))*log(-(2*b^2*d^3 - 4*a*b*d^2*e + 2*a^2*d*e^2 - sqrt(2)*(b^2*d^2*e
^2 - 2*a*b*d*e^3 + a^2*e^4 - 4*(b^2*d^3 - 2*a*b*d^2*e + a^2*d*e^2)*f - (2*c
^3*d^4*e^2 - 3*b*c^2*d^3*e^3 - 2*a*b*c*d*e^5 + a^2*c*e^6 + 8*a^3*d^2*f^4 +
(b^2*c + 3*a*c^2)*d^2*e^4 - 2*(2*a^2*b*d^2*e + 3*a^3*d*e^2 - 4*(a*b^2 - 3*a
^2*c)*d^3)*f^3 + (5*a^2*b*d*e^3 + a^3*e^4 - 8*(b^2*c - 3*a*c^2)*d^4 + 4*(b^
3 - 2*a*b*c)*d^3*e - 2*(5*a*b^2 - 11*a^2*c)*d^2*e^2)*f^2 - (8*c^3*d^5 - 12*
b*c^2*d^4*e + a^2*b*e^5 + 2*(b^2*c + 9*a*c^2)*d^3*e^2 + (b^3 - 10*a*b*c)*d^
2*e^3 - 2*(a*b^2 - 4*a^2*c)*d*e^4)*f)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/
(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^
5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a
^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*
b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c
^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e
^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*
d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c -
5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*x^2 + b*x +
a)*sqrt((2*c*d^2 - b*d*e + a*e^2 - 2*a*d*f + (c^2*d^2*e^2 - b*c*d*e^3 + a*
c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*
c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((b^2*d^2 - 2
*a*b*d*e + a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*
c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^
2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*
a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 -
(8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c
+ 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*
e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)
*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f))
)/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 -
4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a
*c)*d*e^2)*f)) + (4*b*c*d^3 + a*b*d*e^2 - (b^2 + 4*a*c)*d^2*e)*x - (2*a*c^2
*d^3*e^2 - 2*a*b*c*d^2*e^3 + 2*a^2*c*d*e^4 - 8*a^3*d^2*f^3 + 2*(4*a^2*b*d^2
*e + a^3*d*e^2 - 4*(a*b^2 - 2*a^2*c)*d^3)*f^2 - 2*(4*a*c^2*d^4 - 4*a*b*c*d^
3*e + a^2*b*d*e^3 - (a*b^2 - 6*a^2*c)*d^2*e^2)*f + (b*c^2*d^3*e^2 - b^2*c*d
^2*e^3 + a*b*c*d*e^4 - 4*a^2*b*d^2*f^3 + (4*a*b^2*d^2*e + a^2*b*d*e^2 - 4*(
b^3 - 2*a*b*c)*d^3)*f^2 - (4*b*c^2*d^4 - 4*b^2*c*d^3*e + a*b^2*d*e^3 - (b^3
- 6*a*b*c)*d^2*e^2)*f)*x)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(c^4*d^4*e^
2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^
2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*
f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b
*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8
*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b
^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c
^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*
d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x) + 1/4*sqrt(2)*sqrt((2*c*d^

2 - b*d*e + a*e^2 - 2*a*d*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*
f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*
d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e
^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*
d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 -
 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4
*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2
*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d
^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*
c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*
c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 -
 b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)
*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f))*l
og(-(2*b^2*d^3 - 4*a*b*d^2*e + 2*a^2*d*e^2 + sqrt(2)*(b^2*d^2*e^2 - 2*a*b*d
*e^3 + a^2*e^4 - 4*(b^2*d^3 - 2*a*b*d^2*e + a^2*d*e^2)*f + (2*c^3*d^4*e^2 -
 3*b*c^2*d^3*e^3 - 2*a*b*c*d*e^5 + a^2*c*e^6 + 8*a^3*d^2*f^4 + (b^2*c + 3*a
*c^2)*d^2*e^4 - 2*(2*a^2*b*d^2*e + 3*a^3*d*e^2 - 4*(a*b^2 - 3*a^2*c)*d^3)*f
^3 + (5*a^2*b*d*e^3 + a^3*e^4 - 8*(b^2*c - 3*a*c^2)*d^4 + 4*(b^3 - 2*a*b*c)
*d^3*e - 2*(5*a*b^2 - 11*a^2*c)*d^2*e^2)*f^2 - (8*c^3*d^5 - 12*b*c^2*d^4*e
+ a^2*b*e^5 + 2*(b^2*c + 9*a*c^2)*d^3*e^2 + (b^3 - 10*a*b*c)*d^2*e^3 - 2*(a
*b^2 - 4*a^2*c)*d*e^4)*f)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(c^4*d^4*e^2
 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2
 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f
^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*
c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*
(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^
3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^
3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d
^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*x^2 + b*x + a)*sqrt((2*
c*d^2 - b*d*e + a*e^2 - 2*a*d*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^
2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*
b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((b^2*d^2 - 2*a*b*d*e + a
^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*
a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b
^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3
 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2
 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^
2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2
*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (
b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e
^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*
a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f
)) + (4*b*c*d^3 + a*b*d*e^2 - (b^2 + 4*a*c)*d^2*e)*x + (2*a*c^2*d^3*e^2 - 2
*a*b*c*d^2*e^3 + 2*a^2*c*d*e^4 - 8*a^3*d^2*f^3 + 2*(4*a^2*b*d^2*e + a^3*d*e
^2 - 4*(a*b^2 - 2*a^2*c)*d^3)*f^2 - 2*(4*a*c^2*d^4 - 4*a*b*c*d^3*e + a^2*b*

d*e^3 - (a*b^2 - 6*a^2*c)*d^2*e^2)*f + (b*c^2*d^3*e^2 - b^2*c*d^2*e^3 + a*b
*c*d*e^4 - 4*a^2*b*d^2*f^3 + (4*a*b^2*d^2*e + a^2*b*d*e^2 - 4*(b^3 - 2*a*b*
c)*d^3)*f^2 - (4*b*c^2*d^4 - 4*b^2*c*d^3*e + a*b^2*d*e^3 - (b^3 - 6*a*b*c)*
d^2*e^2)*f)*x)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*
d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)
*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3
*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e +
(a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*
b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b
*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a
^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*
(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x) - 1/4*sqrt(2)*sqrt((2*c*d^2 - b*d*e +
a*e^2 - 2*a*d*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b
*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*
e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(c^4*d^4
*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2
*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^
2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^
2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4
- 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(
a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*
b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^
2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 +
a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 -
(4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f))*log(-(2*b^2*d
^3 - 4*a*b*d^2*e + 2*a^2*d*e^2 - sqrt(2)*(b^2*d^2*e^2 - 2*a*b*d*e^3 + a^2*e
^4 - 4*(b^2*d^3 - 2*a*b*d^2*e + a^2*d*e^2)*f + (2*c^3*d^4*e^2 - 3*b*c^2*d^3
*e^3 - 2*a*b*c*d*e^5 + a^2*c*e^6 + 8*a^3*d^2*f^4 + (b^2*c + 3*a*c^2)*d^2*e^
4 - 2*(2*a^2*b*d^2*e + 3*a^3*d*e^2 - 4*(a*b^2 - 3*a^2*c)*d^3)*f^3 + (5*a^2*
b*d*e^3 + a^3*e^4 - 8*(b^2*c - 3*a*c^2)*d^4 + 4*(b^3 - 2*a*b*c)*d^3*e - 2*(
5*a*b^2 - 11*a^2*c)*d^2*e^2)*f^2 - (8*c^3*d^5 - 12*b*c^2*d^4*e + a^2*b*e^5
+ 2*(b^2*c + 9*a*c^2)*d^3*e^2 + (b^3 - 10*a*b*c)*d^2*e^3 - 2*(a*b^2 - 4*a^2
*c)*d*e^4)*f)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d
^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*
d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*
b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (
a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b
*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*
c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^
2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(
a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*x^2 + b*x + a)*sqrt((2*c*d^2 - b*d*
e + a*e^2 - 2*a*d*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4
*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e +
a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(c^4
*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 +

```
(b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c
)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3
- a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*
d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 +
 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5
- 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*
b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e
^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^
2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)) + (4*b*c*
d^3 + a*b*d*e^2 - (b^2 + 4*a*c)*d^2*e)*x + (2*a*c^2*d^3*e^2 - 2*a*b*c*d^2*e
^3 + 2*a^2*c*d*e^4 - 8*a^3*d^2*f^3 + 2*(4*a^2*b*d^2*e + a^3*d*e^2 - 4*(a*b^
2 - 2*a^2*c)*d^3)*f^2 - 2*(4*a*c^2*d^4 - 4*a*b*c*d^3*e + a^2*b*d*e^3 - (a*b
^2 - 6*a^2*c)*d^2*e^2)*f + (b*c^2*d^3*e^2 - b^2*c*d^2*e^3 + a*b*c*d*e^4 - 4
*a^2*b*d^2*f^3 + (4*a*b^2*d^2*e + a^2*b*d*e^2 - 4*(b^3 - 2*a*b*c)*d^3)*f^2
- (4*b*c^2*d^4 - 4*b^2*c*d^3*e + a*b^2*d*e^3 - (b^3 - 6*a*b*c)*d^2*e^2)*f)*
x)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*
a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (
8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(
b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6
*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e
 - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 -
(a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 +
 (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2
*a^2*c^2)*d*e^4)*f)))/x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.01, size = 1516, normalized size = 3.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] -1/2/(-4*d*f+e^2)^(1/2)/f*2^(1/2)/((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2
)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln(((b*f-c*e-(-4*d*f+e^2)^(1
/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*
d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-b*e*f-
```

```
2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*
(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(
1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/
f))*e-1/2/f*2^(1/2)/((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-
4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln(((b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*
(e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)
*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2-
(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*
d*f+e^2)^(1/2))/f)^2*c+4*(b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e
^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d
*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))-1/2/f*2^(1
/2)/((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)
*c*e)/f^2)^(1/2)*ln(((b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)
^(1/2))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^
2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1
/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2)
)/f)^2*c+4*(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)
/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)
*c*e)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))+1/2/(-4*d*f+e^2)^(1/2)
/f*2^(1/2)/((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2
)^(1/2)*c*e)/f^2)^(1/2)*ln(((b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d
*f+e^2)^(1/2))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4
*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+
e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2
)^(1/2))/f)^2*c+4*(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1
/2))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2
)^(1/2)*c*e)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*e
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(x/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a + bx + cx^2}\left(d + ex + fx^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x/(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

## 3.117 $\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx$

**Optimal**. Leaf size=374

$$\frac{\sqrt{2}\,f\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\,\sqrt{a+bx+cx^2}\,\sqrt{2af^2+\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\,\sqrt{2af^2+\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}-\frac{\sqrt{2}\,f\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-\right.}}{2\sqrt{2}\,\sqrt{a+bx+cx^2}\,\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-be}}\right.}{\sqrt{e^2-4df}\,\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-}}$$

**Rubi** [A] time = 0.31, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {983, 724, 206}

$$\frac{\sqrt{2}\,f\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\,\sqrt{a+bx+cx^2}\,\sqrt{2af^2+\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\,\sqrt{2af^2+\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}-\frac{\sqrt{2}\,f\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\,\sqrt{a+bx+cx^2}\,\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\,\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -((Sqrt[2]*f*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - S
qrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (
c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[e^2 - 4*d*f]*S
qrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])) +
(Sqrt[2]*f*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqr
t[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*
e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqr
t[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 983

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[1/((
b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c)/q, Int[1/((b + q
 + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e - b*f, 0] && PosQ[b
^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx = \frac{(2f)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx}{\sqrt{e^2-4df}} - \frac{(2f)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx}{\sqrt{e^2-4df}}$$

$$= -\frac{(4f)\,\text{Subst}\left(\int \frac{1}{16af^2-8bf\left(e-\sqrt{e^2-4df}\right)+4c\left(e-\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{4af-b\left(e-\sqrt{e^2-4df}\right.}{}\right.}{\sqrt{e^2-4df}}$$

$$= -\frac{\sqrt{2}\,f\tanh^{-1}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}} + \frac{\sqrt{2}\,f\tan}{\sqrt{e^2-4d}}$$

**Mathematica [A]** time = 0.79, size = 376, normalized size = 1.01

$$\frac{\sqrt{2}f\left(\frac{\tanh^{-1}\left(\frac{4af-b\left(\sqrt{e^2-4df}+e-2fx\right)-2cx\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\tanh^{-1}\left(\frac{4af+b\left(\sqrt{e^2-4df}-e+2fx\right)+2cx\left(\sqrt{e^2-4df}-e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af+b\sqrt{e^2-4df}+b(-e)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{f\left(2af+b\sqrt{e^2-4df}+b(-e)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] (Sqrt[2]*f*(ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^
2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])
+ f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])]/Sqrt[c*(e^
2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))] -
 ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f
] + 2*f*x))/(2*Sqrt[2]*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e
```

```
^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + x*(b + c*x)])]/Sqrt[f*(-(b*e) +
 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]))/Sq
rt[e^2 - 4*d*f]
```

**IntegrateAlgebraic [C]** time = 0.00, size = 211, normalized size = 0.56

$$-\text{RootSum}\left[\#1^4 f - 2\#1^3\sqrt{c}\,e - 2\#1^2 af + \#1^2 be + 4\#1^2 cd + 2\#1 a\sqrt{c}\,e - 4\#1 b\sqrt{c}\,d + a^2 f - abe + b^2 d\&, \frac{b\log\left(-\#1 + \sqrt{a + bx + cx^2} - \sqrt{c}\,x\right) - 2\#1\sqrt{c}\log\left(-\#1 + \sqrt{a + bx + cx^2} - \sqrt{c}\,x\right)}{-2\#1^3 f + 3\#1^2\sqrt{c}\,e + 2\#1 af - \#1 be - 4\#1 cd - a\sqrt{c}\,e + 2b\sqrt{c}\,d}\&\right]$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -RootSum[b^2*d - a*b*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*
d*#1^2 + b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (b*Log[-(Sqr
t[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*Sqrt[c]*Log[-(Sqrt[c]*x) + Sqrt[a
 + b*x + c*x^2] - #1]*#1)/(2*b*Sqrt[c]*d - a*Sqrt[c]*e - 4*c*d*#1 - b*e*#1
+ 2*a*f*#1 + 3*Sqrt[c]*e*#1^2 - 2*f*#1^3) & ]
```

**fricas [B]** time = 17.21, size = 11287, normalized size = 30.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f + (c^2*d^2*e^2 - b*c*d*
e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f
^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2
*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^
5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e
+ a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^
2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^
2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20
*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2
*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 +
6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*
e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a
^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b
^2 - 6*a*c)*d*e^2)*f))*log((2*(b^2*d - a*b*e)*f^2 + sqrt(2)*(c^2*d*e^3 - 4*
a*b*d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*b*e^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^
2 + a*c*e^3)*f - (c^3*d^3*e^3 - b*c^2*d^2*e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^
2 - a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3*e^3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*
a*b^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^4 - a^2*b*e^4 - 4*(3*b^2*c - a*c^2)*
d^3*e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3*a*b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^
3*d^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e^4 - a^2*c*e^5 - (3*b^2*c - 5*a*c^2)
*d^2*e^3)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^
```

3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d
^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b
*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a
^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*
c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c
)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2
*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a
*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*x^2 + b*x + a)*sqrt((c*e^2 + 2*a*f^2
 - (2*c*d + b*e)*f + (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*
a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a
*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*
d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (
b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)
*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 -
 a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d
^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 +
2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 -
 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b
*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^
3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2
 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)) - 2*(b*c*d
*e - a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^2 - (4*c^2*d*e - b*c*e^2)*f)*x - (8*
a^3*d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 - 4*(a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*
a*c^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 - (a*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a
*c^2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)*f + (4*a^2*b*d*f^4 - (4*a*b^2*d*e +
 a^2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3 + (4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*
b^2*e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 - (b*c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c
*e^4)*f)*x)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3
*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^
2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*
e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^
2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c
^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)
*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*
b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*
b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x) - 1/4*sqrt(2)*sqrt((c*e^2 + 2*a*f^2 - (2*
c*d + b*e)*f + (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*
e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3
 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^
2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^
2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*
f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b
*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8
*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b
^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c

```
^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*
d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*
c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*
c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f))*log((2*(b^2*d -
a*b*e)*f^2 - sqrt(2)*(c^2*d*e^3 - 4*a*b*d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*
b*e^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^2 + a*c*e^3)*f - (c^3*d^3*e^3 - b*c^2*d
^2*e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^2 - a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3
*e^3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*a*b^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^
4 - a^2*b*e^4 - 4*(3*b^2*c - a*c^2)*d^3*e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3
*a*b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^3*d^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e
^4 - a^2*c*e^5 - (3*b^2*c - 5*a*c^2)*d^2*e^3)*f)*sqrt((c^2*e^2 - 2*b*c*e*f
+ b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 -
 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^
2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*
d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*
c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2
*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2
- 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2
+ (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*
x^2 + b*x + a)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f + (c^2*d^2*e^2 - b*c
*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2
)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((
c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d
*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d
*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a
*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d
*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 -
 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2
+ 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2
 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)
*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e
+ a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 -
 (b^2 - 6*a*c)*d*e^2)*f)) - 2*(b*c*d*e - a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^
2 - (4*c^2*d*e - b*c*e^2)*f)*x - (8*a^3*d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 -
4*(a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*a*c^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 -
 (a*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a*c^2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)
*f + (4*a^2*b*d*f^4 - (4*a*b^2*d*e + a^2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3
 + (4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*b^2*e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 -
(b*c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c*e^4)*f)*x)*sqrt((c^2*e^2 - 2*b*c*e*f +
 b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 -
4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2
*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d
^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c
^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*
c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 -
```

```
 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 +
 (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x) + 1/4
*sqrt(2)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f - (c^2*d^2*e^2 - b*c*d*e^3
 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2
- (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^
2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 +
 a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a
^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c
 + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*
f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*
b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^
3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a
*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4
)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*
e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2
- 6*a*c)*d*e^2)*f))*log((2*(b^2*d - a*b*e)*f^2 + sqrt(2)*(c^2*d*e^3 - 4*a*b
*d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*b*e^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^2 +
 a*c*e^3)*f + (c^3*d^3*e^3 - b*c^2*d^2*e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^2 -
 a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3*e^3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*a*b
^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^4 - a^2*b*e^4 - 4*(3*b^2*c - a*c^2)*d^3
*e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3*a*b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^3*d
^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e^4 - a^2*c*e^5 - (3*b^2*c - 5*a*c^2)*d^
2*e^3)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e
^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*
e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^
3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*
b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2
)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d
*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*
c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^
2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*x^2 + b*x + a)*sqrt((c*e^2 + 2*a*f^2 -
(2*c*d + b*e)*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b
*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*
e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4
*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2
*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^
2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^
2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4
- 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(
a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*
b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^
2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 +
 a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 -
(4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)) - 2*(b*c*d*e
- a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^2 - (4*c^2*d*e - b*c*e^2)*f)*x + (8*a^3
```

```
*d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 - 4*(a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*a*c
^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 - (a*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a*c^
2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)*f + (4*a^2*b*d*f^4 - (4*a*b^2*d*e + a^
2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3 + (4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*b^2
*e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 - (b*c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c*e^
4)*f)*x)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^
3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e
^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3
 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b
^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)
*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*
e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c
*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2
*c - 2*a^2*c^2)*d*e^4)*f)))/x) - 1/4*sqrt(2)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d
 + b*e)*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e +
 a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 -
(b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 -
 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 +
 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4
 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)
*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b
^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3
- 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*
d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2
*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e
^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2
*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f))*log((2*(b^2*d - a*b
*e)*f^2 - sqrt(2)*(c^2*d*e^3 - 4*a*b*d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*b*e
^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^2 + a*c*e^3)*f + (c^3*d^3*e^3 - b*c^2*d^2*
e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^2 - a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3*e^
3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*a*b^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^4 -
 a^2*b*e^4 - 4*(3*b^2*c - a*c^2)*d^3*e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3*a*
b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^3*d^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e^4
- a^2*c*e^5 - (3*b^2*c - 5*a*c^2)*d^2*e^3)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b
^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*
a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b
^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3
 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2
 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^
2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2
*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (
b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*x^2
 + b*x + a)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f - (c^2*d^2*e^2 - b*c*d*
e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f
^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2
```

```
*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^
5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e
+ a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^
2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^
2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20
*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2
*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 +
6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*
e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a
^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b
^2 - 6*a*c)*d*e^2)*f)) - 2*(b*c*d*e - a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^2 -
 (4*c^2*d*e - b*c*e^2)*f)*x + (8*a^3*d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 - 4*(
a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*a*c^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 - (a
*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a*c^2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)*f
+ (4*a^2*b*d*f^4 - (4*a*b^2*d*e + a^2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3 +
(4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*b^2*e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 - (b*
c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c*e^4)*f)*x)*sqrt((c^2*e^2 - 2*b*c*e*f + b^
2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a
^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^
2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3
- 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2
- 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2
)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*
(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b
^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.01, size = 761, normalized size = 2.03

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] 1/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/
2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln(((b*f-c*e-(-4*d*f+e^2)^(1/2)*c
```

```
)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e
^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-b*e*f-2*c*d
*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+
(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*
b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))-1
/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2
)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln(((b*f-c*e+(-4*d*f+e^2)^(1/2)*c)
*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e
^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-b*e*f-2*c*d
*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-
e+(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2
)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)
)
```

**maxima** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

**mupad** [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

**sympy** [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
[Out] Integral(1/(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

## 3.118 $\int \frac{1}{x\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx$

**Optimal**. Leaf size=451

$$\frac{f\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\,d\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}-\frac{f\left(e-\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{4af\cdots}{2\sqrt{2}\sqrt{a+b\cdots}}\right)}{\sqrt{2}\,d\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-\cdots}}}$$

**Rubi** [A] time = 2.63, antiderivative size = 451, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6728, 724, 206, 1032}

$$\frac{f\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\,d\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}-\frac{f\left(e-\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\,d\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -(ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]/(Sqrt[a]*d)) + (f*
(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f
 - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f +
2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*
d*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sq
rt[e^2 - 4*d*f]]) - (f*(e - Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e + Sqrt
[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e
^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*
x + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2
*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx = \int \left(\frac{1}{dx\sqrt{a+bx+cx^2}} + \frac{-e-fx}{d\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\right)dx$$

$$= \frac{\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{d} + \frac{\int \frac{-e-fx}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx}{d}$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d} - \frac{\left(f\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2f\cdots\right.}}{d}$$

$$= -\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\,d} + \frac{\left(2f\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\right)\text{Subst}\left(\int \frac{\cdots}{16af^2-8bf\left(e+\sqrt{e^2-\cdots}\right.}\right.}{\cdots}$$

$$= -\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\,d} + \frac{f\left(1+\frac{e}{\sqrt{e^2-4df}}\right)\tanh^{-1}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\cdots\right.}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2a\cdots}}\right.}{\sqrt{2}\,d\sqrt{ce^2-2cdf-bef+2af^2-(c\cdots}}$$

**Mathematica [A]** time = 2.41, size = 450, normalized size = 1.00

$$\frac{\frac{\sqrt{2}f\left(\frac{\left(\sqrt{e^2-4df}-e\right)\tanh^{-1}\left(\frac{4af-b\left(\sqrt{e^2-4df}+e-2fx\right)-2cx\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{4af+b\left(\sqrt{e^2-4df}-e+2fx\right)+2cx\left(\sqrt{e^2-4df}-e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af+b\sqrt{e^2-4df}+b(-e)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{f\left(2af+b\sqrt{e^2-4df}+b(-e)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}}-\frac{2\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{\sqrt{a}}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ((-2*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/Sqrt[a] + (Sqr
t[2]*f*(((-e + Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*
f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f +
 e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(
b + c*x)])])/Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e +
 Sqrt[e^2 - 4*d*f]))] + ((e + Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f + 2*c*(-e +
 Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt
[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4
*d*f])]*Sqrt[a + x*(b + c*x)])])/Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*
f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])])))/Sqrt[e^2 - 4*d*f])/(2*d)
```

**IntegrateAlgebraic [C]** time = 0.54, size = 326, normalized size = 0.72

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}-\frac{\sqrt{a+bx+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d}-\frac{\text{RootSum}\left[\#1^4 f-2\#1^3\sqrt{c}\,e-2\#1^2 af+\#1^2 be+4\#1^2 cd+2\#1 a\sqrt{c}\,e-4\#1 b\sqrt{c}\,d+a^2 f-abe+b^2 d\&,\frac{\#1^2 f\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)-2\#1\sqrt{c}\,e\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)+be\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)-af\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)}{2\#1^3 f-3\#1^2\sqrt{c}\,e-2\#1 af+\#1 be+4\#1 cd+a\sqrt{c}\,e-2b\sqrt{c}\,d}\&\right]}{d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] (2*ArcTanh[(Sqrt[c]*x)/Sqrt[a] - Sqrt[a + b*x + c*x^2]/Sqrt[a]])/(Sqrt[a]*d
) - RootSum[b^2*d - a*b*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4
*c*d*#1^2 + b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (b*e*Log[
-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*f*Log[-(Sqrt[c]*x) + Sqrt[a
+ b*x + c*x^2] - #1] - 2*Sqrt[c]*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2]
 - #1]*#1 + f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(-2*b*Sq
rt[c]*d + a*Sqrt[c]*e + 4*c*d*#1 + b*e*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2
*f*#1^3) & ]/d
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.02, size = 859, normalized size = 1.90

$$-\frac{2\sqrt{2}\, f \ln\left(\frac{\frac{\left(bf-ce-\sqrt{-4df+e^2}\,c\right)\left(x+\frac{e+\sqrt{-4df+e^2}}{2f}\right)}{f}+\frac{2af^2-bef-2cdf+ce^2-\sqrt{-4df+e^2}\,bf+\sqrt{-4df+e^2}\,ce}{f^2}+\frac{\sqrt{2}\sqrt{\frac{2af^2-bef-2cdf+ce^2-\sqrt{-4df+e^2}\,bf+\sqrt{-4df+e^2}\,ce}{f^2}}\sqrt{4\left(x+\frac{e+\sqrt{-4df+e^2}}{2f}\right)^2 c+\frac{4\left(bf-ce-\sqrt{-4df+e^2}\,c\right)\left(x+\frac{e+\sqrt{-4df+e^2}}{2f}\right)}{f}+\frac{4af^2-2bef-4cdf+2ce^2-2\sqrt{-4df+e^2}\,bf+2\sqrt{-4df+e^2}\,ce}{f^2}}}{2}}{x+\frac{e+\sqrt{-4df+e^2}}{2f}}\right)}{\left(e+\sqrt{-4df+e^2}\right)\sqrt{-4df+e^2}\,\sqrt{\frac{2af^2-bef-2cdf+ce^2-\sqrt{-4df+e^2}\,bf+\sqrt{-4df+e^2}\,ce}{f^2}}}-\frac{2\sqrt{2}\, f \ln\left(\frac{\frac{\left(bf-ce+\sqrt{-4df+e^2}\,c\right)\left(x-\frac{-e+\sqrt{-4df+e^2}}{2f}\right)}{f}+\frac{2af^2-bef-2cdf+ce^2+\sqrt{-4df+e^2}\,bf-\sqrt{-4df+e^2}\,ce}{f^2}+\frac{\sqrt{2}\sqrt{\frac{2af^2-bef-2cdf+ce^2+\sqrt{-4df+e^2}\,bf-\sqrt{-4df+e^2}\,ce}{f^2}}\sqrt{4\left(x-\frac{-e+\sqrt{-4df+e^2}}{2f}\right)^2 c+\frac{4\left(bf-ce+\sqrt{-4df+e^2}\,c\right)\left(x-\frac{-e+\sqrt{-4df+e^2}}{2f}\right)}{f}+\frac{4af^2-2bef-4cdf+2ce^2+2\sqrt{-4df+e^2}\,bf-2\sqrt{-4df+e^2}\,ce}{f^2}}}{2}}{x-\frac{-e+\sqrt{-4df+e^2}}{2f}}\right)}{\left(-e+\sqrt{-4df+e^2}\right)\sqrt{-4df+e^2}\,\sqrt{\frac{2af^2-bef-2cdf+ce^2+\sqrt{-4df+e^2}\,bf-\sqrt{-4df+e^2}\,ce}{f^2}}}+\frac{4f\ln\left(\frac{bx+2a+2\sqrt{cx^2+bx+a}\,\sqrt{a}}{x}\right)}{\left(-e+\sqrt{-4df+e^2}\right)\left(e+\sqrt{-4df+e^2}\right)\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] -2*f/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-b*e*f-2*c*
d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln(((b*
f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*f^2-b*e
*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(
1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2
)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b*f-c*e-(-4*d*
f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f
+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+1/2*(e
+(-4*d*f+e^2)^(1/2))/f))-2*f/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*2^(
1/2)/((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2
)*c*e)/f^2)^(1/2)*ln(((b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2
)^(1/2))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e
^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(
1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2
))/f)^2*c+4*(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f
)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2
)*c*e)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))+4*f/(-e+(-4*d*f+e^2)^
(1/2))/(e+(-4*d*f+e^2)^(1/2))/a^(1/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(
1/2))/x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+bx+a}\,\left(fx^2+ex+d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*x), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\,\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(1/(x*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

## 3.119 $\int \frac{1}{x^2 \sqrt{a+bx+cx^2} \left(d+ex+fx^2\right)} \, dx$

**Optimal**. Leaf size=543

$$\frac{b \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d} - \frac{f\left(e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}} + - f$$

**Rubi [A]** time = 4.59, antiderivative size = 543, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6728, 730, 724, 206, 1032}

$$\frac{b \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d} - \frac{f\left(e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}} + \frac{f\left(-e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} + \frac{e \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d^2} - \frac{\sqrt{a+bx+cx^2}}{adx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -(Sqrt[a + b*x + c*x^2]/(a*d*x)) + (b*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a
 + b*x + c*x^2])])/(2*a^(3/2)*d) + (e*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a
 + b*x + c*x^2])])/(Sqrt[a]*d^2) - (f*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*A
rcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*
f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqr
t[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqr
t[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*
f]))]) + (f*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e + Sqr
t[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*
e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b
*x + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f
+ 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+bx+cx^2}\,\left(d+ex+fx^2\right)}\,dx = \int\left(\frac{1}{dx^2\sqrt{a+bx+cx^2}} - \frac{e}{d^2x\sqrt{a+bx+cx^2}} + \frac{e^2-df+efx}{d^2\sqrt{a+bx+cx^2}\,\left(d+ex\right.}\right.$$

$$= \frac{\int \frac{e^2-df+efx}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx}{d^2} + \frac{\int \frac{1}{x^2\sqrt{a+bx+cx^2}}\,dx}{d} - \frac{e\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{d^2}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{adx} - \frac{b\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{2ad} + \frac{(2e)\,\mathrm{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+}{\sqrt{a+bx}}\right.}{d^2}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{adx} + \frac{e\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\,\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\,d^2} + \frac{b\,\mathrm{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, -\right.}{ad}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{adx} + \frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\,\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d} + \frac{e\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\,\sqrt{a+bx+cx^2}}\right.}{\sqrt{a}\,d^2}$$

**Mathematica [A]** time = 1.41, size = 533, normalized size = 0.98

$$-\frac{-\frac{bd\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\,\sqrt{a+x(b+cx)}}\right)}{a^{3/2}} + \frac{\sqrt{2}\,f\left(e\sqrt{e^2-4df}+2df-e^2\right)\tanh^{-1}\left(\frac{4af-b\left(\sqrt{e^2-4df}+e-2fx\right)-2cx\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\,\sqrt{a+x(b+cx)}\,\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\,\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}} + \frac{\sqrt{2}\,f\left(\frac{e^2-2df}{\sqrt{e^2-4df}}+e\right)\tanh^{-1}\left(\frac{4af+b\left(\sqrt{e^2-4df}-e+2fx\right)+2cx\left(\sqrt{e^2-4df}-e\right)}{2\sqrt{2}\,\sqrt{a+x(b+cx)}\,\sqrt{f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}} + \frac{2d\sqrt{a+x(b+cx)}}{ax} - \frac{2e\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\,\sqrt{a+x(b+cx)}}\right)}{\sqrt{a}}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -1/2*((2*d*Sqrt[a + x*(b + c*x)])/(a*x) - (b*d*ArcTanh[(2*a + b*x)/(2*Sqrt[
a]*Sqrt[a + x*(b + c*x)])])/a^(3/2) - (2*e*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*S
qrt[a + x*(b + c*x)])])/Sqrt[a] + (Sqrt[2]*f*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4
*d*f])*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4
*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(
2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/(Sqrt[e^2 - 4*
d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^
2 - 4*d*f]))]) + (Sqrt[2]*f*(e + (e^2 - 2*d*f)/Sqrt[e^2 - 4*d*f])*ArcTanh[(
4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x)
)/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e
+ Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*(e^2 - 2*d*f - e*Sqr
t[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))])/d^2
```

**IntegrateAlgebraic [C]** time = 0.77, size = 430, normalized size = 0.79

$$\frac{\text{RootSum}\left[\#1^4 f-2\#1^3\sqrt{c}e-2\#1^2af+\#1^2be+4\#1^2cd+2\#1a\sqrt{c}e-4\#1b\sqrt{c}d+a^2f-abe+b^2d\&,\frac{\#1^2ef\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-bdf\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+2\#1\sqrt{c}df\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+be^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2\#1\sqrt{c}e^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-aef\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)}{2\#1^3f-3\#1^2\sqrt{c}e-2\#1af+\#1be+4\#1cd+a\sqrt{c}e-2b\sqrt{c}d}\&\right]}{d^2}+\frac{(2ae+bd)\tanh^{-1}\left(\frac{\sqrt{a+bx+cx^2}-\sqrt{c}x}{\sqrt{a}}\right)}{a^{3/2}d^2}-\frac{\sqrt{a+bx+cx^2}}{adx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -(Sqrt[a + b*x + c*x^2]/(a*d*x)) + ((b*d + 2*a*e)*ArcTanh[(-(Sqrt[c]*x) + S
qrt[a + b*x + c*x^2])/Sqrt[a]])/(a^(3/2)*d^2) + RootSum[b^2*d - a*b*e + a^2
*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#1
^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (b*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x +
 c*x^2] - #1] - b*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*e*
f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*Sqrt[c]*e^2*Log[-(Sqrt
[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*Sqrt[c]*d*f*Log[-(Sqrt[c]*x) +
Sqrt[a + b*x + c*x^2] - #1]*#1 + e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^
2] - #1]*#1^2)/(-2*b*Sqrt[c]*d + a*Sqrt[c]*e + 4*c*d*#1 + b*e*#1 - 2*a*f*#1
 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]/d^2
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.02, size = 983, normalized size = 1.81

$$\frac{16ef^2\ln\left(\frac{bx+2a+2\sqrt{cx^2+bx+a}\,\sqrt{a}}{x}\right)}{\left(-e+\sqrt{-4df+e^2}\right)^2\left(e+\sqrt{-4df+e^2}\right)^2\sqrt{a}}+\frac{4\sqrt{2}\,f^2\ln\left(\frac{\frac{\left(bf-ce-\sqrt{-4df+e^2}\,c\right)\left(x+\frac{e+\sqrt{-4df+e^2}}{2f}\right)}{f}+\frac{2af^2-bef-2cdf+ce^2-\sqrt{-4df+e^2}\,bf+\sqrt{-4df+e^2}\,ce}{f^2}+\frac{\sqrt{2}\sqrt{\frac{2af^2-bef-2cdf+ce^2-\sqrt{-4df+e^2}\,bf+\sqrt{-4df+e^2}\,ce}{f^2}}\sqrt{4\left(x+\frac{e+\sqrt{-4df+e^2}}{2f}\right)^2c+\frac{4\left(bf-ce-\sqrt{-4df+e^2}\,c\right)\left(x+\frac{e+\sqrt{-4df+e^2}}{2f}\right)}{f}+\frac{4af^2-2bef-4cdf+2ce^2-2\sqrt{-4df+e^2}\,bf+2\sqrt{-4df+e^2}\,ce}{f^2}}}{2}}{x+\frac{e+\sqrt{-4df+e^2}}{2f}}\right)}{\left(e+\sqrt{-4df+e^2}\right)^2\sqrt{-4df+e^2}\,\sqrt{\frac{2af^2-bef-2cdf+ce^2-\sqrt{-4df+e^2}\,bf+\sqrt{-4df+e^2}\,ce}{f^2}}}-\frac{4\sqrt{2}\,f^2\ln\left(\frac{\frac{\left(bf-ce+\sqrt{-4df+e^2}\,c\right)\left(x-\frac{-e+\sqrt{-4df+e^2}}{2f}\right)}{f}+\frac{2af^2-bef-2cdf+ce^2+\sqrt{-4df+e^2}\,bf-\sqrt{-4df+e^2}\,ce}{f^2}+\frac{\sqrt{2}\sqrt{\frac{2af^2-bef-2cdf+ce^2+\sqrt{-4df+e^2}\,bf-\sqrt{-4df+e^2}\,ce}{f^2}}\sqrt{4\left(x-\frac{-e+\sqrt{-4df+e^2}}{2f}\right)^2c+\frac{4\left(bf-ce+\sqrt{-4df+e^2}\,c\right)\left(x-\frac{-e+\sqrt{-4df+e^2}}{2f}\right)}{f}+\frac{4af^2-2bef-4cdf+2ce^2+2\sqrt{-4df+e^2}\,bf-2\sqrt{-4df+e^2}\,ce}{f^2}}}{2}}{x-\frac{-e+\sqrt{-4df+e^2}}{2f}}\right)}{\left(-e+\sqrt{-4df+e^2}\right)^2\sqrt{-4df+e^2}\,\sqrt{\frac{2af^2-bef-2cdf+ce^2+\sqrt{-4df+e^2}\,bf-\sqrt{-4df+e^2}\,ce}{f^2}}}-\frac{2bf\ln\left(\frac{bx+2a+2\sqrt{cx^2+bx+a}\,\sqrt{a}}{x}\right)}{\left(-e+\sqrt{-4df+e^2}\right)\left(e+\sqrt{-4df+e^2}\right)a^{\frac{3}{2}}}+\frac{4\sqrt{cx^2+bx+a}\,f}{\left(-e+\sqrt{-4df+e^2}\right)\left(e+\sqrt{-4df+e^2}\right)ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] 4*f^2/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-b*e*f-2
*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((
(b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*f^2-
b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*
2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(
1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b*f-c*e-(-4
*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*c*
d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+1/2
*(e+(-4*d*f+e^2)^(1/2))/f))+4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/
2))/a/x*(c*x^2+b*x+a)^(1/2)-2*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/
2))*b/a^(3/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-4*f^2/(-e+(-4*d
*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(
-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln(((b*f-c*e+(-4*d
*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*f^2-b*e*f-2*c*d*f
+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a
*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2
)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b*f-c*e+(-4*d*f+e^2)^(1
/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(
-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*
f+e^2)^(1/2))/f))+16*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+(-4*d*f+e^2)^(1/2))
^2/a^(1/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*x^2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(1/(x^2*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{a+bx+cx^2}\,\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
[Out] Integral(1/(x**2*sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

## 3.120 $\int \frac{1}{x^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx$

**Optimal**. Leaf size=679

$$-\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}d}-\frac{be\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d^2}+\frac{3b\sqrt{a+bx+cx^2}}{4a^2dx}-\frac{\left(e^2-df\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx}}\right)}{\sqrt{a}\,d^3}$$

**Rubi [A]** time = 11.23, antiderivative size = 679, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {6728, 744, 806, 724, 206, 730, 1032}

$$-\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}d}-\frac{be\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d^2}+\frac{3b\sqrt{a+bx+cx^2}}{4a^2dx}+\frac{f\left(-\left(e^2-df\right)\left(e-\sqrt{e^2-4df}\right)-4def+2e^3\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{f\left(-\left(e^2-df\right)\left(\sqrt{e^2-4df}+e\right)-4def+2e^3\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{\left(e^2-df\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\,d^3}+\frac{e\sqrt{a+bx+cx^2}}{ad^2x}-\frac{\sqrt{a+bx+cx^2}}{2adx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -Sqrt[a + b*x + c*x^2]/(2*a*d*x^2) + (3*b*Sqrt[a + b*x + c*x^2])/(4*a^2*d*x
) + (e*Sqrt[a + b*x + c*x^2])/(a*d^2*x) - ((3*b^2 - 4*a*c)*ArcTanh[(2*a + b
*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*a^(5/2)*d) - (b*e*ArcTanh[(2*a +
 b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*a^(3/2)*d^2) - ((e^2 - d*f)*Ar
cTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(Sqrt[a]*d^3) + (f*(2
*e^3 - 4*d*e*f - (e^2 - d*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e
 - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*S
qrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt
[a + b*x + c*x^2])])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f -
b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - (f*(2*e^3 - 4*d*e*f - (
e^2 - d*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*
f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*
f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])
])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 +
(c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
```

```
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx = \int \left(\frac{1}{dx^3\sqrt{a+bx+cx^2}} - \frac{e}{d^2x^2\sqrt{a+bx+cx^2}} + \frac{e^2-df}{d^3x\sqrt{a+bx+cx^2}} + \cdots\right.$$

$$= \frac{\int \frac{-e\left(e^2-2df\right)-f\left(e^2-df\right)x}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx}{d^3} + \frac{\int \frac{1}{x^3\sqrt{a+bx+cx^2}}\,dx}{d} - \frac{e\int \frac{1}{x^2\sqrt{a+bx+cx^2}}\,dx}{d^2} + \frac{\left(e^2 \cdots\right.}{\cdots}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{2adx^2} + \frac{e\sqrt{a+bx+cx^2}}{ad^2x} - \frac{\int \frac{\frac{3b}{2}+cx}{x^2\sqrt{a+bx+cx^2}}\,dx}{2ad} + \frac{(be)\int \frac{1}{x\sqrt{a+bx \cdots}}}{2ad^2 \cdots}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{2adx^2} + \frac{3b\sqrt{a+bx+cx^2}}{4a^2dx} + \frac{e\sqrt{a+bx+cx^2}}{ad^2x} - \frac{\left(e^2-df\right)\tan \cdots}{\cdots}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{2adx^2} + \frac{3b\sqrt{a+bx+cx^2}}{4a^2dx} + \frac{e\sqrt{a+bx+cx^2}}{ad^2x} - \frac{be\tanh^{-1}\left(\cdots\right.}{2a^3 \cdots}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{2adx^2} + \frac{3b\sqrt{a+bx+cx^2}}{4a^2dx} + \frac{e\sqrt{a+bx+cx^2}}{ad^2x} - \frac{\left(3b^2-4ac\right)\cdots}{\cdots}$$

**Mathematica [A]** time = 2.11, size = 669, normalized size = 0.99

$$\frac{\frac{d^2\left(\left(4acx-3b^2x\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)+6\sqrt{a}\,b\sqrt{a+x(b+cx)}\right)}{a^{5/2}x} - \frac{4bde\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{a^{3/2}} - \frac{4d^2\sqrt{a+x(b+cx)}}{ax^2} - \frac{8\left(e^2-df\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{\sqrt{a}} + \frac{4\sqrt{2}f\left(\frac{e\left(e^2-3df\right)}{\sqrt{e^2-4df}}-df+e^2\right)\tanh^{-1}\left(\frac{4af+b\left(\sqrt{e^2-4df}-e+2fx\right)+2cx\left(\sqrt{e^2-4df}-e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}} - \frac{4\sqrt{2}f\left(-e^2\sqrt{e^2-4df}+df\sqrt{e^2-4df}-3def+e^3\right)\tanh^{-1}\left(\frac{4af-b\left(\sqrt{e^2-4df}+e-2fx\right)-2cx\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}} + \frac{8de\sqrt{a+x(b+cx)}}{ax}}{8d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ((-4*d^2*Sqrt[a + x*(b + c*x)])/(a*x^2) + (8*d*e*Sqrt[a + x*(b + c*x)])/(a*
x) - (4*b*d*e*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/a^(3/
2) - (8*(e^2 - d*f)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])
/Sqrt[a] + (d^2*(6*Sqrt[a]*b*Sqrt[a + x*(b + c*x)] + (-3*b^2*x + 4*a*c*x)*A
```

```
rcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])]))/(a^(5/2)*x) - (4*Sq
rt[2]*f*(e^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*d*f] + d*f*Sqrt[e^2 - 4*d*f])*Arc
Tanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*
f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*
(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/(Sqrt[e^2 - 4*d*f]*Sqrt[
c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]
))]) + (4*Sqrt[2]*f*(e^2 - d*f + (e*(e^2 - 3*d*f))/Sqrt[e^2 - 4*d*f])*ArcTa
nh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*
f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*
(-e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*(e^2 - 2*d*f - e
*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))])/(8*d^3)
```

**IntegrateAlgebraic [C]** time = 1.48, size = 558, normalized size = 0.82

$$-\frac{\text{RootSum}\left[\#1^4 f-2\#1^3\sqrt{c}e-2\#1^2af+\#1^2be+4\#1^2cd+2\#1a\sqrt{c}e-4\#1b\sqrt{c}d+a^2f-abe+b^2d\&,\frac{-\#1^2df^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+\#1^2e^2f\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2bdef\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+4\#1\sqrt{c}def\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+adf^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+be^3\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2\#1\sqrt{c}e^3\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-ae^2f\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)}{2\#1^3f-3\#1^2\sqrt{c}e-2\#1af+\#1be+4\#1cd+a\sqrt{c}e-2b\sqrt{c}d}\&\right]}{d^3}+\frac{\sqrt{a+bx+cx^2}\,(-2ad+4aex+3bdx)}{4a^2d^2x^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx+cx^2}-\sqrt{c}x}{\sqrt{a}}\right)\left(8a^2df-8a^2e^2-4abde+4acd^2-3b^2d^2\right)}{4a^{5/2}d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ((-2*a*d + 3*b*d*x + 4*a*e*x)*Sqrt[a + b*x + c*x^2])/(4*a^2*d^2*x^2) + ((-3
*b^2*d^2 + 4*a*c*d^2 - 4*a*b*d*e - 8*a^2*e^2 + 8*a^2*d*f)*ArcTanh[(-(Sqrt[c
]*x) + Sqrt[a + b*x + c*x^2])/Sqrt[a]])/(4*a^(5/2)*d^3) - RootSum[b^2*d - a
*b*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2
- 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (b*e^3*Log[-(Sqrt[c]*x) + Sqrt
[a + b*x + c*x^2] - #1] - 2*b*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2
] - #1] - a*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a*d*f^2*
Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*Sqrt[c]*e^3*Log[-(Sqrt[c
]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 4*Sqrt[c]*d*e*f*Log[-(Sqrt[c]*x) +
Sqrt[a + b*x + c*x^2] - #1]*#1 + e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*
x^2] - #1]*#1^2 - d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2
)/(-2*b*Sqrt[c]*d + a*Sqrt[c]*e + 4*c*d*#1 + b*e*#1 - 2*a*f*#1 - 3*Sqrt[c]*
e*#1^2 + 2*f*#1^3) & ]/d^3
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.02, size = 1296, normalized size = 1.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] -8*f^3/(e+(-4*d*f+e^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-b*e*f-
2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln(
((b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*f^2
-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2
*2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^
(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b*f-c*e-(-
4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*c
*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f))+16*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+(-4*d*f+
e^2)^(1/2))^2/a/x*(c*x^2+b*x+a)^(1/2)-8*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+
(-4*d*f+e^2)^(1/2))^2*b/a^(3/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/
x)+2*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a/x^2*(c*x^2+b*x+a)^(
1/2)-3*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*b/a^2/x*(c*x^2+b*x+
a)^(1/2)+3/2*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*b^2/a^(5/2)*l
n((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-2*f/(-e+(-4*d*f+e^2)^(1/2))/(e
+(-4*d*f+e^2)^(1/2))*c/a^(3/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x
)-8*f^3/(-e+(-4*d*f+e^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-b*e*
f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*l
n(((b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*
f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2+
1/2*2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^
2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b*f-c*
e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*
f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/
(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))-64*f^4/(-e+(-4*d*f+e^2)^(1/2))^3/(e+(-4*
d*f+e^2)^(1/2))^3/a^(1/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)*d+6
4*f^3/(-e+(-4*d*f+e^2)^(1/2))^3/(e+(-4*d*f+e^2)^(1/2))^3/a^(1/2)*ln((b*x+2*
a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)*e^2
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*x^3), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
[Out] int(1/(x^3*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{a+bx+cx^2}\,\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
[Out] Integral(1/(x**3*sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

## 3.121 $\int \frac{x^3}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx$

**Optimal**. Leaf size=779

$$\frac{2\left(cx\left(\left(e^2-df\right)(abf-2ace+bcd)-de\left(-c(2af+be)+b^2f+2c^2d\right)\right)-\left(adf-ae^2+bde\right)\left(-c(2af+be)+b^2f+\cdots\right.}{f^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

**Rubi [A]** time = 14.17, antiderivative size = 779, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {6728, 613, 636, 1016, 1032, 724, 206}

$$\frac{2\left(cx\left(\left(e^2-df\right)(abf-2ace+bcd)-de\left(-c(2af+be)+b^2f+2c^2d\right)\right)-\left(adf-ae^2+bde\right)\left(-c(2af+be)+b^2f+2c^2d\right)+cde(abf-2ace+bcd)\right)}{f^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}+\frac{2e(b+2cx)}{f^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{2(2a+bx)}{f\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{\left(\left(e-\sqrt{e^2-4df}\right)\left(a\left(e^2-df\right)-bde+cd^2\right)+2df(bd-ae)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{\left(\left(\sqrt{e^2-4df}+e\right)\left(a\left(e^2-df\right)-bde+cd^2\right)+2df(bd-ae)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (2*(2*a + b*x))/((b^2 - 4*a*c)*f*Sqrt[a + b*x + c*x^2]) + (2*e*(b + 2*c*x))
/((b^2 - 4*a*c)*f^2*Sqrt[a + b*x + c*x^2]) + (2*(c*d*e*(b*c*d - 2*a*c*e + a
*b*f) - (b*d*e - a*e^2 + a*d*f)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*((b
*c*d - 2*a*c*e + a*b*f)*(e^2 - d*f) - d*e*(2*c^2*d + b^2*f - c*(b*e + 2*a*f
)))*x))/((b^2 - 4*a*c)*f^2*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[a
 + b*x + c*x^2]) + ((2*d*(b*d - a*e)*f + (e - Sqrt[e^2 - 4*d*f])*(c*d^2 - b
*d*e + a*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f
- c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2
*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*S
qrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c
*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - ((2*d*(b*d - a*e
)*f + (e + Sqrt[e^2 - 4*d*f])*(c*d^2 - b*d*e + a*(e^2 - d*f)))*ArcTanh[(4*a
*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*
Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d
*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f
)*Sqrt[e^2 - 4*d*f]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1016

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)
*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*
c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))
 - h*(b*c*d - 2*a*c*e + a*b*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d -
a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e
))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
```

```
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx = \int \left(-\frac{e}{f^2\left(a+bx+cx^2\right)^{3/2}} + \frac{x}{f\left(a+bx+cx^2\right)^{3/2}} + \frac{de+\left(e^2-d\right.}{f^2\left(a+bx+cx^2\right)^{3/2}\left(\right.}\right.$$

$$= \frac{\int \frac{de+\left(e^2-df\right)x}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx}{f^2} - \frac{e\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}}\,dx}{f^2} + \frac{\int \frac{x}{\left(a+bx+cx^2\right)^{3/2}}\,dx}{f}$$

$$= \frac{2(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}} + \frac{2e(b+2cx)}{\left(b^2-4ac\right)f^2\sqrt{a+bx+cx^2}} + \frac{2\left(cde(bcd\right.}{}$$

$$= \frac{2(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}} + \frac{2e(b+2cx)}{\left(b^2-4ac\right)f^2\sqrt{a+bx+cx^2}} + \frac{2\left(cde(bcd\right.}{}$$

$$= \frac{2(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}} + \frac{2e(b+2cx)}{\left(b^2-4ac\right)f^2\sqrt{a+bx+cx^2}} + \frac{2\left(cde(bcd\right.}{}$$

$$= \frac{2(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}} + \frac{2e(b+2cx)}{\left(b^2-4ac\right)f^2\sqrt{a+bx+cx^2}} + \frac{2\left(cde(bcd\right.}{}$$

**Mathematica [A]** time = 2.62, size = 1066, normalized size = 1.37

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

[Out] ((4*(2*a^3*f + b^3*d*x + a^2*(-2*c*d - b*e + 2*c*e*x + b*f*x) + a*b*(-3*c*d
*x + b*(d - e*x))))/((b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]) + (Sqrt[2]*(c*d^2
*(-e + Sqrt[e^2 - 4*d*f]) + b*d*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + a*(-e
^3 + 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f]))*Log[-e + Sqr
t[e^2 - 4*d*f] - 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^
2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]) + (Sqrt[2]*(c*d^2*(e
 + Sqrt[e^2 - 4*d*f]) - b*d*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + a*(e^3 -
3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f]))*Log[e + Sqrt[e^2
- 4*d*f] + 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*
d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]) - (Sqrt[2]*(c*d^2*(e + Sqrt
[e^2 - 4*d*f]) - b*d*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + a*(e^3 - 3*d*e*f
 + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f]))*Log[-4*a*f + 2*c*e*x + 2
*c*Sqrt[e^2 - 4*d*f]*x + b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x) - 2*Sqrt[2]*Sqrt
[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f
]))]*Sqrt[a + x*(b + c*x)]])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqr
t[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]) - (Sqrt[2]*(c*d^2
*(-e + Sqrt[e^2 - 4*d*f]) + b*d*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + a*(-e
^3 + 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f]))*Log[b*(-e +
Sqrt[e^2 - 4*d*f] + 2*f*x) + 2*(2*a*f - c*e*x + c*Sqrt[e^2 - 4*d*f]*x + Sqr
t[2]*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqr
t[e^2 - 4*d*f])]*Sqrt[a + x*(b + c*x)])])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 -
2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]))/(2
*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f)))

**IntegrateAlgebraic [C]** time = 1.69, size = 729, normalized size = 0.94

$$\frac{\text{RootSum}\left[\#1^4 f-2\#1^3\sqrt{c}\,e-2\#1^2 af+\#1^2 be+4\#1^2 cd+2\#1 a\sqrt{c}\,e-4\#1 b\sqrt{c}\,d+a^2 f-abe+b^2 d\&,\frac{-\#1^2 cd^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+\#1^2 bde\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+\#1^2 adf\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-\#1^2 ae^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-a^2 df\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+a^2e^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+b^2d^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+acd^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2\#1 b\sqrt{c}\,d^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2abde\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+2\#1 a\sqrt{c}\,de\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)}{-2\#1^3 f+3\#1^2\sqrt{c}\,e+2\#1 af-\#1 be-4\#1 cd-a\sqrt{c}\,e+2b\sqrt{c}\,d}\&\right]}{a^2f^2-abef-2acdf+ace^2+b^2df-bcde+c^2d^2}+\frac{2\left(2a^3f-a^2be+a^2bfx-2a^2cd+2a^2cex+ab^2d-ab^2ex-3abcdx+b^3dx\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(a^2f^2-abef-2acdf+ace^2+b^2df-bcde+c^2d^2\right)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]

[Out] (2*(a*b^2*d - 2*a^2*c*d - a^2*b*e + 2*a^3*f + b^3*d*x - 3*a*b*c*d*x - a*b^2
*e*x + 2*a^2*c*e*x + a^2*b*f*x))/((b^2 - 4*a*c)*(c^2*d^2 - b*c*d*e + a*c*e^
2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2)*Sqrt[a + b*x + c*x^2]) + RootS
um[b^2*d - a*b*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2
 + b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (b^2*d^2*Log[-(Sqr
t[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a*c*d^2*Log[-(Sqrt[c]*x) + Sqrt[a +
 b*x + c*x^2] - #1] - 2*a*b*d*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] -
#1] + a^2*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a^2*d*f*Log[
-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*b*Sqrt[c]*d^2*Log[-(Sqrt[c]*
x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*a*Sqrt[c]*d*e*Log[-(Sqrt[c]*x) + Sq
rt[a + b*x + c*x^2] - #1]*#1 - c*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^
2] - #1]*#1^2 + b*d*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 -
 a*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + a*d*f*Log[-(Sq
rt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(2*b*Sqrt[c]*d - a*Sqrt[c]*e -

```
4*c*d*#1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*#1^2 - 2*f*#1^3) & ]/(c^2*d^2 -
b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.04, size = 14651, normalized size = 18.81

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
[Out] int(x^3/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
[Out] Timed out
```

## 3.122 $\int \frac{x^2}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

**Optimal**. Leaf size=609

$$-\frac{2\left(cx\left(-abe-2a(cd-af)+b^2d\right)+a(abf-2ace+bcd)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{f\left(2d(cd-af)-\left(e-\sqrt{e^2-4df}\right)(bd-ae)\right)\tanh^{-1}}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{}}$$

**Rubi [A]** time = 5.84, antiderivative size = 609, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1061, 1032, 724, 206}

$$-\frac{2\left(cx\left(-abe-2a(cd-af)+b^2d\right)+a(abf-2ace+bcd)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{f\left(2d(cd-af)-\left(e-\sqrt{e^2-4df}\right)(bd-ae)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}+\frac{f\left(2d(cd-af)-\left(\sqrt{e^2-4df}+e\right)(bd-ae)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (-2*(a*(b*c*d - 2*a*c*e + a*b*f) + c*(b^2*d - a*b*e - 2*a*(c*d - a*f))*x))/
((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[a + b*x + c*x
^2]) - (f*(2*d*(c*d - a*f) - (b*d - a*e)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(
4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/
(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 -
4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2
- (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e -
b*f)*Sqrt[e^2 - 4*d*f]]) + (f*(2*d*(c*d - a*f) - (b*d - a*e)*(e + Sqrt[e^2
- 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqr
t[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*
e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*
d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f
 + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1061

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) +
(e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p +
1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b)
*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a
*f)) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)
^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d -
a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^q*Simp[(-2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1) + (b^2*(C*d + A*f) - b*((Plus[A])*c*e + a*C*e) + 2*(A*c*(c*
d - a*f) - a*(c*C*d - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)
*(2*a*c*e - b*(c*d + a*f)) + (A*b)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
+ q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b)*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(A*c*e + a*C*e)
 + 2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4
)))*x - c*f*(b^2*(C*d + A*f) - b*(A*c*e + a*C*e) + 2*(A*c*(c*d - a*f) - a*(
c*C*d - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1]
&& NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && I
LtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx = -\frac{2\left(a(bcd-2ace+abf)+c\left(b^2d-abe-2a(cd-af)\right)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} - \frac{2\int \frac{-\frac{1}{2}(\cdots}{\cdots}}{\left(b^2-4a\cdots\right.}$$

$$= -\frac{2\left(a(bcd-2ace+abf)+c\left(b^2d-abe-2a(cd-af)\right)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} + \frac{\left(f\left(2d(c\cdots\right.\right.}{\cdots}$$

$$= -\frac{2\left(a(bcd-2ace+abf)+c\left(b^2d-abe-2a(cd-af)\right)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} - \frac{\left(2f\left(2d(\cdots\right.\right.}{\cdots}$$

$$= -\frac{2\left(a(bcd-2ace+abf)+c\left(b^2d-abe-2a(cd-af)\right)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} - \frac{f\left(2d(cd\cdots\right.}{\sqrt{2}\sqrt{e^2\cdots}}$$

**Mathematica** [A] time = 6.59, size = 1097, normalized size = 1.80

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (-2*(e - (e^2 - 2*d*f)/Sqrt[e^2 - 4*d*f])*(2*b^2*f - 4*a*c*f - b*c*(e - Sqr
t[e^2 - 4*d*f]) + 2*c*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)*(a + b*x + c*x^2
))/((b^2 - 4*a*c)*f*(4*a*f^2 - 2*b*f*(e - Sqrt[e^2 - 4*d*f]) + c*(e - Sqrt[
e^2 - 4*d*f])^2)*(a + x*(b + c*x))^(3/2)) - (2*(e + (e^2 - 2*d*f)/Sqrt[e^2
- 4*d*f])*(2*b^2*f - 4*a*c*f - b*c*(e + Sqrt[e^2 - 4*d*f]) + 2*c*(b*f - c*(
e + Sqrt[e^2 - 4*d*f]))*x)*(a + b*x + c*x^2))/((b^2 - 4*a*c)*f*(4*a*f^2 - 2
*b*f*(e + Sqrt[e^2 - 4*d*f]) + c*(e + Sqrt[e^2 - 4*d*f])^2)*(a + x*(b + c*x
))^(3/2)) + (4*(b + 2*c*x)*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(3/2))/
(c*f*(a + x*(b + c*x))^(3/2)*Sqrt[1 - (b + 2*c*x)^2/(b^2 - 4*a*c)]) + (16*S
qrt[2]*f*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b
*f*Sqrt[e^2 - 4*d*f]]*(e + (-e^2 + 2*d*f)/Sqrt[e^2 - 4*d*f])*(a + b*x + c*x
^2)^(3/2)*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) - (-2*b*f + 2*c*(e - S
qrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c
*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/((4*
a*f^2 - 2*b*f*(e - Sqrt[e^2 - 4*d*f]) + c*(e - Sqrt[e^2 - 4*d*f])^2)*(16*a*
f^2 - 8*b*f*(e - Sqrt[e^2 - 4*d*f]) + 4*c*(e - Sqrt[e^2 - 4*d*f])^2)*(a + x
*(b + c*x))^(3/2)) + (16*Sqrt[2]*f*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 +
```

```
c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*(e - (-e^2 + 2*d*f)/Sqrt[e^
2 - 4*d*f])*(a + b*x + c*x^2)^(3/2)*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*
f]) - (-2*b*f + 2*c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c
*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*Sqr
t[a + b*x + c*x^2]])])/((4*a*f^2 - 2*b*f*(e + Sqrt[e^2 - 4*d*f]) + c*(e + Sq
rt[e^2 - 4*d*f])^2)*(16*a*f^2 - 8*b*f*(e + Sqrt[e^2 - 4*d*f]) + 4*c*(e + Sq
rt[e^2 - 4*d*f])^2)*(a + x*(b + c*x))^(3/2))
```

**IntegrateAlgebraic [C]** time = 1.40, size = 570, normalized size = 0.94

$$\frac{\text{RootSum}\left[\#1^4 f-2\#1^3\sqrt{c}e-2\#1^2 af+\#1^2 be+4\#1^2 cd+2\#1 a\sqrt{c}e-4\#1 b\sqrt{c}d+a^2 f-abe+b^2 d\&,\frac{\#1^2 bdf\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-\#1^2 aef\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+a^2 ef\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2\#1 c^{3/2}d^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+bcd^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2abdf\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+2\#1 a\sqrt{c}df\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)}{-2\#1^3 f+3\#1^2\sqrt{c}e+2\#1 af-\#1 be-4\#1 cd-a\sqrt{c}e+2b\sqrt{c}d}\&\right]}{-a^2f^2+abef+2acdf-ace^2+b^2(-d)f+bcde-c^2d^2}-\frac{2\left(a^2bf-2a^2ce+2a^2cfx+abcd-abcex-2ac^2dx+b^2cdx\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(a^2f^2-abef-2acdf+ace^2+b^2df-bcde+c^2d^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (-2*(a*b*c*d - 2*a^2*c*e + a^2*b*f + b^2*c*d*x - 2*a*c^2*d*x - a*b*c*e*x +
2*a^2*c*f*x))/((b^2 - 4*a*c)*(c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c
*d*f - a*b*e*f + a^2*f^2)*Sqrt[a + b*x + c*x^2]) + RootSum[b^2*d - a*b*e +
a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f
*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (b*c*d^2*Log[-(Sqrt[c]*x) + Sqrt[a +
b*x + c*x^2] - #1] - 2*a*b*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #
1] + a^2*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*c^(3/2)*d^2
*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*a*Sqrt[c]*d*f*Log[-(
Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + b*d*f*Log[-(Sqrt[c]*x) + Sqrt
[a + b*x + c*x^2] - #1]*#1^2 - a*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^
2] - #1]*#1^2)/(2*b*Sqrt[c]*d - a*Sqrt[c]*e - 4*c*d*#1 - b*e*#1 + 2*a*f*#1
+ 3*Sqrt[c]*e*#1^2 - 2*f*#1^3) & ]/(-(c^2*d^2) + b*c*d*e - a*c*e^2 - b^2*d*
f + 2*a*c*d*f + a*b*e*f - a^2*f^2)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$sage_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage2
```

**maple [B]** time = 0.02, size = 11341, normalized size = 18.62

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{{\left(c\,x^2+b\,x+a\right)}^{3/2}\,\left(f\,x^2+e\,x+d\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(x^2/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

## 3.123 $\int \frac{x}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)} \, dx$

**Optimal**. Leaf size=609

$$\frac{2\left(a\left(-2acf+b^2f-bce+2c^2d\right)+cx(abf-2ace+bcd)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}+\frac{f\left(2d(ce-bf)-\left(e-\sqrt{e^2-4df}\right)(cd-af)\right)\tanh^{-1}}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{}}$$

**Rubi [A]** time = 5.64, antiderivative size = 609, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1016, 1032, 724, 206}

$$\frac{2\left(a\left(-2acf+b^2f-bce+2c^2d\right)+cx(abf-2ace+bcd)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}+\frac{f\left(2d(ce-bf)-\left(e-\sqrt{e^2-4df}\right)(cd-af)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{f\left(2d(ce-bf)-\left(\sqrt{e^2-4df}+e\right)(cd-af)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (2*(a*(2*c^2*d - b*c*e + b^2*f - 2*a*c*f) + c*(b*c*d - 2*a*c*e + a*b*f)*x))
/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[a + b*x + c*
x^2]) + (f*(2*d*(c*e - b*f) - (c*d - a*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[
(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)
/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 -
 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e -
 b*f)*Sqrt[e^2 - 4*d*f]]) - (f*(2*d*(c*e - b*f) - (c*d - a*f)*(e + Sqrt[e^2
 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sq
rt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c
*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4
*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*
f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1016

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)
*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*
c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))
 - h*(b*c*d - 2*a*c*e + a*b*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d -
a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e
))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx = \frac{2\left(a\left(2c^2d-bce+b^2f-2acf\right)+c(bcd-2ace+abf)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} - \frac{2\int \frac{\frac{1}{2}(\cdots}{\cdots}}{\left(b^2-4a\cdots\right.}$$

$$= \frac{2\left(a\left(2c^2d-bce+b^2f-2acf\right)+c(bcd-2ace+abf)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} - \frac{\left(f\left(2d(c\cdots\right.\right.}{\cdots}$$

$$= \frac{2\left(a\left(2c^2d-bce+b^2f-2acf\right)+c(bcd-2ace+abf)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} + \frac{\left(2f\left(2d(\cdots\right.\right.}{\cdots}$$

$$= \frac{2\left(a\left(2c^2d-bce+b^2f-2acf\right)+c(bcd-2ace+abf)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} + \frac{f\left(2d(ce\cdots\right.}{\sqrt{2}\sqrt{e^2\cdots}}$$

**Mathematica** [A]   time = 5.68, size = 770, normalized size = 1.26

$$\frac{2\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\left(2c\left(cx\left(\sqrt{e^2-4df}-e\right)-2af\right)+2b^2f+bc\left(\sqrt{e^2-4df}-e+2fx\right)\right)}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(4af^2+2bf\left(\sqrt{e^2-4df}-e\right)+c\left(e-\sqrt{e^2-4df}\right)^2\right)}+\frac{2\left(\frac{e}{\sqrt{e^2-4df}}+1\right)\left(-2c\left(2af+cx\left(\sqrt{e^2-4df}+e\right)\right)+2b^2f-bc\left(\sqrt{e^2-4df}+e-2fx\right)\right)}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(4af^2-2bf\left(\sqrt{e^2-4df}+e\right)+c\left(\sqrt{e^2-4df}+e\right)^2\right)}-\frac{\sqrt{2}f^2\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{4af-b\left(\sqrt{e^2-4df}+e-2fx\right)-2cx\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\left(f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)\right)^{3/2}}-\frac{\sqrt{2}f^2\left(\sqrt{e^2-4df}-e\right)\sqrt{f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af+b\left(\sqrt{e^2-4df}-e+2fx\right)+2cx\left(\sqrt{e^2-4df}-e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\left(f\left(b\left(e-\sqrt{e^2-4df}\right)-2af\right)+c\left(e\sqrt{e^2-4df}+2df-e^2\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (2*(1 - e/Sqrt[e^2 - 4*d*f])*(2*b^2*f + b*c*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x
) + 2*c*(-2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)))/((b^2 - 4*a*c)*(4*a*f^2 +
 c*(e - Sqrt[e^2 - 4*d*f])^2 + 2*b*f*(-e + Sqrt[e^2 - 4*d*f]))*Sqrt[a + x*(
b + c*x)]) + (2*(1 + e/Sqrt[e^2 - 4*d*f])*(2*b^2*f - b*c*(e + Sqrt[e^2 - 4*
d*f] - 2*f*x) - 2*c*(2*a*f + c*(e + Sqrt[e^2 - 4*d*f])*x)))/((b^2 - 4*a*c)*
(4*a*f^2 - 2*b*f*(e + Sqrt[e^2 - 4*d*f]) + c*(e + Sqrt[e^2 - 4*d*f])^2)*Sqr
t[a + x*(b + c*x)]) - (Sqrt[2]*f^2*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f -
 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt
[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2
 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/(Sqrt[e^2 - 4*d*f]*(c*(e^2 - 2*d*f +
e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f])))^(3/2)) - (Sqr
t[2]*f^2*(-e + Sqrt[e^2 - 4*d*f])*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]
) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]*ArcTanh[(4*a*f + 2*c*(-e + Sqrt
[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e
^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]
*Sqrt[a + x*(b + c*x)])])/(Sqrt[e^2 - 4*d*f]*(c*(-e^2 + 2*d*f + e*Sqrt[e^2
- 4*d*f]) + f*(-2*a*f + b*(e - Sqrt[e^2 - 4*d*f])))^2)
```

**IntegrateAlgebraic [C]** time = 1.36, size = 608, normalized size = 1.00

$$\frac{\operatorname{RootSum}\left[\#1^4 f-2\#1^3\sqrt{c}e-2\#1^2af+\#1^2be+4\#1^2cd+2\#1a\sqrt{c}e-4\#1b\sqrt{c}d+a^2f-abe+b^2d\&,\frac{-\#1^2cdf\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+\#1^2af^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-a^2f^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+b^2df\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+2\#1c^{3/2}de\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-bcde\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2\#1b\sqrt{c}df\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+acdf\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)}{-2\#1^3f+3\#1^2\sqrt{c}e+2\#1af-\#1be-4\#1cd-a\sqrt{c}e+2b\sqrt{c}d}\&\right]}{-a^2f^2+abef+2acdf-ace^2+b^2(-d)f+bcde-c^2d^2}+\frac{2\left(2a^2cf-ab^2f+abce-abcfx-2ac^2d+2ac^2ex-bc^2dx\right)}{\left(4ac-b^2\right)\sqrt{a+bx+cx^2}\left(a^2f^2-abef-2acdf+ace^2+b^2df-bcde+c^2d^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (2*(-2*a*c^2*d + a*b*c*e - a*b^2*f + 2*a^2*c*f - b*c^2*d*x + 2*a*c^2*e*x -
a*b*c*f*x))/((-b^2 + 4*a*c)*(c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*
d*f - a*b*e*f + a^2*f^2)*Sqrt[a + b*x + c*x^2]) + RootSum[b^2*d - a*b*e + a
^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*
#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (-(b*c*d*e*Log[-(Sqrt[c]*x) + Sqrt[a +
 b*x + c*x^2] - #1]) + b^2*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #
1] + a*c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a^2*f^2*Log[-
(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*c^(3/2)*d*e*Log[-(Sqrt[c]*x)
+ Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*b*Sqrt[c]*d*f*Log[-(Sqrt[c]*x) + Sqrt[
a + b*x + c*x^2] - #1]*#1 - c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2]
- #1]*#1^2 + a*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(2*
b*Sqrt[c]*d - a*Sqrt[c]*e - 4*c*d*#1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*#1^2
 - 2*f*#1^3) & ]/(-(c^2*d^2) + b*c*d*e - a*c*e^2 - b^2*d*f + 2*a*c*d*f + a*
b*e*f - a^2*f^2)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$sage_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage2
```

**maple [B]** time = 0.02, size = 7163, normalized size = 11.76

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(x/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

## 3.124 $\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx$

**Optimal**. Leaf size=666

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{f\left(f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\ \cdots}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd\ \cdots\right.}$$

**Rubi [A]** time = 1.75, antiderivative size = 666, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {974, 1032, 724, 206}

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^2ce+b^3(-f)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{f\left(f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (2*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f) - c*(2*c^2*d - b*c*e +
b^2*f - 2*a*c*f)*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f
))*Sqrt[a + b*x + c*x^2]) - (f*(c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(
2*a*f - b*(e + Sqrt[e^2 - 4*d*f])))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*
f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*
f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])
])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqr
t[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*
f]))]) + (f*(c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt
[e^2 - 4*d*f])))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e
 + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2
 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^
2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*(e^2 - 2*d*f +
e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx = \frac{2\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)-c\left(2c^2d-bce+b^2f-2acf\right)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} +$$

$$= \frac{2\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)-c\left(2c^2d-bce+b^2f-2acf\right)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} +$$

$$= \frac{2\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)-c\left(2c^2d-bce+b^2f-2acf\right)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} -$$

$$= \frac{2\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)-c\left(2c^2d-bce+b^2f-2acf\right)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} -$$

**<u>Mathematica</u> [A]** time = 5.04, size = 700, normalized size = 1.05

$$\frac{2f\left(-\frac{2\left(-2c\left(2af+cx\left(\sqrt{e^2-4df}+e\right)\right)+2b^2f-bc\left(\sqrt{e^2-4df}+e-2fx\right)\right)}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(4af^2-2bf\left(\sqrt{e^2-4df}+e\right)+c\left(\sqrt{e^2-4df}+e\right)^2\right)}+\frac{2c\left(cx\left(\sqrt{e^2-4df}-e\right)-2af\right)+2b^2f+bc\left(\sqrt{e^2-4df}-e+2fx\right)}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)}+\frac{\sqrt{2}f^2\tanh^{-1}\left(\frac{4af-b\left(\sqrt{e^2-4df}+e-2fx\right)-2cx\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\left(f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)\right)^{3/2}}-\frac{\sqrt{2}f^2\sqrt{f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af+b\left(\sqrt{e^2-4df}-e+2fx\right)+2cx\left(\sqrt{e^2-4df}-e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\left(f\left(b\left(e-\sqrt{e^2-4df}\right)-2af\right)+c\left(e\sqrt{e^2-4df}+2df-e^2\right)\right)^2}\right)}{\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (2*f*((2*b^2*f + b*c*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x) + 2*c*(-2*a*f + c*(-e
 + Sqrt[e^2 - 4*d*f])*x))/((b^2 - 4*a*c)*(c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d
*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f])))*Sqrt[a + x*(b + c*x)]) - (2*
(2*b^2*f - b*c*(e + Sqrt[e^2 - 4*d*f] - 2*f*x) - 2*c*(2*a*f + c*(e + Sqrt[e
^2 - 4*d*f])*x)))/((b^2 - 4*a*c)*(4*a*f^2 - 2*b*f*(e + Sqrt[e^2 - 4*d*f]) +
 c*(e + Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + x*(b + c*x)]) + (Sqrt[2]*f^2*ArcTanh
[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x)
)/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e +
 Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/(c*(e^2 - 2*d*f + e*Sqrt[e^2
 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f])))^(3/2) - (Sqrt[2]*f^2*Sq
rt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*
d*f]))]*ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2
- 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) +
f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/(c*(-e^2 +
 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(-2*a*f + b*(e - Sqrt[e^2 - 4*d*f])))^2))
/Sqrt[e^2 - 4*d*f]
```

**IntegrateAlgebraic [C]** time = 0.00, size = 730, normalized size = 1.10

$$\frac{\text{RootSum}\left[\#1^4 f-2\#1^3\sqrt{c}\,e-2\#1^2 af+\#1^2 be+4\#1^2 cd+2\#1 a\sqrt{c}\,e-4\#1 b\sqrt{c}\,d+a^2 f-abe+b^2 d\&,\frac{-\#1^2 cef\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)+\#1^2 bf^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)+b^2 ef\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)-2\#1 c^{3/2} df\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)+2\#1 c^{3/2} e^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)+bcdf\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)-bce^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)-2\#1 b\sqrt{c}\,ef\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)+acef\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)-2abf^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)+2\#1 a\sqrt{c}\,f^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)}{-2\#1^3 f+3\#1^2\sqrt{c}\,e+2\#1 af-\#1 be-4\#1 cd-a\sqrt{c}\,e+2b\sqrt{c}\,d}\&\right]}{a^2f^2-abef-2acdf+ace^2+b^2df-bcde+c^2d^2}-\frac{2\left(-3abcf+2ac^2e-2ac^2fx+b^3f-b^2ce+b^2cfx+bc^2d-bc^2ex+2c^3dx\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(a^2f^2-abef-2acdf+ace^2+b^2df-bcde+c^2d^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (-2*(b*c^2*d - b^2*c*e + 2*a*c^2*e + b^3*f - 3*a*b*c*f + 2*c^3*d*x - b*c^2*
e*x + b^2*c*f*x - 2*a*c^2*f*x))/((b^2 - 4*a*c)*(c^2*d^2 - b*c*d*e + a*c*e^2
 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2)*Sqrt[a + b*x + c*x^2]) + RootSu
m[b^2*d - a*b*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2
+ b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (-(b*c*e^2*Log[-(Sq
rt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]) + b*c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a
 + b*x + c*x^2] - #1] + b^2*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] -
#1] + a*c*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*a*b*f^2*Lo
g[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*c^(3/2)*e^2*Log[-(Sqrt[c]*
x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*c^(3/2)*d*f*Log[-(Sqrt[c]*x) + Sqrt
[a + b*x + c*x^2] - #1]*#1 - 2*b*Sqrt[c]*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*
x + c*x^2] - #1]*#1 + 2*a*Sqrt[c]*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x
^2] - #1]*#1 - c*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 +
b*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(2*b*Sqrt[c]*d -
 a*Sqrt[c]*e - 4*c*d*#1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*#1^2 - 2*f*#1^3)
& ]/(c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$sage_2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage2
```

**maple [B]** time = 0.02, size = 4099, normalized size = 6.15

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] -2/(-4*d*f+e^2)^(1/2)/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(
-4*d*f+e^2)^(1/2)*c*e)*f^2/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e-(
-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-
2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)-4*f
/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e
)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2)*c^2/f^2-b^2)/((x+1/2*(e+(-4*d*f
+e^2)^(1/2))/f)^2*c+(b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(
1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+
e^2)^(1/2)*c*e)/f^2)^(1/2)*x*c^2+4/(-4*d*f+e^2)^(1/2)*f^2/(2*a*f^2-b*e*f-2*
c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f
+c^2*e^2/f^2-(-4*d*f+e^2)*c^2/f^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*
c+(b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*
a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^
2)^(1/2)*x*b*c-4/(-4*d*f+e^2)^(1/2)*f/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+
e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f
+e^2)*c^2/f^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e-(-4*d*f+e
^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+
c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*x*c^2*e-2*f
/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e
)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2)*c^2/f^2-b^2)/((x+1/2*(e+(-4*d*f
+e^2)^(1/2))/f)^2*c+(b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(
1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+
e^2)^(1/2)*c*e)/f^2)^(1/2)*b*c+2/(-4*d*f+e^2)^(1/2)*f^2/(2*a*f^2-b*e*f-2*c*
d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c
^2*e^2/f^2-(-4*d*f+e^2)*c^2/f^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+
(b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*
f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)
^(1/2)*b^2-2/(-4*d*f+e^2)^(1/2)*f/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)
^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2
)*c^2/f^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e-(-4*d*f+e^2)^
(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^
2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*b*c*e+2/(-4*d*f
+e^2)^(1/2)/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2
)^(1/2)*c*e)*f^2*2^(1/2)/((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b
*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln(((b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x
+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^
(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c
*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e
+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*
d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+
(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))+2/(-4
```

```
*d*f+e^2)^(1/2)/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f
+e^2)^(1/2)*c*e)*f^2/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e+(-4*d*
f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*
d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)-4*f/(2*
a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/(4
*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2)*c^2/f^2-b^2)/((x-1/2*(-e+(-4*d*f+e^
2)^(1/2))/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/
2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^
2)^(1/2)*c*e)/f^2)^(1/2)*x*c^2-4/(-4*d*f+e^2)^(1/2)*f^2/(2*a*f^2-b*e*f-2*c*
d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c
^2*e^2/f^2-(-4*d*f+e^2)*c^2/f^2-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c
+(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*
a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^
2)^(1/2)*x*b*c+4/(-4*d*f+e^2)^(1/2)*f/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+
e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f
+e^2)*c^2/f^2-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e+(-4*d*f+
e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*
f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*x*c^2*e-2
*f/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c
*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2)*c^2/f^2-b^2)/((x-1/2*(-e+(-4*
d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^
2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*
d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*b*c-2/(-4*d*f+e^2)^(1/2)*f^2/(2*a*f^2-b*e*f-
2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d
/f+c^2*e^2/f^2-(-4*d*f+e^2)*c^2/f^2-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)
^2*c+(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+1/2
*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e
)/f^2)^(1/2)*b^2+2/(-4*d*f+e^2)^(1/2)*f/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*
f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d
*f+e^2)*c^2/f^2-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e+(-4*d*
f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*
d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*b*c*e-2
/(-4*d*f+e^2)^(1/2)/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4
*d*f+e^2)^(1/2)*c*e)*f^2*2^(1/2)/((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)
^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln(((b*f-c*e+(-4*d*f+e^2)^(1/
2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*
d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-b*e*f-
2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*
(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1
/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)
^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2
))/f))
```

**maxima** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

**mupad** [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

## 3.125 $\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx$

**Optimal**. Leaf size=816

$$\frac{2\left(b^2+cxb-2ac\right)}{a\left(b^2-4ac\right)d\sqrt{cx^2+bx+a}}-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{cx^2+bx+a}}\right)}{a^{3/2}d}+\frac{f\left(\left(e-\sqrt{e^2-4df}\right)\left(f(be-af)-c\left(e^2-df\right)\right)-2\left(f\left(be\right.\right.\right.}{\sqrt{2}\,d\sqrt{e^2-4df}\,\left((cd-af)^2-(bd\right.}$$

**Rubi** [A] time = 15.92, antiderivative size = 814, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {6728, 740, 12, 724, 206, 1016, 1032}

$$\frac{2\left(b^2+cxb-2ac\right)}{a\left(b^2-4ac\right)d\sqrt{cx^2+bx+a}}-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{cx^2+bx+a}}\right)}{a^{3/2}d}-\frac{f\left(2f\left(be^2-afe-bdf\right)-2c\left(e^3-2def\right)-\left(e-\sqrt{e^2-4df}\right)\left(f(be-af)-c\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-bfe+2af^2-2cdf-(ce-bf)\sqrt{e^2-4df}}\sqrt{cx^2+bx+a}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{ce^2-bfe+2af^2-2cdf-(ce-bf)\sqrt{e^2-4df}}}+\frac{f\left(2f\left(be^2-afe-bdf\right)-2c\left(e^3-2def\right)-\left(e+\sqrt{e^2-4df}\right)\left(f(be-af)-c\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-bfe+2af^2-2cdf+(ce-bf)\sqrt{e^2-4df}}\sqrt{cx^2+bx+a}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{ce^2-bfe+2af^2-2cdf+(ce-bf)\sqrt{e^2-4df}}}+\frac{2\left(ce(2ace-b(cd+af))+(be-af)\left(fb^2+2c^2d-c(be+2af)\right)+c\left(2dec^2-b\left(e^2+df\right)c+bf(be-af)\right)x\right)}{\left(b^2-4ac\right)d\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{cx^2+bx+a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*d*Sqrt[a + b*x + c*x^2]) + (2*(c
*e*(2*a*c*e - b*(c*d + a*f)) + (b*e - a*f)*(2*c^2*d + b^2*f - c*(b*e + 2*a*
f)) + c*(2*c^2*d*e + b*f*(b*e - a*f) - b*c*(e^2 + d*f))*x))/((b^2 - 4*a*c)*
d*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[a + b*x + c*x^2]) - ArcTan
h[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]/(a^(3/2)*d) - (f*(2*f*(b*e
^2 - b*d*f - a*e*f) - 2*c*(e^3 - 2*d*e*f) - (e - Sqrt[e^2 - 4*d*f])*(f*(b*e
 - a*f) - c*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b
*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f
+ 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2
]*d*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2
- 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) + (f*(2*f*(b*
e^2 - b*d*f - a*e*f) - 2*c*(e^3 - 2*d*e*f) - (e + Sqrt[e^2 - 4*d*f])*(f*(b*
e - a*f) - c*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(
b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f
 + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[
2]*d*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2
 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1016

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)
*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*
c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))
 - h*(b*c*d - 2*a*c*e + a*b*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d -
a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e
))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx=\int \left(\frac{1}{dx\left(a+bx+cx^2\right)^{3/2}}+\frac{-e-fx}{d\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\right)dx$$

$$=\frac{\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}}\,dx}{d}+\frac{\int \frac{-e-fx}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx}{d}$$

$$=\frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}}+\frac{2\left(ce(2ace-b(cd+af))+(be-af)\left(\right.\right.}{\left(b^2-4ac\right)d\left(\right.}$$

$$=\frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}}+\frac{2\left(ce(2ace-b(cd+af))+(be-af)\left(\right.\right.}{\left(b^2-4ac\right)d\left(\right.}$$

$$=\frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}}+\frac{2\left(ce(2ace-b(cd+af))+(be-af)\left(\right.\right.}{\left(b^2-4ac\right)d\left(\right.}$$

$$=\frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}}+\frac{2\left(ce(2ace-b(cd+af))+(be-af)\left(\right.\right.}{\left(b^2-4ac\right)d\left(\right.}$$

**Mathematica [A]** time = 6.56, size = 1121, normalized size = 1.37

$$\frac{16\sqrt{2}\left(\frac{ef}{\sqrt{e^2-4df}}+f\right)\sqrt{ce^2-bfe-c\sqrt{e^2-4df}\,e+2af^2-2cdf+bf\sqrt{e^2-4df}}\,\left(cx^2+bx+a\right)^{3/2}\tanh^{-1}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)-\left(2c\left(e-\sqrt{e^2-4df}\right)-2bf\right)x}{2\sqrt{2}\sqrt{ce^2-bfe-c\sqrt{e^2-4df}\,e+2af^2-2cdf+bf\sqrt{e^2-4df}}\,\sqrt{cx^2+bx+a}}\right)f^2}{d\left(4af^2-2b\left(e-\sqrt{e^2-4df}\right)f+c\left(e-\sqrt{e^2-4df}\right)^2\right)\left(16af^2-8b\left(e-\sqrt{e^2-4df}\right)f+4c\left(e-\sqrt{e^2-4df}\right)^2\right)(a+x(b+cx))^{3/2}}-\frac{16\sqrt{2}\left(\frac{ef}{\sqrt{e^2-4df}}-f\right)\sqrt{ce^2-bfe+c\sqrt{e^2-4df}\,e+2af^2-2cdf-bf\sqrt{e^2-4df}}\,\left(cx^2+bx+a\right)^{3/2}\tanh^{-1}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)-\left(2c\left(e+\sqrt{e^2-4df}\right)-2bf\right)x}{2\sqrt{2}\sqrt{ce^2-bfe+c\sqrt{e^2-4df}\,e+2af^2-2cdf-bf\sqrt{e^2-4df}}\,\sqrt{cx^2+bx+a}}\right)f^2}{d\left(4af^2-2b\left(e+\sqrt{e^2-4df}\right)f+c\left(e+\sqrt{e^2-4df}\right)^2\right)\left(16af^2-8b\left(e+\sqrt{e^2-4df}\right)f+4c\left(e+\sqrt{e^2-4df}\right)^2\right)(a+x(b+cx))^{3/2}}-\frac{2\left(\frac{e}{\sqrt{e^2-4df}}+1\right)\left(2fb^2-c\left(e-\sqrt{e^2-4df}\right)b-4acf+2c\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x\right)\left(cx^2+bx+a\right)f}{\left(b^2-4ac\right)d\left(4af^2-2b\left(e-\sqrt{e^2-4df}\right)f+c\left(e-\sqrt{e^2-4df}\right)^2\right)(a+x(b+cx))^{3/2}}-\frac{2\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\left(2fb^2-c\left(e+\sqrt{e^2-4df}\right)b-4acf+2c\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x\right)\left(cx^2+bx+a\right)f}{\left(b^2-4ac\right)d\left(4af^2-2b\left(e+\sqrt{e^2-4df}\right)f+c\left(e+\sqrt{e^2-4df}\right)^2\right)(a+x(b+cx))^{3/2}}-\frac{\left(cx^2+bx+a\right)^{3/2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{cx^2+bx+a}}\right)}{a^{3/2}d(a+x(b+cx))^{3/2}}+\frac{2\left(b^2+cxb-2ac\right)\left(cx^2+bx+a\right)}{a\left(b^2-4ac\right)d(a+x(b+cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x)*(a + b*x + c*x^2))/(a*(b^2 - 4*a*c)*d*(a + x*(b + 
c*x))^(3/2)) - (2*f*(1 + e/Sqrt[e^2 - 4*d*f])*(2*b^2*f - 4*a*c*f - b*c*(e - 
 Sqrt[e^2 - 4*d*f]) + 2*c*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)*(a + b*x + c
*x^2))/((b^2 - 4*a*c)*d*(4*a*f^2 - 2*b*f*(e - Sqrt[e^2 - 4*d*f]) + c*(e - S
qrt[e^2 - 4*d*f])^2)*(a + x*(b + c*x))^(3/2)) - (2*f*(1 - e/Sqrt[e^2 - 4*d*
f])*(2*b^2*f - 4*a*c*f - b*c*(e + Sqrt[e^2 - 4*d*f]) + 2*c*(b*f - c*(e + Sq
rt[e^2 - 4*d*f]))*x)*(a + b*x + c*x^2))/((b^2 - 4*a*c)*d*(4*a*f^2 - 2*b*f*(
e + Sqrt[e^2 - 4*d*f]) + c*(e + Sqrt[e^2 - 4*d*f])^2)*(a + x*(b + c*x))^(3/
2)) - ((a + b*x + c*x^2)^(3/2)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x 
+ c*x^2])])/(a^(3/2)*d*(a + x*(b + c*x))^(3/2)) + (16*Sqrt[2]*f^2*(f + (e*f
)/Sqrt[e^2 - 4*d*f])*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 
- 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*(a + b*x + c*x^2)^(3/2)*ArcTanh[(4*a*f - 
b*(e - Sqrt[e^2 - 4*d*f]) - (-2*b*f + 2*c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sq
rt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*
Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(d*(4*a*f^2 - 2*b*f*(e - Sqrt[e
^2 - 4*d*f]) + c*(e - Sqrt[e^2 - 4*d*f])^2)*(16*a*f^2 - 8*b*f*(e - Sqrt[e^2
 - 4*d*f]) + 4*c*(e - Sqrt[e^2 - 4*d*f])^2)*(a + x*(b + c*x))^(3/2)) - (16*
Sqrt[2]*f^2*(-f + (e*f)/Sqrt[e^2 - 4*d*f])*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2
*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*(a + b*x + c*x^2)^(
3/2)*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) - (-2*b*f + 2*c*(e + Sqrt[e
^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*Sq
rt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(d*(4*a*f
^2 - 2*b*f*(e + Sqrt[e^2 - 4*d*f]) + c*(e + Sqrt[e^2 - 4*d*f])^2)*(16*a*f^2
 - 8*b*f*(e + Sqrt[e^2 - 4*d*f]) + 4*c*(e + Sqrt[e^2 - 4*d*f])^2)*(a + x*(b
 + c*x))^(3/2))
```

**IntegrateAlgebraic [C]** time = 6.98, size = 1054, normalized size = 1.29

$$\frac{2\left(fb^4-ceb^3+cfxb^3+c^2db^2-4acfb^2-c^2exb^2+3ac^2eb+c^3dxb-3ac^2fxb-2ac^3d+2a^2c^2f+2ac^3ex\right)}{a\left(b^2-4ac\right)\left(dfb^2-cdeb-aefb+c^2d^2+ace^2+a^2f^2-2acdf\right)\sqrt{cx^2+bx+a}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}-\frac{\sqrt{cx^2+bx+a}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{\text{RootSum}\left[f\#1^4-2\sqrt{c}\,e\#1^3+4cd\#1^2+be\#1^2-2af\#1^2-4b\sqrt{c}\,d\#1+2a\sqrt{c}\,e\#1+b^2d-abe+a^2f\&,\ \ldots\right]}{d\left(dfb^2-cdeb-aefb+c^2d^2+ace^2+a^2f^2-2acdf\right)}$$

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (2*(b^2*c^2*d - 2*a*c^3*d - b^3*c*e + 3*a*b*c^2*e + b^4*f - 4*a*b^2*c*f + 2
*a^2*c^2*f + b*c^3*d*x - b^2*c^2*e*x + 2*a*c^3*e*x + b^3*c*f*x - 3*a*b*c^2*
f*x))/(a*(b^2 - 4*a*c)*(c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f -
 a*b*e*f + a^2*f^2)*Sqrt[a + b*x + c*x^2]) + (2*ArcTanh[(Sqrt[c]*x)/Sqrt[a]
```

```
 - Sqrt[a + b*x + c*x^2]/Sqrt[a]])/(a^(3/2)*d) - RootSum[b^2*d - a*b*e + a^
2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#
1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (-(b*c*e^3*Log[-(Sqrt[c]*x) + Sqrt[a +
b*x + c*x^2] - #1]) + 2*b*c*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2]
- #1] + b^2*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a*c*e^2*
f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - b^2*d*f^2*Log[-(Sqrt[c]*
x) + Sqrt[a + b*x + c*x^2] - #1] - a*c*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*
x + c*x^2] - #1] - 2*a*b*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #
1] + a^2*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*c^(3/2)*e^3
*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 4*c^(3/2)*d*e*f*Log[-(
Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*b*Sqrt[c]*e^2*f*Log[-(Sqrt[
c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*b*Sqrt[c]*d*f^2*Log[-(Sqrt[c]*x)
 + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*a*Sqrt[c]*e*f^2*Log[-(Sqrt[c]*x) + Sq
rt[a + b*x + c*x^2] - #1]*#1 - c*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*
x^2] - #1]*#1^2 + c*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1
^2 + b*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 - a*f^3*Lo
g[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(2*b*Sqrt[c]*d - a*Sqrt[
c]*e - 4*c*d*#1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*#1^2 - 2*f*#1^3) & ]/(d*(
c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage0*x
```

**maple [B]** time = 0.04, size = 4594, normalized size = 5.63

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] 4*f^3/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/(2*a*f^2-b*e*f-2*c*d*f+c*e^
2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/((x+1/2*(e+(-4*d*f+e^2)^(1
/2))/f)^2*c+(b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)
/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/
2)*c*e)/f^2)^(1/2)+8*f^2/(e+(-4*d*f+e^2)^(1/2))/(2*a*f^2-b*e*f-2*c*d*f+c*e^
2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f
^2-(-4*d*f+e^2)*c^2/f^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e
-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*
f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*x
*c^2-8*f^3/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/(2*a*f^2-b*e*f-2*c*d*f
+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*
e^2/f^2-(-4*d*f+e^2)*c^2/f^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*
f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2
-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1
/2)*x*b*c+8*f^2/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/(2*a*f^2-b*e*f-2*
c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f
+c^2*e^2/f^2-(-4*d*f+e^2)*c^2/f^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*
c+(b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*
a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^
2)^(1/2)*x*c^2*e+4*f^2/(e+(-4*d*f+e^2)^(1/2))/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-
(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2
-(-4*d*f+e^2)*c^2/f^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e-(
-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-
2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*b*c
-4*f^3/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/(2*a*f^2-b*e*f-2*c*d*f+c*e
^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/
f^2-(-4*d*f+e^2)*c^2/f^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*
e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e
*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*
b^2+4*f^2/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/(2*a*f^2-b*e*f-2*c*d*f+
c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e
^2/f^2-(-4*d*f+e^2)*c^2/f^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f
-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-
b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/
2)*b*c*e-4*f^3/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/(2*a*f^2-b*e*f-2*c
*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/((2*a*f^2
-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1
/2)*ln(((b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+(
2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/
f^2+1/2*2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*
f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b*f
-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*
e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)
)/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))+4*f^3/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e
^2)^(1/2)/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^
```

```
(1/2)*c*e)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^(1/
2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+
(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)-8*f^2/(-e+(-4*d*f
+e^2)^(1/2))/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^
2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2)*c^2/f^2-b^2)/((x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+
(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2
)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*x*c^2-8*f^3/(-e+(-4*d*f+e^2)^(1/2)
)/(-4*d*f+e^2)^(1/2)/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-
4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2)*c^2/f^2-b^2
)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-
1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e
^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*x*b*c+8*f^2/(-e+(-4*d*f+e^
2)^(1/2))/(-4*d*f+e^2)^(1/2)/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2
)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2)*c^2
/f^2-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^(1/2
)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(
-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*x*c^2*e-4*f^2/(-e+
(-4*d*f+e^2)^(1/2))/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4
*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2)*c^2/f^2-b^2)
/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1
/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^
2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*b*c-4*f^3/(-e+(-4*d*f+e^2)^
(1/2))/(-4*d*f+e^2)^(1/2)/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b
*f-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2)*c^2/f^
2-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^(1/2)*c
)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*
d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*b^2+4*f^2/(-e+(-4*d*f
+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(
1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2)*
c^2/f^2-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^(
1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^
2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*b*c*e-4*f^3/(-e
+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*
f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/((2*a*f^2-b*e*f-2*c*d*f+c*
e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln(((b*f-c*e+
(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*f^2-b*e*f-2*
c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*
((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e
)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b*f-c*e+(-4*d*f+e^
2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*
e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x-1/2*(-e+(
-4*d*f+e^2)^(1/2))/f))-4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a
/(c*x^2+b*x+a)^(1/2)+8*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*b/a
/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*c*x+4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f
```

```
+e^2)^(1/2))*b^2/a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+4*f/(-e+(-4*d*f+e^2)^(1/
2))/(e+(-4*d*f+e^2)^(1/2))/a^(3/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2
))/x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{3}{2}}\left(fx^2+ex+d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate(1/((c*x^2 + b*x + a)^(3/2)*(f*x^2 + e*x + d)*x), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
[Out] int(1/(x*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
[Out] Integral(1/(x*(a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

## 3.126 $\int \frac{x^4}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx$

**Optimal**. Leaf size=140

$$-\frac{1}{4}\sqrt{-x^2-4x-3}\,x+\frac{5}{2}\sqrt{-x^2-4x-3}+\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{5}{4}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)+$$

**Rubi [A]** time = 0.50, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 14, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {6728, 619, 216, 640, 742, 1028, 986, 12, 1026, 1161, 618, 204, 1027, 206}

$$-\frac{1}{4}\sqrt{-x^2-4x-3}\,x+\frac{5}{2}\sqrt{-x^2-4x-3}+\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{5}{4}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)+\frac{11}{2}\sin^{-1}(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] (5*Sqrt[-3 - 4*x - x^2])/2 - (x*Sqrt[-3 - 4*x - x^2])/4 + (11*ArcSin[2 + x]
)/2 + ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/(2*Sqrt[2]) - ArcT
an[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/(2*Sqrt[2]) - (5*ArcTanh[x/S
qrt[-3 - 4*x - x^2]])/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 986

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
 b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1026

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 -
 4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1027

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[g, Subst[Int[1/(a + (c*d - a*f)
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
 EqQ[2*h*d - g*e, 0]
```

Rule 1028

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(2*h*d - g*e)/e, Int[1/((a + b
*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g
, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
 && NeQ[2*h*d - g*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx = \int\left(\frac{5}{4\sqrt{-3-4x-x^2}} - \frac{x}{\sqrt{-3-4x-x^2}} + \frac{x^2}{2\sqrt{-3-4x-x^2}} - \frac{}{4\sqrt{-3-4x}}\right.$$

$$= -\left(\frac{1}{4}\int \frac{15+8x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx\right) + \frac{1}{2}\int \frac{x^2}{\sqrt{-3-4x-x^2}}\,dx + \frac{5}{4}$$

$$= \sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} - \frac{1}{4}\int \frac{3+6x}{\sqrt{-3-4x-x^2}}\,dx + \frac{1}{2}\int \frac{}{\sqrt{-}}$$

$$= \frac{5}{2}\sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} + \frac{5}{4}\sin^{-1}(2+x) + \frac{1}{8}\int \frac{}{\sqrt{-3-4x}}$$

$$= \frac{5}{2}\sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} + \frac{13}{4}\sin^{-1}(2+x) - \tanh^{-1}\left(\frac{}{\sqrt{-3}}\right.$$

$$= \frac{5}{2}\sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} + \frac{11}{2}\sin^{-1}(2+x) - \frac{5}{4}\tanh^{-1}\left(\frac{}{\sqrt{-3}}\right.$$

$$= \frac{5}{2}\sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} + \frac{11}{2}\sin^{-1}(2+x) - \frac{5}{4}\tanh^{-1}\left(\frac{}{\sqrt{-3}}\right.$$

$$= \frac{5}{2}\sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} + \frac{11}{2}\sin^{-1}(2+x) - \frac{5}{4}\tanh^{-1}\left(\frac{}{\sqrt{-3}}\right.$$

$$= \frac{5}{2}\sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} + \frac{11}{2}\sin^{-1}(2+x) + \frac{\tan^{-1}\left(\frac{1-\frac{3+}{\sqrt{-3-4}}}{\sqrt{2}}\right.}{2\sqrt{2}}$$

**Mathematica** [C] time = 0.53, size = 210, normalized size = 1.50

$$\frac{1}{24}\left(-6\sqrt{-x^2-4x-3}\,x+60\sqrt{-x^2-4x-3}-\sqrt{1-2i\sqrt{2}}\left(4\sqrt{2}+7i\right)\tanh^{-1}\left(\frac{-i\sqrt{2}x+2x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right)-\sqrt{1+2i\sqrt{2}}\left(4\sqrt{2}-7i\right)\tanh^{-1}\left(\frac{\left(2+i\sqrt{2}\right)x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right)+132\sin^{-1}(x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]

[Out] (60*Sqrt[-3 - 4*x - x^2] - 6*x*Sqrt[-3 - 4*x - x^2] + 132*ArcSin[2 + x] - S
qrt[1 - (2*I)*Sqrt[2]]*(7*I + 4*Sqrt[2])*ArcTanh[(2 - (2*I)*Sqrt[2] + 2*x -
 I*Sqrt[2]*x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])] - Sqrt[1 + (2
```

```
*I)*Sqrt[2]]*(-7*I + 4*Sqrt[2])*ArcTanh[(2 + (2*I)*Sqrt[2] + (2 + I*Sqrt[2]
)*x)/(Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])])/24
```

**IntegrateAlgebraic [A]** time = 0.42, size = 108, normalized size = 0.77

$$\frac{1}{4}\sqrt{-x^2-4x-3}\,(10-x)-\frac{\tan^{-1}\left(\frac{\sqrt{2}x+\frac{3}{\sqrt{2}}}{\sqrt{-x^2-4x-3}}\right)}{2\sqrt{2}}-11\tan^{-1}\left(\frac{\sqrt{-x^2-4x-3}}{x+3}\right)-\frac{5}{4}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] ((10 - x)*Sqrt[-3 - 4*x - x^2])/4 - ArcTan[(3/Sqrt[2] + Sqrt[2]*x)/Sqrt[-3
 - 4*x - x^2]]/(2*Sqrt[2]) - 11*ArcTan[Sqrt[-3 - 4*x - x^2]/(3 + x)] - (5*Ar
cTanh[x/Sqrt[-3 - 4*x - x^2]])/4
```

**fricas [A]** time = 1.28, size = 178, normalized size = 1.27

$$-\frac{1}{4}\sqrt{-x^2-4x-3}\,(x-10)+\frac{1}{8}\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)+\frac{1}{8}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x-3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)-\frac{11}{2}\arctan\left(\frac{\sqrt{-x^2-4x-3}\,(x+2)}{x^2+4x+3}\right)+\frac{5}{16}\log\left(-\frac{2\sqrt{-x^2-4x-3}\,x+4x+3}{x^2}\right)-\frac{5}{16}\log\left(\frac{2\sqrt{-x^2-4x-3}\,x-4x-3}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-x^2 - 4*x - 3)*(x - 10) + 1/8*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*
sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) + 1/8*sqrt(2)*arctan(-1/2*(sqrt(2)
*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) - 11/2*arctan(sqrt(-x^2 - 4
*x - 3)*(x + 2)/(x^2 + 4*x + 3)) + 5/16*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*
x + 3)/x^2) - 5/16*log((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)
```

**giac [A]** time = 0.20, size = 188, normalized size = 1.34

$$-\frac{1}{4}\sqrt{-x^2-4x-3}\,(x-10)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right)+\frac{11}{2}\arcsin(x+2)-\frac{5}{8}\log\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{3\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+1\right)+\frac{5}{8}\log\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(-x^2 - 4*x - 3)*(x - 10) + 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqr
t(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/4*sqrt(2)*arctan(1/2*sqrt(2)*((sqr
t(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 11/2*arcsin(x + 2) - 5/8*log(2*(sqrt
(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 +
1) + 5/8*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) -
 1)^2/(x + 2)^2 + 3)
```

**maple [A]** time = 0.02, size = 159, normalized size = 1.14

$$-\frac{\sqrt{-x^{2}-4x-3}\,x}{4}+\frac{11\arcsin\left(x+2\right)}{2}+\frac{5\sqrt{-x^{2}-4x-3}}{2}+\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^{2}}{\left(-x-\frac{3}{2}\right)^{2}}-12}\,\left(5\,\mathrm{arctanh}\left(\frac{3x}{\left(-x-\frac{3}{2}\right)\sqrt{\frac{3x^{2}}{\left(-x-\frac{3}{2}\right)^{2}}-12}}\right)+\sqrt{2}\,\arctan\left(\frac{\sqrt{\frac{3x^{2}}{\left(-x-\frac{3}{2}\right)^{2}}-12}\,\sqrt{2}}{6}\right)\right)}{24\sqrt{\frac{\frac{x^{2}}{\left(-x-\frac{3}{2}\right)^{2}}-4}{\left(\frac{x}{-x-\frac{3}{2}}+1\right)^{2}}}\,\left(\frac{x}{-x-\frac{3}{2}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)
```

```
[Out] -1/4*x*(-x^2-4*x-3)^(1/2)+5/2*(-x^2-4*x-3)^(1/2)+11/2*arcsin(2+x)+1/24*3^(1
/2)*4^(1/2)*(3/(-x-3/2)^2*x^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3/(-x-3/2)^2*x
^2-12)^(1/2)*2^(1/2))+5*arctanh(3/(-x-3/2)/(3/(-x-3/2)^2*x^2-12)^(1/2)*x))/
((1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)*x+1)^2)^(1/2)/(1/(-x-3/2)*x+1)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}}{\left(2\,x^{2}+4\,x+3\right)\sqrt{-x^{2}-4\,x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{4}}{\sqrt{-x^{2}-4\,x-3}\,\left(2\,x^{2}+4\,x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)
```

```
[Out] int(x^4/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}}{\sqrt{-\left(x+1\right)\left(x+3\right)}\,\left(2x^{2}+4x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
[Out] Integral(x**4/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

## 3.127 $\int \frac{x^3}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx$

**Optimal**. Leaf size=115

$$-\frac{1}{2}\sqrt{-x^2-4x-3}+\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{2\sqrt{2}}+\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)-2\sin^{-1}(x+2)$$

**Rubi [A]** time = 0.42, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {6728, 619, 216, 640, 1028, 986, 12, 1026, 1161, 618, 204, 1027, 206}

$$-\frac{1}{2}\sqrt{-x^2-4x-3}+\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{2\sqrt{2}}+\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)-2\sin^{-1}(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] -Sqrt[-3 - 4*x - x^2]/2 - 2*ArcSin[2 + x] + ArcTan[(1 - (3 + x)/Sqrt[-3 - 4
*x - x^2])/Sqrt[2]]/(2*Sqrt[2]) - ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])
/Sqrt[2]]/(2*Sqrt[2]) + ArcTanh[x/Sqrt[-3 - 4*x - x^2]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 986

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
 b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1026

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 -
 4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1027

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[g, Subst[Int[1/(a + (c*d - a*f)
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
```

```
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
 EqQ[2*h*d - g*e, 0]
```

Rule 1028

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(2*h*d - g*e)/e, Int[1/((a + b
*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g
, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
 && NeQ[2*h*d - g*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx = \int \left(-\frac{1}{\sqrt{-3-4x-x^2}} + \frac{x}{2\sqrt{-3-4x-x^2}} + \frac{6+5x}{2\sqrt{-3-4x-x^2}\left(3+4x+2\right.}\right.$$

$$= \frac{1}{2}\int \frac{x}{\sqrt{-3-4x-x^2}}\,dx + \frac{1}{2}\int \frac{6+5x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx - \int$$

$$= -\frac{1}{2}\sqrt{-3-4x-x^2} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx, x, -4-2x\right) - \frac{5}{8}\int \frac{}{\sqrt{-3}}$$

$$= -\frac{1}{2}\sqrt{-3-4x-x^2} - \sin^{-1}(2+x) + \frac{1}{8}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}$$

$$= -\frac{1}{2}\sqrt{-3-4x-x^2} - 2\sin^{-1}(2+x) + \frac{5}{4}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{1}{2}\int$$

$$= -\frac{1}{2}\sqrt{-3-4x-x^2} - 2\sin^{-1}(2+x) + \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + 4\,\text{Sub}$$

$$= -\frac{1}{2}\sqrt{-3-4x-x^2} - 2\sin^{-1}(2+x) + \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) - \frac{1}{6}\,\text{Sub}$$

$$= -\frac{1}{2}\sqrt{-3-4x-x^2} - 2\sin^{-1}(2+x) + \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{1}{3}\,\text{Sub}$$

$$= -\frac{1}{2}\sqrt{-3-4x-x^2} - 2\sin^{-1}(2+x) + \frac{\tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{\tan^{-1}\left(\frac{1+\frac{}{\sqrt{}}}{}\right)}{2\sqrt{2}}$$

**Mathematica** [C] time = 0.44, size = 192, normalized size = 1.67

$$\frac{1}{8}\left(-4\left(\sqrt{-x^2-4x-3}+4\sin^{-1}(x+2)\right)+\frac{\left(5\sqrt{2}-2i\right)\tanh^{-1}\left(\frac{i\sqrt{2}x+2x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right)}{\sqrt{1-2i\sqrt{2}}}+\frac{\left(5\sqrt{2}+2i\right)\tanh^{-1}\left(\frac{\left(2-i\sqrt{2}\right)x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right)}{\sqrt{1+2i\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] (-4*(Sqrt[-3 - 4*x - x^2] + 4*ArcSin[2 + x]) + ((-2*I + 5*Sqrt[2])*ArcTanh[
(2 + (2*I)*Sqrt[2] + 2*x + I*Sqrt[2]*x)/(Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 -
```

```
4*x - x^2])])/Sqrt[1 - (2*I)*Sqrt[2]] + ((2*I + 5*Sqrt[2])*ArcTanh[(2 - (2*
I)*Sqrt[2] + (2 - I*Sqrt[2])*x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^
2])])/Sqrt[1 + (2*I)*Sqrt[2]])/8
```

**IntegrateAlgebraic [A]** time = 0.37, size = 99, normalized size = 0.86

$$-\frac{1}{2}\sqrt{-x^2-4x-3}-\frac{\tan^{-1}\left(\frac{\sqrt{2}x+\frac{3}{\sqrt{2}}}{\sqrt{-x^2-4x-3}}\right)}{2\sqrt{2}}+4\tan^{-1}\left(\frac{\sqrt{-x^2-4x-3}}{x+3}\right)+\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] -1/2*Sqrt[-3 - 4*x - x^2] - ArcTan[(3/Sqrt[2] + Sqrt[2]*x)/Sqrt[-3 - 4*x -
x^2]]/(2*Sqrt[2]) + 4*ArcTan[Sqrt[-3 - 4*x - x^2]/(3 + x)] + ArcTanh[x/Sqrt
[-3 - 4*x - x^2]]
```

**fricas [A]** time = 1.40, size = 175, normalized size = 1.52

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)+\frac{1}{8}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x-3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)-\frac{1}{2}\sqrt{-x^2-4x-3}+2\arctan\left(\frac{\sqrt{-x^2-4x-3}(x+2)}{x^2+4x+3}\right)-\frac{1}{4}\log\left(-\frac{2\sqrt{-x^2-4x-3}x+4x+3}{x^2}\right)+\frac{1}{4}\log\left(\frac{2\sqrt{-x^2-4x-3}x-4x-3}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x +
3)) + 1/8*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/
(2*x + 3)) - 1/2*sqrt(-x^2 - 4*x - 3) + 2*arctan(sqrt(-x^2 - 4*x - 3)*(x +
2)/(x^2 + 4*x + 3)) - 1/4*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) +
1/4*log((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)
```

**giac [A]** time = 0.25, size = 185, normalized size = 1.61

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right)-\frac{1}{2}\sqrt{-x^2-4x-3}-2\arcsin(x+2)+\frac{1}{2}\log\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{3\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+1\right)-\frac{1}{2}\log\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
+ 1/4*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
- 1/2*sqrt(-x^2 - 4*x - 3) - 2*arcsin(x + 2) + 1/2*log(2*(sqrt(-x^2 - 4*x -
 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) - 1/2*log(
2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)
^2 + 3)
```

**maple [A]** time = 0.02, size = 144, normalized size = 1.25

$$-2\arcsin\left(x+2\right)-\frac{\sqrt{-x^{2}-4x-3}}{2}+\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^{2}}{\left(-x-\frac{3}{2}\right)^{2}}-12}\,\left(-4\,\mathrm{arctanh}\left(\frac{3x}{\left(-x-\frac{3}{2}\right)\sqrt{\frac{3x^{2}}{\left(-x-\frac{3}{2}\right)^{2}}-12}}\right)+\sqrt{2}\,\arctan\left(\frac{\sqrt{\frac{3x^{2}}{\left(-x-\frac{3}{2}\right)^{2}}-12}\,\sqrt{2}}{6}\right)\right)}{24\sqrt{\frac{\frac{x^{2}}{\left(-x-\frac{3}{2}\right)^{2}}-4}{\left(\frac{x}{-x-\frac{3}{2}}+1\right)^{2}}}\,\left(\frac{x}{-x-\frac{3}{2}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)
```

```
[Out] -1/2*(-x^2-4*x-3)^(1/2)-2*arcsin(x+2)+1/24*3^(1/2)*4^(1/2)*(3/(-x-3/2)^2*x^
2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3/(-x-3/2)^2*x^2-12)^(1/2)*2^(1/2))-4*arct
anh(3/(-x-3/2)/(3/(-x-3/2)^2*x^2-12)^(1/2)*x))/((1/(-x-3/2)^2*x^2-4)/(1/(-x
-3/2)*x+1)^2)^(1/2)/(1/(-x-3/2)*x+1)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\left(2\,x^{2}+4\,x+3\right)\sqrt{-x^{2}-4\,x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3}}{\sqrt{-x^{2}-4\,x-3}\,\left(2\,x^{2}+4\,x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)
```

```
[Out] int(x^3/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\sqrt{-\left(x+1\right)\left(x+3\right)}\,\left(2x^{2}+4x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

## 3.128 $\int \frac{x^2}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx$

**Optimal**. Leaf size=98

$$-\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)+\frac{1}{2}\sin^{-1}(x+2)$$

**Rubi [A]** time = 0.20, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1077, 619, 216, 1028, 986, 12, 1026, 1161, 618, 204, 1027, 206}

$$-\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)+\frac{1}{2}\sin^{-1}(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] ArcSin[2 + x]/2 - ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[2
] + ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[2] - ArcTanh[x/
Sqrt[-3 - 4*x - x^2]]/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 986

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
 b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1026

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 -
 4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1027

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[g, Subst[Int[1/(a + (c*d - a*f)
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
 EqQ[2*h*d - g*e, 0]
```

Rule 1028

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(2*h*d - g*e)/e, Int[1/((a + b
```

```
*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g
, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
 && NeQ[2*h*d - g*e, 0]
```

Rule 1077

```
Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C - b*C*x)/((a + b*x + c*x^2)*Sqrt[d
 + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\int \frac{x^2}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx = \frac{1}{2}\int \frac{1}{\sqrt{-3-4x-x^2}}\,dx + \frac{1}{2}\int \frac{-3-4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx$$

$$= -\left(\frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx, x, -4-2x\right)\right) + \frac{1}{2}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\left(3+4x \cdots\right.}$$

$$= \frac{1}{2}\sin^{-1}(2+x) - \frac{1}{4}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx + \frac{1}{4}\int -\frac{\cdots}{\sqrt{-3-\cdots}}$$

$$= \frac{1}{2}\sin^{-1}(2+x) - \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{3}{2}\text{Subst}\left(\int \frac{1}{3-3x^2}\,dx, x, \cdots\right.$$

$$= \frac{1}{2}\sin^{-1}(2+x) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) - 8\,\text{Subst}\left(\int \frac{1+3x^2}{-4-8x^2-36\cdots}\right.$$

$$= \frac{1}{2}\sin^{-1}(2+x) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{1}{3}\text{Subst}\left(\int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2}\,d\cdots\right.$$

$$= \frac{1}{2}\sin^{-1}(2+x) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) - \frac{2}{3}\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx, x\cdots\right.$$

$$= \frac{1}{2}\sin^{-1}(2+x) - \frac{\tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{1}{2}\tanh^{-1}\left(\cdots\right.$$

**Mathematica [C]** time = 0.19, size = 159, normalized size = 1.62

$$\frac{1}{4}\left(-i\sqrt{1-2i\sqrt{2}}\,\tanh^{-1}\left(\frac{i\sqrt{2}x+2x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}}\,\sqrt{-x^2-4x-3}}\right)+i\sqrt{1+2i\sqrt{2}}\,\tanh^{-1}\left(\frac{\left(2-i\sqrt{2}\right)x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}}\,\sqrt{-x^2-4x-3}}\right)+2\sin^{-1}(x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] (2*ArcSin[2 + x] - I*Sqrt[1 - (2*I)*Sqrt[2]]*ArcTanh[(2 + (2*I)*Sqrt[2] + 2
*x + I*Sqrt[2]*x)/(Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])] + I*Sqrt[
1 + (2*I)*Sqrt[2]]*ArcTanh[(2 - (2*I)*Sqrt[2] + (2 - I*Sqrt[2])*x)/(Sqrt[2
+ (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])])/4
```

**IntegrateAlgebraic [A]** time = 0.33, size = 82, normalized size = 0.84

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x+\frac{3}{\sqrt{2}}}{\sqrt{-x^2-4x-3}}\right)}{\sqrt{2}}-\tan^{-1}\left(\frac{\sqrt{-x^2-4x-3}}{x+3}\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] ArcTan[(3/Sqrt[2] + Sqrt[2]*x)/Sqrt[-3 - 4*x - x^2]]/Sqrt[2] - ArcTan[Sqrt[
-3 - 4*x - x^2]/(3 + x)] - ArcTanh[x/Sqrt[-3 - 4*x - x^2]]/2
```

**fricas [A]** time = 1.04, size = 161, normalized size = 1.64

$$-\frac{1}{4}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x+3\,\sqrt{2}\,\sqrt{-x^2-4\,x-3}}{2\,(2\,x+3)}\right)-\frac{1}{4}\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\,x-3\,\sqrt{2}\,\sqrt{-x^2-4\,x-3}}{2\,(2\,x+3)}\right)-\frac{1}{2}\,\arctan\left(\frac{\sqrt{-x^2-4\,x-3}\,(x+2)}{x^2+4\,x+3}\right)+\frac{1}{8}\,\log\left(-\frac{2\,\sqrt{-x^2-4\,x-3}\,x+4\,x+3}{x^2}\right)-\frac{1}{8}\,\log\left(\frac{2\,\sqrt{-x^2-4\,x-3}\,x-4\,x-3}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x +
 3)) - 1/4*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))
/(2*x + 3)) - 1/2*arctan(sqrt(-x^2 - 4*x - 3)*(x + 2)/(x^2 + 4*x + 3)) + 1/
8*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) - 1/8*log((2*sqrt(-x^2 - 4
*x - 3)*x - 4*x - 3)/x^2)
```

**giac [B]** time = 0.23, size = 171, normalized size = 1.74

$$-\frac{1}{2}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+1\right)\right)-\frac{1}{2}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\frac{\sqrt{-x^2-4\,x-3}-1}{x+2}+1\right)\right)+\frac{1}{2}\,\arcsin\left(x+2\right)-\frac{1}{4}\,\log\left(\frac{2\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+\frac{3\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{(x+2)^2}+1\right)+\frac{1}{4}\,\log\left(\frac{2\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+\frac{\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{(x+2)^2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
 - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
 + 1/2*arcsin(x + 2) - 1/4*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sq
rt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) + 1/4*log(2*(sqrt(-x^2 - 4*x - 3)
- 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)
```

**maple [A]** time = 0.01, size = 130, normalized size = 1.33

$$\frac{\arcsin\left(x+2\right)}{2}-\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^{2}}{\left(-x-\frac{3}{2}\right)^{2}}-12}\,\left(-\operatorname{arctanh}\left(\frac{3x}{\left(-x-\frac{3}{2}\right)\sqrt{\frac{3x^{2}}{\left(-x-\frac{3}{2}\right)^{2}}-12}}\right)+\sqrt{2}\,\arctan\left(\frac{\sqrt{\frac{3x^{2}}{\left(-x-\frac{3}{2}\right)^{2}}-12}\,\sqrt{2}}{6}\right)\right)}{12\sqrt{\frac{\frac{x^{2}}{\left(-x-\frac{3}{2}\right)^{2}}-4}{\left(\frac{x}{-x-\frac{3}{2}}+1\right)^{2}}}\,\left(\frac{x}{-x-\frac{3}{2}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)
```

```
[Out] 1/2*arcsin(x+2)-1/12*3^(1/2)*4^(1/2)*(3/(-x-3/2)^2*x^2-12)^(1/2)*(2^(1/2)*a
rctan(1/6*(3/(-x-3/2)^2*x^2-12)^(1/2)*2^(1/2))-arctanh(3/(-x-3/2)/(3/(-x-3/
2)^2*x^2-12)^(1/2)*x))/((1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)*x+1)^2)^(1/2)/(1/(
-x-3/2)*x+1)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(2\,x^{2}+4\,x+3\right)\sqrt{-x^{2}-4\,x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2}}{\sqrt{-x^{2}-4\,x-3}\,\left(2\,x^{2}+4\,x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)
```

```
[Out] int(x^2/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\sqrt{-\left(x+1\right)\left(x+3\right)}\,\left(2x^{2}+4x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

## 3.129 $\int \frac{x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx$

**Optimal**. Leaf size=68

$$\frac{\tan^{-1}\left(\frac{\frac{3\sqrt{-x-1}}{\sqrt{x+3}}+1}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)}{\sqrt{2}}$$

**Rubi [A]** time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1026, 1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[2] - ArcTan[(1 + (3
 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[2]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1026

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 -
 4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\int \frac{x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx = 8\,\text{Subst}\left(\int \frac{1+3x^2}{-4-8x^2-36x^4}\,dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right)$$

$$= -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2}\,dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right)\right) - \frac{1}{3}\,\text{Subst}\left(\int \frac{\phantom{1}}{\frac{1}{3}+\frac{2}{\phantom{3}}}\right.$$

$$= \frac{2}{3}\,\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx, x, \frac{2}{3}\left(-1+\frac{3+x}{\sqrt{-3-4x-x^2}}\right)\right) + \frac{2}{3}\,\text{Subst}\left(\int \frac{\phantom{1}}{-\frac{8}{9}}\right.$$

$$= \frac{\tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{\sqrt{2}}$$

**Mathematica [C]** time = 0.17, size = 174, normalized size = 2.56

$$\frac{\left(1-i\sqrt{2}\right)\sqrt{1-2i\sqrt{2}}\,\tanh^{-1}\left(\frac{\left(2-i\sqrt{2}\right)x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}}\,\sqrt{-x^2-4x-3}}\right)+\left(1+i\sqrt{2}\right)\sqrt{1+2i\sqrt{2}}\,\tanh^{-1}\left(\frac{\left(2+i\sqrt{2}\right)x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}}\,\sqrt{-x^2-4x-3}}\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] ((1 - I*Sqrt[2])*Sqrt[1 - (2*I)*Sqrt[2]]*ArcTanh[(2 - (2*I)*Sqrt[2] + (2 -
I*Sqrt[2])*x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])] + (1 + I*Sqrt
[2])*Sqrt[1 + (2*I)*Sqrt[2]]*ArcTanh[(2 + (2*I)*Sqrt[2] + (2 + I*Sqrt[2])*x
)/(Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])])/(6*Sqrt[2])
```

**IntegrateAlgebraic** [A] time = 0.25, size = 38, normalized size = 0.56

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x+\frac{3}{\sqrt{2}}}{\sqrt{-x^2-4x-3}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
[Out] -(ArcTan[(3/Sqrt[2] + Sqrt[2]*x)/Sqrt[-3 - 4*x - x^2]]/Sqrt[2])
```

**fricas** [A] time = 0.87, size = 50, normalized size = 0.74

$$\frac{1}{4}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(6\,x^2+20\,x+15\right)\sqrt{-x^2-4\,x-3}}{4\left(2\,x^3+11\,x^2+18\,x+9\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*arctan(1/4*sqrt(2)*(6*x^2 + 20*x + 15)*sqrt(-x^2 - 4*x - 3)/(2*
x^3 + 11*x^2 + 18*x + 9))
```

**giac** [A] time = 0.18, size = 68, normalized size = 1.00

$$\frac{1}{2}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4\,x-3}\,-1\right)}{x+2}+1\right)\right)+\frac{1}{2}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\frac{\sqrt{-x^2-4\,x-3}\,-1}{x+2}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
+ 1/2*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
```

**maple** [A] time = 0.01, size = 92, normalized size = 1.35

$$\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\,\sqrt{2}}{6}\right)}{12\,\sqrt{\frac{\frac{x^2}{\left(-x-\frac{3}{2}\right)^2}-4}{\left(\frac{x}{-x-\frac{3}{2}}+1\right)^2}}\,\left(\frac{x}{-x-\frac{3}{2}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)
```

```
[Out] 1/12*3^(1/2)*4^(1/2)/((1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)*x+1)^2)^(1/2)/(1/(-x
-3/2)*x+1)*(3/(-x-3/2)^2*x^2-12)^(1/2)*2^(1/2)*arctan(1/6*(3/(-x-3/2)^2*x^2
-12)^(1/2)*2^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(2\,x^2+4\,x+3\right)\sqrt{-x^2-4\,x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{-x^2-4\,x-3}\,\left(2\,x^2+4\,x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)
```

```
[Out] int(x/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-\left(x+1\right)\left(x+3\right)}\,\left(2x^2+4x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

## 3.130 $\int \frac{1}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx$

**Optimal**. Leaf size=95

$$-\frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)+\frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)+\frac{1}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

**Rubi [A]** time = 0.11, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {986, 12, 1026, 1161, 618, 204, 1027, 206}

$$-\frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)+\frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)+\frac{1}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] -(Sqrt[2]*ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/3 + (Sqrt[2]*
ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/3 + ArcTanh[x/Sqrt[-3 -
 4*x - x^2]]/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 986

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
 b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1026

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 -
 4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1027

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[g, Subst[Int[1/(a + (c*d - a*f)
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
 EqQ[2*h*d - g*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx = -\left(\frac{1}{6}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx\right) + \frac{1}{6}\int -\frac{4x}{\sqrt{-3-4x-x^2}\left(3+\right.}$$

$$= -\left(\frac{2}{3}\int \frac{x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx\right) + \text{Subst}\left(\int \frac{1}{3-3x^2}\,dx, x, \frac{}{\sqrt{-}}\right.$$

$$= \frac{1}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) - \frac{16}{3}\,\text{Subst}\left(\int \frac{1+3x^2}{-4-8x^2-36x^4}\,dx, x, \frac{1}{\sqrt{-3-}}\right.$$

$$= \frac{1}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{2}{9}\,\text{Subst}\left(\int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2}\,dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-}}\right.$$

$$= \frac{1}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) - \frac{4}{9}\,\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx, x, \frac{2}{3}\left(-1+\frac{3}{\sqrt{-3-}}\right.\right.$$

$$= -\frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) + \frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) + \frac{1}{3}\tanh^{-1}$$

**Mathematica [C]** time = 0.10, size = 150, normalized size = 1.58

$$\frac{1}{6}i\left(\sqrt{1-2i\sqrt{2}}\tanh^{-1}\left(\frac{\left(2-i\sqrt{2}\right)x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right)-\sqrt{1+2i\sqrt{2}}\tanh^{-1}\left(\frac{\left(2+i\sqrt{2}\right)x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] (I/6)*(Sqrt[1 - (2*I)*Sqrt[2]]*ArcTanh[(2 - (2*I)*Sqrt[2] + (2 - I*Sqrt[2])
*x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])] - Sqrt[1 + (2*I)*Sqrt[2
]]*ArcTanh[(2 + (2*I)*Sqrt[2] + (2 + I*Sqrt[2])*x)/(Sqrt[2 - (4*I)*Sqrt[2]]
*Sqrt[-3 - 4*x - x^2])])
```

**IntegrateAlgebraic [A]** time = 0.31, size = 62, normalized size = 0.65

$$\frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x+\frac{3}{\sqrt{2}}}{\sqrt{-x^2-4x-3}}\right)+\frac{1}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] (Sqrt[2]*ArcTan[(3/Sqrt[2] + Sqrt[2]*x)/Sqrt[-3 - 4*x - x^2]])/3 + ArcTanh[
x/Sqrt[-3 - 4*x - x^2]]/3
```

**fricas [A]** time = 0.78, size = 132, normalized size = 1.39

$$-\frac{1}{6}\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)-\frac{1}{6}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x-3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)-\frac{1}{12}\log\left(-\frac{2\sqrt{-x^2-4x-3}x+4x+3}{x^2}\right)+\frac{1}{12}\log\left(\frac{2\sqrt{-x^2-4x-3}x-4x-3}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x +
 3)) - 1/6*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))
/(2*x + 3)) - 1/12*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) + 1/12*lo
g((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)
```

**giac [B]** time = 0.19, size = 165, normalized size = 1.74

$$-\frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)-\frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right)+\frac{1}{6}\log\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{3\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+1\right)-\frac{1}{6}\log\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
 - 1/3*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
 + 1/6*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) -
 1)^2/(x + 2)^2 + 1) - 1/6*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt
(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)
```

**maple [A]** time = 0.01, size = 121, normalized size = 1.27

$$-\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\,\left(\operatorname{arctanh}\left(\frac{3x}{\left(-x-\frac{3}{2}\right)\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}}\right)+\sqrt{2}\,\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\,\sqrt{2}}{6}\right)\right)}{18\sqrt{\frac{\frac{x^2}{\left(-x-\frac{3}{2}\right)^2}-4}{\left(\frac{x}{-x-\frac{3}{2}}+1\right)^2}}\,\left(\frac{x}{-x-\frac{3}{2}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)
```

[Out] -1/18*3^(1/2)*4^(1/2)*(3/(-x-3/2)^2*x^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3/(-x-3/2)^2*x^2-12)^(1/2)*2^(1/2))+arctanh(3/(-x-3/2)/(3/(-x-3/2)^2*x^2-12)^(1/2)*x))/((1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)*x+1)^2)^(1/2)/(1/(-x-3/2)*x+1)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2\,x^2+4\,x+3\right)\sqrt{-x^2-4\,x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-x^2-4\,x-3}\,\left(2\,x^2+4\,x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)

[Out] int(1/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(x+1\right)\left(x+3\right)}\,\left(2x^2+4x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)

[Out] Integral(1/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)

## 3.131 $\int \frac{1}{x\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx$

**Optimal**. Leaf size=130

$$-\frac{\tan^{-1}\left(\frac{2x+3}{\sqrt{3}\sqrt{-x^2-4x-3}}\right)}{3\sqrt{3}}+\frac{1}{9}\sqrt{2}\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)-\frac{1}{9}\sqrt{2}\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)-\frac{4}{9}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

**Rubi [A]** time = 0.42, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {6728, 724, 204, 1028, 986, 12, 1026, 1161, 618, 1027, 206}

$$-\frac{\tan^{-1}\left(\frac{2x+3}{\sqrt{3}\sqrt{-x^2-4x-3}}\right)}{3\sqrt{3}}+\frac{1}{9}\sqrt{2}\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)-\frac{1}{9}\sqrt{2}\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)-\frac{4}{9}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] -ArcTan[(3 + 2*x)/(Sqrt[3]*Sqrt[-3 - 4*x - x^2])]/(3*Sqrt[3]) + (Sqrt[2]*Ar
cTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/9 - (Sqrt[2]*ArcTan[(1 +
(3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/9 - (4*ArcTanh[x/Sqrt[-3 - 4*x - x^
2]])/9
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 986

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
 b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1026

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 -
 4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1027

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[g, Subst[Int[1/(a + (c*d - a*f)
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
 EqQ[2*h*d - g*e, 0]
```

Rule 1028

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(2*h*d - g*e)/e, Int[1/((a + b
*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g
, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

```
&& NeQ[2*h*d - g*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx = \int \left(\frac{1}{3x\sqrt{-3-4x-x^2}} - \frac{2(2+x)}{3\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\right) dx$$

$$= \frac{1}{3}\int \frac{1}{x\sqrt{-3-4x-x^2}}\,dx - \frac{2}{3}\int \frac{2+x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx$$

$$= \frac{1}{6}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx - \frac{1}{3}\int \frac{1}{\sqrt{-3-4x-x^2}\left(3+4x+\right.}$$

$$= -\frac{\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{1}{18}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx - \frac{1}{18}$$

$$= -\frac{\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} - \frac{1}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{2}{9}\int \frac{}{\sqrt{-3-4x}}$$

$$= -\frac{\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} - \frac{4}{9}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{16}{9}\,\text{Subst}\left(\int \frac{}{-4}\right.$$

$$= -\frac{\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} - \frac{4}{9}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) - \frac{2}{27}\,\text{Subst}\left(\int \frac{}{\frac{1}{3}}\right.$$

$$= -\frac{\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} - \frac{4}{9}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{4}{27}\,\text{Subst}\left(\int \frac{}{-}\right.$$

$$= -\frac{\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{1}{9}\sqrt{2}\tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) - \frac{1}{9}\sqrt{2}\tan^{-1}\left(\frac{1-}{}\right.$$

**<u>Mathematica</u>** [C] time = 0.44, size = 200, normalized size = 1.54

$$\frac{1}{54}\left(-6\sqrt{3}\tan^{-1}\left(\frac{2x+3}{\sqrt{3}\sqrt{-x^2-4x-3}}\right) - 3\sqrt{1-2i\sqrt{2}}\left(\sqrt{2}+2i\right)\tanh^{-1}\left(\frac{\left(2-i\sqrt{2}\right)x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right) - 3\sqrt{1+2i\sqrt{2}}\left(\sqrt{2}-2i\right)\tanh^{-1}\left(\frac{\left(2+i\sqrt{2}\right)x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] (-6*Sqrt[3]*ArcTan[(3 + 2*x)/(Sqrt[3]*Sqrt[-3 - 4*x - x^2])] - 3*Sqrt[1 - (
2*I)*Sqrt[2]]*(2*I + Sqrt[2])*ArcTanh[(2 - (2*I)*Sqrt[2] + (2 - I*Sqrt[2])*
x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])] - 3*Sqrt[1 + (2*I)*Sqrt[
```

```
2]]*(-2*I + Sqrt[2])*ArcTanh[(2 + (2*I)*Sqrt[2] + (2 + I*Sqrt[2])*x)/(Sqrt[
2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])])/54
```

**IntegrateAlgebraic [A]** time = 0.33, size = 97, normalized size = 0.75

$$-\frac{1}{9}\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}x+\frac{3}{\sqrt{2}}}{\sqrt{-x^2-4x-3}}\right)+\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{-x^2-4x-3}}{x+3}\right)}{3\sqrt{3}}-\frac{4}{9}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] -1/9*(Sqrt[2]*ArcTan[(3/Sqrt[2] + Sqrt[2]*x)/Sqrt[-3 - 4*x - x^2]]) + (2*Ar
cTan[(Sqrt[3]*Sqrt[-3 - 4*x - x^2])/(3 + x)])/(3*Sqrt[3]) - (4*ArcTanh[x/Sq
rt[-3 - 4*x - x^2]])/9
```

**fricas [A]** time = 1.05, size = 170, normalized size = 1.31

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-x^2-4x-3}(2x+3)}{3(x^2+4x+3)}\right)+\frac{1}{18}\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)+\frac{1}{18}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x-3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)+\frac{1}{9}\log\left(-\frac{2\sqrt{-x^2-4x-3}x+4x+3}{x^2}\right)-\frac{1}{9}\log\left(\frac{2\sqrt{-x^2-4x-3}x-4x-3}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(-x^2 - 4*x - 3)*(2*x + 3)/(x^2 + 4*x +
3)) + 1/18*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/
(2*x + 3)) + 1/18*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*
x - 3))/(2*x + 3)) + 1/9*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) - 1
/9*log((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)
```

**giac [A]** time = 0.20, size = 199, normalized size = 1.53

$$\frac{1}{9}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)+\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)+\frac{1}{9}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right)-\frac{2}{9}\log\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{3\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+1\right)+\frac{2}{9}\log\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/9*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
+ 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)
) + 1/9*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)
) - 2/9*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3)
- 1)^2/(x + 2)^2 + 1) + 2/9*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqr
t(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)
```

**maple [A]** time = 0.02, size = 152, normalized size = 1.17

$$\frac{\sqrt{3}\ \arctan\left(\frac{(-4x-6)\sqrt{3}}{6\sqrt{-x^2-4x-3}}\right)}{9}+\frac{\sqrt{3}\ \sqrt{4}\ \sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\ \left(4\,\mathrm{arctanh}\left(\frac{3x}{\left(-x-\frac{3}{2}\right)\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}}\right)+\sqrt{2}\ \arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\ \sqrt{2}}{6}\right)\right)}{54\sqrt{\frac{\frac{x^2}{\left(-x-\frac{3}{2}\right)^2}-4}{\left(\frac{x}{-x-\frac{3}{2}}+1\right)^2}}\ \left(\frac{x}{-x-\frac{3}{2}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)
```

```
[Out] 1/54*3^(1/2)*4^(1/2)*(3/(-x-3/2)^2*x^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3/(-x
-3/2)^2*x^2-12)^(1/2)*2^(1/2))+4*arctanh(3/(-x-3/2)/(3/(-x-3/2)^2*x^2-12)^(
1/2)*x))/((1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)*x+1)^2)^(1/2)/(1/(-x-3/2)*x+1)+1
/9*3^(1/2)*arctan(1/6*(-6-4*x)*3^(1/2)/(-x^2-4*x-3)^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2\,x^2+4\,x+3\right)\sqrt{-x^2-4\,x-3}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)*x), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\sqrt{-x^2-4\,x-3}\ \left(2\,x^2+4\,x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)
```

```
[Out] int(1/(x*(- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{-(x+1)(x+3)}\ \left(2x^2+4x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

## 3.132 $\int \frac{1}{x^2\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx$

**Optimal**. Leaf size=151

$$\frac{\sqrt{-x^2-4x-3}}{9x}+\frac{2\tan^{-1}\left(\frac{2x+3}{\sqrt{3}\sqrt{-x^2-4x-3}}\right)}{3\sqrt{3}}+\frac{2}{27}\sqrt{2}\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)-\frac{2}{27}\sqrt{2}\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)+\frac{10}{27}\tanh$$

**Rubi [A]** time = 0.45, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6728, 730, 724, 204, 1028, 986, 12, 1026, 1161, 618, 1027, 206}

$$\frac{\sqrt{-x^2-4x-3}}{9x}+\frac{2\tan^{-1}\left(\frac{2x+3}{\sqrt{3}\sqrt{-x^2-4x-3}}\right)}{3\sqrt{3}}+\frac{2}{27}\sqrt{2}\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)-\frac{2}{27}\sqrt{2}\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)+\frac{10}{27}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] Sqrt[-3 - 4*x - x^2]/(9*x) + (2*ArcTan[(3 + 2*x)/(Sqrt[3]*Sqrt[-3 - 4*x - x
^2])])/(3*Sqrt[3]) + (2*Sqrt[2]*ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/S
qrt[2]])/27 - (2*Sqrt[2]*ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]
)/27 + (10*ArcTanh[x/Sqrt[-3 - 4*x - x^2]])/27
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 986

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
 b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1026

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 -
 4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1027

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[g, Subst[Int[1/(a + (c*d - a*f)
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
```

```
EqQ[2*h*d - g*e, 0]
```

Rule 1028

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(2*h*d - g*e)/e, Int[1/((a + b
*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g
, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
 && NeQ[2*h*d - g*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx = \int \left(\frac{1}{3x^2\sqrt{-3-4x-x^2}} - \frac{4}{9x\sqrt{-3-4x-x^2}} + \frac{2(5+4x)}{9\sqrt{-3-4x-x^2}\,\left(3+4\right.}\right.$$

$$= \frac{2}{9}\int \frac{5+4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx + \frac{1}{3}\int \frac{1}{x^2\sqrt{-3-4x-x^2}}\,dx -$$

$$= \frac{\sqrt{-3-4x-x^2}}{9x} - \frac{2}{9}\int \frac{1}{x\sqrt{-3-4x-x^2}}\,dx - \frac{2}{9}\int \frac{1}{\sqrt{-3-4x-x^2}\,(3}$$

$$= \frac{\sqrt{-3-4x-x^2}}{9x} + \frac{4\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{9\sqrt{3}} + \frac{1}{27}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\,(3}$$

$$= \frac{\sqrt{-3-4x-x^2}}{9x} + \frac{2\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{4}{9}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right.$$

$$= \frac{\sqrt{-3-4x-x^2}}{9x} + \frac{2\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{10}{27}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-}}\right.$$

$$= \frac{\sqrt{-3-4x-x^2}}{9x} + \frac{2\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{10}{27}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-}}\right.$$

$$= \frac{\sqrt{-3-4x-x^2}}{9x} + \frac{2\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{10}{27}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-}}\right.$$

$$= \frac{\sqrt{-3-4x-x^2}}{9x} + \frac{2\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{2}{27}\sqrt{2}\,\tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-}}}{\sqrt{2}}\right.$$

**Mathematica** [C] time = 0.41, size = 225, normalized size = 1.49

$$\frac{3\left(\sqrt{-x^2-4x-3}+2\sqrt{3}\,x\tan^{-1}\left(\frac{2x+3}{\sqrt{3}\sqrt{-x^2-4x-3}}\right)\right)+\sqrt{1-2i\sqrt{2}}\,\left(2\sqrt{2}+i\right)x\tanh^{-1}\left(\frac{-i\sqrt{2}\,x+2x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}}\,\sqrt{-x^2-4x-3}}\right)+\sqrt{1+2i\sqrt{2}}\,\left(2\sqrt{2}-i\right)x\tanh^{-1}\left(\frac{\left(2+i\sqrt{2}\right)x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}}\,\sqrt{-x^2-4x-3}}\right)}{27x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] (3*(Sqrt[-3 - 4*x - x^2] + 2*Sqrt[3]*x*ArcTan[(3 + 2*x)/(Sqrt[3]*Sqrt[-3 - 
4*x - x^2])]) + Sqrt[1 - (2*I)*Sqrt[2]]*(I + 2*Sqrt[2])*x*ArcTanh[(2 - (2*I
```

```
)*Sqrt[2] + 2*x - I*Sqrt[2]*x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2
])] + Sqrt[1 + (2*I)*Sqrt[2]]*(-I + 2*Sqrt[2])*x*ArcTanh[(2 + (2*I)*Sqrt[2]
 + (2 + I*Sqrt[2])*x)/(Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])]])/(27*
x)
```

**IntegrateAlgebraic [A]** time = 0.43, size = 118, normalized size = 0.78

$$\frac{\sqrt{-x^2-4x-3}}{9x}-\frac{2}{27}\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}x+\frac{3}{\sqrt{2}}}{\sqrt{-x^2-4x-3}}\right)-\frac{4\tan^{-1}\left(\frac{\sqrt{3}\sqrt{-x^2-4x-3}}{x+3}\right)}{3\sqrt{3}}+\frac{10}{27}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] Sqrt[-3 - 4*x - x^2]/(9*x) - (2*Sqrt[2]*ArcTan[(3/Sqrt[2] + Sqrt[2]*x)/Sqrt
[-3 - 4*x - x^2]])/27 - (4*ArcTan[(Sqrt[3]*Sqrt[-3 - 4*x - x^2])/(3 + x)])/
(3*Sqrt[3]) + (10*ArcTanh[x/Sqrt[-3 - 4*x - x^2]])/27
```

**fricas [A]** time = 1.09, size = 194, normalized size = 1.28

$$-\frac{12\sqrt{3}x\arctan\left(\frac{\sqrt{3}\sqrt{-x^2-4x-3}(2x+3)}{3(x^2+4x+3)}\right)-2\sqrt{2}x\arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)-2\sqrt{2}x\arctan\left(-\frac{\sqrt{2}x-3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)+5x\log\left(-\frac{2\sqrt{-x^2-4x-3}x+4x+3}{x^2}\right)-5x\log\left(\frac{2\sqrt{-x^2-4x-3}x-4x-3}{x^2}\right)-6\sqrt{-x^2-4x-3}}{54x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/54*(12*sqrt(3)*x*arctan(1/3*sqrt(3)*sqrt(-x^2 - 4*x - 3)*(2*x + 3)/(x^2
+ 4*x + 3)) - 2*sqrt(2)*x*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x
 - 3))/(2*x + 3)) - 2*sqrt(2)*x*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^
2 - 4*x - 3))/(2*x + 3)) + 5*x*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^
2) - 5*x*log((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2) - 6*sqrt(-x^2 - 4*x
- 3))/x
```

**giac [B]** time = 0.48, size = 269, normalized size = 1.78

$$\frac{2}{27}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)-\frac{4}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)+\frac{2}{27}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right)-\frac{\frac{\sqrt{-x^2-4x-3}-1}{x+2}+2}{18\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+\frac{\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+1\right)}+\frac{5}{27}\log\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{3\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+1\right)-\frac{5}{27}\log\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
```

```
[Out] 2/27*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
 - 4/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1
)) + 2/27*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) +
1)) - 1/18*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 2)/((sqrt(-x^2 - 4*x - 3)
```

```
- 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) + 5/27*log(2*(sq
rt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2
+ 1) - 5/27*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3
) - 1)^2/(x + 2)^2 + 3)
```

**maple [A]** time = 0.02, size = 169, normalized size = 1.12

$$-\frac{2\sqrt{3}\,\arctan\left(\frac{(-4x-6)\sqrt{3}}{6\sqrt{-x^2-4x-3}}\right)}{9}+\frac{\sqrt{-x^2-4x-3}}{9x}+\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\,\left(-5\,\mathrm{arctanh}\left(\frac{3x}{\left(-x-\frac{3}{2}\right)\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}}\right)+\sqrt{2}\,\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\,\sqrt{2}}{6}\right)\right)}{81\sqrt{\frac{\frac{x^2}{\left(-x-\frac{3}{2}\right)^2}-4}{\left(\frac{x}{-x-\frac{3}{2}}+1\right)^2}}\,\left(\frac{x}{-x-\frac{3}{2}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)
```

```
[Out] 1/81*3^(1/2)*4^(1/2)*(3/(-x-3/2)^2*x^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3/(-x
-3/2)^2*x^2-12)^(1/2)*2^(1/2))-5*arctanh(3/(-x-3/2)/(3/(-x-3/2)^2*x^2-12)^(
1/2)*x))/((1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)*x+1)^2)^(1/2)/(1/(-x-3/2)*x+1)+1
/9*(-x^2-4*x-3)^(1/2)/x-2/9*3^(1/2)*arctan(1/6*(-4*x-6)*3^(1/2)/(-x^2-4*x-3
)^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2\,x^2+4\,x+3\right)\sqrt{-x^2-4\,x-3}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)*x^2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\sqrt{-x^2-4\,x-3}\,\left(2\,x^2+4\,x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)
```

```
[Out] int(1/(x^2*(- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{-\left(x+1\right)\left(x+3\right)}\,\left(2x^2+4x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

## 3.133 $\int (2+3x)^2 \left(30+31x-12x^2\right)^2 \sqrt{6+17x+12x^2}\, dx$

**Optimal**. Leaf size=149

$$-\frac{1}{32}(10-3x)\left(12x^2+17x+6\right)^{7/2}-\frac{873\left(12x^2+17x+6\right)^{7/2}}{1792}+\frac{25091(24x+17)\left(12x^2+17x+6\right)^{5/2}}{24576}-\frac{125455(24x+}{}$$

**Rubi** [A] time = 0.09, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1002, 742, 640, 612, 621, 206}

$$-\frac{1}{32}(10-3x)\left(12x^2+17x+6\right)^{7/2}-\frac{873\left(12x^2+17x+6\right)^{7/2}}{1792}+\frac{25091(24x+17)\left(12x^2+17x+6\right)^{5/2}}{24576}-\frac{125455(24x+17)\left(12x^2+17x+6\right)^{3/2}}{4718592}+\frac{125455(24x+17)\sqrt{12x^2+17x+6}}{150994944}-\frac{125455\tanh^{-1}\left(\frac{24x+17}{4\sqrt{3}\sqrt{12x^2+17x+6}}\right)}{603979776\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)^2*(30 + 31*x - 12*x^2)^2*Sqrt[6 + 17*x + 12*x^2],x]
```

```
[Out] (125455*(17 + 24*x)*Sqrt[6 + 17*x + 12*x^2])/150994944 - (125455*(17 + 24*x
)*(6 + 17*x + 12*x^2)^(3/2))/4718592 + (25091*(17 + 24*x)*(6 + 17*x + 12*x^
2)^(5/2))/24576 - (873*(6 + 17*x + 12*x^2)^(7/2))/1792 - ((10 - 3*x)*(6 + 1
7*x + 12*x^2)^(7/2))/32 - (125455*ArcTanh[(17 + 24*x)/(4*Sqrt[3]*Sqrt[6 + 1
7*x + 12*x^2])])/(603979776*Sqrt[3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1002

```
Int[((g_) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.
) + (e_.)*(x_) + (f_.)*(x_)^2)^(m_.), x_Symbol] :> Int[((d*g)/a + (f*h*x)/c
)^m*(a + b*x + c*x^2)^(m + p), x] /; FreeQ[{a, b, c, d, e, f, g, h, p}, x]
&& EqQ[c*g^2 - b*g*h + a*h^2, 0] && EqQ[c^2*d*g^2 - a*c*e*g*h + a^2*f*h^2,
0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int (2+3x)^2\left(30+31x-12x^2\right)^2\sqrt{6+17x+12x^2}\,dx &= \int (10-3x)^2\left(6+17x+12x^2\right)^{5/2}\,dx \\
&= -\frac{1}{32}(10-3x)\left(6+17x+12x^2\right)^{7/2}+\frac{1}{96}\int\left(11331-\frac{785}{2}\right. \\
&= -\frac{873\left(6+17x+12x^2\right)^{7/2}}{1792}-\frac{1}{32}(10-3x)\left(6+17x+12x^2\right. \\
&= \frac{25091(17+24x)\left(6+17x+12x^2\right)^{5/2}}{24576}-\frac{873\left(6+17x+1\right.}{1792} \\
&= -\frac{125455(17+24x)\left(6+17x+12x^2\right)^{3/2}}{4718592}+\frac{25091(17+2}{} \\
&= \frac{125455(17+24x)\sqrt{6+17x+12x^2}}{150994944}-\frac{125455(17+24x)}{471} \\
&= \frac{125455(17+24x)\sqrt{6+17x+12x^2}}{150994944}-\frac{125455(17+24x)}{471} \\
&= \frac{125455(17+24x)\sqrt{6+17x+12x^2}}{150994944}-\frac{125455(17+24x)}{471}
\end{aligned}
$$

**Mathematica** [A]  time = 0.19, size = 87, normalized size = 0.58

$$
\frac{12\sqrt{12x^2+17x+6}\left(171228266496x^7-732816211968x^6-1190083166208x^5+3438453030912x^4+8974844476416x^3+7899203409792x^2+3132157281976x+474999091769\right)-878185\sqrt{3}\tanh^{-1}\left(\frac{24x+17}{4\sqrt{36x^2+51x+18}}\right)}{12683575296}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)^2*(30 + 31*x - 12*x^2)^2*Sqrt[6 + 17*x + 12*x^2],x]
```

```
[Out] (12*Sqrt[6 + 17*x + 12*x^2]*(474999091769 + 3132157281976*x + 7899203409792
*x^2 + 8974844476416*x^3 + 3438453030912*x^4 - 1190083166208*x^5 - 73281621
1968*x^6 + 171228266496*x^7) - 878185*Sqrt[3]*ArcTanh[(17 + 24*x)/(4*Sqrt[1
8 + 51*x + 36*x^2])])/12683575296
```

**IntegrateAlgebraic** [A]  time = 0.73, size = 92, normalized size = 0.62

$$
\frac{\sqrt{12x^2+17x+6}\left(171228266496x^7-732816211968x^6-1190083166208x^5+3438453030912x^4+8974844476416x^3+7899203409792x^2+3132157281976x+474999091769\right)}{1056964608}-\frac{125455\tanh^{-1}\left(\frac{2\sqrt{12x^2+17x+6}}{\sqrt{3}(4x+3)}\right)}{301989888\sqrt{3}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 3*x)^2*(30 + 31*x - 12*x^2)^2*Sqrt[6 + 17*x + 12*x^
2],x]
```

```
[Out] (Sqrt[6 + 17*x + 12*x^2]*(474999091769 + 3132157281976*x + 7899203409792*x^
2 + 8974844476416*x^3 + 3438453030912*x^4 - 1190083166208*x^5 - 73281621196
8*x^6 + 171228266496*x^7))/1056964608 - (125455*ArcTanh[(2*Sqrt[6 + 17*x +
12*x^2])/(Sqrt[3]*(3 + 4*x))])/(301989888*Sqrt[3])
```

**fricas** [A] time = 0.95, size = 88, normalized size = 0.59

$$\frac{1}{1056964608}\left(171228266496\,x^{7}-732816211968\,x^{6}-1190083166208\,x^{5}+3438453030912\,x^{4}+8974844476416\,x^{3}+7899203409792\,x^{2}+3132157281976\,x+474999091769\right)\sqrt{12\,x^{2}+17\,x+6}+\frac{125455}{3623878656}\sqrt{3}\log\left(-8\sqrt{3}\sqrt{12\,x^{2}+17\,x+6}\,(24\,x+17)+1152\,x^{2}+1632\,x+577\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2*(-12*x^2+31*x+30)^2*(12*x^2+17*x+6)^(1/2),x, algorithm=
"fricas")
```

```
[Out] 1/1056964608*(171228266496*x^7 - 732816211968*x^6 - 1190083166208*x^5 + 343
8453030912*x^4 + 8974844476416*x^3 + 7899203409792*x^2 + 3132157281976*x +
474999091769)*sqrt(12*x^2 + 17*x + 6) + 125455/3623878656*sqrt(3)*log(-8*sq
rt(3)*sqrt(12*x^2 + 17*x + 6)*(24*x + 17) + 1152*x^2 + 1632*x + 577)
```

**giac** [A] time = 0.30, size = 85, normalized size = 0.57

$$\frac{1}{1056964608}\left(8\left(48\left(24\left(96\left(24\left(48\left(168\,x-719\right)x-56047\right)x+3886417\right)x+973832951\right)x+20570842213\right)x+391519660247\right)x+474999091769\right)\sqrt{12\,x^{2}+17\,x+6}+\frac{125455}{1811939328}\sqrt{3}\log\left(\left|-4\sqrt{3}\left(2\sqrt{3}x-\sqrt{12\,x^{2}+17\,x+6}\right)-17\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2*(-12*x^2+31*x+30)^2*(12*x^2+17*x+6)^(1/2),x, algorithm=
"giac")
```

```
[Out] 1/1056964608*(8*(48*(24*(96*(24*(48*(168*x - 719)*x - 56047)*x + 3886417)*x
 + 973832951)*x + 20570842213)*x + 391519660247)*x + 474999091769)*sqrt(12*
x^2 + 17*x + 6) + 125455/1811939328*sqrt(3)*log(abs(-4*sqrt(3)*(2*sqrt(3)*x
 - sqrt(12*x^2 + 17*x + 6)) - 17))
```

**maple** [A] time = 0.02, size = 147, normalized size = 0.99

$$\frac{27\left(12x^{2}+17x+6\right)^{\frac{3}{2}}x^{5}}{2}-\frac{8613\left(12x^{2}+17x+6\right)^{\frac{3}{2}}x^{4}}{112}+\frac{14991\left(12x^{2}+17x+6\right)^{\frac{3}{2}}x^{3}}{1792}+\frac{4267751\left(12x^{2}+17x+6\right)^{\frac{3}{2}}x^{2}}{14336}+\frac{129220757\left(12x^{2}+17x+6\right)^{\frac{3}{2}}x}{458752}-\frac{125455\sqrt{12}\,\ln\left(\frac{\left(12x+\frac{17}{2}\right)\sqrt{12}}{12}+\sqrt{12x^{2}+17x+6}\right)}{3623878656}+\frac{2473875847\left(12x^{2}+17x+6\right)^{\frac{3}{2}}}{33030144}+\frac{125455\,(24x+17)\sqrt{12x^{2}+17x+6}}{150994944}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)^2*(-12*x^2+31*x+30)^2*(12*x^2+17*x+6)^(1/2),x)
```

```
[Out] 27/2*x^5*(12*x^2+17*x+6)^(3/2)-8613/112*x^4*(12*x^2+17*x+6)^(3/2)+14991/179
2*x^3*(12*x^2+17*x+6)^(3/2)+4267751/14336*x^2*(12*x^2+17*x+6)^(3/2)+1292207
57/458752*x*(12*x^2+17*x+6)^(3/2)+2473875847/33030144*(12*x^2+17*x+6)^(3/2)
-125455/3623878656*ln(1/12*(17/2+12*x)*12^(1/2)+(12*x^2+17*x+6)^(1/2))*12^(
1/2)+125455/150994944*(17+24*x)*(12*x^2+17*x+6)^(1/2)
```

**maxima [A]** time = 1.00, size = 155, normalized size = 1.04

$$\frac{27}{2}\left(12x^2+17x+6\right)^{\frac{3}{2}}x^5-\frac{8613}{112}\left(12x^2+17x+6\right)^{\frac{3}{2}}x^4+\frac{14991}{1792}\left(12x^2+17x+6\right)^{\frac{3}{2}}x^3+\frac{4267751}{14336}\left(12x^2+17x+6\right)^{\frac{3}{2}}x^2+\frac{129220757}{458752}\left(12x^2+17x+6\right)^{\frac{3}{2}}x+\frac{2473875847}{33030144}\left(12x^2+17x+6\right)^{\frac{3}{2}}+\frac{125455}{6291456}\sqrt{12x^2+17x+6}\,x-\frac{125455}{1811939328}\sqrt{3}\log\left(4\sqrt{3}\sqrt{12x^2+17x+6}+24x+17\right)+\frac{2132735}{150994944}\sqrt{12x^2+17x+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2*(-12*x^2+31*x+30)^2*(12*x^2+17*x+6)^(1/2),x, algorithm=
"maxima")
```

```
[Out] 27/2*(12*x^2 + 17*x + 6)^(3/2)*x^5 - 8613/112*(12*x^2 + 17*x + 6)^(3/2)*x^4
 + 14991/1792*(12*x^2 + 17*x + 6)^(3/2)*x^3 + 4267751/14336*(12*x^2 + 17*x
+ 6)^(3/2)*x^2 + 129220757/458752*(12*x^2 + 17*x + 6)^(3/2)*x + 2473875847/
33030144*(12*x^2 + 17*x + 6)^(3/2) + 125455/6291456*sqrt(12*x^2 + 17*x + 6)
*x - 125455/1811939328*sqrt(3)*log(4*sqrt(3)*sqrt(12*x^2 + 17*x + 6) + 24*x
 + 17) + 2132735/150994944*sqrt(12*x^2 + 17*x + 6)
```

**mupad [B]** time = 5.08, size = 187, normalized size = 1.26

$$\frac{4267751\,x^2\left(12x^2+17x+6\right)^{3/2}}{14336}+\frac{14991\,x^3\left(12x^2+17x+6\right)^{3/2}}{1792}-\frac{8613\,x^4\left(12x^2+17x+6\right)^{3/2}}{112}+\frac{27\,x^5\left(12x^2+17x+6\right)^{3/2}}{2}-\frac{146030443\sqrt{12}\,\ln\left(\sqrt{12x^2+17x+6}+\frac{\sqrt{12}\left(12x+\frac{17}{2}\right)}{12}\right)}{88080384}+\frac{438091329\left(\frac{x}{2}+\frac{17}{48}\right)\sqrt{12x^2+17x+6}}{229376}+\frac{2473875847\sqrt{12x^2+17x+6}\left(1152x^2+408x-291\right)}{3170893824}+\frac{129220757\,x\left(12x^2+17x+6\right)^{3/2}}{458752}+\frac{42055889399\sqrt{12}\,\ln\left(2\sqrt{12x^2+17x+6}+\frac{\sqrt{12}\,(24x+17)}{12}\right)}{25367150592}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 2)^2*(17*x + 12*x^2 + 6)^(1/2)*(31*x - 12*x^2 + 30)^2,x)
```

```
[Out] (4267751*x^2*(17*x + 12*x^2 + 6)^(3/2))/14336 + (14991*x^3*(17*x + 12*x^2 +
 6)^(3/2))/1792 - (8613*x^4*(17*x + 12*x^2 + 6)^(3/2))/112 + (27*x^5*(17*x
+ 12*x^2 + 6)^(3/2))/2 - (146030443*12^(1/2)*log((17*x + 12*x^2 + 6)^(1/2)
+ (12^(1/2)*(12*x + 17/2))/12))/88080384 + (438091329*(x/2 + 17/48)*(17*x +
 12*x^2 + 6)^(1/2))/229376 + (2473875847*(17*x + 12*x^2 + 6)^(1/2)*(408*x +
 1152*x^2 - 291))/3170893824 + (129220757*x*(17*x + 12*x^2 + 6)^(3/2))/4587
52 + (42055889399*12^(1/2)*log(2*(17*x + 12*x^2 + 6)^(1/2) + (12^(1/2)*(24*
x + 17))/12))/25367150592
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{(3x+2)\,(4x+3)}\;(3x-10)^2\,(3x+2)^2\,(4x+3)^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**2*(-12*x**2+31*x+30)**2*(12*x**2+17*x+6)**(1/2),x)
```

```
[Out] Integral(sqrt((3*x + 2)*(4*x + 3))*(3*x - 10)**2*(3*x + 2)**2*(4*x + 3)**2,
 x)
```

## 3.134 $\int (2+3x)\left(30+31x-12x^2\right)\sqrt{6+17x+12x^2}\,dx$

**Optimal**. Leaf size=103

$$-\frac{1}{20}\left(12x^2+17x+6\right)^{5/2}+\frac{97}{768}(24x+17)\left(12x^2+17x+6\right)^{3/2}-\frac{97(24x+17)\sqrt{12x^2+17x+6}}{24576}+\frac{97\tanh^{-1}\left(\frac{24x+17}{4\sqrt{3}\sqrt{12x^2+17x+6}}\right)}{98304\sqrt{3}}$$

**Rubi [A]** time = 0.04, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1002, 640, 612, 621, 206}

$$-\frac{1}{20}\left(12x^2+17x+6\right)^{5/2}+\frac{97}{768}(24x+17)\left(12x^2+17x+6\right)^{3/2}-\frac{97(24x+17)\sqrt{12x^2+17x+6}}{24576}+\frac{97\tanh^{-1}\left(\frac{24x+17}{4\sqrt{3}\sqrt{12x^2+17x+6}}\right)}{98304\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)*(30 + 31*x - 12*x^2)*Sqrt[6 + 17*x + 12*x^2],x]
```

```
[Out] (-97*(17 + 24*x)*Sqrt[6 + 17*x + 12*x^2])/24576 + (97*(17 + 24*x)*(6 + 17*x
 + 12*x^2)^(3/2))/768 - (6 + 17*x + 12*x^2)^(5/2)/20 + (97*ArcTanh[(17 + 24
*x)/(4*Sqrt[3]*Sqrt[6 + 17*x + 12*x^2])])/(98304*Sqrt[3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1002

```
Int[((g_) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.
) + (e_.)*(x_) + (f_.)*(x_)^2)^(m_.), x_Symbol] :> Int[((d*g)/a + (f*h*x)/c
)^m*(a + b*x + c*x^2)^(m + p), x] /; FreeQ[{a, b, c, d, e, f, g, h, p}, x]
&& EqQ[c*g^2 - b*g*h + a*h^2, 0] && EqQ[c^2*d*g^2 - a*c*e*g*h + a^2*f*h^2,
0] && IntegerQ[m]
```

Rubi steps

$$\int (2+3x)\left(30+31x-12x^2\right)\sqrt{6+17x+12x^2}\,dx = \int (10-3x)\left(6+17x+12x^2\right)^{3/2}\,dx$$
$$= -\frac{1}{20}\left(6+17x+12x^2\right)^{5/2}+\frac{97}{8}\int \left(6+17x+12x^2\right)^{3/2}\,dx$$
$$= \frac{97}{768}(17+24x)\left(6+17x+12x^2\right)^{3/2}-\frac{1}{20}\left(6+17x+12x^2\right)^{5}$$
$$= -\frac{97(17+24x)\sqrt{6+17x+12x^2}}{24576}+\frac{97}{768}(17+24x)\left(6+17x\right.$$
$$= -\frac{97(17+24x)\sqrt{6+17x+12x^2}}{24576}+\frac{97}{768}(17+24x)\left(6+17x\right.$$
$$= -\frac{97(17+24x)\sqrt{6+17x+12x^2}}{24576}+\frac{97}{768}(17+24x)\left(6+17x\right.$$

**Mathematica [A]** time = 0.03, size = 72, normalized size = 0.70

$$\frac{485\sqrt{3}\tanh^{-1}\left(\frac{24x+17}{4\sqrt{36x^2+51x+18}}\right)+12\sqrt{12x^2+17x+6}\left(-884736x^4+1963008x^3+6837888x^2+5455144x+1353611\right)}{1474560}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)*(30 + 31*x - 12*x^2)*Sqrt[6 + 17*x + 12*x^2],x]
```

```
[Out] (12*Sqrt[6 + 17*x + 12*x^2]*(1353611 + 5455144*x + 6837888*x^2 + 1963008*x^
3 - 884736*x^4) + 485*Sqrt[3]*ArcTanh[(17 + 24*x)/(4*Sqrt[18 + 51*x + 36*x^
2])])/1474560
```

**IntegrateAlgebraic [A]** time = 0.44, size = 77, normalized size = 0.75

$$\frac{97\tanh^{-1}\left(\frac{2\sqrt{12x^2+17x+6}}{\sqrt{3}(4x+3)}\right)}{49152\sqrt{3}}+\frac{\sqrt{12x^2+17x+6}\left(-884736x^4+1963008x^3+6837888x^2+5455144x+1353611\right)}{122880}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 3*x)*(30 + 31*x - 12*x^2)*Sqrt[6 + 17*x + 12*x^2],x
]
```

```
[Out] (Sqrt[6 + 17*x + 12*x^2]*(1353611 + 5455144*x + 6837888*x^2 + 1963008*x^3 -
 884736*x^4))/122880 + (97*ArcTanh[(2*Sqrt[6 + 17*x + 12*x^2])/(Sqrt[3]*(3
+ 4*x))])/(49152*Sqrt[3])
```

**fricas [A]** time = 0.87, size = 73, normalized size = 0.71

$$-\frac{1}{122880}\left(884736\,x^4-1963008\,x^3-6837888\,x^2-5455144\,x-1353611\right)\sqrt{12\,x^2+17\,x+6}+\frac{97}{589824}\sqrt{3}\log\left(8\sqrt{3}\sqrt{12\,x^2+17\,x+6}(24\,x+17)+1152\,x^2+1632\,x+577\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(-12*x^2+31*x+30)*(12*x^2+17*x+6)^(1/2),x, algorithm="fri
cas")
```

```
[Out] -1/122880*(884736*x^4 - 1963008*x^3 - 6837888*x^2 - 5455144*x - 1353611)*sq
rt(12*x^2 + 17*x + 6) + 97/589824*sqrt(3)*log(8*sqrt(3)*sqrt(12*x^2 + 17*x
+ 6)*(24*x + 17) + 1152*x^2 + 1632*x + 577)
```

**giac [A]** time = 0.20, size = 70, normalized size = 0.68

$$-\frac{1}{122880}(8(48(72(32\,x-71)x-17807)x-681893)x-1353611)\sqrt{12\,x^2+17\,x+6}-\frac{97}{294912}\sqrt{3}\log\left(\left|-4\sqrt{3}\left(2\sqrt{3}x-\sqrt{12\,x^2+17\,x+6}\right)-17\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(-12*x^2+31*x+30)*(12*x^2+17*x+6)^(1/2),x, algorithm="gia
c")
```

```
[Out] -1/122880*(8*(48*(72*(32*x - 71)*x - 17807)*x - 681893)*x - 1353611)*sqrt(1
2*x^2 + 17*x + 6) - 97/294912*sqrt(3)*log(abs(-4*sqrt(3)*(2*sqrt(3)*x - sqr
t(12*x^2 + 17*x + 6)) - 17))
```

**maple [A]** time = 0.01, size = 96, normalized size = 0.93

$$-\frac{3\left(12x^2+17x+6\right)^{\frac{3}{2}}x^2}{5}+\frac{349\left(12x^2+17x+6\right)^{\frac{3}{2}}x}{160}+\frac{97\sqrt{12}\,\ln\left(\frac{\left(12x+\frac{17}{2}\right)\sqrt{12}}{12}+\sqrt{12x^2+17x+6}\right)}{589824}+\frac{7093\left(12x^2+17x+6\right)^{\frac{3}{2}}}{3840}-\frac{97(24x+17)\sqrt{12x^2+17x+6}}{24576}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)*(-12*x^2+31*x+30)*(12*x^2+17*x+6)^(1/2),x)
```

```
[Out] -3/5*(12*x^2+17*x+6)^(3/2)*x^2+349/160*(12*x^2+17*x+6)^(3/2)*x+7093/3840*(1
2*x^2+17*x+6)^(3/2)-97/24576*(24*x+17)*(12*x^2+17*x+6)^(1/2)+97/589824*12^(
1/2)*ln(1/12*(12*x+17/2)*12^(1/2)+(12*x^2+17*x+6)^(1/2))
```

**maxima [A]** time = 0.98, size = 104, normalized size = 1.01

$$-\frac{3}{5}\left(12x^2+17x+6\right)^{\frac{3}{2}}x^2+\frac{349}{160}\left(12x^2+17x+6\right)^{\frac{3}{2}}x+\frac{7093}{3840}\left(12x^2+17x+6\right)^{\frac{3}{2}}-\frac{97}{1024}\sqrt{12x^2+17x+6}\,x+\frac{97}{294912}\sqrt{3}\log\left(4\sqrt{3}\sqrt{12x^2+17x+6}+24x+17\right)-\frac{1649}{24576}\sqrt{12x^2+17x+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(-12*x^2+31*x+30)*(12*x^2+17*x+6)^(1/2),x, algorithm="max
ima")
```

```
[Out] -3/5*(12*x^2 + 17*x + 6)^(3/2)*x^2 + 349/160*(12*x^2 + 17*x + 6)^(3/2)*x +
7093/3840*(12*x^2 + 17*x + 6)^(3/2) - 97/1024*sqrt(12*x^2 + 17*x + 6)*x + 9
7/294912*sqrt(3)*log(4*sqrt(3)*sqrt(12*x^2 + 17*x + 6) + 24*x + 17) - 1649/
24576*sqrt(12*x^2 + 17*x + 6)
```

**mupad [B]** time = 4.69, size = 136, normalized size = 1.32

$$\frac{3753\left(\frac{x}{2}+\frac{17}{48}\right)\sqrt{12x^2+17x+6}}{80}-\frac{417\sqrt{12}\,\ln\left(\sqrt{12x^2+17x+6}+\frac{\sqrt{12}\left(12x+\frac{17}{2}\right)}{12}\right)}{10240}-\frac{3x^2\left(12x^2+17x+6\right)^{3/2}}{5}+\frac{7093\sqrt{12x^2+17x+6}\left(1152x^2+408x-291\right)}{368640}+\frac{349x\left(12x^2+17x+6\right)^{3/2}}{160}+\frac{120581\sqrt{12}\,\ln\left(2\sqrt{12x^2+17x+6}+\frac{\sqrt{12}\,(24x+17)}{12}\right)}{2949120}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 2)*(17*x + 12*x^2 + 6)^(1/2)*(31*x - 12*x^2 + 30),x)
```

```
[Out] (3753*(x/2 + 17/48)*(17*x + 12*x^2 + 6)^(1/2))/80 - (417*12^(1/2)*log((17*x
 + 12*x^2 + 6)^(1/2) + (12^(1/2)*(12*x + 17/2))/12))/10240 - (3*x^2*(17*x +
 12*x^2 + 6)^(3/2))/5 + (7093*(17*x + 12*x^2 + 6)^(1/2)*(408*x + 1152*x^2 -
 291))/368640 + (349*x*(17*x + 12*x^2 + 6)^(3/2))/160 + (120581*12^(1/2)*lo
g(2*(17*x + 12*x^2 + 6)^(1/2) + (12^(1/2)*(24*x + 17))/12))/2949120
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-152x\sqrt{12x^2+17x+6}\right)dx-\int\left(-69x^2\sqrt{12x^2+17x+6}\right)dx-\int 36x^3\sqrt{12x^2+17x+6}\,dx-\int\left(-60\sqrt{12x^2+17x+6}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(-12*x**2+31*x+30)*(12*x**2+17*x+6)**(1/2),x)
```

```
[Out] -Integral(-152*x*sqrt(12*x**2 + 17*x + 6), x) - Integral(-69*x**2*sqrt(12*x
**2 + 17*x + 6), x) - Integral(36*x**3*sqrt(12*x**2 + 17*x + 6), x) - Integ
ral(-60*sqrt(12*x**2 + 17*x + 6), x)
```

## 3.135 $\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)(30+31x-12x^2)}\,dx$

**Optimal**. Leaf size=28

$$\frac{1}{42}\tanh^{-1}\left(\frac{291x+206}{84\sqrt{12x^2+17x+6}}\right)$$

**Rubi** [A] time = 0.05, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {1002, 724, 206}

$$\frac{1}{42}\tanh^{-1}\left(\frac{291x+206}{84\sqrt{12x^2+17x+6}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)*(30 + 31*x - 12*x^2)),x]
```

```
[Out] ArcTanh[(206 + 291*x)/(84*Sqrt[6 + 17*x + 12*x^2])]/42
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1002

```
Int[((g_) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.
) + (e_.)*(x_) + (f_.)*(x_)^2)^(m_.), x_Symbol] :> Int[((d*g)/a + (f*h*x)/c
)^m*(a + b*x + c*x^2)^(m + p), x] /; FreeQ[{a, b, c, d, e, f, g, h, p}, x]
&& EqQ[c*g^2 - b*g*h + a*h^2, 0] && EqQ[c^2*d*g^2 - a*c*e*g*h + a^2*f*h^2,
0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)\left(30+31x-12x^2\right)}\,dx = \int \frac{1}{(10-3x)\sqrt{6+17x+12x^2}}\,dx$$
$$= -\left(2\,\text{Subst}\left(\int \frac{1}{7056-x^2}\,dx, x, \frac{-206-291x}{\sqrt{6+17x+12x^2}}\right)\right)$$
$$= \frac{1}{42}\tanh^{-1}\left(\frac{206+291x}{84\sqrt{6+17x+12x^2}}\right)$$

**Mathematica** [A] time = 0.10, size = 37, normalized size = 1.32

$$\frac{1}{42}\log\left(84\sqrt{12x^2+17x+6}+291x+206\right)-\frac{1}{42}\log(10-3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)*(30 + 31*x - 12*x^2)),x]
[Out] -1/42*Log[10 - 3*x] + Log[206 + 291*x + 84*Sqrt[6 + 17*x + 12*x^2]]/42
```

**IntegrateAlgebraic** [A] time = 0.30, size = 30, normalized size = 1.07

$$\frac{1}{21}\tanh^{-1}\left(\frac{6\sqrt{12x^2+17x+6}}{7(3x+2)}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)*(30 + 31*x - 12*x^2))
,x]
[Out] ArcTanh[(6*Sqrt[6 + 17*x + 12*x^2])/(7*(2 + 3*x))]/21
```

**fricas** [B] time = 0.63, size = 53, normalized size = 1.89

$$\frac{1}{84}\log\left(\frac{291\,x+84\,\sqrt{12\,x^2+17\,x+6}+206}{x}\right)-\frac{1}{84}\log\left(\frac{291\,x-84\,\sqrt{12\,x^2+17\,x+6}+206}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)/(-12*x^2+31*x+30),x, algorithm="fri
cas")
[Out] 1/84*log((291*x + 84*sqrt(12*x^2 + 17*x + 6) + 206)/x) - 1/84*log((291*x -
84*sqrt(12*x^2 + 17*x + 6) + 206)/x)
```

**giac [B]** time = 0.27, size = 63, normalized size = 2.25

$$\frac{1}{42}\,\log\left(\left|-6\,\sqrt{3}x+20\,\sqrt{3}+3\,\sqrt{12\,x^2+17\,x+6}+42\right|\right)-\frac{1}{42}\,\log\left(\left|-6\,\sqrt{3}x+20\,\sqrt{3}+3\,\sqrt{12\,x^2+17\,x+6}-42\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)/(-12*x^2+31*x+30),x, algorithm="gia
c")
```

```
[Out] 1/42*log(abs(-6*sqrt(3)*x + 20*sqrt(3) + 3*sqrt(12*x^2 + 17*x + 6) + 42)) -
 1/42*log(abs(-6*sqrt(3)*x + 20*sqrt(3) + 3*sqrt(12*x^2 + 17*x + 6) - 42))
```

**maple [B]** time = 0.02, size = 163, normalized size = 5.82

$$\frac{\operatorname{arctanh}\left(\frac{97x+\frac{206}{3}}{28\sqrt{97x+12\left(x-\frac{10}{3}\right)^2-\frac{382}{3}}}\right)}{42}-\frac{97\sqrt{12}\,\ln\left(\frac{\left(12x+\frac{17}{2}\right)\sqrt{12}}{12}+\sqrt{97x+12\left(x-\frac{10}{3}\right)^2-\frac{382}{3}}\right)}{14112}+\frac{\sqrt{12}\,\ln\left(\frac{\left(12x+\frac{17}{2}\right)\sqrt{12}}{12}+\sqrt{x+12\left(x+\frac{2}{3}\right)^2+\frac{2}{3}}\right)}{288}+\frac{\sqrt{12}\,\ln\left(\frac{\left(12x+\frac{17}{2}\right)\sqrt{12}}{12}+\sqrt{-x+12\left(x+\frac{3}{4}\right)^2-\frac{3}{4}}\right)}{294}-\frac{4\sqrt{-x+12\left(x+\frac{3}{4}\right)^2-\frac{3}{4}}}{49}+\frac{\sqrt{x+12\left(x+\frac{2}{3}\right)^2+\frac{2}{3}}}{12}-\frac{\sqrt{97x+12\left(x-\frac{10}{3}\right)^2-\frac{382}{3}}}{588}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((12*x^2+17*x+6)^(1/2)/(2+3*x)/(-12*x^2+31*x+30),x)
```

```
[Out] -4/49*(12*(x+3/4)^2-x-3/4)^(1/2)+1/294*ln(1/12*(12*x+17/2)*12^(1/2)+(12*(x+
3/4)^2-x-3/4)^(1/2))*12^(1/2)+1/12*(12*(x+2/3)^2+x+2/3)^(1/2)+1/288*ln(1/12
*(12*x+17/2)*12^(1/2)+(12*(x+2/3)^2+x+2/3)^(1/2))*12^(1/2)-1/588*(12*(x-10/
3)^2+97*x-382/3)^(1/2)-97/14112*ln(1/12*(12*x+17/2)*12^(1/2)+(12*(x-10/3)^2
+97*x-382/3)^(1/2))*12^(1/2)+1/42*arctanh(1/28*(206/3+97*x)/(12*(x-10/3)^2+
97*x-382/3)^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sqrt{12\,x^2+17\,x+6}}{\left(12\,x^2-31\,x-30\right)(3\,x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)/(-12*x^2+31*x+30),x, algorithm="max
ima")
```

```
[Out] -integrate(sqrt(12*x^2 + 17*x + 6)/((12*x^2 - 31*x - 30)*(3*x + 2)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.04

$$\int\frac{\sqrt{12\,x^2+17\,x+6}}{(3\,x+2)\,\left(-12\,x^2+31\,x+30\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((17*x + 12*x^2 + 6)^(1/2)/((3*x + 2)*(31*x - 12*x^2 + 30)),x)
[Out] int((17*x + 12*x^2 + 6)^(1/2)/((3*x + 2)*(31*x - 12*x^2 + 30)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{12x^2+17x+6}}{36x^3-69x^2-152x-60}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x**2+17*x+6)**(1/2)/(2+3*x)/(-12*x**2+31*x+30),x)
[Out] -Integral(sqrt(12*x**2 + 17*x + 6)/(36*x**3 - 69*x**2 - 152*x - 60), x)
```

## 3.136 $\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^2\left(30+31x-12x^2\right)^2}\,dx$

**Optimal**. Leaf size=84

$$-\frac{388x+275}{98(10-3x)\sqrt{12x^2+17x+6}}+\frac{3137\sqrt{12x^2+17x+6}}{38416(10-3x)}+\frac{97\tanh^{-1}\left(\frac{291x+206}{84\sqrt{12x^2+17x+6}}\right)}{3226944}$$

**Rubi [A]** time = 0.08, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1002, 740, 806, 724, 206}

$$-\frac{388x+275}{98(10-3x)\sqrt{12x^2+17x+6}}+\frac{3137\sqrt{12x^2+17x+6}}{38416(10-3x)}+\frac{97\tanh^{-1}\left(\frac{291x+206}{84\sqrt{12x^2+17x+6}}\right)}{3226944}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)^2*(30 + 31*x - 12*x^2)^2),x]
```

```
[Out] -(275 + 388*x)/(98*(10 - 3*x)*Sqrt[6 + 17*x + 12*x^2]) + (3137*Sqrt[6 + 17*
x + 12*x^2])/(38416*(10 - 3*x)) + (97*ArcTanh[(206 + 291*x)/(84*Sqrt[6 + 17
*x + 12*x^2])])/3226944
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
```

```
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1002

```
Int[((g_) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.
) + (e_.)*(x_) + (f_.)*(x_)^2)^(m_.), x_Symbol] :> Int[((d*g)/a + (f*h*x)/c
)^m*(a + b*x + c*x^2)^(m + p), x] /; FreeQ[{a, b, c, d, e, f, g, h, p}, x]
&& EqQ[c*g^2 - b*g*h + a*h^2, 0] && EqQ[c^2*d*g^2 - a*c*e*g*h + a^2*f*h^2,
0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^2\left(30+31x-12x^2\right)^2}\,dx = \int \frac{1}{(10-3x)^2\left(6+17x+12x^2\right)^{3/2}}\,dx$$

$$= -\frac{275+388x}{98(10-3x)\sqrt{6+17x+12x^2}} - \frac{1}{882}\int \frac{-\frac{14859}{2}-10476x}{(10-3x)^2\sqrt{6+17x+12x^2}}\,dx$$

$$= -\frac{275+388x}{98(10-3x)\sqrt{6+17x+12x^2}} + \frac{3137\sqrt{6+17x+12x^2}}{38416(10-3x)} + \frac{97\int \frac{1}{(10-3x)\sqrt{6\ldots}}}{768\ldots}$$

$$= -\frac{275+388x}{98(10-3x)\sqrt{6+17x+12x^2}} + \frac{3137\sqrt{6+17x+12x^2}}{38416(10-3x)} - \frac{97\,\mathrm{Subst}\left(\int \frac{\ldots}{7\ldots}\right.}{\ldots}$$

$$= -\frac{275+388x}{98(10-3x)\sqrt{6+17x+12x^2}} + \frac{3137\sqrt{6+17x+12x^2}}{38416(10-3x)} + \frac{97\tanh^{-1}\left(\frac{\ldots}{84\ldots}\right.}{3226\ldots}$$

**Mathematica [A]** time = 0.24, size = 114, normalized size = 1.36

$$\frac{\sqrt{12x^2+17x+6}\left(97\left(36x^3-69x^2-152x-60\right)\tanh^{-1}\left(\frac{7\sqrt{3x+2}}{6\sqrt{4x+3}}\right)-42\sqrt{3x+2}\sqrt{4x+3}\left(37644x^2-98767x-88978\right)\right)}{1613472(3x-10)(3x+2)^{3/2}(4x+3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)^2*(30 + 31*x - 12*x^2)^2),x]
```

```
[Out] (Sqrt[6 + 17*x + 12*x^2]*(-42*Sqrt[2 + 3*x]*Sqrt[3 + 4*x]*(-88978 - 98767*x
 + 37644*x^2) + 97*(-60 - 152*x - 69*x^2 + 36*x^3)*ArcTanh[(7*Sqrt[2 + 3*x]
)/(6*Sqrt[3 + 4*x])]))/(1613472*(-10 + 3*x)*(2 + 3*x)^(3/2)*(3 + 4*x)^(3/2)
)
```

**IntegrateAlgebraic [A]** time = 0.37, size = 80, normalized size = 0.95

$$\frac{\sqrt{12x^2+17x+6}\left(-37644x^2+98767x+88978\right)}{38416(3x-10)(3x+2)(4x+3)}+\frac{97\tanh^{-1}\left(\frac{6\sqrt{12x^2+17x+6}}{7(3x+2)}\right)}{1613472}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)^2*(30 + 31*x - 12*x^2
)^2),x]
```

```
[Out] ((88978 + 98767*x - 37644*x^2)*Sqrt[6 + 17*x + 12*x^2])/(38416*(-10 + 3*x)*
(2 + 3*x)*(3 + 4*x)) + (97*ArcTanh[(6*Sqrt[6 + 17*x + 12*x^2])/(7*(2 + 3*x)
)])/1613472
```

**fricas [A]** time = 0.98, size = 126, normalized size = 1.50

$$\frac{97\left(36x^3-69x^2-152x-60\right)\log\left(\frac{291x+84\sqrt{12x^2+17x+6}+206}{x}\right)-97\left(36x^3-69x^2-152x-60\right)\log\left(\frac{291x-84\sqrt{12x^2+17x+6}+206}{x}\right)-168\left(37644x^2-98767x-88978\right)\sqrt{12x^2+17x+6}}{6453888\left(36x^3-69x^2-152x-60\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^2/(-12*x^2+31*x+30)^2,x, algorithm=
"fricas")
```

```
[Out] 1/6453888*(97*(36*x^3 - 69*x^2 - 152*x - 60)*log((291*x + 84*sqrt(12*x^2 +
17*x + 6) + 206)/x) - 97*(36*x^3 - 69*x^2 - 152*x - 60)*log((291*x - 84*sqr
t(12*x^2 + 17*x + 6) + 206)/x) - 168*(37644*x^2 - 98767*x - 88978)*sqrt(12*
x^2 + 17*x + 6))/(36*x^3 - 69*x^2 - 152*x - 60)
```

**giac [B]** time = 0.27, size = 159, normalized size = 1.89

$$\frac{1}{9680832}\sqrt{3}\left(\sqrt{3}\left(175672\sqrt{3}+97\log\left(\frac{7\sqrt{3}-12}{7\sqrt{3}+12}\right)\right)\mathrm{sgn}\left(\frac{1}{3x+2}\right)-\left(97\sqrt{3}\log\left(\frac{\left|-28\sqrt{3}+24\sqrt{\frac{1}{3x+2}+4}\right|}{4\left(7\sqrt{3}+6\sqrt{\frac{1}{3x+2}+4}\right)}\right)+134456\sqrt{\frac{1}{3x+2}+4}+\frac{28\left(\frac{221183}{3x+2}-18436\right)}{12\left(\frac{1}{3x+2}+4\right)^{\frac{3}{2}}-49\sqrt{\frac{1}{3x+2}+4}}\right)\mathrm{sgn}\left(\frac{1}{3x+2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^2/(-12*x^2+31*x+30)^2,x, algorithm=
"giac")
```

```
[Out] 1/9680832*sqrt(3)*(sqrt(3)*(175672*sqrt(3) + 97*log((7*sqrt(3) - 12)/(7*sqr
t(3) + 12)))*sgn(1/(3*x + 2)) - (97*sqrt(3)*log(1/4*abs(-28*sqrt(3) + 24*sq
rt(1/(3*x + 2) + 4))/(7*sqrt(3) + 6*sqrt(1/(3*x + 2) + 4))) + 134456*sqrt(1
/(3*x + 2) + 4) + 28*(221183/(3*x + 2) - 18436)/(12*(1/(3*x + 2) + 4)^(3/2)
 - 49*sqrt(1/(3*x + 2) + 4)))*sgn(1/(3*x + 2)))
```

**maple [B]** time = 0.02, size = 245, normalized size = 2.92

$$\frac{97\,\mathrm{arctanh}\left(\frac{97x+\frac{206}{3}}{28\sqrt{97x+12\left(x-\frac{10}{3}\right)^2-\frac{382}{3}}}\right)}{3226944}-\frac{7057\sqrt{12}\,\ln\left(\frac{\left(12x+\frac{17}{2}\right)\sqrt{12}}{12}+\sqrt{97x+12\left(x-\frac{10}{3}\right)^2-\frac{382}{3}}\right)}{813189888}+\frac{\sqrt{12}\,\ln\left(\frac{\left(12x+\frac{17}{2}\right)\sqrt{12}}{12}+\sqrt{x+12\left(x+\frac{2}{3}\right)^2+\frac{2}{3}}\right)}{6912}-\frac{16\sqrt{12}\,\ln\left(\frac{\left(12x+\frac{17}{2}\right)\sqrt{12}}{12}+\sqrt{-x+12\left(x+\frac{3}{4}\right)^2-\frac{3}{4}}\right)}{117649}+\frac{384\sqrt{-x+12\left(x+\frac{3}{4}\right)^2-\frac{3}{4}}}{117649}+\frac{\sqrt{x+12\left(x+\frac{2}{3}\right)^2+\frac{2}{3}}}{288}-\frac{97\sqrt{97x+12\left(x-\frac{10}{3}\right)^2-\frac{382}{3}}}{45177216}-\frac{\left(x+12\left(x+\frac{2}{3}\right)^2+\frac{2}{3}\right)^{\frac{3}{2}}}{72\left(x+\frac{2}{3}\right)^2}+\frac{32\left(-x+12\left(x+\frac{3}{4}\right)^2-\frac{3}{4}\right)^{\frac{3}{2}}}{2401\left(x+\frac{3}{4}\right)^2}-\frac{\left(97x+12\left(x-\frac{10}{3}\right)^2-\frac{382}{3}\right)^{\frac{3}{2}}}{67765824\left(x-\frac{10}{3}\right)}+\frac{(24x+17)\sqrt{97x+12\left(x-\frac{10}{3}\right)^2-\frac{382}{3}}}{135531648}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((12*x^2+17*x+6)^(1/2)/(2+3*x)^2/(-12*x^2+31*x+30)^2,x)
```

```
[Out] 384/117649*(-x+12*(x+3/4)^2-3/4)^(1/2)-16/117649*12^(1/2)*ln(1/12*(12*x+17/
2)*12^(1/2)+(-x+12*(x+3/4)^2-3/4)^(1/2))+1/288*(x+12*(x+2/3)^2+2/3)^(1/2)+1
/6912*12^(1/2)*ln(1/12*(12*x+17/2)*12^(1/2)+(x+12*(x+2/3)^2+2/3)^(1/2))-97/
45177216*(97*x+12*(x-10/3)^2-382/3)^(1/2)-7057/813189888*12^(1/2)*ln(1/12*(
12*x+17/2)*12^(1/2)+(97*x+12*(x-10/3)^2-382/3)^(1/2))+97/3226944*arctanh(1/
28*(97*x+206/3)/(97*x+12*(x-10/3)^2-382/3)^(1/2))-1/72/(x+2/3)^2*(x+12*(x+2
/3)^2+2/3)^(3/2)+32/2401/(x+3/4)^2*(-x+12*(x+3/4)^2-3/4)^(3/2)-1/67765824/(
x-10/3)*(97*x+12*(x-10/3)^2-382/3)^(3/2)+1/135531648*(24*x+17)*(97*x+12*(x-
10/3)^2-382/3)^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{12\,x^2+17\,x+6}}{\left(12\,x^2-31\,x-30\right)^2\left(3\,x+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^2/(-12*x^2+31*x+30)^2,x, algorithm=
"maxima")
```

```
[Out] integrate(sqrt(12*x^2 + 17*x + 6)/((12*x^2 - 31*x - 30)^2*(3*x + 2)^2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{12\,x^2+17\,x+6}}{{\left(3\,x+2\right)}^2\,{\left(-12\,x^2+31\,x+30\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((17*x + 12*x^2 + 6)^(1/2)/((3*x + 2)^2*(31*x - 12*x^2 + 30)^2),x)
```

```
[Out] int((17*x + 12*x^2 + 6)^(1/2)/((3*x + 2)^2*(31*x - 12*x^2 + 30)^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(3x+2)(4x+3)}}{(3x-10)^2(3x+2)^2(4x+3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x**2+17*x+6)**(1/2)/(2+3*x)**2/(-12*x**2+31*x+30)**2,x)
```

```
[Out] Integral(sqrt((3*x + 2)*(4*x + 3))/((3*x - 10)**2*(3*x + 2)**2*(4*x + 3)**2
), x)
```

## 3.137 $\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^3\left(30+31x-12x^2\right)^3}\,dx$

**Optimal**. Leaf size=139

$$-\frac{388x+275}{294(10-3x)^2\left(12x^2+17x+6\right)^{3/2}}-\frac{1634466587\sqrt{12x^2+17x+6}}{7589772288(10-3x)}-\frac{50555899\sqrt{12x^2+17x+6}}{19361664(10-3x)^2}+\frac{10425}{8232(10-3x}$$

**Rubi** [A] time = 0.12, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206, Rules used = {1002, 740, 822, 834, 806, 724, 206}

$$-\frac{388x+275}{294(10-3x)^2\left(12x^2+17x+6\right)^{3/2}}-\frac{1634466587\sqrt{12x^2+17x+6}}{7589772288(10-3x)}-\frac{50555899\sqrt{12x^2+17x+6}}{19361664(10-3x)^2}+\frac{1042556x+738029}{8232(10-3x)^2\sqrt{12x^2+17x+6}}+\frac{40325\tanh^{-1}\left(\frac{291x+206}{84\sqrt{12x^2+17x+6}}\right)}{637540872192}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)^3*(30 + 31*x - 12*x^2)^3),x]
```

```
[Out] -(275 + 388*x)/(294*(10 - 3*x)^2*(6 + 17*x + 12*x^2)^(3/2)) + (738029 + 104
2556*x)/(8232*(10 - 3*x)^2*Sqrt[6 + 17*x + 12*x^2]) - (50555899*Sqrt[6 + 17
*x + 12*x^2])/(19361664*(10 - 3*x)^2) - (1634466587*Sqrt[6 + 17*x + 12*x^2]
)/(7589772288*(10 - 3*x)) + (40325*ArcTanh[(206 + 291*x)/(84*Sqrt[6 + 17*x
+ 12*x^2])])/637540872192
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

### Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
```

```
e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1002

```
Int[((g_) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.
) + (e_.)*(x_) + (f_.)*(x_)^2)^(m_.), x_Symbol] :> Int[((d*g)/a + (f*h*x)/c
```

```
)^m*(a + b*x + c*x^2)^(m + p), x] /; FreeQ[{a, b, c, d, e, f, g, h, p}, x]
&& EqQ[c*g^2 - b*g*h + a*h^2, 0] && EqQ[c^2*d*g^2 - a*c*e*g*h + a^2*f*h^2,
0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^3\left(30+31x-12x^2\right)^3}\,dx = \int \frac{1}{(10-3x)^3\left(6+17x+12x^2\right)^{5/2}}\,dx$$

$$= -\frac{275+388x}{294(10-3x)^2\left(6+17x+12x^2\right)^{3/2}} - \frac{\int \frac{\frac{109953}{2}-41904x}{(10-3x)^3\left(6+17x+12x^2\right)^{3/2}}\,dx}{2646}$$

$$= -\frac{275+388x}{294(10-3x)^2\left(6+17x+12x^2\right)^{3/2}} + \frac{738029+1042556x}{8232(10-3x)^2\sqrt{6+17x+12x^2}} +$$

$$= -\frac{275+388x}{294(10-3x)^2\left(6+17x+12x^2\right)^{3/2}} + \frac{738029+1042556x}{8232(10-3x)^2\sqrt{6+17x+12x^2}} -$$

$$= -\frac{275+388x}{294(10-3x)^2\left(6+17x+12x^2\right)^{3/2}} + \frac{738029+1042556x}{8232(10-3x)^2\sqrt{6+17x+12x^2}} -$$

$$= -\frac{275+388x}{294(10-3x)^2\left(6+17x+12x^2\right)^{3/2}} + \frac{738029+1042556x}{8232(10-3x)^2\sqrt{6+17x+12x^2}} -$$

$$= -\frac{275+388x}{294(10-3x)^2\left(6+17x+12x^2\right)^{3/2}} + \frac{738029+1042556x}{8232(10-3x)^2\sqrt{6+17x+12x^2}} -$$

**Mathematica [A]** time = 0.37, size = 131, normalized size = 0.94

$$\frac{\sqrt{12x^2+17x+6}\left(40325\left(-36x^3+69x^2+152x+60\right)^2\tanh^{-1}\left(\frac{7\sqrt{3x+2}}{6\sqrt{4x+3}}\right)+42\sqrt{3x+2}\sqrt{4x+3}\left(706089565584x^5-3206824169544x^4-1096520427663x^3+9848047480070x^2+10124325497244x+2773753482408\right)\right)}{318770436096(10-3x)^2(3x+2)^{5/2}(4x+3)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)^3*(30 + 31*x - 12*x^2)^3),x]
```

```
[Out] (Sqrt[6 + 17*x + 12*x^2]*(42*Sqrt[2 + 3*x]*Sqrt[3 + 4*x]*(2773753482408 + 1
0124325497244*x + 9848047480070*x^2 - 1096520427663*x^3 - 3206824169544*x^4
```

```
 + 706089565584*x^5) + 40325*(60 + 152*x + 69*x^2 - 36*x^3)^2*ArcTanh[(7*Sq
rt[2 + 3*x])/(6*Sqrt[3 + 4*x])]))/(318770436096*(10 - 3*x)^2*(2 + 3*x)^(5/2
)*(3 + 4*x)^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.39, size = 95, normalized size = 0.68

$$\frac{40325\tanh^{-1}\left(\frac{6\sqrt{12x^2+17x+6}}{7(3x+2)}\right)}{318770436096}+\frac{\sqrt{12x^2+17x+6}\left(706089565584x^5-3206824169544x^4-1096520427663x^3+9848047480070x^2+10124325497244x+2773753482408\right)}{7589772288(3x-10)^2(3x+2)^2(4x+3)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)^3*(30 + 31*x - 12*x^2
)^3),x]
```

```
[Out] (Sqrt[6 + 17*x + 12*x^2]*(2773753482408 + 10124325497244*x + 9848047480070*
x^2 - 1096520427663*x^3 - 3206824169544*x^4 + 706089565584*x^5))/(758977228
8*(-10 + 3*x)^2*(2 + 3*x)^2*(3 + 4*x)^2) + (40325*ArcTanh[(6*Sqrt[6 + 17*x
+ 12*x^2])/(7*(2 + 3*x))])/318770436096
```

**fricas [A]** time = 1.74, size = 186, normalized size = 1.34

$$\frac{40325\left(1296x^6-4968x^5-6183x^4+16656x^3+31384x^2+18240x+3600\right)\log\left(\frac{291x+84\sqrt{12x^2+17x+6}+206}{x}\right)-40325\left(1296x^6-4968x^5-6183x^4+16656x^3+31384x^2+18240x+3600\right)\log\left(\frac{291x-84\sqrt{12x^2+17x+6}+206}{x}\right)+168\left(706089565584x^5-3206824169544x^4-1096520427663x^3+9848047480070x^2+10124325497244x+2773753482408\right)\sqrt{12x^2+17x+6}}{1275081744384\left(1296x^6-4968x^5-6183x^4+16656x^3+31384x^2+18240x+3600\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^3/(-12*x^2+31*x+30)^3,x, algorithm=
"fricas")
```

```
[Out] 1/1275081744384*(40325*(1296*x^6 - 4968*x^5 - 6183*x^4 + 16656*x^3 + 31384*
x^2 + 18240*x + 3600)*log((291*x + 84*sqrt(12*x^2 + 17*x + 6) + 206)/x) - 4
0325*(1296*x^6 - 4968*x^5 - 6183*x^4 + 16656*x^3 + 31384*x^2 + 18240*x + 36
00)*log((291*x - 84*sqrt(12*x^2 + 17*x + 6) + 206)/x) + 168*(706089565584*x
^5 - 3206824169544*x^4 - 1096520427663*x^3 + 9848047480070*x^2 + 1012432549
7244*x + 2773753482408)*sqrt(12*x^2 + 17*x + 6))/(1296*x^6 - 4968*x^5 - 618
3*x^4 + 16656*x^3 + 31384*x^2 + 18240*x + 3600)
```

**giac [A]** time = 0.26, size = 232, normalized size = 1.67

$$\frac{\sqrt{3}\left(282273\sqrt{3}\left(2\sqrt{3}x-\sqrt{12x^2+17x+6}\right)^3-11460924\left(2\sqrt{3}x-\sqrt{12x^2+17x+6}\right)^2-37551180\sqrt{3}\left(2\sqrt{3}x-\sqrt{12x^2+17x+6}\right)-83365264\right)}{159385218048\left(3\left(2\sqrt{3}x-\sqrt{12x^2+17x+6}\right)^2-40\sqrt{3}\left(2\sqrt{3}x-\sqrt{12x^2+17x+6}\right)-188\right)^2}+\frac{(8(2860316794x+6078171227)x+34383350229)x+8090114146}{2213683584\left(12x^2+17x+6\right)^{\frac{3}{2}}}+\frac{40325}{637540872192}\log\left(\left|-6\sqrt{3}x+20\sqrt{3}+3\sqrt{12x^2+17x+6}+42\right|\right)-\frac{40325}{637540872192}\log\left(\left|-6\sqrt{3}x+20\sqrt{3}+3\sqrt{12x^2+17x+6}-42\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^3/(-12*x^2+31*x+30)^3,x, algorithm=
"giac")
```

```
[Out] 1/159385218048*sqrt(3)*(282273*sqrt(3)*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x +
6))^3 - 11460924*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x + 6))^2 - 37551180*sqrt(
```

```
3)*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x + 6)) - 83365264)/(3*(2*sqrt(3)*x - sq
rt(12*x^2 + 17*x + 6))^2 - 40*sqrt(3)*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x + 6
)) - 188)^2 + 1/2213683584*((8*(2860316794*x + 6078171227)*x + 34383350229)
*x + 8090114146)/(12*x^2 + 17*x + 6)^(3/2) + 40325/637540872192*log(abs(-6*
sqrt(3)*x + 20*sqrt(3) + 3*sqrt(12*x^2 + 17*x + 6) + 42)) - 40325/637540872
192*log(abs(-6*sqrt(3)*x + 20*sqrt(3) + 3*sqrt(12*x^2 + 17*x + 6) - 42))
```

**maple [B]** time = 0.02, size = 306, normalized size = 2.20

$$\frac{40325\,\mathrm{arctanh}\left(\frac{97x+\frac{206}{3}}{28\sqrt{97x+12\left(x-\frac{10}{3}\right)^2-\frac{382}{3}}}\right)}{637540872192}-\frac{570457\sqrt{12}\,\ln\left(\frac{\left(12x+\frac{17}{2}\right)\sqrt{12}}{12}+\sqrt{97x+12\left(x-\frac{10}{3}\right)^2-\frac{382}{3}}\right)}{31239502737408}-\frac{23\sqrt{12}\,\ln\left(\frac{\left(12x+\frac{17}{2}\right)\sqrt{12}}{12}+\sqrt{x+12\left(x+\frac{2}{3}\right)^2+\frac{2}{3}}\right)}{110592}+\frac{58752\sqrt{12}\,\ln\left(\frac{\left(12x+\frac{17}{2}\right)\sqrt{12}}{12}+\sqrt{-x+12\left(x+\frac{3}{4}\right)^2-\frac{3}{4}}\right)}{282475249}-\frac{\left(x+12\left(x+\frac{2}{3}\right)^2+\frac{2}{3}\right)^{\frac{3}{2}}}{2592\left(x+\frac{2}{3}\right)^3}-\frac{1261\left(97x+12\left(x-\frac{10}{3}\right)^2-\frac{382}{3}\right)^{\frac{3}{2}}}{31239502737408\left(x-\frac{10}{3}\right)}+\frac{1261\left(24x+17\right)\sqrt{97x+12\left(x-\frac{10}{3}\right)^2-\frac{382}{3}}}{62479005474816}+\frac{\left(97x+12\left(x-\frac{10}{3}\right)^2-\frac{382}{3}\right)^{\frac{3}{2}}}{79692609024\left(x-\frac{10}{3}\right)^2}-\frac{230400\left(-x+12\left(x+\frac{3}{4}\right)^2-\frac{3}{4}\right)^{\frac{3}{2}}}{5764801\left(x+\frac{3}{4}\right)^2}-\frac{128\left(-x+12\left(x+\frac{3}{4}\right)^2-\frac{3}{4}\right)^{\frac{3}{2}}}{352947\left(x+\frac{3}{4}\right)^3}+\frac{47\left(x+12\left(x+\frac{2}{3}\right)^2+\frac{2}{3}\right)^{\frac{3}{2}}}{1152\left(x+\frac{2}{3}\right)^2}-\frac{40325\sqrt{97x+12\left(x-\frac{10}{3}\right)^2-\frac{382}{3}}}{8925572210688}-\frac{23\sqrt{x+12\left(x+\frac{2}{3}\right)^2+\frac{2}{3}}}{4608}-\frac{1410048\sqrt{-x+12\left(x+\frac{3}{4}\right)^2-\frac{3}{4}}}{282475249}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((12*x^2+17*x+6)^(1/2)/(2+3*x)^3/(-12*x^2+31*x+30)^3,x)
```

```
[Out] -1/2592/(x+2/3)^3*(x+12*(x+2/3)^2+2/3)^(3/2)-1261/31239502737408/(x-10/3)*(
97*x+12*(x-10/3)^2-382/3)^(3/2)+1261/62479005474816*(24*x+17)*(97*x+12*(x-1
0/3)^2-382/3)^(1/2)+1/79692609024/(x-10/3)^2*(97*x+12*(x-10/3)^2-382/3)^(3/
2)-230400/5764801/(x+3/4)^2*(-x+12*(x+3/4)^2-3/4)^(3/2)-128/352947/(x+3/4)^
3*(-x+12*(x+3/4)^2-3/4)^(3/2)-570457/31239502737408*12^(1/2)*ln(1/12*(12*x+
17/2)*12^(1/2)+(97*x+12*(x-10/3)^2-382/3)^(1/2))+47/1152/(x+2/3)^2*(x+12*(x
+2/3)^2+2/3)^(3/2)-23/110592*12^(1/2)*ln(1/12*(12*x+17/2)*12^(1/2)+(x+12*(x
+2/3)^2+2/3)^(1/2))+58752/282475249*12^(1/2)*ln(1/12*(12*x+17/2)*12^(1/2)+(
-x+12*(x+3/4)^2-3/4)^(1/2))-40325/8925572210688*(97*x+12*(x-10/3)^2-382/3)^
(1/2)+40325/637540872192*arctanh(1/28*(97*x+206/3)/(97*x+12*(x-10/3)^2-382/
3)^(1/2))-23/4608*(x+12*(x+2/3)^2+2/3)^(1/2)-1410048/282475249*(-x+12*(x+3/
4)^2-3/4)^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{12\,x^2+17\,x+6}}{\left(12\,x^2-31\,x-30\right)^3\left(3\,x+2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^3/(-12*x^2+31*x+30)^3,x, algorithm=
"maxima")
```

```
[Out] -integrate(sqrt(12*x^2 + 17*x + 6)/((12*x^2 - 31*x - 30)^3*(3*x + 2)^3), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{12\,x^2+17\,x+6}}{{\left(3\,x+2\right)}^3\,{\left(-12\,x^2+31\,x+30\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((17*x + 12*x^2 + 6)^(1/2)/((3*x + 2)^3*(31*x - 12*x^2 + 30)^3),x)
```

```
[Out] int((17*x + 12*x^2 + 6)^(1/2)/((3*x + 2)^3*(31*x - 12*x^2 + 30)^3), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{12x^2 + 17x + 6}}{46656x^9 - 268272x^8 - 76788x^7 + 1703619x^6 + 1218456x^5 - 3669588x^4 - 6898688x^3 - 4903920x^2 - 1641600x - 216000}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x**2+17*x+6)**(1/2)/(2+3*x)**3/(-12*x**2+31*x+30)**3,x)
```

```
[Out] -Integral(sqrt(12*x**2 + 17*x + 6)/(46656*x**9 - 268272*x**8 - 76788*x**7 +
 1703619*x**6 + 1218456*x**5 - 3669588*x**4 - 6898688*x**3 - 4903920*x**2 -
 1641600*x - 216000), x)
```

## 3.138 $\int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx$

**Optimal**. Leaf size=15

$$\frac{3}{5}\left(x^2-3x\right)^{5/3}$$

**Rubi** [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {629}

$$\frac{3}{5}\left(x^2-3x\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 2*x)*(-3*x + x^2)^(2/3),x]
```

```
[Out] (3*(-3*x + x^2)^(5/3))/5
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx=\frac{3}{5}\left(-3x+x^2\right)^{5/3}$$

**Mathematica** [A] time = 0.01, size = 13, normalized size = 0.87

$$\frac{3}{5}((x-3)x)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 2*x)*(-3*x + x^2)^(2/3),x]
```

```
[Out] (3*((-3 + x)*x)^(5/3))/5
```

**IntegrateAlgebraic** [A] time = 0.02, size = 13, normalized size = 0.87

$$\frac{3}{5}((x-3)x)^{5/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-3 + 2*x)*(-3*x + x^2)^(2/3),x]
```

```
[Out] (3*((-3 + x)*x)^(5/3))/5
```

**fricas [A]** time = 0.60, size = 11, normalized size = 0.73

$$\frac{3}{5} \left(x^2 - 3\,x\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)*(x^2-3*x)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

**giac [A]** time = 0.15, size = 11, normalized size = 0.73

$$\frac{3}{5} \left(x^2 - 3\,x\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)*(x^2-3*x)^(2/3),x, algorithm="giac")
```

```
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

**maple [A]** time = 0.00, size = 16, normalized size = 1.07

$$\frac{3\,(x-3)\left(x^2-3x\right)^{\frac{2}{3}} x}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+2*x)*(x^2-3*x)^(2/3),x)
```

```
[Out] 3/5*(x-3)*x*(x^2-3*x)^(2/3)
```

**maxima [A]** time = 0.43, size = 11, normalized size = 0.73

$$\frac{3}{5} \left(x^2 - 3\,x\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)*(x^2-3*x)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

**mupad [B]** time = 3.72, size = 15, normalized size = 1.00

$$\frac{3\,x\,{\left(x^2-3\,x\right)}^{2/3}\,\left(x-3\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - 3)*(x^2 - 3*x)^(2/3),x)
[Out] (3*x*(x^2 - 3*x)^(2/3)*(x - 3))/5
```

**sympy [B]** time = 0.31, size = 31, normalized size = 2.07

$$\frac{3x^{2}\left(x^{2}-3x\right)^{\frac{2}{3}}}{5}-\frac{9x\left(x^{2}-3x\right)^{\frac{2}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)*(x**2-3*x)**(2/3),x)
[Out] 3*x**2*(x**2 - 3*x)**(2/3)/5 - 9*x*(x**2 - 3*x)**(2/3)/5
```

## 3.139 $\int ((-3+x)x)^{2/3}(-3+2x)\,dx$

**Optimal**. Leaf size=16

$$\frac{3}{5}(-((3-x)x))^{5/3}$$

**Rubi [A]** time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1588}

$$\frac{3}{5}(-(3-x)x)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[((-3 + x)*x)^(2/3)*(-3 + 2*x),x]
```

```
[Out] (3*(-((3 - x)*x))^(5/3))/5
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int ((-3+x)x)^{2/3}(-3+2x)\,dx = \frac{3}{5}(-((3-x)x))^{5/3}$$

**Mathematica [A]** time = 0.00, size = 13, normalized size = 0.81

$$\frac{3}{5}((x-3)x)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-3 + x)*x)^(2/3)*(-3 + 2*x),x]
```

```
[Out] (3*((-3 + x)*x)^(5/3))/5
```

**IntegrateAlgebraic** [A] time = 0.01, size = 13, normalized size = 0.81

$$\frac{3}{5}((x-3)x)^{5/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-3 + x)*x)^(2/3)*(-3 + 2*x),x]
[Out] (3*((-3 + x)*x)^(5/3))/5
```

**fricas** [A] time = 0.67, size = 11, normalized size = 0.69

$$\frac{3}{5}\left(x^2-3\,x\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-3+x)*x)^(2/3)*(-3+2*x),x, algorithm="fricas")
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

**giac** [A] time = 0.15, size = 11, normalized size = 0.69

$$\frac{3}{5}\left(x^2-3\,x\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-3+x)*x)^(2/3)*(-3+2*x),x, algorithm="giac")
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

**maple** [A] time = 0.00, size = 14, normalized size = 0.88

$$\frac{3\,(x-3)\,((x-3)\,x)^{\frac{2}{3}}\,x}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x-3)*x)^(2/3)*(-3+2*x),x)
[Out] 3/5*(x-3)*x*((x-3)*x)^(2/3)
```

**maxima** [A] time = 0.43, size = 9, normalized size = 0.56

$$\frac{3}{5}\,((x-3)x)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-3+x)*x)^(2/3)*(-3+2*x),x, algorithm="maxima")
```

```
[Out] 3/5*((x - 3)*x)^(5/3)
```

**mupad [B]** time = 3.66, size = 13, normalized size = 0.81

$$\frac{3\,x\,(x\,\left(x-3\right))^{2/3}\,\left(x-3\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - 3)*(x*(x - 3))^(2/3),x)
```

```
[Out] (3*x*(x*(x - 3))^(2/3)*(x - 3))/5
```

**sympy [A]** time = 4.25, size = 10, normalized size = 0.62

$$\frac{3\left(x\left(x-3\right)\right)^{\frac{5}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-3+x)*x)**(2/3)*(-3+2*x),x)
```

```
[Out] 3*(x*(x - 3))**(5/3)/5
```

## 3.140 $\int \frac{x(9-9x+2x^2)}{\sqrt[3]{-3x+x^2}}\,dx$

**Optimal**. Leaf size=15

$$\frac{3}{5}\left(x^2-3x\right)^{5/3}$$

**Rubi [A]** time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1631, 629}

$$\frac{3}{5}\left(x^2-3x\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(9 - 9*x + 2*x^2))/(-3*x + x^2)^(1/3),x]
```

```
[Out] (3*(-3*x + x^2)^(5/3))/5
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1631

```
Int[(Pq_)*((e_.)*(x_))^(m_.)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[e, Int[(e*x)^(m - 1)*PolynomialQuotient[Pq, b + c*x, x]*(b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{b, c, e, m, p}, x] && PolyQ[Pq, x] && EqQ[Polyn
omialRemainder[Pq, b + c*x, x], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(9-9x+2x^2\right)}{\sqrt[3]{-3x+x^2}}\,dx &= \int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx\\ &= \frac{3}{5}\left(-3x+x^2\right)^{5/3}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 13, normalized size = 0.87

$$\frac{3}{5}((x-3)x)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(9 - 9*x + 2*x^2))/(-3*x + x^2)^(1/3),x]
[Out] (3*((-3 + x)*x)^(5/3))/5
```

**IntegrateAlgebraic** [A] time = 0.02, size = 13, normalized size = 0.87

$$\frac{3}{5}((x-3)x)^{5/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(9 - 9*x + 2*x^2))/(-3*x + x^2)^(1/3),x]
[Out] (3*((-3 + x)*x)^(5/3))/5
```

**fricas** [A] time = 0.99, size = 11, normalized size = 0.73

$$\frac{3}{5}\left(x^2-3\,x\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^2-9*x+9)/(x^2-3*x)^(1/3),x, algorithm="fricas")
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

**giac** [A] time = 0.16, size = 11, normalized size = 0.73

$$\frac{3}{5}\left(x^2-3\,x\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^2-9*x+9)/(x^2-3*x)^(1/3),x, algorithm="giac")
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

**maple** [A] time = 0.00, size = 20, normalized size = 1.33

$$\frac{3\,(x-3)^2\,x^2}{5\left(x^2-3x\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*x^2-9*x+9)/(x^2-3*x)^(1/3),x)
```

```
[Out] 3/5*(x-3)^2*x^2/(x^2-3*x)^(1/3)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - 9x + 9)x}{(x^2 - 3x)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^2-9*x+9)/(x^2-3*x)^(1/3),x, algorithm="maxima")
[Out] integrate((2*x^2 - 9*x + 9)*x/(x^2 - 3*x)^(1/3), x)
```

**mupad [B]** time = 3.68, size = 15, normalized size = 1.00

$$\frac{3x(x^2 - 3x)^{2/3}(x - 3)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(2*x^2 - 9*x + 9))/(x^2 - 3*x)^(1/3),x)
[Out] (3*x*(x^2 - 3*x)^(2/3)*(x - 3))/5
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(x - 3)(2x - 3)}{\sqrt[3]{x(x - 3)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x**2-9*x+9)/(x**2-3*x)**(1/3),x)
[Out] Integral(x*(x - 3)*(2*x - 3)/(x*(x - 3))**(1/3), x)
```

## 3.141 $\int \frac{x(9-9x+2x^2)}{\sqrt[3]{(-3+x)x}} \, dx$

**Optimal**. Leaf size=15

$$\frac{3}{5}\left(x^2-3x\right)^{5/3}$$

**Rubi** [A] time = 0.06, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1985, 1631, 629}

$$\frac{3}{5}\left(x^2-3x\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(9 - 9*x + 2*x^2))/((-3 + x)*x)^(1/3),x]
```

```
[Out] (3*(-3*x + x^2)^(5/3))/5
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1631

```
Int[(Pq_)*((e_.)*(x_))^(m_.)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[e, Int[(e*x)^(m - 1)*PolynomialQuotient[Pq, b + c*x, x]*(b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{b, c, e, m, p}, x] && PolyQ[Pq, x] && EqQ[Polyn
omialRemainder[Pq, b + c*x, x], 0]
```

Rule 1985

```
Int[(u_)^(p_.)*(v_)^(q_.)*(z_)^(m_.), x_Symbol] :> Int[ExpandToSum[z, x]^m*
ExpandToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{m, p, q}, x] && Linea
rQ[z, x] && QuadraticQ[{u, v}, x] &&  !(LinearMatchQ[z, x] && QuadraticMatc
hQ[{u, v}, x])
```

Rubi steps

$$\int \frac{x\left(9-9x+2x^2\right)}{\sqrt[3]{(-3+x)x}}\,dx = \int \frac{x\left(9-9x+2x^2\right)}{\sqrt[3]{-3x+x^2}}\,dx$$
$$= \int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx$$
$$= \frac{3}{5}\left(-3x+x^2\right)^{5/3}$$

**Mathematica** [A] time = 0.00, size = 13, normalized size = 0.87

$$\frac{3}{5}((x-3)x)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(9 - 9*x + 2*x^2))/((-3 + x)*x)^(1/3),x]
[Out] (3*((-3 + x)*x)^(5/3))/5
```

**IntegrateAlgebraic** [A] time = 0.01, size = 13, normalized size = 0.87

$$\frac{3}{5}((x-3)x)^{5/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(9 - 9*x + 2*x^2))/((-3 + x)*x)^(1/3),x]
[Out] (3*((-3 + x)*x)^(5/3))/5
```

**fricas** [A] time = 1.02, size = 11, normalized size = 0.73

$$\frac{3}{5}\left(x^2-3\,x\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^2-9*x+9)/((-3+x)*x)^(1/3),x, algorithm="fricas")
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

**giac** [A] time = 0.20, size = 11, normalized size = 0.73

$$\frac{3}{5}\left(x^2-3\,x\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^2-9*x+9)/((-3+x)*x)^(1/3),x, algorithm="giac")
```

```
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

**maple [A]** time = 0.00, size = 18, normalized size = 1.20

$$\frac{3 (x-3)^{2} x^{2}}{5 ((x-3) x)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*x^2-9*x+9)/((x-3)*x)^(1/3),x)
```

```
[Out] 3/5*(x-3)^2*x^2/((x-3)*x)^(1/3)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2 x^{2}-9 x+9\right) x}{((x-3) x)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^2-9*x+9)/((-3+x)*x)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 - 9*x + 9)*x/((x - 3)*x)^(1/3), x)
```

**mupad [B]** time = 3.61, size = 13, normalized size = 0.87

$$\frac{3 x (x (x-3))^{2/3} (x-3)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(2*x^2 - 9*x + 9))/(x*(x - 3))^(1/3),x)
```

```
[Out] (3*x*(x*(x - 3))^(2/3)*(x - 3))/5
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x (x-3) (2x-3)}{\sqrt[3]{x (x-3)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x**2-9*x+9)/((-3+x)*x)**(1/3),x)
```

```
[Out] Integral(x*(x - 3)*(2*x - 3)/(x*(x - 3))**(1/3), x)
```

## 3.142 $\int \frac{g+hx}{\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}\left(g^2+3h^2x^2\right)}\,dx$

**Optimal**. Leaf size=242

$$\frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\,\log\left(g^2+3h^2x^2\right)}{6\,2^{2/3}h\sqrt[3]{9cx^2-\frac{cg^2}{h^2}}}-\frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\,\log\left(\left(1-\frac{3hx}{g}\right)^{2/3}+\sqrt[3]{2}\sqrt[3]{\frac{3hx}{g}+1}\right)}{2\,2^{2/3}h\sqrt[3]{9cx^2-\frac{cg^2}{h^2}}}+\frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\left(1-\frac{3hx}{g}\right)^{2/3}}{\sqrt{3}\sqrt[3]{\frac{3hx}{g}+1}}\right)}{2^{2/3}\sqrt{3}\,h\sqrt[3]{9cx^2-\frac{cg^2}{h^2}}}$$

**Rubi** [A] time = 0.09, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1009, 1008}

$$\frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\,\log\left(g^2+3h^2x^2\right)}{6\,2^{2/3}h\sqrt[3]{9cx^2-\frac{cg^2}{h^2}}}-\frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\,\log\left(\left(1-\frac{3hx}{g}\right)^{2/3}+\sqrt[3]{2}\sqrt[3]{\frac{3hx}{g}+1}\right)}{2\,2^{2/3}h\sqrt[3]{9cx^2-\frac{cg^2}{h^2}}}+\frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\left(1-\frac{3hx}{g}\right)^{2/3}}{\sqrt{3}\sqrt[3]{\frac{3hx}{g}+1}}\right)}{2^{2/3}\sqrt{3}\,h\sqrt[3]{9cx^2-\frac{cg^2}{h^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)/((-((c*g^2)/h^2) + 9*c*x^2)^(1/3)*(g^2 + 3*h^2*x^2)),x]
```

```
[Out] ((1 - (9*h^2*x^2)/g^2)^(1/3)*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - (3*h*x)/g)^(2
/3))/(Sqrt[3]*(1 + (3*h*x)/g)^(1/3))]])/(2^(2/3)*Sqrt[3]*h*(-((c*g^2)/h^2) +
 9*c*x^2)^(1/3)) + ((1 - (9*h^2*x^2)/g^2)^(1/3)*Log[g^2 + 3*h^2*x^2])/(6*2^
(2/3)*h*(-((c*g^2)/h^2) + 9*c*x^2)^(1/3)) - ((1 - (9*h^2*x^2)/g^2)^(1/3)*Lo
g[(1 - (3*h*x)/g)^(2/3) + 2^(1/3)*(1 + (3*h*x)/g)^(1/3)])/(2*2^(2/3)*h*(-((
c*g^2)/h^2) + 9*c*x^2)^(1/3))
```

Rule 1008

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)^(1/3)*((d_) + (f_.)*(x_)^2))
, x_Symbol] :> Simp[(Sqrt[3]*h*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - (3*h*x)/g)^
(2/3))/(Sqrt[3]*(1 + (3*h*x)/g)^(1/3))]])/(2^(2/3)*a^(1/3)*f), x] + (-Simp[(
3*h*Log[(1 - (3*h*x)/g)^(2/3) + 2^(1/3)*(1 + (3*h*x)/g)^(1/3)])/(2^(5/3)*a^
(1/3)*f), x] + Simp[(h*Log[d + f*x^2])/(2^(5/3)*a^(1/3)*f), x]) /; FreeQ[{a
, c, d, f, g, h}, x] && EqQ[c*d + 3*a*f, 0] && EqQ[c*g^2 + 9*a*h^2, 0] && G
tQ[a, 0]
```

Rule 1009

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)^(1/3)*((d_) + (f_.)*(x_)^2))
, x_Symbol] :> Dist[(1 + (c*x^2)/a)^(1/3)/(a + c*x^2)^(1/3), Int[(g + h*x)/
```

```
((1 + (c*x^2)/a)^(1/3)*(d + f*x^2)), x], x] /; FreeQ[{a, c, d, f, g, h}, x]
 && EqQ[c*d + 3*a*f, 0] && EqQ[c*g^2 + 9*a*h^2, 0] && !GtQ[a, 0]
```

Rubi steps

$$\int \frac{g+hx}{\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}\left(g^2+3h^2x^2\right)}\,dx = \frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\int \frac{g+hx}{\left(g^2+3h^2x^2\right)\sqrt[3]{1-\frac{9h^2x^2}{g^2}}}\,dx}{\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}}$$

$$= \frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\left(1-\frac{3hx}{g}\right)^{2/3}}{\sqrt{3}\sqrt[3]{1+\frac{3hx}{g}}}\right)}{2^{2/3}\sqrt{3}\,h\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}} + \frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\log\left(g^2+3h^2x^2\right)}{6\,2^{2/3}h\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}} - \ldots$$

**Mathematica [C]** time = 0.56, size = 268, normalized size = 1.11

$$\frac{h^2x\left(-hx\sqrt[3]{1-\frac{9h^2x^2}{g^2}}F_1\left(1;\frac{1}{3},1;2;\frac{9h^2x^2}{g^2},-\frac{3h^2x^2}{g^2}\right)-\frac{2g^5F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{9h^2x^2}{g^2},-\frac{3h^2x^2}{g^2}\right)}{\left(g^2+3h^2x^2\right)\left(g^2F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{9h^2x^2}{g^2},-\frac{3h^2x^2}{g^2}\right)+2h^2x^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{9h^2x^2}{g^2},-\frac{3h^2x^2}{g^2}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{9h^2x^2}{g^2},-\frac{3h^2x^2}{g^2}\right)\right)\right)}\right)\left(c\left(9x^2-\frac{g^2}{h^2}\right)\right)^{2/3}}{2cg^2\left(g^2-9h^2x^2\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g + h*x)/((-((c*g^2)/h^2) + 9*c*x^2)^(1/3)*(g^2 + 3*h^2*x^2)),x]
```

```
[Out] (h^2*x*(c*(-(g^2/h^2) + 9*x^2))^(2/3)*(-(h*x*(1 - (9*h^2*x^2)/g^2)^(1/3)*Ap
pellF1[1, 1/3, 1, 2, (9*h^2*x^2)/g^2, (-3*h^2*x^2)/g^2]) - (2*g^5*AppellF1[
1/2, 1/3, 1, 3/2, (9*h^2*x^2)/g^2, (-3*h^2*x^2)/g^2])/((g^2 + 3*h^2*x^2)*(g
^2*AppellF1[1/2, 1/3, 1, 3/2, (9*h^2*x^2)/g^2, (-3*h^2*x^2)/g^2] + 2*h^2*x^
2*(-AppellF1[3/2, 1/3, 2, 5/2, (9*h^2*x^2)/g^2, (-3*h^2*x^2)/g^2] + AppellF
1[3/2, 4/3, 1, 5/2, (9*h^2*x^2)/g^2, (-3*h^2*x^2)/g^2])))))/(2*c*g^2*(g^2 -
 9*h^2*x^2))
```

**IntegrateAlgebraic [B]** time = 1.08, size = 499, normalized size = 2.06

$$-\frac{\log\left(\sqrt[3]{2}c^{2/3}g^2-6\sqrt[3]{2}c^{2/3}ghx+9\sqrt[3]{2}c^{2/3}h^2x^2-3\,2^{2/3}\sqrt[3]{c}\sqrt[3]{g}h^{5/3}x\sqrt[3]{9cx^2-\frac{cg^2}{h^2}}+2^{2/3}\sqrt[3]{c}g^{4/3}h^{2/3}\sqrt[3]{9cx^2-\frac{cg^2}{h^2}}+2g^{2/3}h^{4/3}\left(9cx^2-\frac{cg^2}{h^2}\right)^{2/3}\right)}{6\,2^{2/3}\sqrt[3]{c}g^{2/3}\sqrt[3]{h}}-\frac{\log\left(\sqrt[3]{g}\sqrt[3]{9cx^2-\frac{cg^2}{h^2}}\right)}{3\,2^{2/3}\sqrt[3]{c}g^{2/3}\sqrt[3]{h}}+\frac{\log\left(g^{2/3}\left(9cx^2-\frac{cg^2}{h^2}\right)^{2/3}\right)}{6\,2^{2/3}\sqrt[3]{c}g^{2/3}\sqrt[3]{h}}+\frac{\log\left(-2\sqrt[3]{g}h^{2/3}\sqrt[3]{9cx^2-\frac{cg^2}{h^2}}+2^{2/3}\sqrt[3]{c}g-3\,2^{2/3}\sqrt[3]{c}hx\right)}{3\,2^{2/3}\sqrt[3]{c}g^{2/3}\sqrt[3]{h}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{g}h^{2/3}\sqrt[3]{9cx^2-\frac{cg^2}{h^2}}}{\sqrt[3]{g}h^{2/3}\sqrt[3]{9cx^2-\frac{cg^2}{h^2}}+2^{2/3}\sqrt[3]{c}g-3\,2^{2/3}\sqrt[3]{c}hx}\right)}{2^{2/3}\sqrt{3}\sqrt[3]{c}g^{2/3}\sqrt[3]{h}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(g + h*x)/((-((c*g^2)/h^2) + 9*c*x^2)^(1/3)*(g^2 + 3*h^2
*x^2)),x]
```

```
[Out] ArcTan[(Sqrt[3]*g^(1/3)*h^(2/3)*(-((c*g^2)/h^2) + 9*c*x^2)^(1/3))/(2^(2/3)*
c^(1/3)*g - 3*2^(2/3)*c^(1/3)*h*x + g^(1/3)*h^(2/3)*(-((c*g^2)/h^2) + 9*c*x
^2)^(1/3))]/(2^(2/3)*Sqrt[3]*c^(1/3)*g^(2/3)*h^(1/3)) - Log[g^(1/3)*(-((c*g
^2)/h^2) + 9*c*x^2)^(1/3)]/(3*2^(2/3)*c^(1/3)*g^(2/3)*h^(1/3)) + Log[g^(2/3
)*(-((c*g^2)/h^2) + 9*c*x^2)^(2/3)]/(6*2^(2/3)*c^(1/3)*g^(2/3)*h^(1/3)) + L
og[2^(2/3)*c^(1/3)*g - 3*2^(2/3)*c^(1/3)*h*x - 2*g^(1/3)*h^(2/3)*(-((c*g^2)
/h^2) + 9*c*x^2)^(1/3)]/(3*2^(2/3)*c^(1/3)*g^(2/3)*h^(1/3)) - Log[2^(1/3)*c
^(2/3)*g^2 - 6*2^(1/3)*c^(2/3)*g*h*x + 9*2^(1/3)*c^(2/3)*h^2*x^2 + 2^(2/3)*
c^(1/3)*g^(4/3)*h^(2/3)*(-((c*g^2)/h^2) + 9*c*x^2)^(1/3) - 3*2^(2/3)*c^(1/3
)*g^(1/3)*h^(5/3)*x*(-((c*g^2)/h^2) + 9*c*x^2)^(1/3) + 2*g^(2/3)*h^(4/3)*(-
((c*g^2)/h^2) + 9*c*x^2)^(2/3)]/(6*2^(2/3)*c^(1/3)*g^(2/3)*h^(1/3))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(-c*g^2/h^2+9*c*x^2)^(1/3)/(3*h^2*x^2+g^2),x, algorithm="
fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{hx+g}{\left(3\,h^2x^2+g^2\right)\left(9\,cx^2-\frac{cg^2}{h^2}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(-c*g^2/h^2+9*c*x^2)^(1/3)/(3*h^2*x^2+g^2),x, algorithm="
giac")
```

```
[Out] integrate((h*x + g)/((3*h^2*x^2 + g^2)*(9*c*x^2 - c*g^2/h^2)^(1/3)), x)
```

**maple [F]** time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{hx+g}{\left(9c\,x^2-\frac{c\,g^2}{h^2}\right)^{\frac{1}{3}}\left(3h^2x^2+g^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)/(-c*g^2/h^2+9*c*x^2)^(1/3)/(3*h^2*x^2+g^2),x)
```

```
[Out] int((h*x+g)/(-c*g^2/h^2+9*c*x^2)^(1/3)/(3*h^2*x^2+g^2),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{hx+g}{\left(3\,h^2x^2+g^2\right)\left(9\,cx^2-\frac{cg^2}{h^2}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(-c*g^2/h^2+9*c*x^2)^(1/3)/(3*h^2*x^2+g^2),x, algorithm="
maxima")
```

```
[Out] integrate((h*x + g)/((3*h^2*x^2 + g^2)*(9*c*x^2 - c*g^2/h^2)^(1/3)), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{g+h\,x}{\left(g^2+3\,h^2\,x^2\right)\,\left(9\,c\,x^2-\frac{c\,g^2}{h^2}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)/((g^2 + 3*h^2*x^2)*(9*c*x^2 - (c*g^2)/h^2)^(1/3)),x)
```

```
[Out] int((g + h*x)/((g^2 + 3*h^2*x^2)*(9*c*x^2 - (c*g^2)/h^2)^(1/3)), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{g+hx}{\sqrt[3]{c\left(-\frac{g}{h}+3x\right)\left(\frac{g}{h}+3x\right)}\,\left(g^2+3h^2x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(-c*g**2/h**2+9*c*x**2)**(1/3)/(3*h**2*x**2+g**2),x)
```

```
[Out] Integral((g + h*x)/((c*(-g/h + 3*x)*(g/h + 3*x))**(1/3)*(g**2 + 3*h**2*x**2
)), x)
```

## 3.143 $\int \frac{g+hx}{\sqrt[3]{\frac{-c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2}\left(\frac{f\left(b^2-\frac{-c^2g^2+bcgh+2b^2h^2}{3h^2}\right)}{c^2}+\frac{bfx}{c}+fx^2\right)}\,dx$

**Optimal**. Leaf size=488

$$\frac{3^{2/3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}}\log\left(\frac{f\left(b^2h^2-bcgh+c^2g^2\right)}{3c^2h^2}+\frac{bfx}{c}+fx^2\right)}{2f\sqrt[3]{-\frac{(cg-2bh)(bh+cg)}{ch^2}+9bx+9cx^2}}-\frac{3\,3^{2/3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}}\log\left(\left(1-\right.\right.}{2f\sqrt[3]{-\frac{(cg-2bh)(bh+cg)}{ch^2}+\ldots}}$$

**Rubi [A]** time = 0.36, antiderivative size = 488, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 104, $\frac{\text{number of rules}}{\text{integrand size}} = 0.019$, Rules used = {1041, 1040}

$$\frac{3^{2/3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}}\log\left(\frac{f\left(b^2h^2-bcgh+c^2g^2\right)}{3c^2h^2}+\frac{bfx}{c}+fx^2\right)}{2f\sqrt[3]{-\frac{(cg-2bh)(bh+cg)}{ch^2}+9bx+9cx^2}}-\frac{3\,3^{2/3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}}\log\left(\left(1-\frac{3h(b+2cx)}{2cg-bh}\right)^{2/3}+\sqrt[3]{2}\sqrt[3]{\frac{3h(b+2cx)}{2cg-bh}+1}\right)}{2f\sqrt[3]{-\frac{(cg-2bh)(bh+cg)}{ch^2}+9bx+9cx^2}}+\frac{3\sqrt[6]{3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\left(1-\frac{3h(b+2cx)}{2cg-bh}\right)^{2/3}}{\sqrt{3}\sqrt[3]{\frac{3h(b+2cx)}{2cg-bh}+1}}\right)}{f\sqrt[3]{-\frac{(cg-2bh)(bh+cg)}{ch^2}+9bx+9cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)/((((-(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(9*c*h^2) + b*x + c*x^2)
^(1/3)*((f*(b^2 - (-(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(3*h^2)))/c^2 + (b*f*x
)/c + f*x^2)),x]
```

```
[Out] (3*3^(1/6)*h*((c*h^2*(((c*g - 2*b*h)*(c*g + b*h))/(c*h^2) - 9*b*x - 9*c*x^2
))/(2*c*g - b*h)^2)^(1/3)*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - (3*h*(b + 2*c*x)
)/(2*c*g - b*h))^(2/3))/(Sqrt[3]*(1 + (3*h*(b + 2*c*x))/(2*c*g - b*h))^(1/3
))])/(f*(-(((c*g - 2*b*h)*(c*g + b*h))/(c*h^2)) + 9*b*x + 9*c*x^2)^(1/3)) +
 (3^(2/3)*h*((c*h^2*(((c*g - 2*b*h)*(c*g + b*h))/(c*h^2) - 9*b*x - 9*c*x^2)
)/(2*c*g - b*h)^2)^(1/3)*Log[(f*(c^2*g^2 - b*c*g*h + b^2*h^2))/(3*c^2*h^2)
+ (b*f*x)/c + f*x^2])/(2*f*(-(((c*g - 2*b*h)*(c*g + b*h))/(c*h^2)) + 9*b*x
+ 9*c*x^2)^(1/3)) - (3*3^(2/3)*h*((c*h^2*(((c*g - 2*b*h)*(c*g + b*h))/(c*h^
2) - 9*b*x - 9*c*x^2))/(2*c*g - b*h)^2)^(1/3)*Log[(1 - (3*h*(b + 2*c*x))/(2
*c*g - b*h))^(2/3) + 2^(1/3)*(1 + (3*h*(b + 2*c*x))/(2*c*g - b*h))^(1/3)])/
(2*f*(-(((c*g - 2*b*h)*(c*g + b*h))/(c*h^2)) + 9*b*x + 9*c*x^2)^(1/3))
```

Rule 1040

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)*((d_.)
+ (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = ((-9*c*h^2)/(2*c*g -
b*h)^2)^(1/3)}, Simp[(Sqrt[3]*h*q*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - (3*h*(b
+ 2*c*x))/(2*c*g - b*h))^(2/3))/(Sqrt[3]*(1 + (3*h*(b + 2*c*x))/(2*c*g - b*
h))^(1/3))])/f, x] + (-Simp[(3*h*q*Log[(1 - (3*h*(b + 2*c*x))/(2*c*g - b*h)
```

```
)^(2/3) + 2^(1/3)*(1 + (3*h*(b + 2*c*x))/(2*c*g - b*h))^(1/3)])/(2*f), x] +
 Simp[(h*q*Log[d + e*x + f*x^2])/(2*f), x])] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x] && EqQ[c*e - b*f, 0] && EqQ[c^2*d - f*(b^2 - 3*a*c), 0] && EqQ[c^2*
g^2 - b*c*g*h - 2*b^2*h^2 + 9*a*c*h^2, 0] && GtQ[(-9*c*h^2)/(2*c*g - b*h)^2
, 0]
```

Rule 1041

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)*((d_.)
+ (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = -(c/(b^2 - 4*a*c))},
Dist[(q*(a + b*x + c*x^2))^(1/3)/(a + b*x + c*x^2)^(1/3), Int[(g + h*x)/((q
*a + b*q*x + c*q*x^2)^(1/3)*(d + e*x + f*x^2)), x], x]] /; FreeQ[{a, b, c,
d, e, f, g, h}, x] && EqQ[c*e - b*f, 0] && EqQ[c^2*d - f*(b^2 - 3*a*c), 0]
&& EqQ[c^2*g^2 - b*c*g*h - 2*b^2*h^2 + 9*a*c*h^2, 0] &&  !GtQ[4*a - b^2/c,
0]
```

Rubi steps

$$\int \frac{g+hx}{\sqrt[3]{\frac{-c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2}\left(\frac{f\left(b^2-\frac{-c^2g^2+bcgh+2b^2h^2}{3h^2}\right)}{c^2}+\frac{bfx}{c}+fx^2\right)}\,dx = \frac{\sqrt[3]{-\frac{c\left(\frac{-c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2\right)}{b^2-\frac{4\left(-c^2g^2+bcgh+2b^2h^2\right)}{9h^2}}}\int \frac{\cdots}{\left(\frac{f\left(b^2-\frac{-c^2g^2+b\cdots}{3\cdots}\right)}{c^2}\cdots\right.}}{\cdots}$$

$$= \frac{3\sqrt[6]{3}\,h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(cg+bh)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}}\;\tan^{-1}\left(\cdots\right.}{f\sqrt[3]{-\frac{(cg-2bh)(cg+bh)}{ch^2}+9bx\cdots}}$$

**Mathematica [F]** time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{g+hx}{\sqrt[3]{\frac{-c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2}\left(\frac{f\left(b^2-\frac{-c^2g^2+bcgh+2b^2h^2}{3h^2}\right)}{c^2}+\frac{bfx}{c}+fx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(g + h*x)/(((-(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(9*c*h^2) + b*x +
c*x^2)^(1/3)*((f*(b^2 - (-(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(3*h^2)))/c^2 +
(b*f*x)/c + f*x^2)),x]
```

```
[Out] Integrate[(g + h*x)/(((-(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(9*c*h^2) + b*x +
c*x^2)^(1/3)*((f*(b^2 - (-(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(3*h^2)))/c^2 +
(b*f*x)/c + f*x^2)), x]
```

**IntegrateAlgebraic [C]** time = 6.23, size = 1038, normalized size = 2.13

$$-\frac{3\sqrt[6]{-3}\sqrt[3]{c}\tanh^{-1}\left(\frac{\frac{2i\sqrt[3]{h}b}{\sqrt{3}\sqrt[3]{c}\sqrt[3]{2cg-bh}}+\frac{i\sqrt{3}c^{2/3}\sqrt[3]{h}x}{\sqrt[3]{2cg-bh}}-\frac{3c^{2/3}\sqrt[3]{h}x}{\sqrt[3]{2cg-bh}}+\frac{i\sqrt[3]{\frac{2b^2}{c}+9xb+\frac{gb}{h}+9cx^2-\frac{cg^2}{h^2}}}{\sqrt{3}}+\frac{cg-2bh}{\sqrt[3]{c}h^{2/3}\sqrt[3]{2cg-bh}}-\frac{ic^{2/3}g}{\sqrt{3}h^{2/3}\sqrt[3]{2cg-bh}}}{\sqrt[3]{\frac{2b^2}{c}+9xb+\frac{gb}{h}+9cx^2-\frac{cg^2}{h^2}}}\right)h^{5/3}}{f(2cg-bh)^{2/3}}+\frac{(-3)^{2/3}\sqrt[3]{c}\log\left(3cg-6i\sqrt{3}bh-6bh-9chx+\sqrt{3}c(3ig-9ihx)+6\sqrt[3]{c}h^{2/3}\sqrt[3]{2cg-bh}\sqrt[3]{\frac{2b^2}{c}+9xb+\frac{gb}{h}+9cx^2-\frac{cg^2}{h^2}}\right)h^{5/3}}{f(2cg-bh)^{2/3}}-\text{[illegible]}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(g + h*x)/(((-(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(9*c*h^2)
 + b*x + c*x^2)^(1/3)*((f*(b^2 - (-(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(3*h^2)
))/c^2 + (b*f*x)/c + f*x^2)),x]
```

```
[Out] (-3*(-3)^(1/6)*c^(1/3)*h^(5/3)*ArcTanh[(((-I)*c^(2/3)*g)/(Sqrt[3]*h^(2/3)*(
2*c*g - b*h)^(1/3)) + ((2*I)*b*h^(1/3))/(Sqrt[3]*c^(1/3)*(2*c*g - b*h)^(1/3
)) + (c*g - 2*b*h)/(c^(1/3)*h^(2/3)*(2*c*g - b*h)^(1/3)) - (3*c^(2/3)*h^(1/
3)*x)/(2*c*g - b*h)^(1/3) + (I*Sqrt[3]*c^(2/3)*h^(1/3)*x)/(2*c*g - b*h)^(1/
3) + (I*((2*b^2)/c - (c*g^2)/h^2 + (b*g)/h + 9*b*x + 9*c*x^2)^(1/3))/Sqrt[3
])/((2*b^2)/c - (c*g^2)/h^2 + (b*g)/h + 9*b*x + 9*c*x^2)^(1/3)])/(f*(2*c*g
- b*h)^(2/3)) + ((-3)^(2/3)*c^(1/3)*h^(5/3)*Log[3*c*g - 6*b*h - (6*I)*Sqrt[
3]*b*h - 9*c*h*x + Sqrt[3]*c*((3*I)*g - (9*I)*h*x) + 6*c^(1/3)*h^(2/3)*(2*c
*g - b*h)^(1/3)*((2*b^2)/c - (c*g^2)/h^2 + (b*g)/h + 9*b*x + 9*c*x^2)^(1/3)
])/(f*(2*c*g - b*h)^(2/3)) - ((-3)^(2/3)*c^(1/3)*h^(5/3)*Log[-9*c^2*g^2 + 3
6*b*c*g*h - 36*b^2*h^2 + (36*I)*Sqrt[3]*b^2*h^2 + 54*c^2*g*h*x - 108*b*c*h^
2*x - 81*c^2*h^2*x^2 + Sqrt[3]*c*((-36*I)*b*g*h + (108*I)*b*h^2*x) - 9*c^(4
/3)*g*h^(2/3)*(2*c*g - b*h)^(1/3)*((2*b^2)/c - (c*g^2)/h^2 + (b*g)/h + 9*b*
x + 9*c*x^2)^(1/3) + 18*b*c^(1/3)*h^(5/3)*(2*c*g - b*h)^(1/3)*((2*b^2)/c -
(c*g^2)/h^2 + (b*g)/h + 9*b*x + 9*c*x^2)^(1/3) + (18*I)*Sqrt[3]*b*c^(1/3)*h
^(5/3)*(2*c*g - b*h)^(1/3)*((2*b^2)/c - (c*g^2)/h^2 + (b*g)/h + 9*b*x + 9*c
*x^2)^(1/3) + 27*c^(4/3)*h^(5/3)*(2*c*g - b*h)^(1/3)*x*((2*b^2)/c - (c*g^2)
/h^2 + (b*g)/h + 9*b*x + 9*c*x^2)^(1/3) + 18*c^(2/3)*h^(4/3)*(2*c*g - b*h)^
(2/3)*((2*b^2)/c - (c*g^2)/h^2 + (b*g)/h + 9*b*x + 9*c*x^2)^(2/3) + Sqrt[3]
*c^2*((9*I)*g^2 - (54*I)*g*h*x + (81*I)*h^2*x^2) + Sqrt[3]*c^(4/3)*((-9*I)*
g*h^(2/3)*(2*c*g - b*h)^(1/3)*((2*b^2)/c - (c*g^2)/h^2 + (b*g)/h + 9*b*x +
9*c*x^2)^(1/3) + (27*I)*h^(5/3)*(2*c*g - b*h)^(1/3)*x*((2*b^2)/c - (c*g^2)/
h^2 + (b*g)/h + 9*b*x + 9*c*x^2)^(1/3))])/(2*f*(2*c*g - b*h)^(2/3))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(1/9*(2*b^2*h^2+b*c*g*h-c^2*g^2)/c/h^2+b*x+c*x^2)^(1/3)/(
f*(b^2+1/3*(-2*b^2*h^2-b*c*g*h+c^2*g^2)/h^2)/c^2+b*f*x/c+f*x^2),x, algorith
m="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\left(hx+g\right)}{\left(cx^2+bx-\frac{c^2g^2-bcgh-2\,b^2h^2}{9\,ch^2}\right)^{\frac{1}{3}}\left(3\,fx^2+\frac{3\,bfx}{c}+\frac{\left(3\,b^2+\frac{c^2g^2-bcgh-2\,b^2h^2}{h^2}\right)f}{c^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(1/9*(2*b^2*h^2+b*c*g*h-c^2*g^2)/c/h^2+b*x+c*x^2)^(1/3)/(
f*(b^2+1/3*(-2*b^2*h^2-b*c*g*h+c^2*g^2)/h^2)/c^2+b*f*x/c+f*x^2),x, algorith
m="giac")
```

```
[Out] integrate(3*(h*x + g)/((c*x^2 + b*x - 1/9*(c^2*g^2 - b*c*g*h - 2*b^2*h^2)/(
c*h^2))^(1/3)*(3*f*x^2 + 3*b*f*x/c + (3*b^2 + (c^2*g^2 - b*c*g*h - 2*b^2*h^
2)/h^2)*f/c^2)), x)
```

**maple [F]** time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{hx+g}{\left(c\,x^2+bx+\frac{2b^2h^2+bcgh-c^2g^2}{9c\,h^2}\right)^{\frac{1}{3}}\left(f\,x^2+\frac{bfx}{c}+\frac{\left(b^2+\frac{-2b^2h^2-bcgh+c^2g^2}{3h^2}\right)f}{c^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)/(1/9*(2*b^2*h^2+b*c*g*h-c^2*g^2)/c/h^2+b*x+c*x^2)^(1/3)/(f*(b^2
+1/3*(-2*b^2*h^2-b*c*g*h+c^2*g^2)/h^2)/c^2+b*f*x/c+f*x^2),x)
```

```
[Out] int((h*x+g)/(1/9*(2*b^2*h^2+b*c*g*h-c^2*g^2)/c/h^2+b*x+c*x^2)^(1/3)/(f*(b^2
+1/3*(-2*b^2*h^2-b*c*g*h+c^2*g^2)/h^2)/c^2+b*f*x/c+f*x^2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$3\int \frac{hx+g}{\left(cx^2+bx-\frac{c^2g^2-bcgh-2\,b^2h^2}{9\,ch^2}\right)^{\frac{1}{3}}\left(3\,fx^2+\frac{3\,bfx}{c}+\frac{\left(3\,b^2+\frac{c^2g^2-bcgh-2\,b^2h^2}{h^2}\right)f}{c^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(1/9*(2*b^2*h^2+b*c*g*h-c^2*g^2)/c/h^2+b*x+c*x^2)^(1/3)/(
f*(b^2+1/3*(-2*b^2*h^2-b*c*g*h+c^2*g^2)/h^2)/c^2+b*f*x/c+f*x^2),x, algorith
m="maxima")
```

```
[Out] 3*integrate((h*x + g)/((c*x^2 + b*x - 1/9*(c^2*g^2 - b*c*g*h - 2*b^2*h^2)/(
c*h^2))^(1/3)*(3*f*x^2 + 3*b*f*x/c + (3*b^2 + (c^2*g^2 - b*c*g*h - 2*b^2*h^
2)/h^2)*f/c^2)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{g+h\,x}{\left(b\,x+c\,x^2+\frac{\frac{2\,b^2\,h^2}{9}+\frac{b\,c\,g\,h}{9}-\frac{c^2\,g^2}{9}}{c\,h^2}\right)^{1/3}\,\left(f\,x^2-\frac{f\,\left(\frac{\frac{2\,b^2\,h^2}{3}+\frac{b\,c\,g\,h}{3}-\frac{c^2\,g^2}{3}}{h^2}-b^2\right)}{c^2}+\frac{b\,f\,x}{c}\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)/((b*x + c*x^2 + ((2*b^2*h^2)/9 - (c^2*g^2)/9 + (b*c*g*h)/9)/(
c*h^2))^(1/3)*(f*x^2 - (f*(((2*b^2*h^2)/3 - (c^2*g^2)/3 + (b*c*g*h)/3)/h^2
- b^2))/c^2 + (b*f*x)/c)),x)
```

```
[Out] int((g + h*x)/((b*x + c*x^2 + ((2*b^2*h^2)/9 - (c^2*g^2)/9 + (b*c*g*h)/9)/(
c*h^2))^(1/3)*(f*x^2 - (f*(((2*b^2*h^2)/3 - (c^2*g^2)/3 + (b*c*g*h)/3)/h^2
- b^2))/c^2 + (b*f*x)/c)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{3\cdot 3^{\frac{2}{3}} c^2 h^2 \left(\int \frac{g}{b^2h^2\sqrt[3]{\frac{2b^2}{c}+\frac{bg}{h}+9bx-\frac{cg^2}{h^2}+9cx^2}-bcgh\sqrt[3]{\frac{2b^2}{c}+\frac{bg}{h}+9bx-\frac{cg^2}{h^2}+9cx^2}+3bch^2x\sqrt[3]{\frac{2b^2}{c}+\frac{bg}{h}+9bx-\frac{cg^2}{h^2}+9cx^2}+c^2g^2\sqrt[3]{\frac{2b^2}{c}+\frac{bg}{h}+9bx-\frac{cg^2}{h^2}+9cx^2}+3c^2h^2x^2\sqrt[3]{\frac{2b^2}{c}+\frac{bg}{h}+9bx-\frac{cg^2}{h^2}+9cx^2}}\, dx + \int \frac{hx}{b^2h^2\sqrt[3]{\frac{2b^2}{c}+\frac{bg}{h}+9bx-\frac{cg^2}{h^2}+9cx^2}-bcgh\sqrt[3]{\frac{2b^2}{c}+\frac{bg}{h}+9bx-\frac{cg^2}{h^2}+9cx^2}+3bch^2x\sqrt[3]{\frac{2b^2}{c}+\frac{bg}{h}+9bx-\frac{cg^2}{h^2}+9cx^2}+c^2g^2\sqrt[3]{\frac{2b^2}{c}+\frac{bg}{h}+9bx-\frac{cg^2}{h^2}+9cx^2}+3c^2h^2x^2\sqrt[3]{\frac{2b^2}{c}+\frac{bg}{h}+9bx-\frac{cg^2}{h^2}+9cx^2}}\, dx\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(1/9*(2*b**2*h**2+b*c*g*h-c**2*g**2)/c/h**2+b*x+c*x**2)**
(1/3)/(f*(b**2+1/3*(-2*b**2*h**2-b*c*g*h+c**2*g**2)/h**2)/c**2+b*f*x/c+f*x*
*2),x)
```

```
[Out] 3*3**(2/3)*c**2*h**2*(Integral(g/(b**2*h**2*(2*b**2/c + b*g/h + 9*b*x - c*g
**2/h**2 + 9*c*x**2)**(1/3) - b*c*g*h*(2*b**2/c + b*g/h + 9*b*x - c*g**2/h*
*2 + 9*c*x**2)**(1/3) + 3*b*c*h**2*x*(2*b**2/c + b*g/h + 9*b*x - c*g**2/h**
2 + 9*c*x**2)**(1/3) + c**2*g**2*(2*b**2/c + b*g/h + 9*b*x - c*g**2/h**2 +
9*c*x**2)**(1/3) + 3*c**2*h**2*x**2*(2*b**2/c + b*g/h + 9*b*x - c*g**2/h**2
 + 9*c*x**2)**(1/3)), x) + Integral(h*x/(b**2*h**2*(2*b**2/c + b*g/h + 9*b*
x - c*g**2/h**2 + 9*c*x**2)**(1/3) - b*c*g*h*(2*b**2/c + b*g/h + 9*b*x - c*
g**2/h**2 + 9*c*x**2)**(1/3) + 3*b*c*h**2*x*(2*b**2/c + b*g/h + 9*b*x - c*g
**2/h**2 + 9*c*x**2)**(1/3) + c**2*g**2*(2*b**2/c + b*g/h + 9*b*x - c*g**2/
h**2 + 9*c*x**2)**(1/3) + 3*c**2*h**2*x**2*(2*b**2/c + b*g/h + 9*b*x - c*g*
*2/h**2 + 9*c*x**2)**(1/3)), x))/f
```

# Chapter 4

# Appendix

## Local contents

## 4.1 Download section

The following zip files contain the raw integrals used in this test.

This is a subset of Rubi test suite thanks to Albert Rich, which includes only the algebraic integrals with elementray optimal antiderivatives. It also includes a subset of a test file provided thanks to Sam Blake.

**Mathematica format** Mathematica_syntax_CAS_integration_elementary_version.zip

**Maple and Mupad format** Maple_syntax_CAS_integration_elementary_version.zip

**Sympy format** SYMPY_syntax_CAS_integration_elementary_version.zip

**Sage math format** SAGE_syntax_CAS_integration_elementary_version.zip

## 4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
```

```
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

### 4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
```

```
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

### 4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
   ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
   (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
   Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

### 4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
```

```
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
```

```
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
   func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
   expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
   is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
   leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
   expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
```

```
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
        else: #result contains complex but optimal is not
            return "C"
    else: # result do not contain complex, this assumes optimal do not as
well
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
else:
    return "C"
```